Schedule...

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
6 Oct	Mon	10	Energy Storage	3.7, 4.1		NO LAB	
7 Oct	Tue					NO LAB	
8 Oct	Wed	11	Dynamic Circuits	4.2 - 4.4			
9 Oct	Thu						
10 Oct	Fri		Recitation		HW 4		
11 Oct	Sat						
12 Oct	Sun						
13 Oct	Mon	12	Exam 1 Review			LAB 4	EXAM 1
14 Oct	Tue						1



Energy

<u>Moro. 7: 48</u>

48 Wherefore, my beloved brethren, pray unto the Father with all the **energy** of heart, that ye may be filled with this love, which he hath bestowed upon all who are true followers of his Son, Jesus Christ; that ye may become the sons of God; that when he shall appear we shall be like him, for we shall see him as he is; that we may have this hope; that we may be purified even as he is pure. Amen.



Lecture 10 – Energy Storage

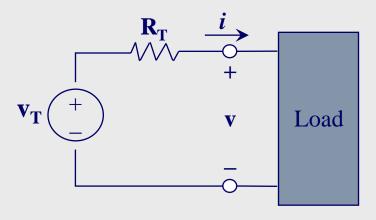
Maximum Power Transfer Capacitors Inductors



3

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 Equivalent circuit representations are important in power transfer analysis

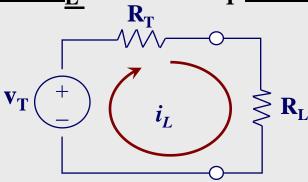


• Ideally all power from source is absorbed by the load

A But some power will be absorbed by internal circuits (represented by \mathbf{R}_{T})



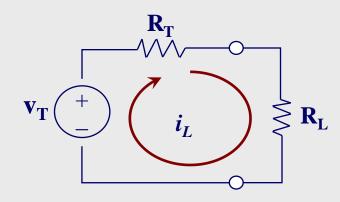
- Efficiently transferring power from source to load means that internal resistances (\mathbf{R}_{T}) must be minimized
 - \wedge i.e. for a given $\mathbf{R}_{\mathbf{L}}$ we want $\mathbf{R}_{\mathbf{T}}$ as small as possible!



For a given $\mathbf{R}_{\underline{\mathbf{T}}}$ there is a specific $\mathbf{R}_{\underline{\mathbf{L}}}$ that will maximize power transfer to the load

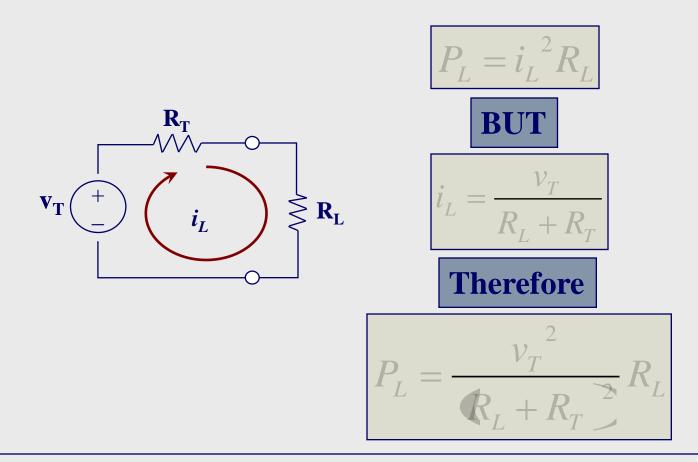


• Consider the power ($\mathbf{P}_{\mathbf{L}}$) absorbed by the load





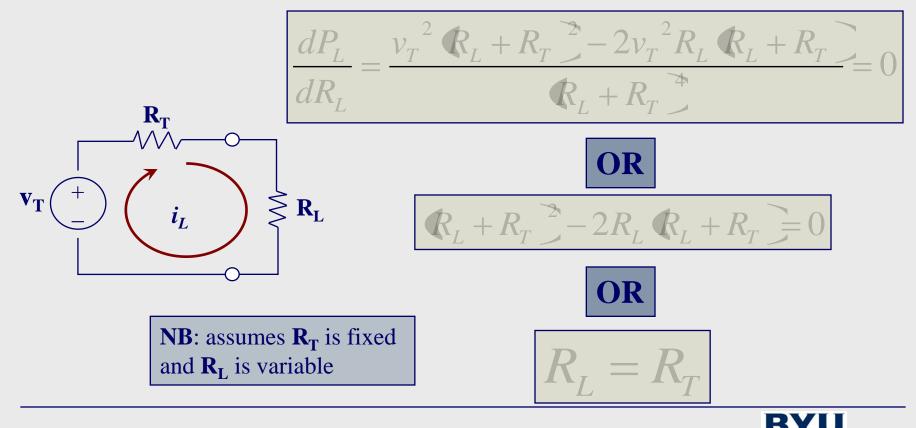
• Consider the power ($\mathbf{P}_{\mathbf{L}}$) absorbed by the load





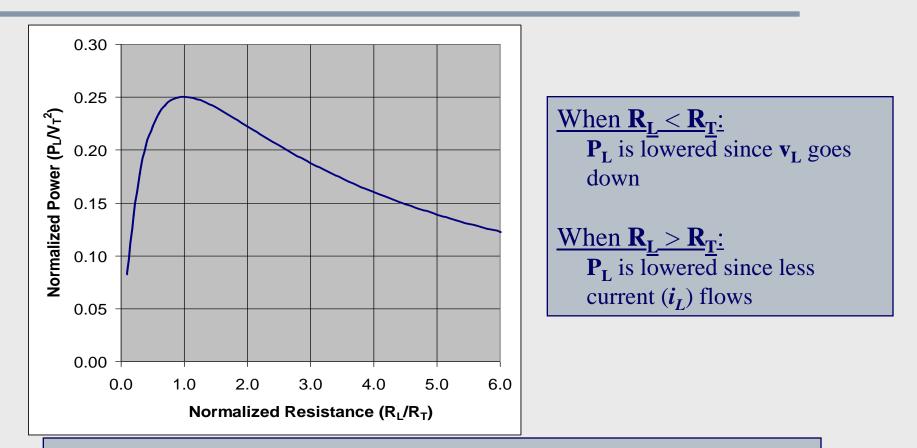
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• The **maximum** power transfer can be found by calculating $dP_L/dR_L = 0$



