CALIBRATION AND OPTIMAL BEAMFORMING FOR A 19 ELEMENT PHASED ARRAY FEED

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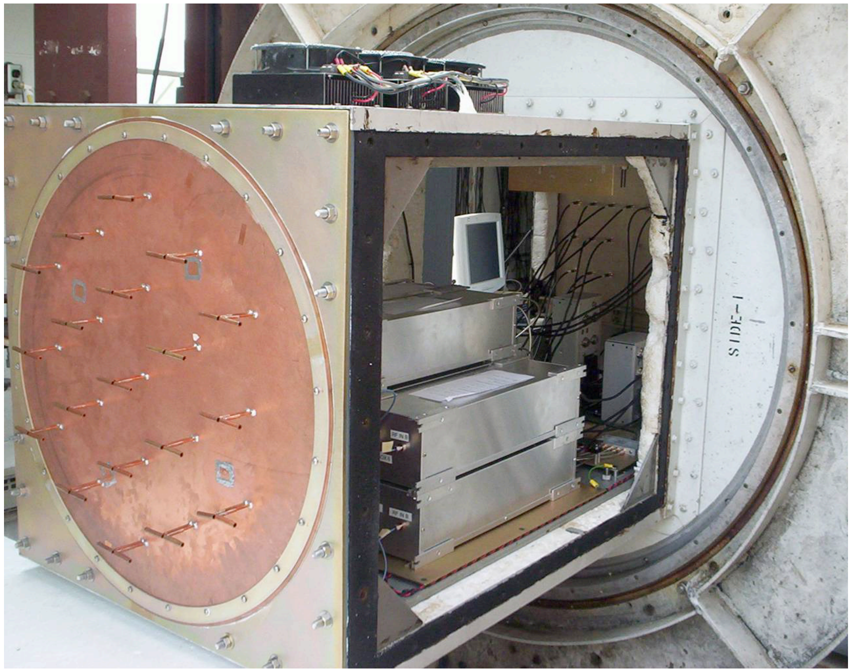
Green Bank 20-Meter Telescope

- 20m, F/D = 0.43, fast slewing.
- Re-commissioned for array feed experiments.
- 3.6m auxiliary antenna tracks RFI for mitigation.

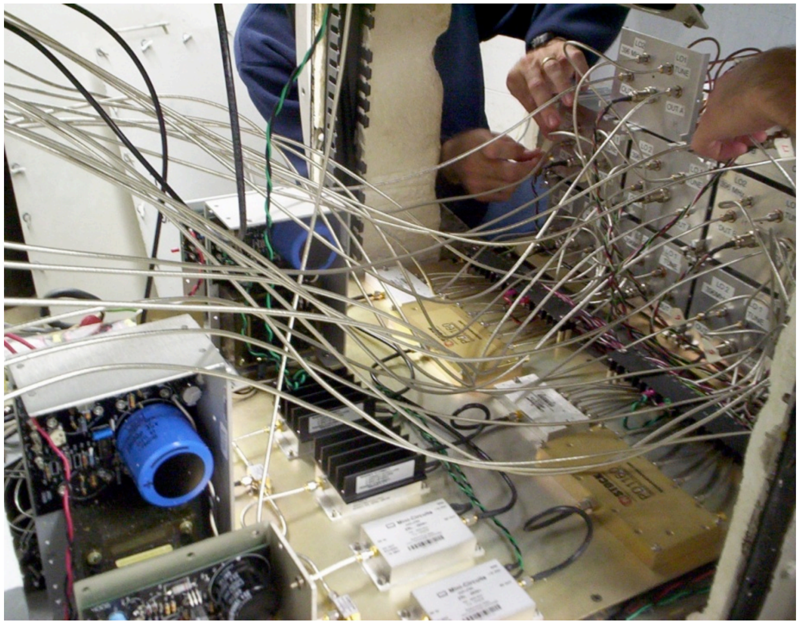
BYU – NRAO 19 element array

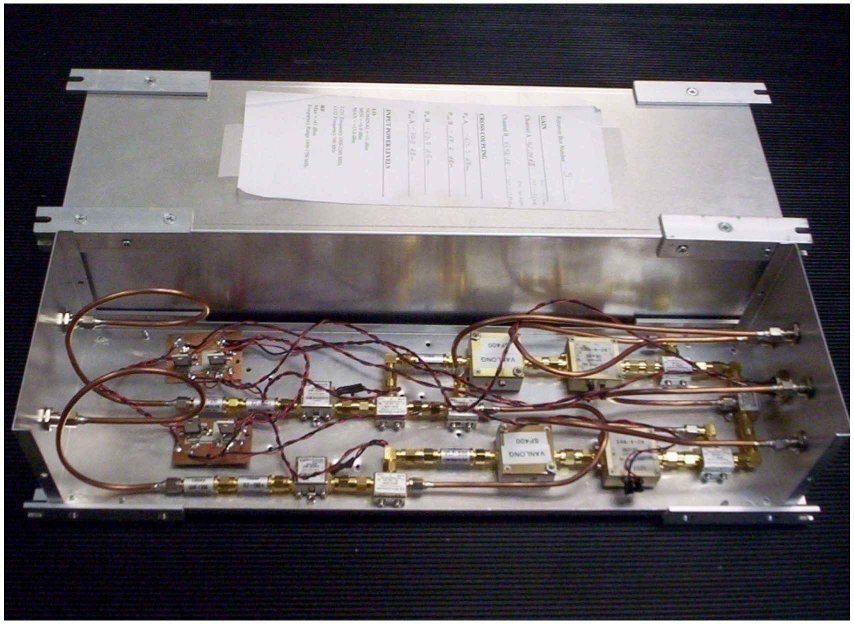


- Hexagonal grid of thickened 1.6 GHz dipoles.
- 0.6 λ spacing, single polarization.
- 0.25 λ offset from ground plane.

