Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
8 Oct	Wed	11	Dynamic Circuits	4.2 – 4.4			
9 Oct	Thu						
10 Oct	Fri		Recitation		HW 5		
11 Oct	Sat						
12 Oct	Sun						
13 Oct	Mon	12	Exam 1 Review			LAB 4	EXAM 1
14 Oct	Tue						
15 Oct	Wed	13	AC Circuit Analysis	4.5			

BYU Electrical Engineering Computer Engineering

Change

1 Corinthians 15:51-57

51 Behold, I shew you a mystery; We shall not all sleep, but we shall all be
changed,

52 In a moment, in the twinkling of an eye, at the last trump: for the trumpet
shall sound, and the dead shall be raised incorruptible, and we shall be
changed.

53 For this corruptible must put on incorruption, and this mortal *must* put on
immortality.

54 So when this corruptible shall have put on incorruption, and this mortal
shall have put on immortality, then shall be brought to pass the saying that is
written, Death is swallowed up in victory.

55 O death, where *is* thy sting? O grave, where *is* thy victory?

56 The sting of death *is* sin; and the strength of sin *is* the law.

57 But thanks *be* to God, which giveth us the victory through our Lord Jesus
Christ.

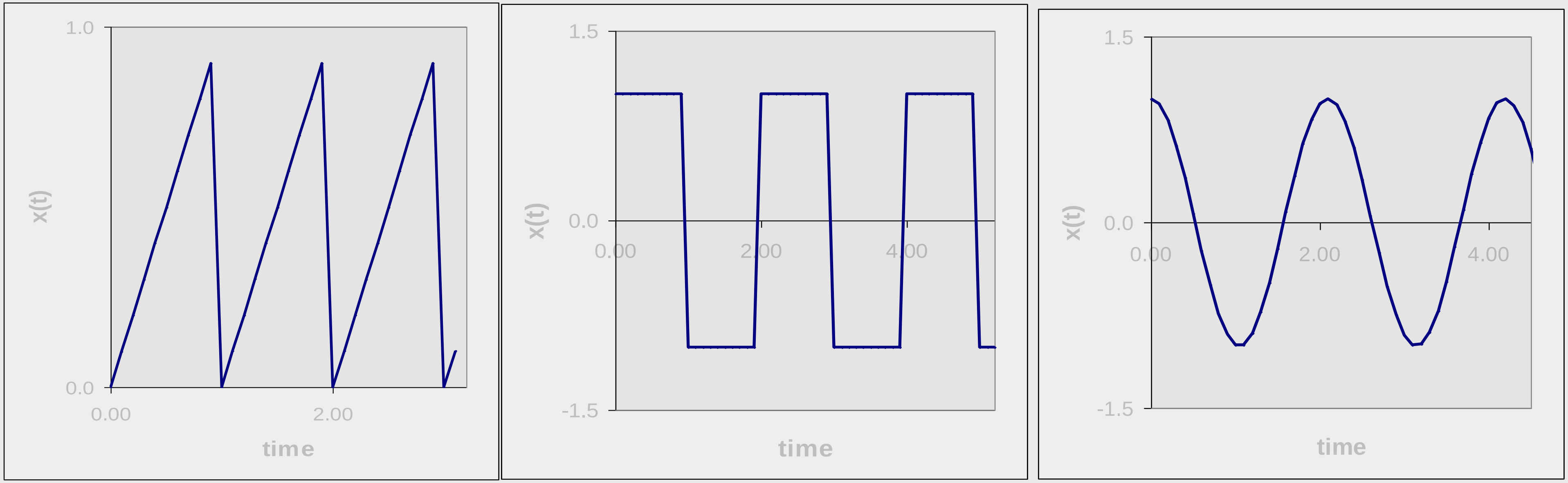
Lecture 11 – Dynamic Circuits

Time-Dependent Sources

Time Dependent Sources

- **Periodic signals**: repeating patterns that appear frequently in practical applications
  - A periodic signal x(t) satisfies the equation:

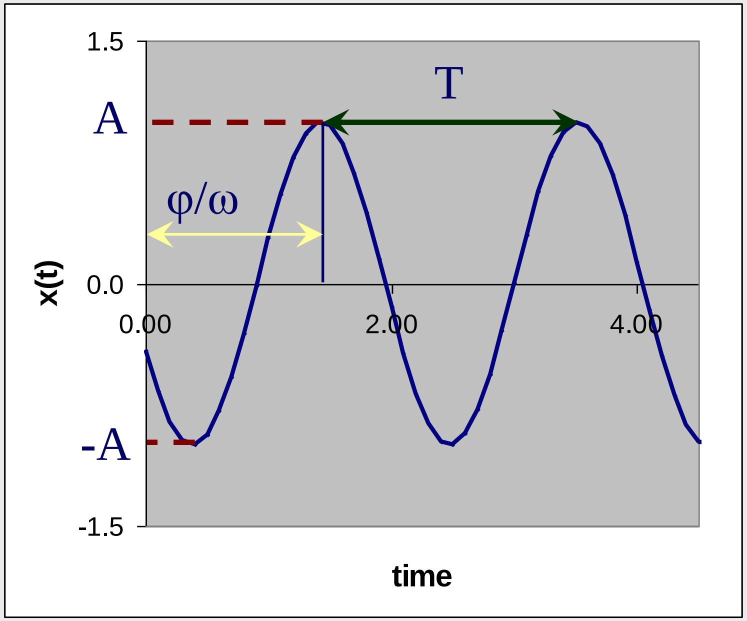
$$x(t) = x(t + nT) \qquad n = 1, 2, 3, \ldots$$



Time Dependent Sources

- **Sinusoidal signal**: a periodic waveform satisfying the following equation:

$$x(t) = A\cos(\omega t + \phi)$$



A – amplitude
ω – radian frequency
φ – phase

Sinusoidal Sources

- Helpful identities:

$$f = \frac{1}{T} \; Hz\,(cycles/s)$$

$$\omega = 2\pi f \quad rad/s$$

$$T = \frac{2\pi}{\omega} \; s$$

$$\phi = 2\pi \frac{\Delta t}{T} \; rad$$

$$= 360 \frac{\Delta t}{T} \; \deg$$

$$\sin(\omega t) = \cos\left(\omega t - \frac{\pi}{2}\right) = \cos\left(\omega t - 90^\circ\right)$$

$$\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right) = \sin\left(\omega t + 90^\circ\right)$$

$$\sin(\omega t \pm \theta) = \cos(\theta)\sin(\omega t) \pm \sin(\theta)\cos(\omega t)$$

$$\cos(\omega t \pm \theta) = \cos(\theta)\cos(\omega t) \mp \sin(\theta)\sin(\omega t)$$

Sinusoidal Sources

- Why sinusoidal sources?

• Sinusoidal AC is the fundamental current type supplied to homes throughout the world by way of power grids

  • **Current war**:

    • late 1880's AC (Westinghouse and Tesla) competed with DC (Edison) for the electric power grid standard

    • Low frequency AC (50 - 60Hz) can be more dangerous than DC

      • Alternating fluctuations can cause the heart to lose coordination (death)

    • High frequency DC can be more dangerous than AC

      • causes muscles to lock in position – preventing victim from releasing conductor

    • DC has serious limitations

      • DC cannot be transmitted over long distances (greater than 1 mile) without serious power losses

      • DC cannot be easily changed to higher or lower voltages

Measuring Signal Strength

- Methods of quantifying the strength of time-varying electric signals:
  - **Average (DC) value**
    - Mean voltage (or current) over a period of time
  - **Root-mean-square (RMS) value**
    - Takes into account the fluctuations of the signal about its average value

Measuring Signal Strength

- **Time – averaged signal strength**: integrate signal **x(t)** over a period (**T**) of time

$$\langle x(t) \rangle = \frac{1}{T} \int_0^T x(\tau) d\tau$$

Measuring Signal Strength

- **Example1**: compute the average value of the signal – **x(t) = 10cos(100t)**

Measuring Signal Strength

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$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100}$$

Measuring Signal Strength

- **Example1**: compute the average value of the signal – **x(t) = 10cos(100t)**

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100}$$

$$\begin{aligned}\langle x(t)\rangle &= \frac{1}{T}\int_0^T x(\tau)d\tau \\ &= \frac{100}{2\pi}\int_0^{2\pi/100} 10\cos(100t)dt \\ &= \frac{10}{2\pi}\langle \sin(2\pi)-\sin(0)\rangle \\ &= 0\end{aligned}$$