Discussion #10 – Energy Storage

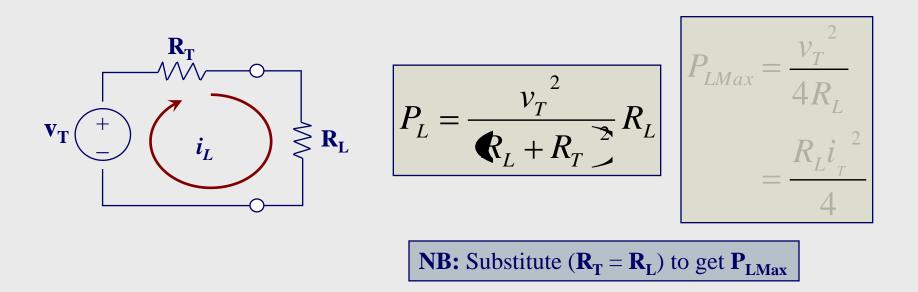
Electrical Engineering Computer Engineering



To transfer maximum power to the load, the source and load resistors must be **matched** (i.e. $\mathbf{R}_{T} = \mathbf{R}_{L}$). Power is attenuated as \mathbf{R}_{L} departs from \mathbf{R}_{T}



<u>Maximum power theorem</u>: maximum power is delivered by a source (represented by its Thévenin equivalent circuit) is attained when the load (\mathbf{R}_{L}) is equal to the Thévenin resistance \mathbf{R}_{T}





10

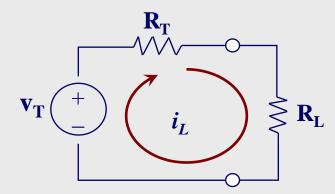
Efficiency of power transfer (η) : the ratio of the power delivered to the load (\mathbf{P}_{out}) , to the power supplied by the source (\mathbf{P}_{in})

$$\eta = \frac{p_{out}}{p_{in}}$$

NB:
$$\mathbf{P}_{out} = \mathbf{P}_{\mathbf{L}}$$

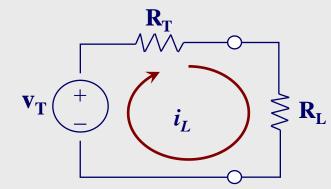


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Efficiency of power transfer (η) : the ratio of the power delivered to the load (\mathbf{P}_{out}) , to the power supplied by the source (\mathbf{P}_{in})



$$P_{in} = i_L v_T$$
$$= v_T \left(\frac{v_T}{R_T + R_L} \right)$$

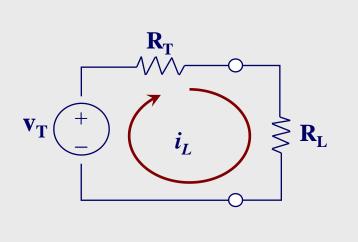
$$P_{inMax} = \frac{v_T^2}{2R_L}$$
$$= \frac{R_L i_T^2}{2}$$

NB: Substitute ($\mathbf{R}_{T} = \mathbf{R}_{L}$) to get \mathbf{P}_{inMax}



13

Efficiency of power transfer (η) : the ratio of the power delivered to the load (\mathbf{P}_{out}) , to the power supplied by the source (\mathbf{P}_{in})

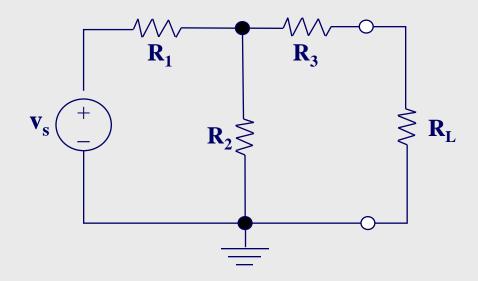


$$P_{\text{max}} = \frac{P_{outMax}}{P_{inMax}}$$
$$= \frac{v_T^2}{4R_L} \cdot \frac{2R_L}{v_T^2}$$
$$= \frac{1}{2}$$

In other words, **at best**, only 50% efficiency can be achieved

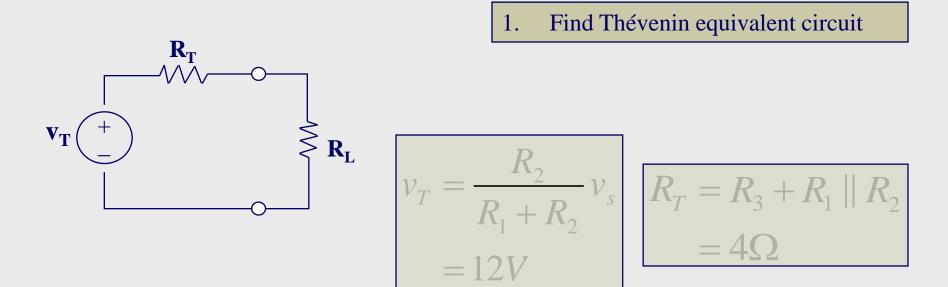


• Example1: Find the maximum power delivered to $\mathbf{R}_{\mathbf{L}}$ • $\mathbf{v}_{\mathbf{s}} = 18 \mathbf{V}, \mathbf{R}_{\mathbf{1}} = 3\Omega, \mathbf{R}_{\mathbf{2}} = 6\Omega, \mathbf{R}_{\mathbf{3}} = 2\Omega$