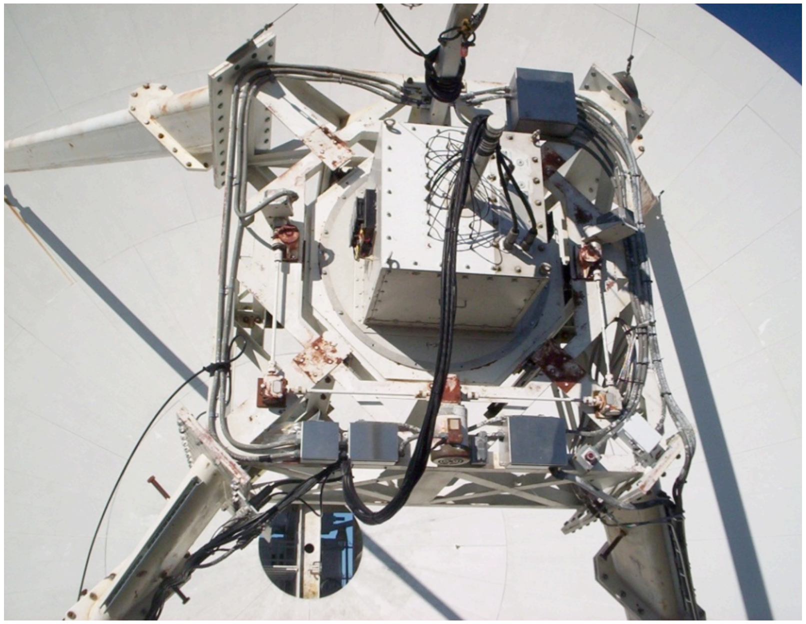
Analog RF front end

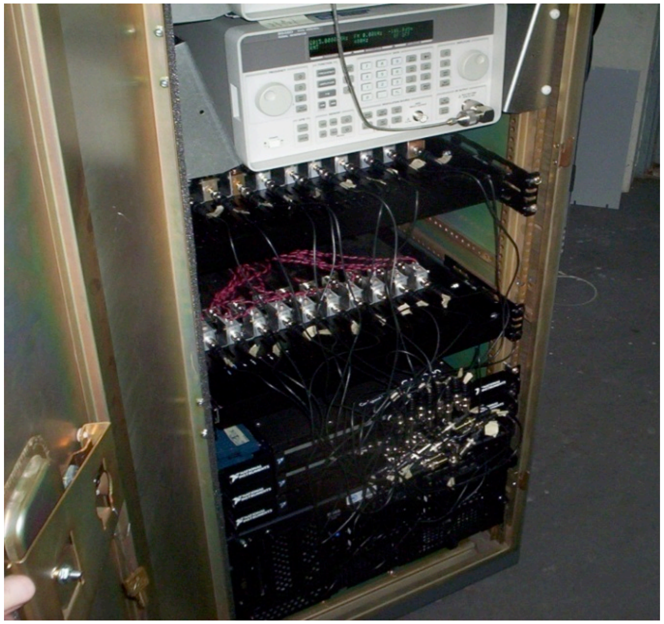
- Downconversion to IF in front-end box behind array.
- 19 double conversion, room temperature analog receivers, $T_{sys} \approx 110K$.
- Remotely tunable from 1200-2000MHz.
- IF bandwidth selectable at 0.5, 1.0, and 5.0 MHz.
- Connectorized modular system facilitates maintenance & channel isolation.