NB: in general, for any sinusoidal signal

$$\langle A\cos(\omega t+\phi)\rangle = 0$$

Measuring Signal Strength

- **Root–mean–square (RMS)**: since a zero average signal strength is not useful, often the RMS value is used instead
  - The RMS value of a signal x(t) is defined as:

$$x(t)_{rms} = \sqrt{\frac{1}{T}\int_0^T x(\tau)^2 d\tau}$$

NB: often $\tilde{x}(t)$ notation is used instead of $x(t)_{rms}$

NB: the **rms** value is simply the **square root** of the average (**mean**) after being **squared** – hence: **root – mean – square**

Measuring Signal Strength

- **Example2**: Compute the rms value of the sinusoidal current $\mathbf{i(t) = I \cos(\omega t)}$

Measuring Signal Strength

- **Example2**: Compute the rms value of the sinusoidal current **i(t) = I cos(ωt)**

$$\cos^2(t) = \frac{\cos(2t)+1}{2}$$

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\cos^2(\omega\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\left[\frac{1}{2}+\frac{1}{2}\cos(2\omega\tau)\right]d\tau}$$

$$= \sqrt{\frac{1}{2}I^2 + \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{I^2}{2}\cos(2\omega\tau)d\tau}$$

$$= \sqrt{\frac{1}{2}I^2 + 0}$$

$$= \frac{I}{\sqrt{2}}$$

Integrating a sinusoidal waveform over 2 periods equals zero

Measuring Signal Strength

- **Example2**: Compute the rms value of the sinusoidal current **i(t) = I cos(ωt)**

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\cos^2(2\omega\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\left[\frac{1}{2}+\frac{1}{2}\cos(2\omega\tau)\right]d\tau}$$

$$= \sqrt{\frac{1}{2}I^2 + \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{I^2}{2}\cos(2\omega\tau)d\tau}$$

$$= \sqrt{\frac{1}{2}I^2 + 0}$$

$$= \frac{I}{\sqrt{2}}$$

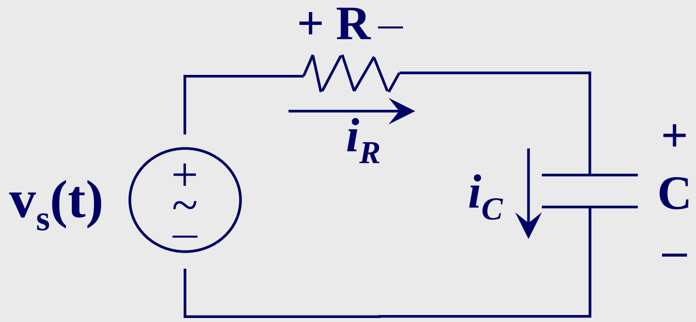
The RMS value of any sinusoid signal is always equal to 0.707 times the peak value (regardless of amplitude or frequency)

Network Analysis with Capacitors and Inductors (Dynamic Circuits)

Differential Equations

Dynamic Circuit Network Analysis

Kirchoff's law's (KCL and KVL) still apply, but they will now produce differential equations.



Dynamic Circuit Network Analysis

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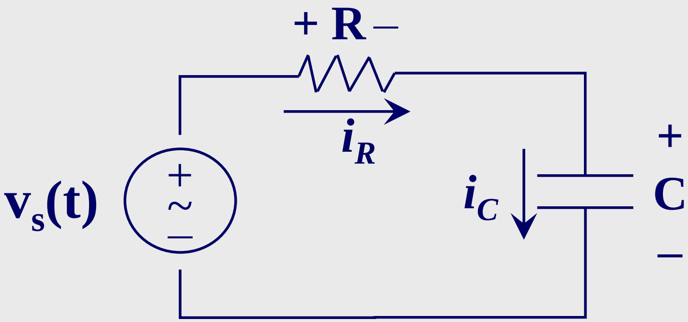