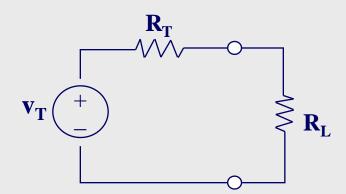


• Example1: Find the maximum power delivered to \mathbf{R}_{L} • $\mathbf{v}_{s} = 18 \text{V}, \mathbf{R}_{1} = 3\Omega, \mathbf{R}_{2} = 6\Omega, \mathbf{R}_{3} = 2\Omega$





• Example1: Find the maximum power delivered to \mathbf{R}_{L} • $\mathbf{v}_{s} = 18V, \mathbf{R}_{1} = 3\Omega, \mathbf{R}_{2} = 6\Omega, \mathbf{R}_{3} = 2\Omega$



1. Find Thévenin equivalent circuit

2. Set
$$\mathbf{R}_{\mathbf{L}} = \mathbf{R}_{\mathbf{T}}$$

3. Calculate
$$\mathbf{P}_{\mathbf{LMax}}$$

$$P_{L \max} = 0.5 P_{in\max}$$
$$= 0.5 \frac{v_T^2}{2R_L}$$
$$= 0.5 \frac{(12)^2}{8}$$
$$= 9W$$



17

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Dynamic Energy Storage Elements

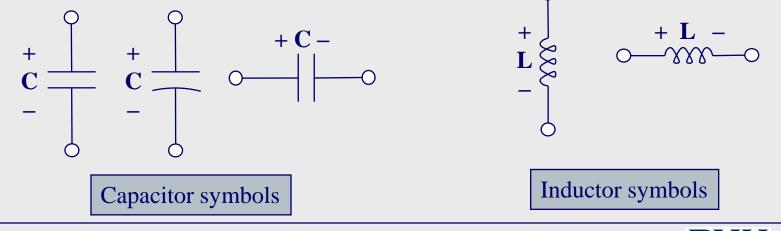
Capacitor and Inductor



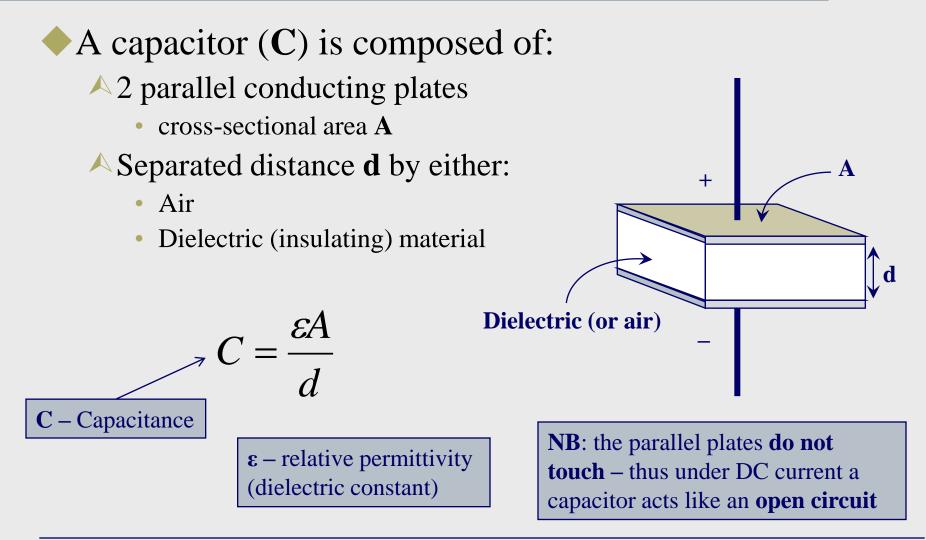
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Storage Elements

To this point, circuits have consisted of either:
DC Sources (active element, provide energy)
Resistors (passive element, absorb energy)
Some passive (non-source) elements can store energy
Capacitors – store voltage
Inductors – store current



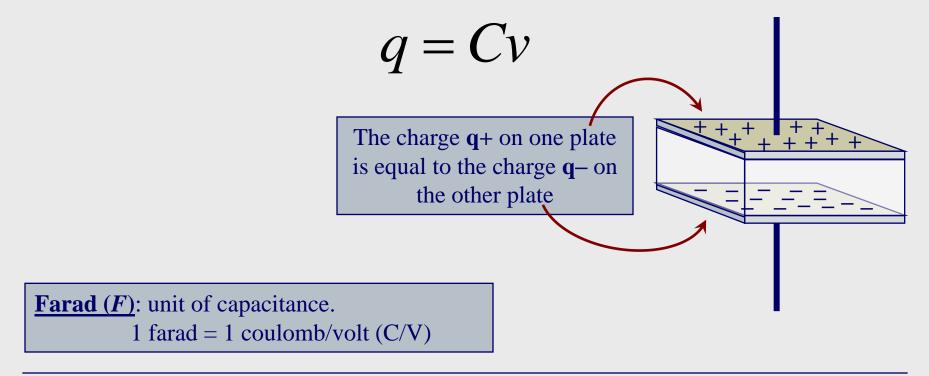
19



20

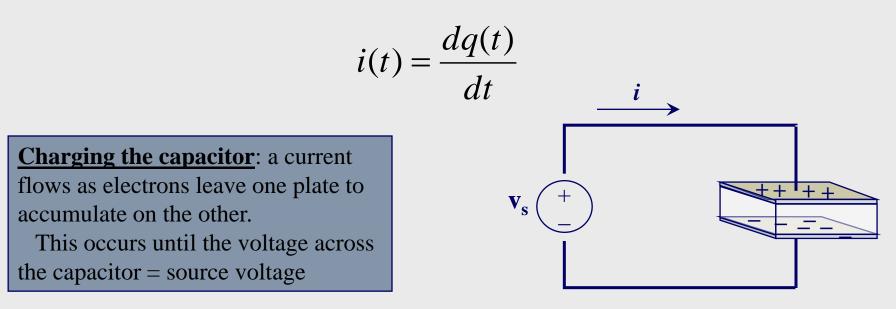
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<u>**Capacitance</u>**: a measure of the ability to store energy in the form of separated charge (or an electric field)</u>