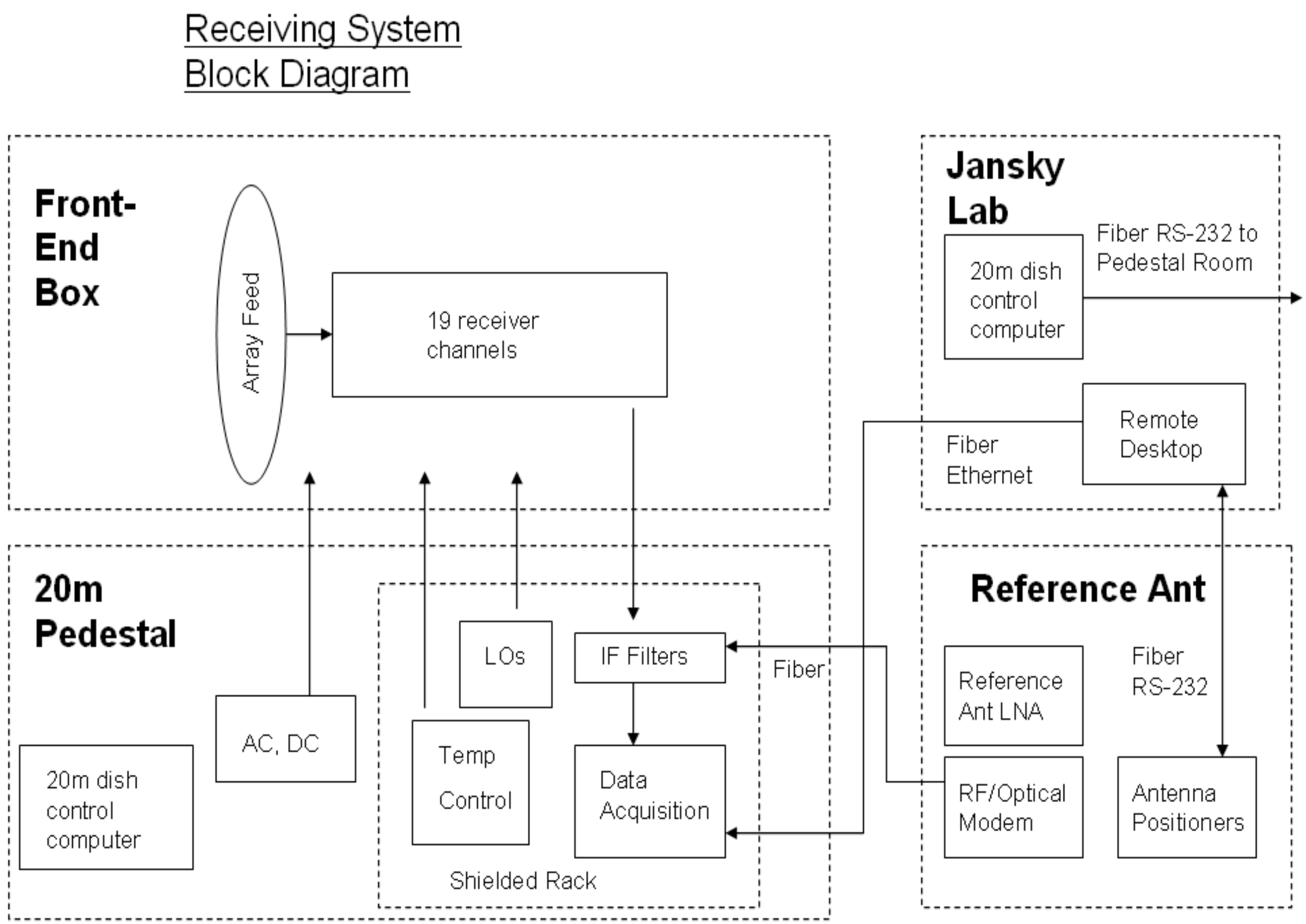
Digital receiver back end



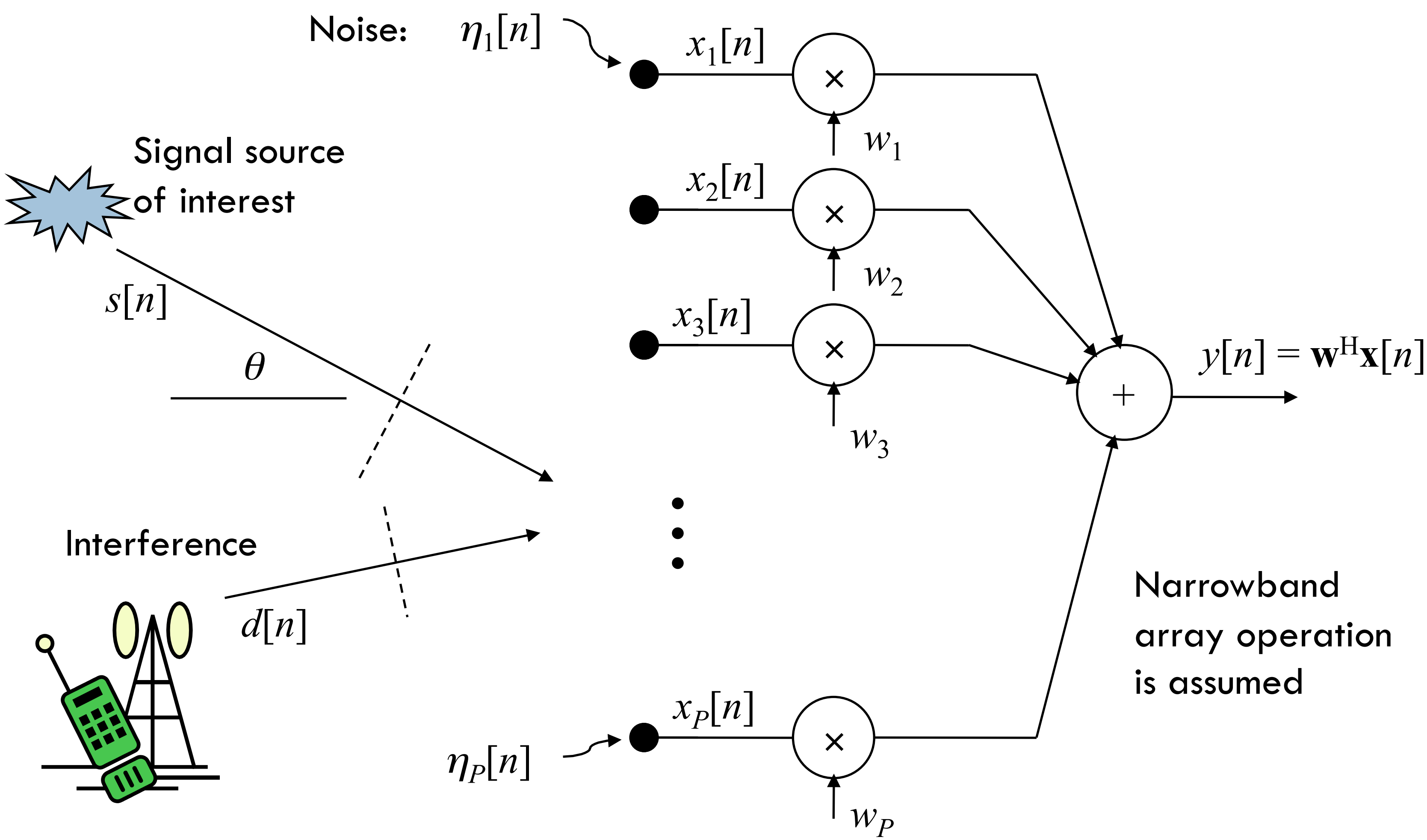


- 3 MHz IF from FEB to RFI shielded digital receiver in pedestal room.
  - 19 thin, inexpensive cables are low loss at this IF.
  - 1st and 2nd LO from base to FEB using low loss microwave coax.
- 20 channel, 12 bit, synchronous sampling at 1.25Msamp/sec.
- Continuous streaming to fast RAID-0 disk array for ~2 hours.
- Digitally mixed to complex baseband in post processing.

System overview



Signal and beamformer model



Signal and beamformer model (2)

- Array sample vector

$$\mathbf{x}[n] = \mathbf{v}_s s[n] + \mathbf{v}_d d[n] + \mathbf{n}[n]$$

- Sample array covariance estimate for *j-th* short-term integration (STI):

$$\hat{\mathbf{R}}_j = \frac{1}{L}\mathbf{X}_j\mathbf{X}_j^H \approx \mathbf{R}_s + \mathbf{R}_d[jL] + \mathbf{R}_\mathbf{n}$$

$$\mathbf{X}_j = \left[\mathbf{x}[jL], \mathbf{x}[jL+1], \cdots, \mathbf{x}[(j+1)L-1]\right].$$

- Beamformer output

$$y[n] = \mathbf{w}_j^H \mathbf{x}[n], \qquad j = \lfloor n/L \rfloor$$

Beamformer Definitions

- Conjugate field match (CFM)
  - $\mathbf{w}_j = \hat{\mathbf{v}}_s$, $\hat{\mathbf{v}}_s$ from calibration data set.
  - A.K.A. phase conjugate weighting, or spatial matched filter.
  - Max SNR solution for i.i.d. noise.
- LCMV
  - $\mathbf{w}_j = \hat{\mathbf{R}}_j^{-1}\mathbf{C}[\mathbf{C}^H\hat{\mathbf{R}}_j^{-1}\mathbf{C}]^{-1}\mathbf{f}$,
  - Columns of $\mathbf{C}$ are distinct calibration constraint vectors.
  - $\mathbf{f}$ is desired response in constraint directions, $\mathbf{w}_j^H\mathbf{C} = \mathbf{f}$.
  - Single mainlobe constraint case (MVDR):

$$\mathbf{w}_j = \frac{\hat{\mathbf{R}}_j^{-1}\hat{\mathbf{v}}_s}{\hat{\mathbf{v}}_s^H\hat{\mathbf{R}}_j^{-1}\hat{\mathbf{v}}_s}$$

Beamformer Definitions (2)

- Max SNR (A.K.A. Max sensitivity)
  - $\mathbf{w}_j$ is the solution to eigenvector problem

$$\hat{\mathbf{R}}_{\mathbf{n}}^{-1}(\hat{\mathbf{v}}_{\mathbf{s}}\hat{\mathbf{v}}_{s}^{H})\mathbf{w} = \lambda_{\mathbf{max}}\mathbf{w}$$

  - $\hat{\mathbf{R}}_{\mathbf{n}}$ estimated from off-source calibration data.
- Subspace projection for interference cancellation.
  - Partitioned eigen decomposition estimates subspace $\mathbf{U}_d$ spanning interference array signature:

$$\hat{\mathbf{R}}_j[\mathbf{U}_d \mid \mathbf{U}_{s+\eta}] = [\mathbf{U}_d \mid \mathbf{U}_{s+\eta}]\Lambda$$

$$\mathbf{w}_j = \mathbf{P}_j\tilde{\mathbf{w}} \text{ where } \mathbf{P}_j = \mathbf{I} - \mathbf{U}_d\mathbf{U}_d^H .$$

  - $\tilde{\mathbf{w}}$ is the precomptued fixed, quiescent, interference-free beamformer.
- Note the lack of any conventional, fixed weight, windowed, or iteratively optimized beamformer.