KVL:

$$-v_S(t)+v_R(t)+v_C(t)=0$$

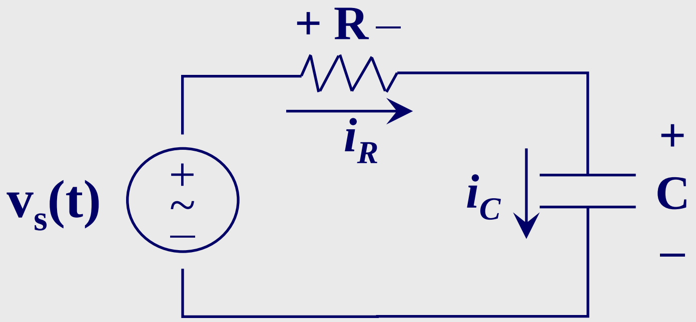
$$-v_S(t)+Ri_C(t)+\frac{1}{C}\int_{-\infty}^{t} i_C(\tau)d\tau=0$$

Differentiate both sides:

$$\frac{di_C(t)}{dt}+\frac{1}{RC}i_C(t)=\frac{1}{R}\frac{dv_S(t)}{dt}$$

Dynamic Circuit Network Analysis

Kirchoff's law's (KCL and KVL) still apply, but they will now produce differential equations.



KCL :

$$i_R = i_C$$

$$\frac{v_R(t)}{R} = C\frac{dv_C(t)}{dt}$$

$$\frac{[v_s(t) - v_C(t)]}{R} = C\frac{dv_C(t)}{dt}$$

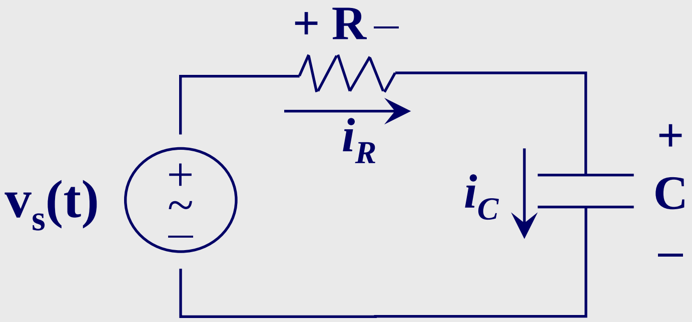
$$\frac{dv_C(t)}{dt} + \frac{1}{RC}v_C(t) = \frac{1}{RC}v_S(t)$$

Sinusoidal Source Responses

- Consider the AC source producing the voltage:

$v_s(t) = V\cos(\omega t)$

$$\frac{dv_C(t)}{dt} + \frac{1}{RC} v_C(t) = \frac{1}{RC} V\cos(\omega t)$$



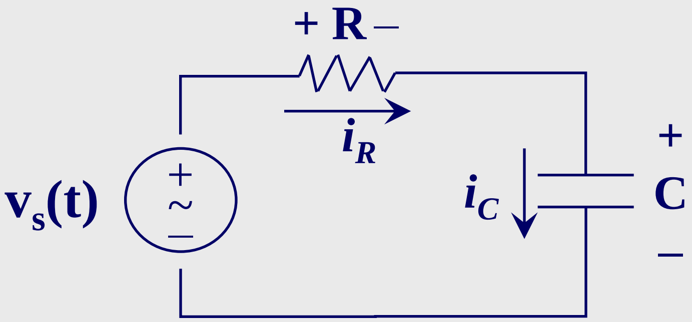
The solution to this diff EQ will be a sinusoid:

$$\begin{aligned} v_C(t) &= A\sin(\omega t) + B\cos(\omega t) \\ &= C\cos(\omega t + \phi) \end{aligned}$$

Sinusoidal Source Responses

- Consider the AC source producing the voltage:

$v_s(t) = V\cos(\omega t)$



Substitute the solution form into the diff EQ:

$$\frac{dv_C(t)}{dt} + \frac{1}{RC} v_C(t) = \frac{1}{RC} V\cos(\omega t)$$

Substitute $v_C(t) = A\sin(\omega\omega t + B\cos(\omega\omega t$

$$\frac{d[A\sin(\omega t) + B\cos(\omega t)]}{dt} +$$

$$\frac{1}{RC}[A\sin(\omega t) + B\cos(\omega t)] = \frac{1}{RC} V\cos(\omega t)$$