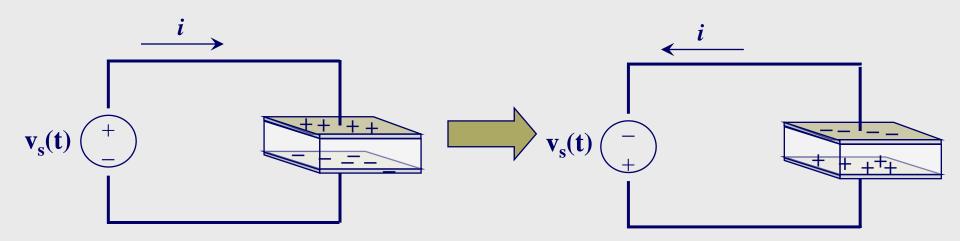


- No current can flow through a capacitor if the voltage across it is constant (i.e. connected to a DC source)
- BUT when a DC source is applied to a capacitor, there is an initial circuit current as the capacitor plates are charged ('charging' the capacitor)



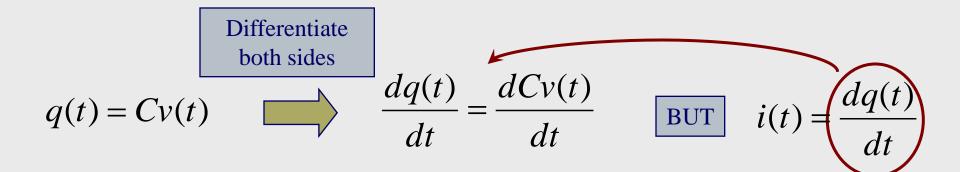


- <u>Capacitor with an AC source</u>: since AC sources periodically reverse voltage directions, the capacitor plates will change charge polarity accordingly
 - With each polarity change, the capacitor goes through a 'charging' state, thus current continuously flows
 - ▲ Just as with a DC source, **no electrons cross between the plates**
 - Just as with a DC source, voltage across a capacitor cannot change instantaneously





♦ i – v characteristic for an ideal capacitor



$$i(t) = C \frac{dv(t)}{dt} \quad \text{OR} \quad v(t) = \frac{1}{C} \int_{-\infty}^{t} i_C(\tau) d\tau$$



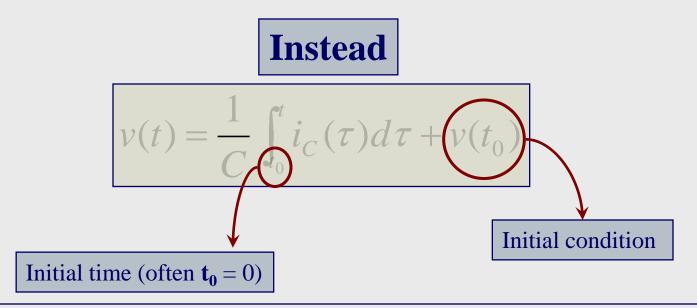
24

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♦ i – v characteristic for an ideal capacitor

$$v(t) = \frac{1}{C} \int_{-\infty}^{t} i_C(\tau) d\tau$$

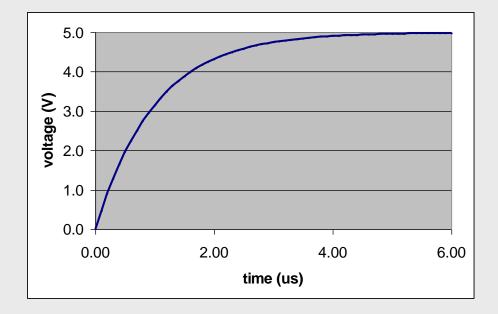
NB: Assumes we know the value of the capacitor from time ($\tau = -\infty$) until time ($\tau = t$)





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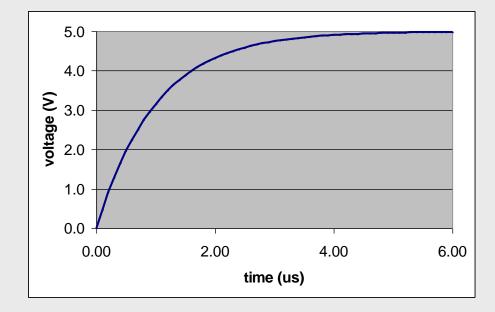
• <u>Example2</u>: calculate the current through the capacitor $C = 0.1 \mu F$ with the voltage as shown: $v(t) = 5(1-e^{-t/10^{-6}})$





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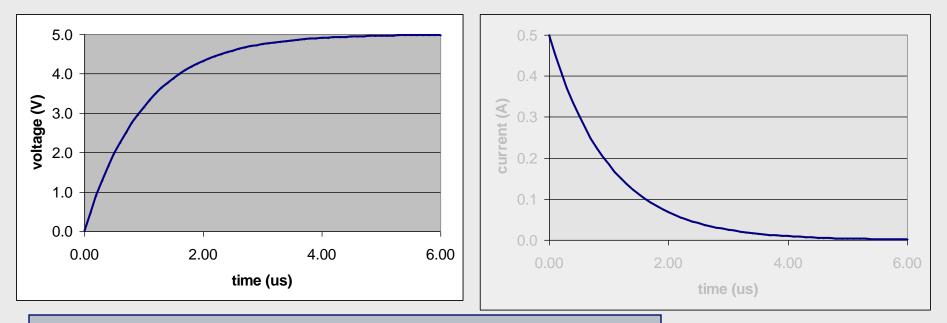


$$i(t) = C \frac{dv(t)}{dt}$$

= $(10^{-7}) \frac{5}{10^{-6}} e^{-t/10^{-6}}$
= $0.5e^{-t/10^{-6}} A$



• <u>Example2</u>: calculate the current through the capacitor $C = 0.1 \mu F$ with the voltage as shown: $v(t) = 5(1-e^{-t/10^{-6}})$