Calibration procedure

1. Estimate $\hat{\mathbf{R}}_{\mathbf{n}}$ while steered to a quiet, off source sky patch.
2. Using the brightest available calibrator source, steer dish to calibration angle $\{\Omega_k \mid 1 \le k \le K\}$ (relative to the source) and estimate $\hat{\mathbf{R}}_{\Omega_k}$.
3. Calibration grid, $\{\Omega_k \mid 1 \le k \le K\}$, includes all multi-beam steerings and mainlobe shape constraints.
4. Calibration steering vector $\hat{\mathbf{v}}_{\Omega_k}$ is the dominant eigenvector of $(\hat{\mathbf{R}}_{\Omega_k} - \hat{\mathbf{R}}_{\mathbf{n}})$.

Beamformer calibration Issues

- Bench-top or antenna range calibration is inadequate.
  - Differential drift between channels: gain, phase, and noise levels.
  - Local environment (e.g. supports) affect element patterns.
  - Variation in element patterns across the array is complex, hard to measure the patterns densely enough.
- Detailed EM model is unsuitable for calibration.
  - Helpful for qualitative analysis and representative studies.
  - Cannot model physical response closely enough for high sensitivity beamforming.

Beamformer calibration Issues (2)

- Strong mutual coupling
  - Complicates element patterns; they are *not* identical.
  - Correlates receiver noise which must be accounted for in beamformer design to optimize sensitivity.
  - Coupled differential LNA noise drift makes fixed beamformer non-optimal, even if element patterns were known exactly.
- Adaptive beamforming calibration is needed:
  - In each (multiply)steered beam direction,
  - In each response constraint direction.
- Deterministic beamforming requires in addition:
  - Calibration over entire spillover region.
  - Responses are too variable to achieve low spillover illumination without dense calibration.

Fixed-adaptive beamforming

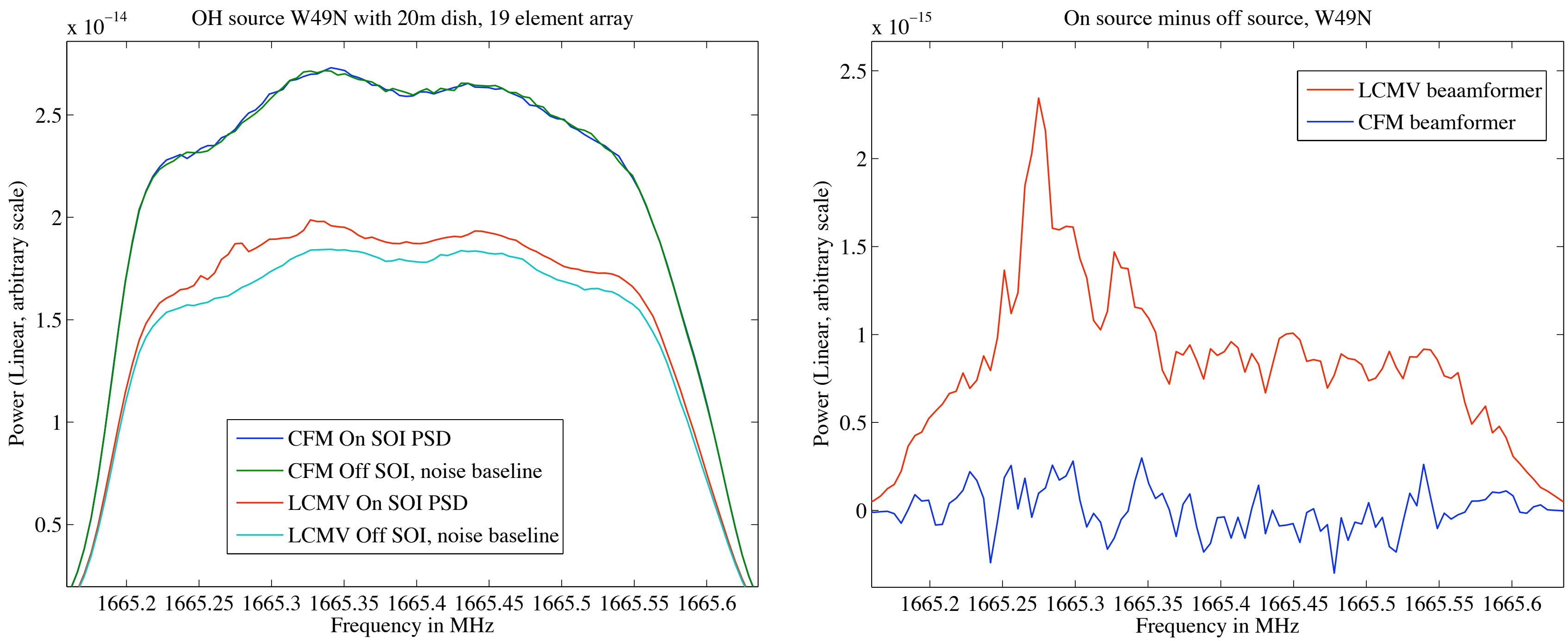
- We don't know how to do conventional, deterministic beamformer design in this calibration environment.
  - Works in simulation, but for real data we lack sufficient information.
  - Much easier with a bare array than with a PAF and dish.
  - Much easier in a lower sensitivity regimes (e.g. non-cryo-cooled LNAs, comm. systems and radar, not dominated by spillover noise).
- Solution:
  Pre-compute fixed-adaptive $\mathbf{w}$ from calibration data only.
- Optimal LCMV, MVDR, and Max SNR solutions automatically minimize T_{sys}.
- Hold $\mathbf{w}$ fixed for observations, i.e. use like a non-adaptive beamformer.
- For subspace projection, use e.g. LCMV to find $\tilde{\mathbf{w}}$.

Calibration and beamforming results

Green Bank 20 Meter Telescope

and EM simulations

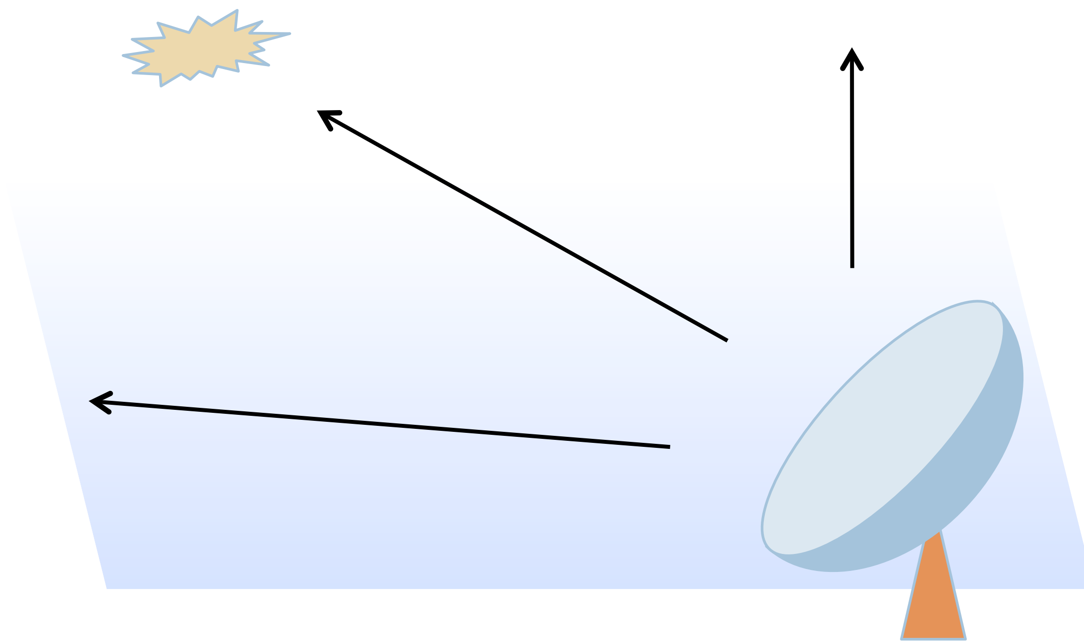
W49N, OH source detection



- Conjugate field match (CFM) and LCMV beamformers were calculated from single pointing Cyg A calibration data.
- 10s observation, on-off source baseline subtraction.
- LCMV permits W49N detection. CFM spillover noise is too high.

