CALIBRATION AND OPTIMAL BEAMFORMING FOR A 19 ELEMENT PHASED ARRAY FEED

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Green Bank 20-Meter Telescope

- 20m, F/D = 0.43, fast slewing.
- Re-commissioned for array feed experiments.
- 3.6m auxiliary antenna tracks RFI for mitigation.
BYU – NRAO 19 element array

- Hexagonal grid of thickened 1.6 GHz dipoles.
- 0.6 $\lambda$ spacing, single polarization.
- 0.25 $\lambda$ offset from ground plane.
Analog RF front end

- Downconversion to IF in front-end box behind array.
- 19 double conversion, room temperature analog receivers, $T_{sys} \approx 110K$.
- Remotely tunable from 1200-2000MHz.
- IF bandwidth selectable at 0.5, 1.0, and 5.0 MHz.
- Connectorized modular system facilitates maintenance & channel isolation.
Digital receiver back end

- 3 MHz IF from FEB to RFI shielded digital receiver in pedestal room.
  - 19 thin, inexpensive cables are low loss at this IF.
  - 1\textsuperscript{st} and 2\textsuperscript{nd} LO from base to FEB using low loss microwave coax.
- 20 channel, 12 bit, synchronous sampling at 1.25Msamp/sec.
- Continuous streaming to fast RAID-0 disk array for ~2 hours.
- Digitally mixed to complex baseband in post processing.
System overview

Receiving System
Block Diagram

Front-End Box
- Array Feed
- 19 receiver channels

20m Pedestal
- 20m dish control computer
- AC, DC
- Temp Control
- Shielded Rack

Jansky Lab
- 20m dish control computer
- Fiber RS-232 to Pedestal Room
- Remote Desktop

Reference Ant
- Reference Ant LNA
- Fiber RS-232
- RF/Optical Modem
- Antenna Positioners
Signal and beamformer model

\[ y[n] = w^H x[n] \]

Noise: \( \eta_1[n] \)  
Signal source of interest: \( s[n] \)  
Interference: \( d[n] \)  
\( \theta \)  
\( x_1[n] \)  
\( x_2[n] \)  
\( x_3[n] \)  
..  
\( x_p[n] \)  
\( \eta_p[n] \)  
\( w_1 \)  
\( w_2 \)  
\( w_3 \)  
..  
\( w_p \)  
Narrowband array operation is assumed
Signal and beamformer model (2)

- **Array sample vector**
  \[ x[n] = v_s s[n] + v_d d[n] + n[n] \]

- **Sample array covariance estimate for \( j \)-th short-term integration (STI):**
  \[ \hat{R}_j = \frac{1}{L} X_j X^H_j \approx R_s + R_d[jL] + R_n \]

  \[ X_j = [x[jL], x[jL+1], \ldots, x[(j+1)L-1]]. \]

- **Beamformer output**
  \[ y[n] = w^H_j x[n], \quad j = \lfloor n/L \rfloor \]
Beamformer Definitions

- **Conjugate field match (CFM)**
  - \( w_j = \hat{v}_s, \ \hat{v}_s \) from calibration data set.
  - A.K.A. phase conjugate weighting, or spatial matched filter.
  - Max SNR solution for i.i.d. noise.

- **LCMV**
  - \( w_j = \hat{R}_j^{-1} [C^H \hat{R}_j^{-1} C]^{-1} f \),
  - Columns of \( C \) are distinct calibration constraint vectors.
  - \( f \) is desired response in constraint directions, \( w_j^H C = f \).
  - Single mainlobe constraint case (MVDR):
    \[
    w_j = \frac{\hat{R}_j^{-1} \hat{v}_s}{\hat{v}_s^H \hat{R}_j^{-1} \hat{v}_s}
    \]
Beamformer Definitions (2)

- **Max SNR (A.K.A. Max sensitivity)**
  - $w_j$ is the solution to eigenvector problem
    \[
    \hat{R}_n^{-1}(\hat{v}_s\hat{v}_s^H)w = \lambda_{\text{max}}w
    \]
  - $\hat{R}_n$ estimated from off-source calibration data.

- **Subspace projection for interference cancellation.**
  - Partitioned eigen decomposition estimates subspace $U_d$ spanning interference array signature:
    \[
    \hat{R}_j[U_d | U_{s+\eta}] = [U_d | U_{s+\eta}]\Lambda
    \]
    \[
    w_j = P_j\tilde{w} \text{ where } P_j = I - U_dU_d^H.
    \]
  - $\tilde{w}$ is the precomputed fixed, quiescent, interference-free beamformer.

- Note the lack of any conventional, fixed weight, windowed, or iteratively optimized beamformer.
Calibration procedure

1. Estimate $\hat{R}_n$ while steered to a quiet, off source sky patch.

2. Using the brightest available calibrator source, steer dish to calibration angle $\{\Omega_k \mid 1 \leq k \leq K\}$ (relative to the source) and estimate $\hat{R}_{\Omega_k}$.

3. Calibration grid, $\{\Omega_k \mid 1 \leq k \leq K\}$, includes all multi-beam steerings and mainlobe shape constraints.

4. Calibration steering vector $\hat{v}_{\Omega_k}$ is the dominant eigenvector of $(\hat{R}_{\Omega_k} - \hat{R}_n)$. 
Beamformer calibration Issues

- Bench-top or antenna range calibration is inadequate.
  - Differential drift between channels: gain, phase, and noise levels.
  - Local environment (e.g. supports) affect element patterns.
  - Variation in element patterns across the array is complex, hard to measure the patterns densely enough.