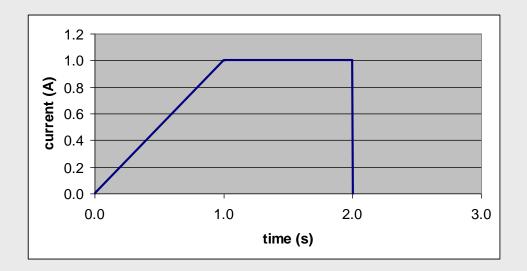
NB: the capacitor's current jumps '**instantaneously**' to 0.5A. The ability of a capacitor's current to change instantaneously is an important property of capacitors

$$i(t) = 0.5e^{-t/10^{-6}}A$$



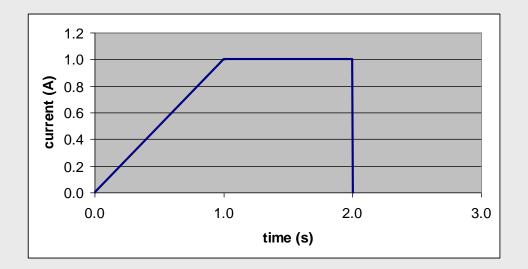
28

• Example3: find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0





• Example3: find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0



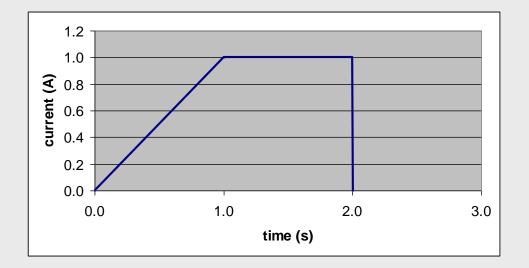
Interval	current $i(t)$
$t \leq 0$	0
$0 < t \le 1$	t
$1 < t \le 2$	1
2 < t	0

$$v(t) = \frac{1}{C} \int_0^t i d\tau + v(0)$$



30

• Example3: find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0



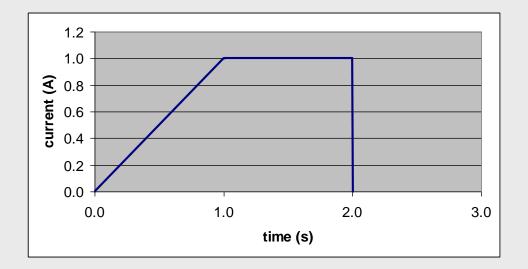
Interval	voltage $v(t)$
$t \leq 0$	0
$0 < t \le 1$	$2\int_0^t \tau d\tau + 0$
$1 < t \le 2$	$2\int_0^t (1)d\tau + v(1)$
2 < t	0 + v(2)

$$v(t) = \frac{1}{C} \int_0^t i d\tau + v(0)$$



31

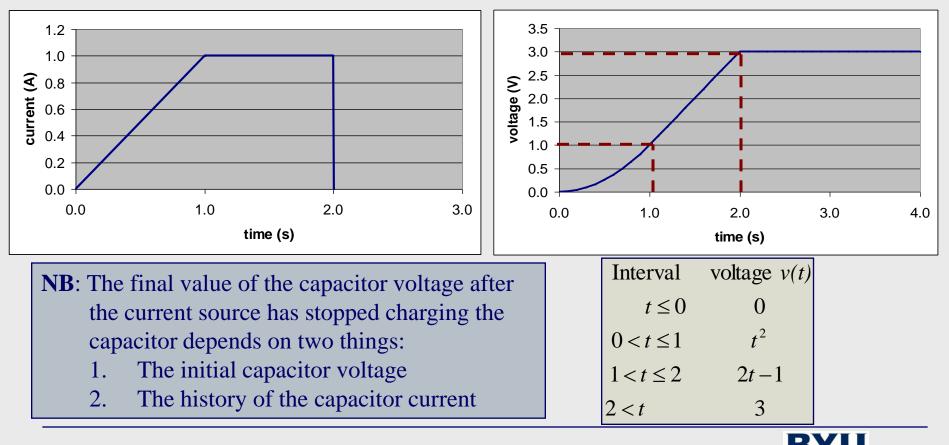
• Example3: find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0



Interval	voltage $v(t)$
$t \leq 0$	0
$0 < t \leq 1$	t^2
$1 < t \le 2$	2t - 1
2 < t	3



• Example3: find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0



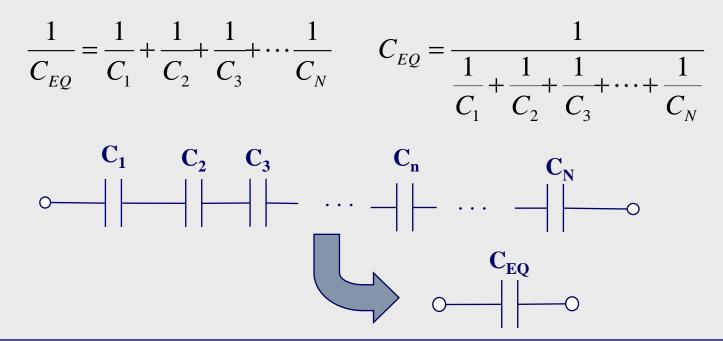
Discussion #10 – Energy Storage

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Series Capacitors

Capacitor Series Rule: two or more circuit elements are said to be in series if the current from one element *exclusively* flows into the next element.