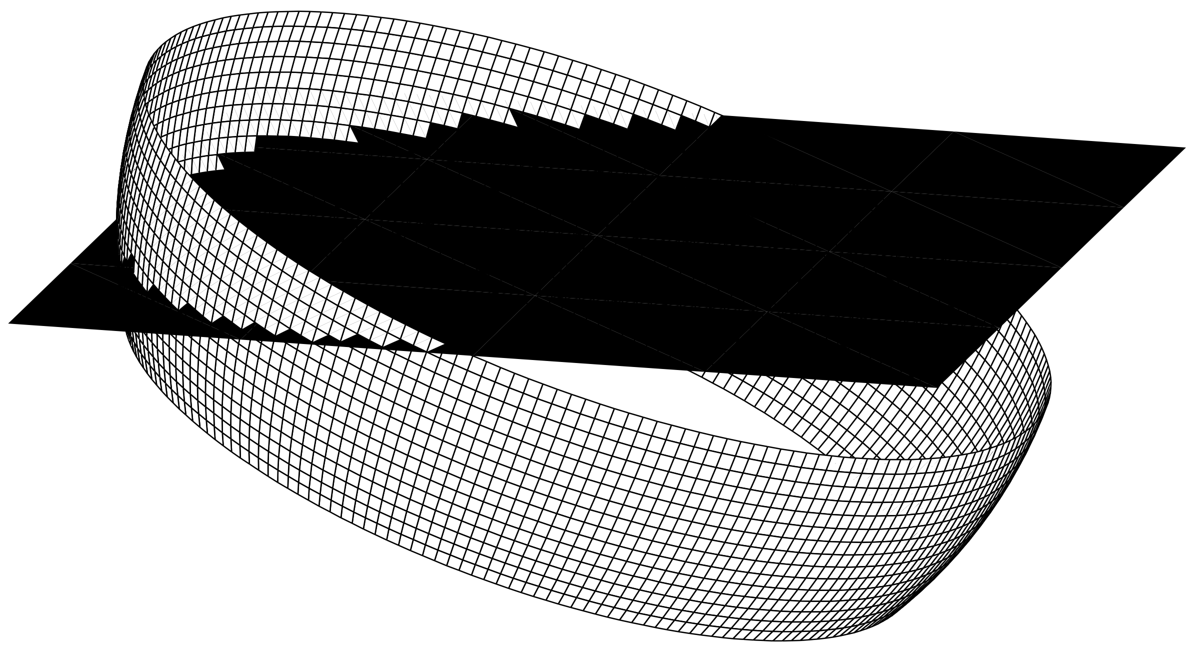
Adapting to spillover noise variation with elevation "tipping"

- At zenith:
  - All spillover sidelobes see warm ground, dominates T_{sys}.
  - 2K-4K sky in main beam.
- Mid elevations:
  - Upper sidelobes see cold sky.
  - 10K-20Ksky in main beam.
- Near horizon:
  - 45K sky dominates T_{sys}.
  - Half of spillover sidelobes see cold sky.