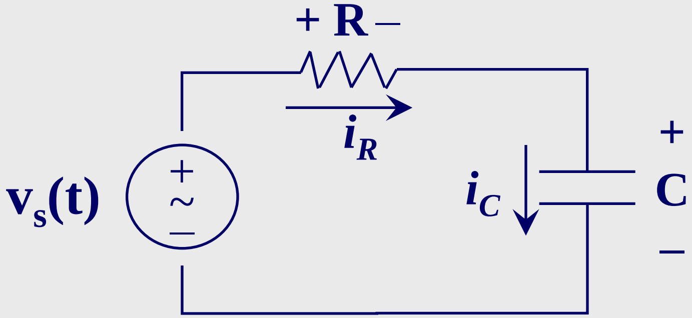
Sinusoidal Source Responses

- Consider the AC source producing the voltage:

$v_s(t) = V\cos(\omega t)$

$$A\omega\cos(\omega t) - B\omega\sin(\omega t) + \frac{1}{RC}\left[A\sin(\omega t) + B\cos(\omega t)\right] = \frac{1}{RC}V\cos(\omega t)$$

$$\left(\frac{A}{RC} - B\omega\right)\sin\omega t + \left(A\omega - \frac{B}{RC} - \frac{V}{RC}\right)\cos\omega t = 0$$



For this equation to hold, both the **sin(ωt)** and **cos(ωt)** coefficients must be zero

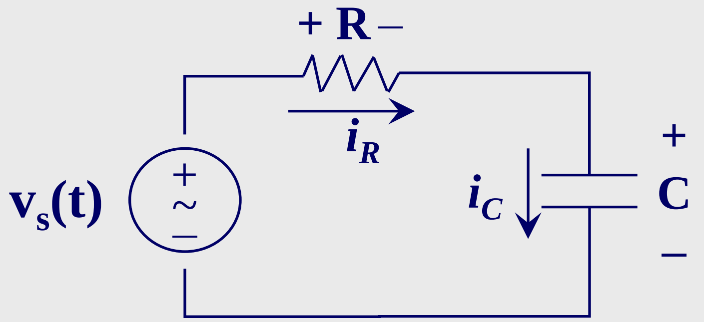
$$\frac{A}{RC} - B\omega = 0$$

$$A\omega + \frac{B}{RC} - \frac{V}{RC} = 0$$

Sinusoidal Source Responses

- Consider the AC source producing the voltage:

$v_s(t) = V\cos(\omega t)$



$$\frac{A}{RC} - B\omega = 0$$

$$A\omega + \frac{B}{RC} - \frac{V}{RC} = 0$$

Solving these equations for **A** and **B** gives:

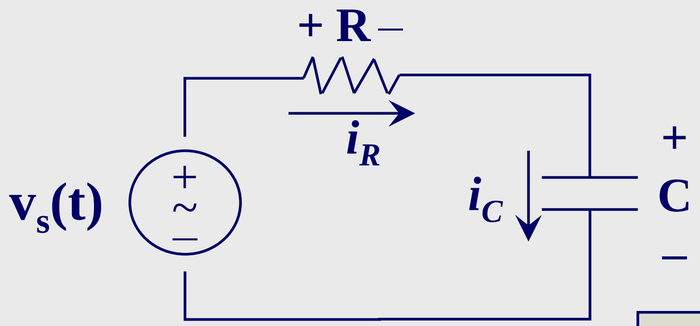
$$A = \frac{V\omega RC}{1+\omega^2(RC)^2}$$

$$B = \frac{V}{1+\omega^2(RC)^2}$$

Sinusoidal Source Responses

- Consider the AC source producing the voltage:

$v_s(t) = V\cos(\omega t)$



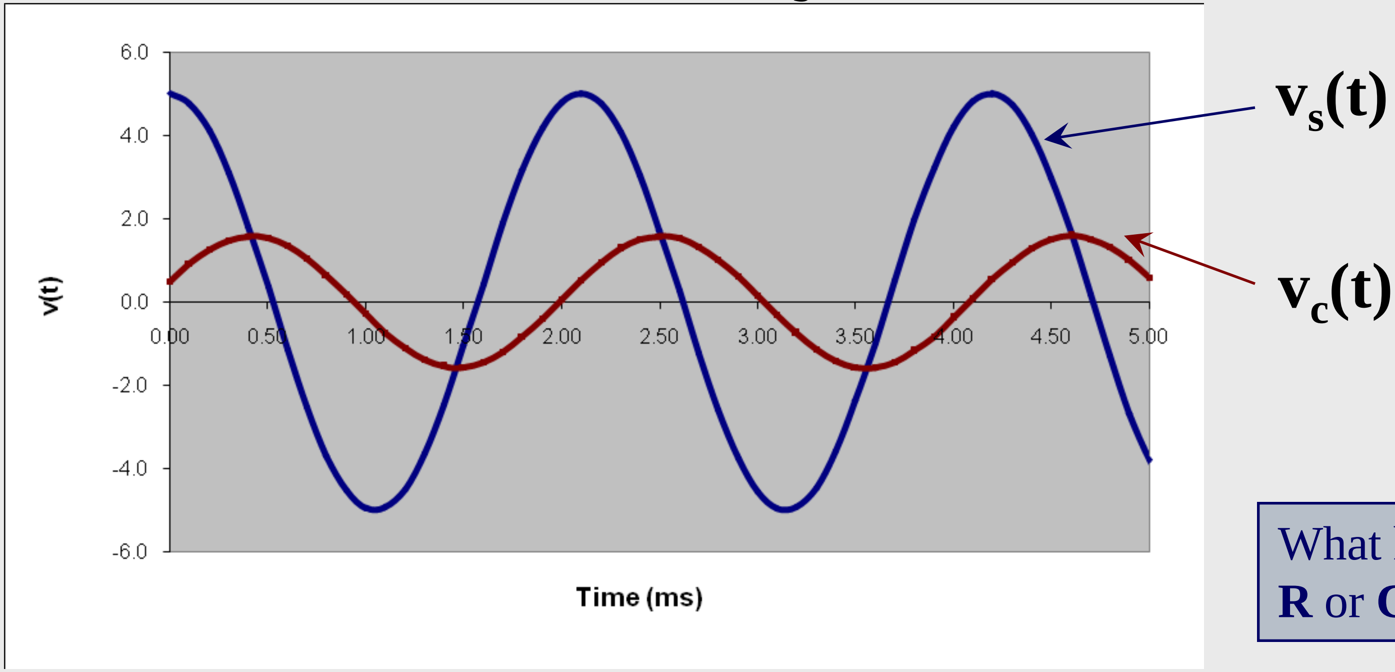
Writing the solution for **$v_C(t)$:**

$$v_C(t) = \frac{V\omega RC}{1+\omega^2(RC)^2}\sin(\omega t) + \frac{V}{1+\omega^2(RC)^2}\cos(\omega t)$$

NB: This is the solution for **a single-order** diff EQ (i.e. with only **one** capacitor)

Sinusoidal Source Responses

v_c(t) has the same **frequency**, but different **amplitude** and different **phase** than **v_s(t)**

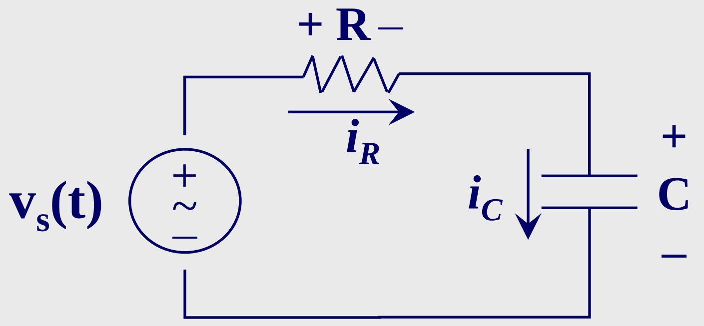


What happens when **R** or **C** is small?

$$v_C(t) = \frac{V\omega RC}{1+\omega^2(RC)^2}\sin(\omega t) + \frac{V}{1+\omega^2(RC)^2}\cos(\omega t)$$

Sinusoidal Source Responses

<u>In a circuit with an AC source</u>: all branch voltages and currents are also **sinusoids** with the **same frequency** as the source. The **amplitudes** of the branch voltages and currents are **scaled versions** of the source amplitude (i.e. not as large as the source) and the branch voltages and currents may be **shifted in phase** with respect to the source.



3 parameters that uniquely identify a sinusoid:
- frequency
- amplitude
- phase