A Capacitors in series add the same way resistors in parallel add



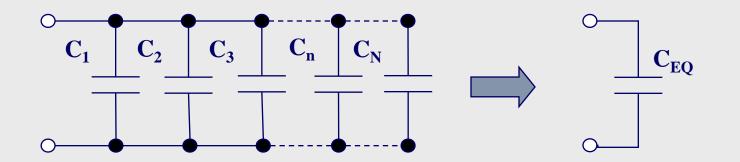


34

Parallel Capacitors

Parallel Rule: two or more circuit elements are said to be in parallel if the elements share the same terminals Capacitors in parallel add the same way resistors in series add

$$C_{EQ} = \sum_{n=1}^{N} C_n$$

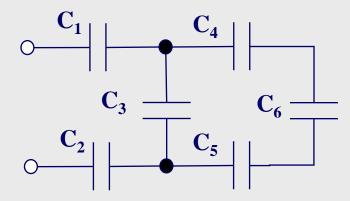




35

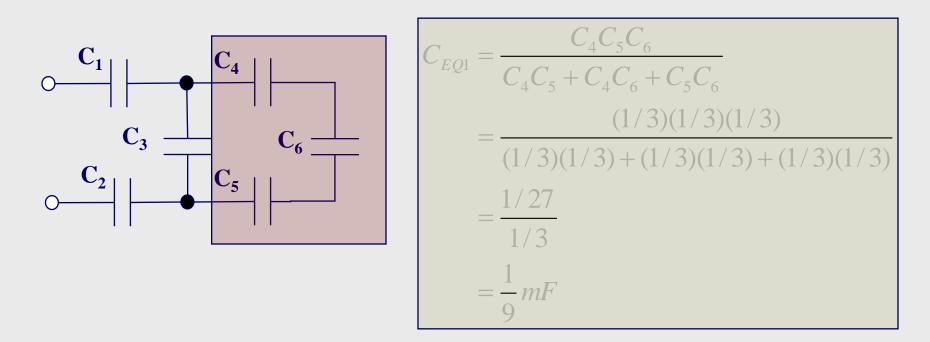
Parallel and Series Capacitors

• Example4: determine the equivalent capacitance C_{EQ} • $C_1 = 2mF$, $C_2 = 2mF$, $C_3 = 1mF$, $C_4 = 1/3mF$, $C_5 = 1/3mF$, • $C_6 = 1/3mF$



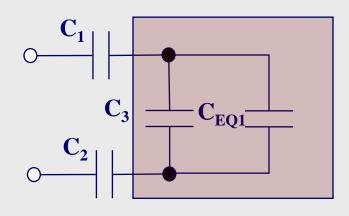


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Example4: determine the equivalent capacitance $C_{EQ} \land C_1 = 2mF, C_2 = 2mF, C_3 = 1mF, C_4 = 1/3mF, C_5 = 1/3mF, C_6 = 1/3mF$



$$C_{EQ2} = C_3 + C_{EQ1}$$
$$= (1) + \left(\frac{1}{9}\right)$$
$$= \frac{10}{9} mF$$

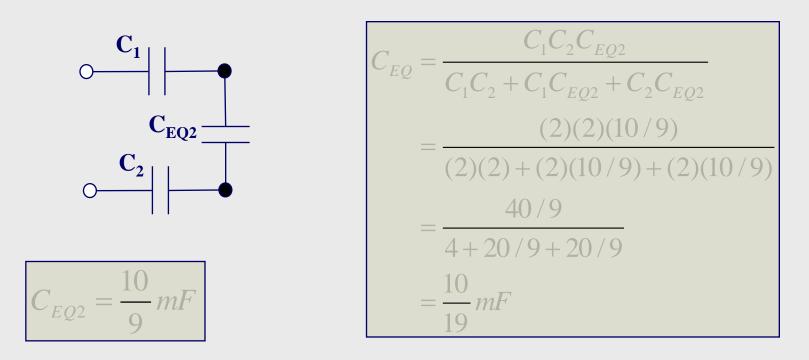
$$C_{EQ1} = \frac{1}{9} mF$$

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38

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Example4: determine the equivalent capacitance $C_{EQ} \land C_1 = 2mF, C_2 = 2mF, C_3 = 1mF, C_4 = 1/3mF, C_5 = 1/3mF, C_6 = 1/3mF$





39

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• Example4: determine the equivalent capacitance C_{EQ} • $C_1 = 2mF$, $C_2 = 2mF$, $C_3 = 1mF$, $C_4 = 1/3mF$, $C_5 = 1/3mF$, • $C_6 = 1/3mF$





<u>**Capacitor energy W**</u>_C(t): can be found by taking the integral of power

▲ Instantaneous power → $P_C = iv$

$$W_{C}(t) = \int_{-\infty}^{t} P_{C}(\tau) d\tau$$

= $\int_{-\infty}^{t} v_{C}(\tau) i_{C}(\tau) d\tau$
= $\int_{-\infty}^{t} v_{C}(\tau) C \frac{dv_{C}(\tau)}{d\tau} d\tau$
= $C \int_{-\infty}^{v(t)} v dv$
= $\frac{1}{2} C v^{2} \Big|_{v(-\infty)}^{v(t)}$