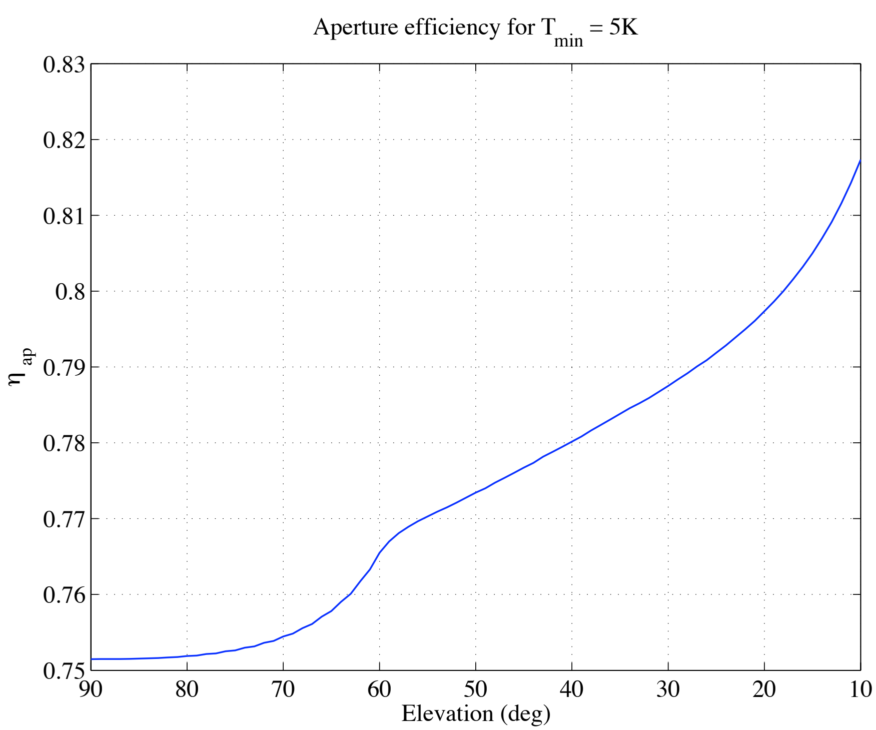


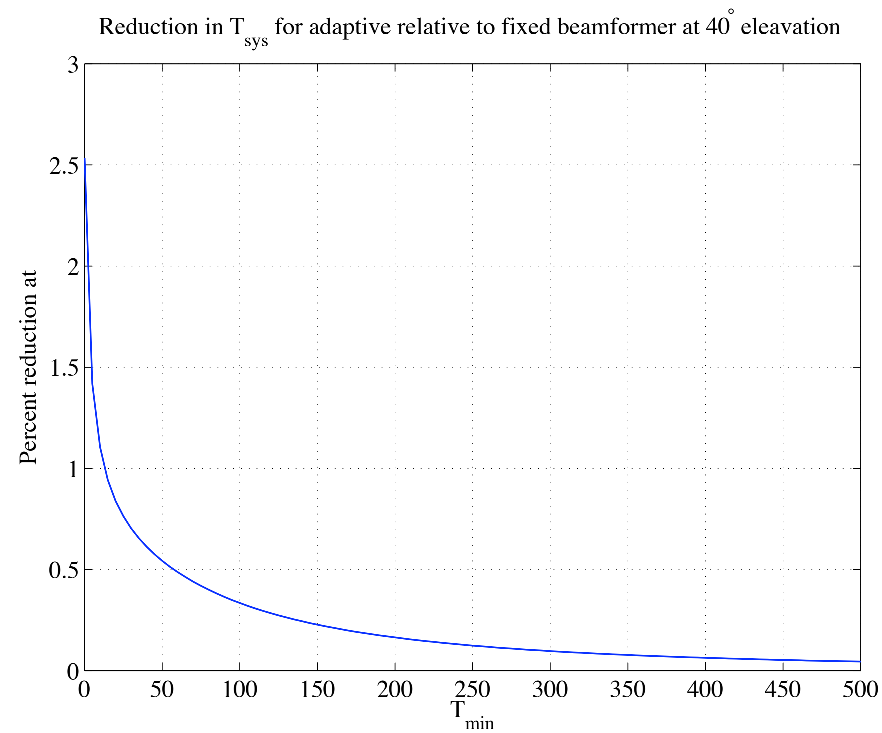
- An adaptive beamformer can exploit changes in spillover and sky noise spatial structure to minimize T_{sys} at each elevation.

Tipping spillover geometry



LCMV tipping simulation results

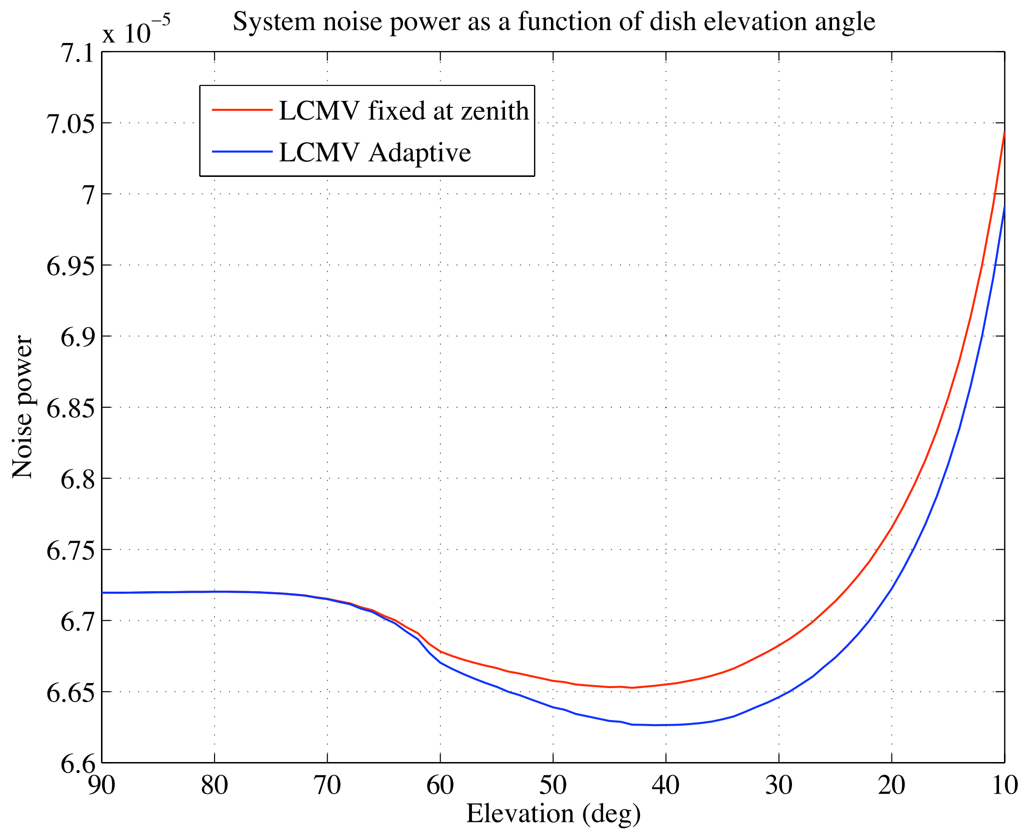




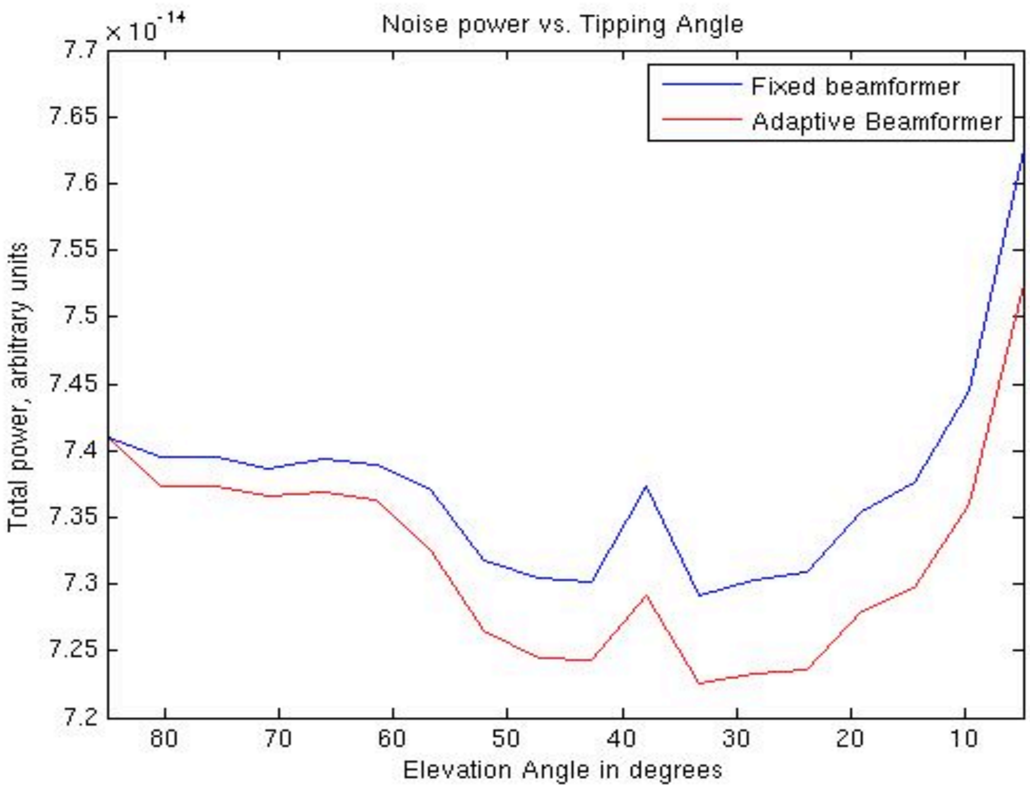
- Aperture efficiency increases as sky temp in main beam grows.
- Beam becomes more directive to narrow the patch of warmer sky in main lobe.
- At typical LNA T_{min} values, T_{sys} is reduced by 1-2 percent.
- 20m, F/D = 0.43 dish with 19 element array was assumed.