- Detailed EM model is unsuitable for calibration.
  - Helpful for qualitative analysis and representative studies.
  - Cannot model physical response closely enough for high sensitivity beamforming.
Beamformer calibration Issues (2)

- **Strong mutual coupling**
  - Complicates element patterns; they are *not* identical.
  - Correlates receiver noise which must be accounted for in beamformer design to optimize sensitivity.
  - Coupled differential LNA noise drift makes fixed beamformer non-optimal, even if element patterns were known exactly.

- **Adaptive beamforming calibration is needed:**
  - In each (multiply)steered beam direction,
  - In each response constraint direction.

- **Deterministic beamforming requires in addition:**
  - Calibration over entire spillover region.
  - Responses are too variable to achieve low spillover illumination without dense calibration.
Fixed-adaptive beamforming

- We don’t know how to do conventional, deterministic beamformer design in this calibration environment.
  - Works in simulation, but for real data we lack sufficient information.
  - Much easier with a bare array than with a PAF and dish.
  - Much easier in a lower sensitivity regimes (e.g. non-cryo-cooled LNAs, comm. systems and radar, not dominated by spillover noise).

- Solution:
  - Pre-compute fixed-adaptive $\mathbf{w}$ from calibration data only.
  - Optimal LCMV, MVDR, and Max SNR solutions automatically minimize $T_{\text{sys}}$.
  - Hold $\mathbf{w}$ fixed for observations, i.e. use like a non-adaptive beamformer.
  - For subspace projection, use e.g. LCMV to find $\mathbf{\tilde{w}}$. 
Calibration and beamforming results

Green Bank 20 Meter Telescope
and EM simulations
Conjugate field match (CFM) and LCMV beamformers were calculated from single pointing Cyg A calibration data.

10s observation, on-off source baseline subtraction.

LCMV permits W49N detection. CFM spillover noise is too high.
Adapting to spillover noise variation with elevation “tipping”

- **At zenith:**
  - All spillover sidelobes see warm ground, dominates $T_{\text{sys}}$.
  - 2K-4K sky in main beam.

- **Mid elevations:**
  - Upper sidelobes see cold sky.
  - 10K-20K sky in main beam.

- **Near horizon:**
  - 45K sky dominates $T_{\text{sys}}$.
  - Half of spillover sidelobes see cold sky.

- An adaptive beamformer can exploit changes in spillover and sky noise spatial structure to minimize $T_{\text{sys}}$ at each elevation.
Tipping spillover geometry
Aperture efficiency increases as sky temp in main beam grows.

Beam becomes more directive to narrow the patch of warmer sky in main lobe.

At typical LNA $T_{\text{min}}$ values, $T_{\text{sys}}$ is reduced by 1-2 percent.

20m, F/D = 0.43 dish with 19 element array was assumed.
Tipping simulation results

- Noise power (arbitrary scale) drops at lower elevations as LCMV adapts to changes in the noise spatial structure.
- This simulation used $T_{\min} = 120K$ to match the experimental system.
Tipping real data results

- Good agreement with simulation.
- Peak at 40 deg. is due to temporary instability in one receiver channel.
- A modest improvement in $T_{sys}$ is possible at mid to low elevations.
Experimental beamformer responses

- Green Bank 20m, 19 element data, F/D = .43.
- Stepped elevation slice across sky through Cyg A.

Beamformers:
- Center element only.
- Conjugate filed match.
- Max SNR (max sensitivity)
Multiple steered beams

- Single 19 element data record combined with multiple sets of beamformer weights.
- Source: Virgo A
- 12 arcmin. steps (1/4 beamwidth) in an azimuth cut.
- Beamwidth is 48 arcmin.
- Beamformer is max SNR:
  - Cyg A calibration
  - multiple calibration pointings.
Sensitivity for off-axis beams

Model Details:
- Element patterns: Ansoft HFSS (FEM)
- Array mutual impedance matrix: HFSS
- Reflector: Physical optics (PO), no blockage or feed support scattering
- Receiver noise: Network model
- LNA noise parameters:
  \[ T_{\text{min}} = 100 \, \text{K}, \quad Z_{\text{opt}} = 45 + j5 \, \Omega, \quad R_n = 5 \, \Omega \]

Boresight Beam (Model)
- Aperture efficiency: 79%
- Spillover efficiency: 97%
- Beam equivalent \( T_{\text{rec}} \): 127K
- Beam equivalent \( T_{\text{sys}} \): 134K
- Sensitivity: 1.85 m\(^2\)/K

Boresight Beam (Experiment)
- Sensitivity: 1.33 m\(^2\)/K
- Aperture efficiency (\( @ T_{\text{sys}} = 134K \)): 57%

- Due to large LNA \( T_{\text{min}} \), the beamformer finds a solution with relatively high aperture efficiency (79%) and low spillover efficiency (97%).
- Beam equivalent \( T_{\text{sys}} \) is higher than the LNA \( T_{\text{min}} \) due to array mutual coupling.
Adaptive noise cancellation

- Subspace projection adaptive canceling beamformer.
- Strong moving CW RFI source in deep sidelobes.
- Dish was stepped in elevation through source, Cyg A
Future work

- Real-data beampattern and sensitivity measurements for $w$ pre-computed from EM models. How close can we get?
- Study performance bounds for deterministic beamformers.
- Lower $T_{rec}, T_{sys}$ experiments, $\sim$35K, July 2008.
- 37 element array soon.
- Larger grids and longer integrations for calibration data.
- RFI mitigation
  - Null depth improvement.
  - Beampattern control and adaptive bias removal.