Since the capacitor is uncharged at $\mathbf{t} = -\infty$, $\mathbf{v}(-\infty) = \mathbf{0}$, thus:

$$W_C(t) = \frac{1}{2}Cv(t)^2 J$$

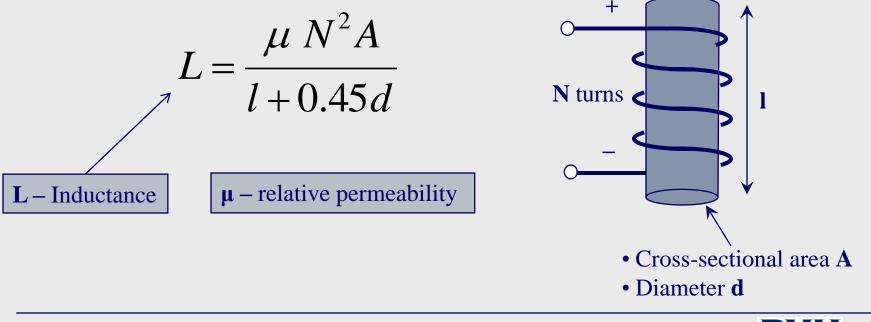


41

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Inductors are made by winding a coil of wire around a core

▲ The core can be an insulator or ferromagnetic material



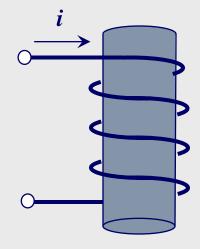


42

Inductance: a measure of the ability of a device to store energy in the form of a magnetic field

Ideally the resistance through an inductor is **zero** (i.e. **no voltage drop**), thus an inductor acts like a **short circuit** in the presence of a **DC source**.

BUT there is an **initial** voltage across the inductor as the current builds up (much like 'charging' with capacitors)

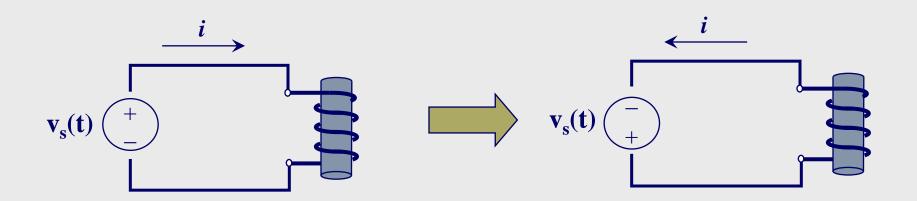


<u>Henry (*H*)</u>: unit of inductance.

1 henry = 1 volt-second/ampere (V-s/A)



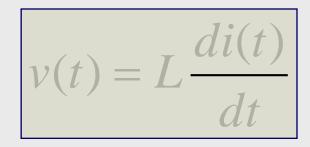
- Inductor with an AC source: since AC sources periodically reverse current directions, the current flow through the inductor also changes
 - ▲ With each current direction change, the current through the inductor must 'build up', thus there is a continual voltage drop across the inductor
 - ▲ Just as with a DC source, current across an inductor cannot change instantaneously





 $\Delta \Delta$

\bullet I – v characteristic for an ideal inductor



$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v_L(\tau) d\tau$$

NB: note the duality between inductors and capacitors

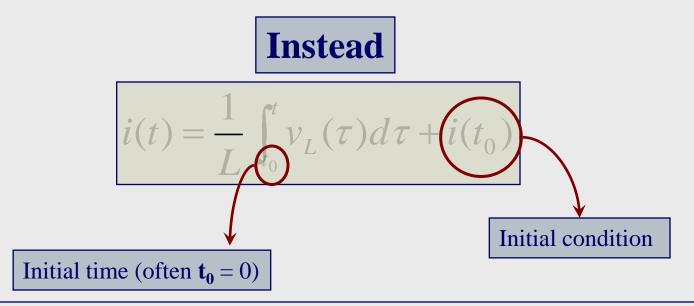




♦ i – v characteristic for an ideal inductor

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v_L(\tau) d\tau$$

NB: Assumes we know the value of the inductor from time ($\tau = -\infty$) until time ($\tau = t$)

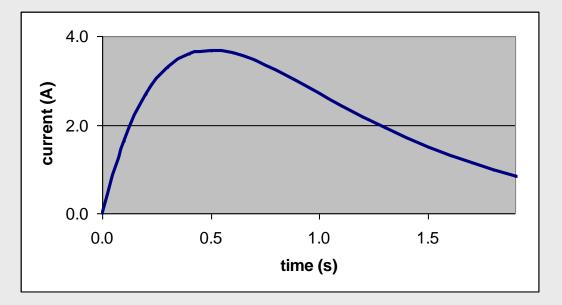




46

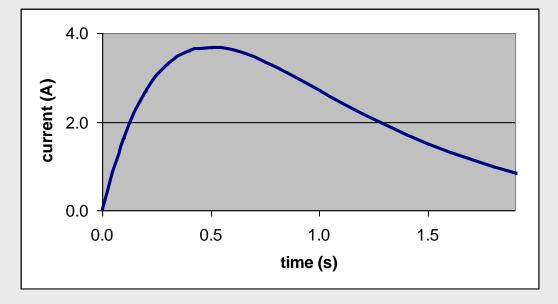
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Example5: find the voltage across an inductor L = 0.1H when the current is: i(t) = 20 t e^{-2t}





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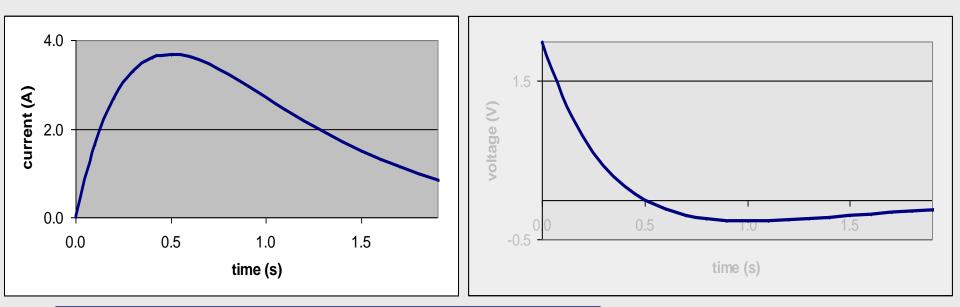


$$v(t) = L \frac{di(t)}{dt}$$

= $0.1 \frac{d}{dt} (20te^{-2t})$
= $2(-2te^{-2t} + e^{-2t})$
= $2e^{-2t} (1-2t)V$