Tipping simulation results

- Noise power (arbitrary scale) drops at lower elevations as LCMV adapts to changes in the noise spatial structure.
- This simulation used T_{min} = 120K to match the experimental system.



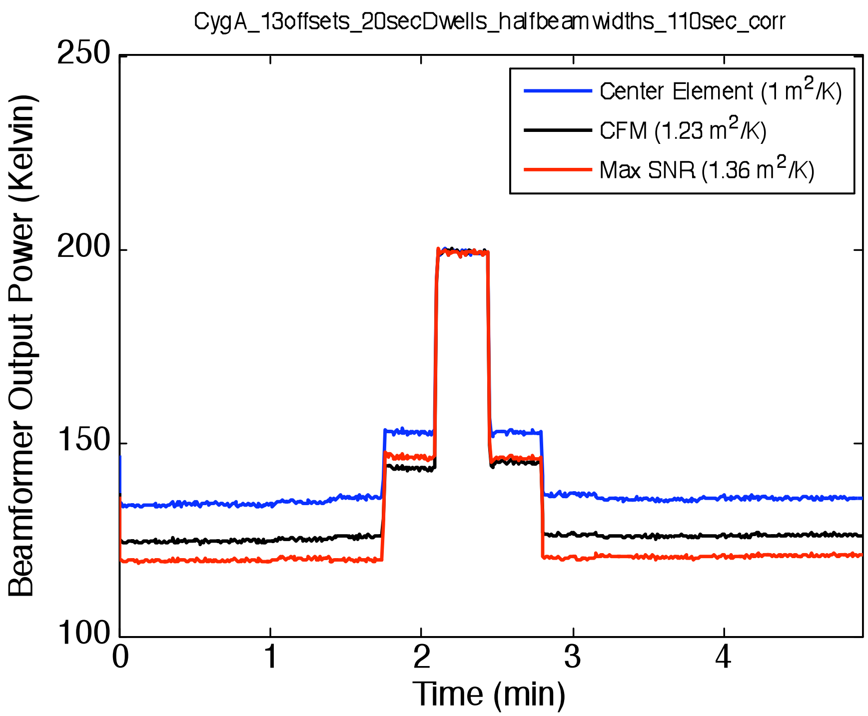
Tipping real data results

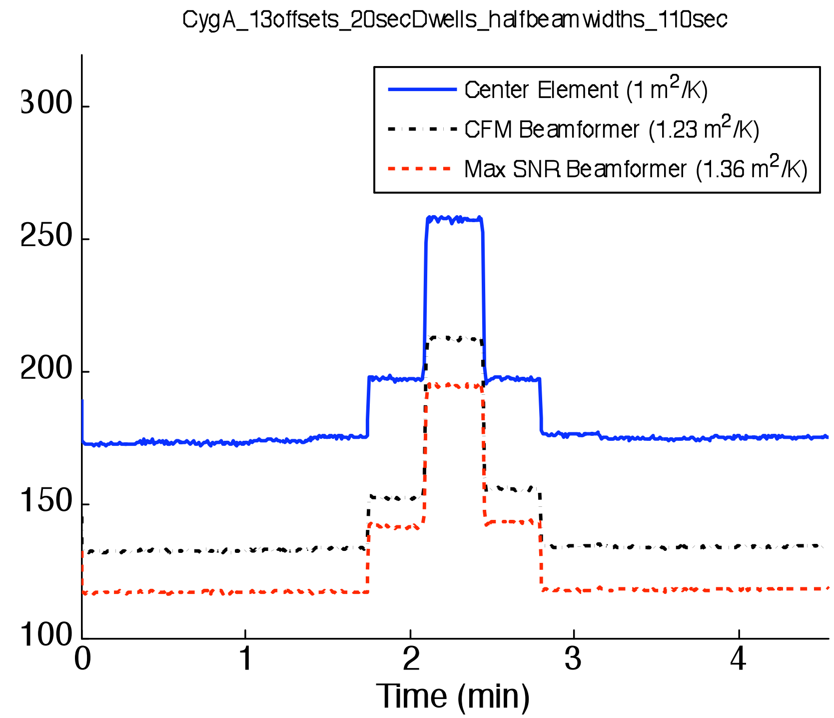


- Good agreement with simulation.
- Peak at 40 deg. is due to temporary instability in one receiver channel.
- A modest improvement in T_{sys} is possible at mid to low elevations.

Experimental beamformer responses

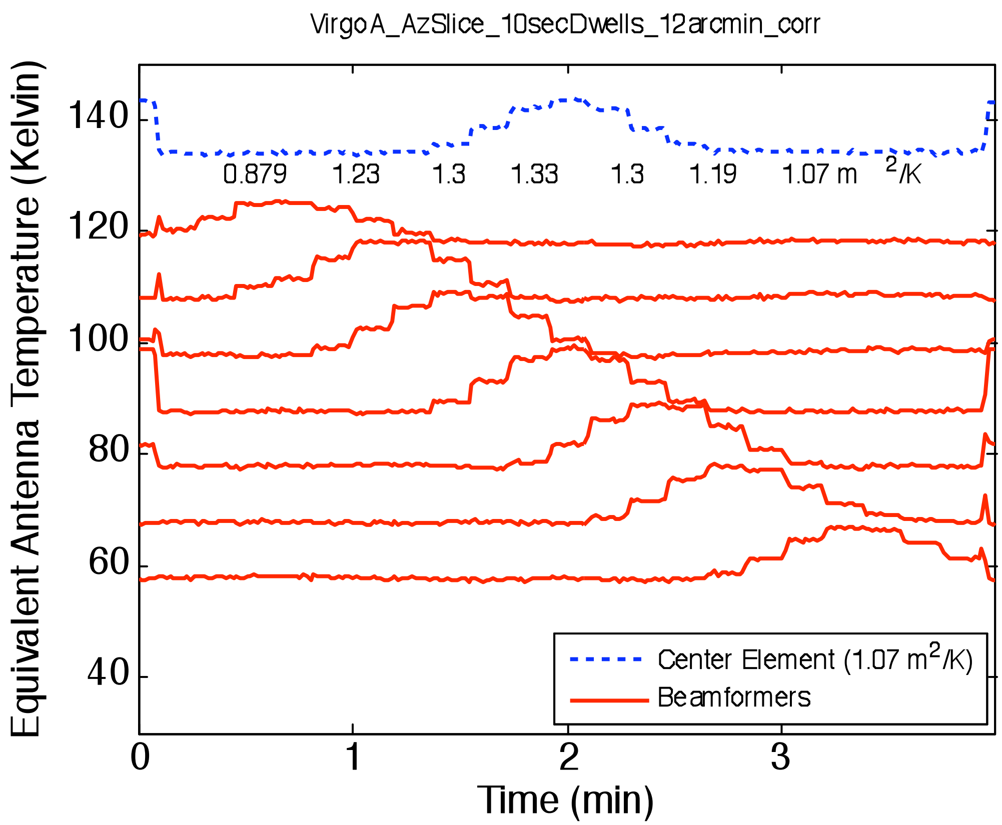
- Green Bank 20m, 19 element data, F/D = .43.
- Stepped elevation slice across sky through Cyg A.
- Beamformers:
  - Center element only.
  - Conjugate filed match.
  - Max SNR (max sensitivity)





Multiple steered beams

- Single 19 element data record combined with multiple sets of beamformer weights.
- Source: Virgo A
- 12 arcmin. steps (1/4 beamwidth) in an azimuth cut.
- Beamwidth is 48 arcmin.
- Beamformer is max SNR:
  - Cyg A calibration
  - multiple calibration pointings.



Sensitivity for off-axis beams

Model Details:

Element patterns: Ansoft HFSS (FEM)

Array mutual impedance matrix: HFSS

Reflector: Physical optics (PO), no blockage or feed support scattering

Receiver noise: Network model

LNA noise parameters:

T_{min} = 100 K, Z_{opt} = 45 + j5 Ω, R_n = 5 Ω

Boresight Beam (Model)

Aperture efficiency: 79%

Spillover efficiency: 97%

Beam equivalent T_{rec}: 127K

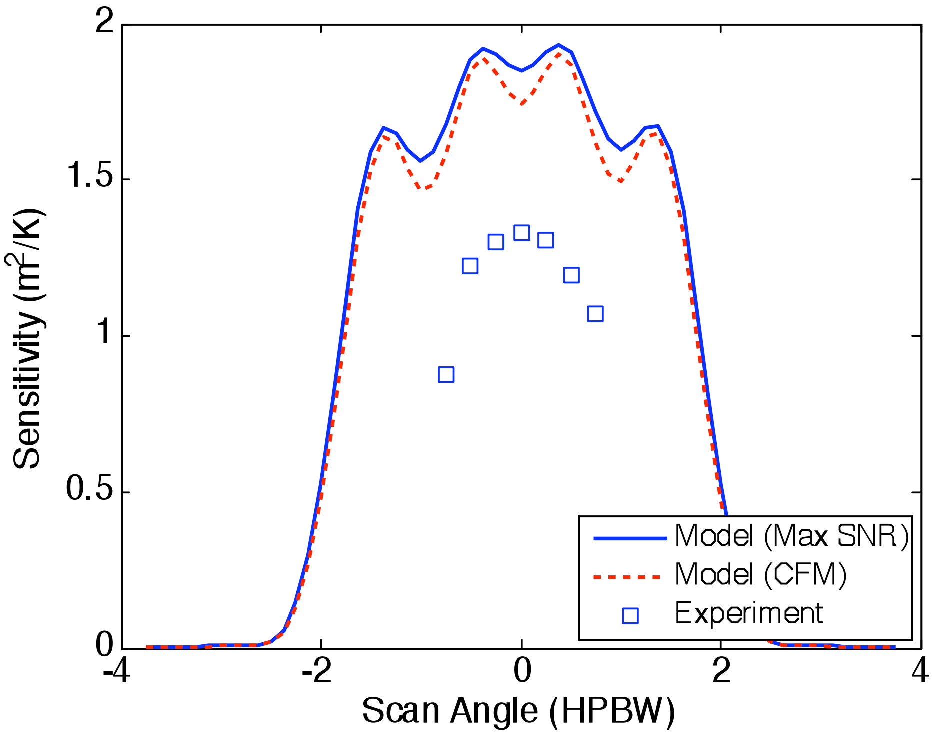
Beam equivalent T_{sys}: 134K

Sensitivity: 1.85 m^2/K

Boresight Beam (Experiment)

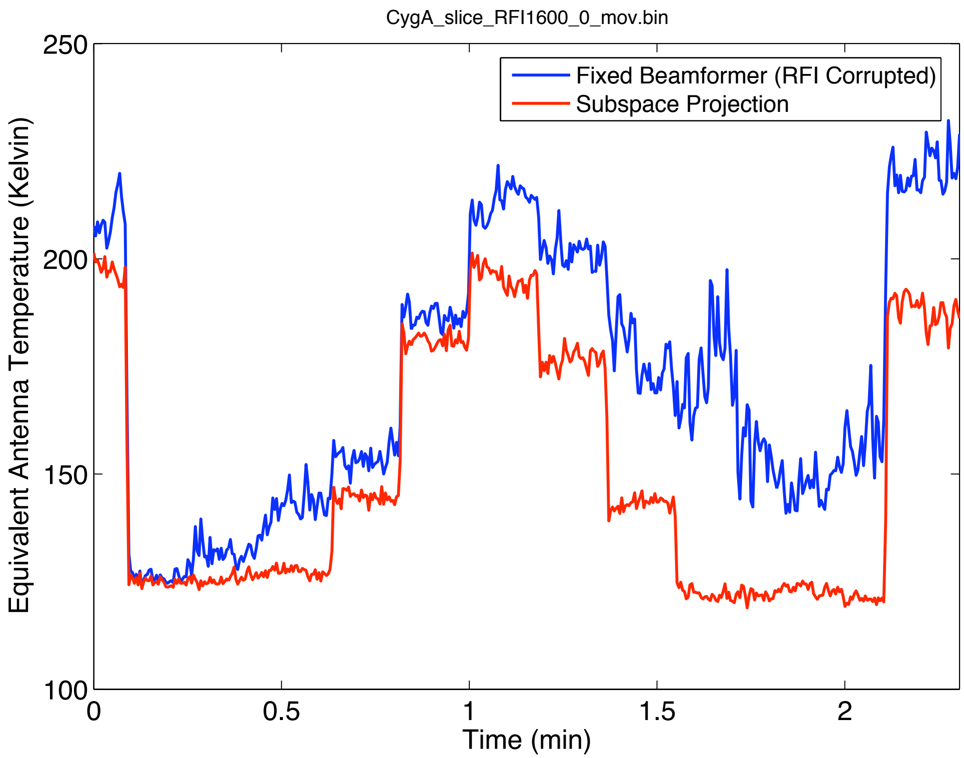
Sensitivity: 1.33 m^2/K

Aperture efficiency (@T_{sys} = 134K): 57%



- Due to large LNA T_{min}, the beamformer finds a solution with relatively high aperture efficiency (79%) and low spillover efficiency (97%).
- Beam equivalent T_{sys} is higher than the LNA T_{min} due to array mutual coupling.

Adaptive noise cancellation



- Subspace projection adaptive canceling beamformer.
- Strong moving CW RFI source in deep sidelobes.
- Dish was stepped in elevation through source, Cyg A

Future work

- Real-data beampattern and sensitivity measurements for $\mathbf{w}$ pre-computed from EM models.
  How close can we get?
- Study performance bounds for deterministic beamformers.
- Lower T_{rec},T_{sys} experiments, ~35K, July 2008.
- 37 element array soon.
- Larger grids and longer integrations for calibration data.
- RFI mitigation
  - Null depth improvement.
  - Beampattern control and adaptive bias removal.