Example5: find the voltage across an inductor L = 0.1H when the current is: i(t) = 20 t e^{-2t}



NB: the inductor's voltage jumps '**instantaneously**' to 2V. The ability of an inductor's voltage to change instantaneously is an important property of inductors

 $v(t) = 2e^{-2t} (1 - 2t)V$



49

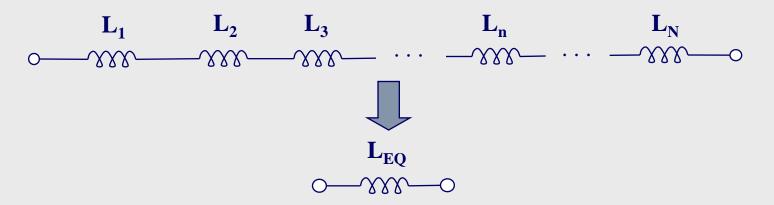
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Series Inductors

Series Rule: two or more circuit elements are said to be **in series** if the current from one element *exclusively* flows into the next element.

▲ Inductors in **series** add the same way resistors in **series** add

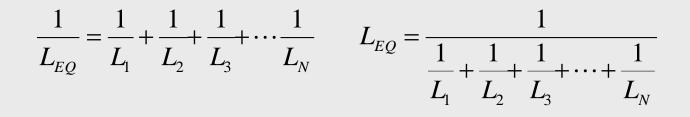
$$L_{EQ} = \sum_{n=1}^{N} L_n$$

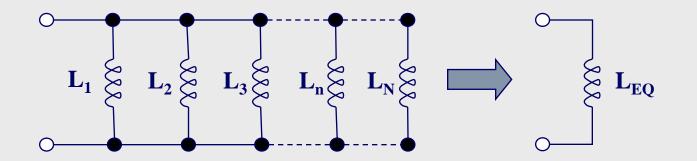




Parallel Inductors

Parallel Rule: two or more circuit elements are said to be in parallel if the elements share the *same* terminals
Inductors in parallel add the same way resistors in parallel add







<u>Capacitor energy $W_{\underline{L}}(t)$ </u>: can be found by taking the integral of power

▲ Instantaneous power → $P_L = iv$

$$W_{L}(t) = \int_{-\infty}^{t} P_{L}(\tau) d\tau$$
$$= \int_{-\infty}^{t} v_{L}(\tau) i_{L}(\tau) d\tau$$
$$= \int_{-\infty}^{t} L \frac{di_{L}(\tau)}{d\tau} i_{L}(\tau) d\tau$$
$$= L \int_{(-\infty)}^{i(t)} i dt$$
$$= \frac{1}{2} L i^{2} \Big|_{i(-\infty)}^{i(t)}$$

Since the inductor is uncharged at $\mathbf{t} = -\infty$, i(- ∞) = 0, thus:

$$W_L(t) = \frac{1}{2} Li(t)^2 J$$



52

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- **Example6**: calculate the power and energy stored in a 0.1-H inductor when:
 - $\land i = 20 t e^{-2t} A$ (i = 0 for t < 0)
 - \wedge V = 2 e^{-2t} (1- 2t)V (for t >= 0)



Example6: calculate the power and energy stored in a 0.1-H inductor when:

$$\land i = 20 t e^{-2t} A (i = 0 \text{ for } t < 0)$$

 \wedge V = 2 e^{-2t} (1- 2t)V (for t >= 0)

$$P_{L}(t) = v_{L}(t)i_{L}(t)$$

= $2e^{-2t}(1-2t)$ $(20te^{-2t})$
= $40te^{-4t}(1-2t)W$



Example6: calculate the **power** and **energy** stored in a 0.1-H inductor when:

$$\land i = 20 t e^{-2t} A (i = 0 \text{ for } t < 0)$$

 \wedge V = 2 e^{-2t} (1- 2t)V (for t >= 0)

$$P_{L}(t) = v_{L}(t)i_{L}(t)$$

= $2e^{-2t}(1-2t)$ $0te^{-2t}$
= $40te^{-4t}(1-2t)W$

$$W_{L}(t) = \frac{1}{2} Li_{L}(t)^{2}$$
$$= \frac{1}{2} (0.1) \ 0 te^{-2t} \ = 20t^{2}e^{-4t} J$$



Ideal Capacitors and Inductors

	Inductors	Capacitors
Passive sign convention	$ \begin{array}{c} + L - \\ \circ \\ i \end{array} $	$\begin{array}{c c} & + \mathbf{C} - \\ & & & \\ & & & \\ & & & \\ & & i \end{array} \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & $
Voltage	$v(t) = L \frac{di(t)}{dt}$	$v(t) = \frac{1}{C} \int_{0}^{t} i_{C}(\tau) d\tau + v(t_{0})$
Current	$i(t) = \frac{1}{L} \int_0^t v_L(\tau) d\tau + i(t_0)$	$i(t) = C \frac{dv(t)}{dt}$
Power	$P_L(t) = Li(t)\frac{di(t)}{dt}$	$P_{C}(t) = Cv(t)\frac{dv(t)}{dt}$



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	Inductors	Capacitors
Energy	$W_L(t) = \frac{1}{2} Li(t)^2$	$W_C(t) = \frac{1}{2}Cv(t)^2$
An instantaneous change is not permitted in:	Current	Voltage
Will permit an instantaneous change in:	Voltage	Current
With DC source element acts as a:	Short Circuit	Open Circuit

