Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
20 Oct	Mon	14	AC Circuit Analysis	4.5		NO LAB	
21 Oct	Tue					NO LAB	
22 Oct	Wed	15	Transient Response 1st Order Circuits	5.4			
23 Oct	Thu						
24 Oct	Fri		Recitation		HW 6		
25 Oct	Sat						
26 Oct	Sun						
27 Oct	Mon	16	Transient Response 2nd Order Circuits	5.5		LAB 5	
28 Oct	Tue						Exam 1

Obedience = Happiness

Mosiah 2:41

41 And moreover, I would desire that ye should consider on the blessed and happy state of those that keep the commandments of God. For behold, they are blessed in all things, both temporal and spiritual; and if they hold out faithful to the end they are received into heaven, that thereby they may dwell with God in a state of never-ending happiness. O remember, remember that these things are true; for the Lord God hath spoken it.

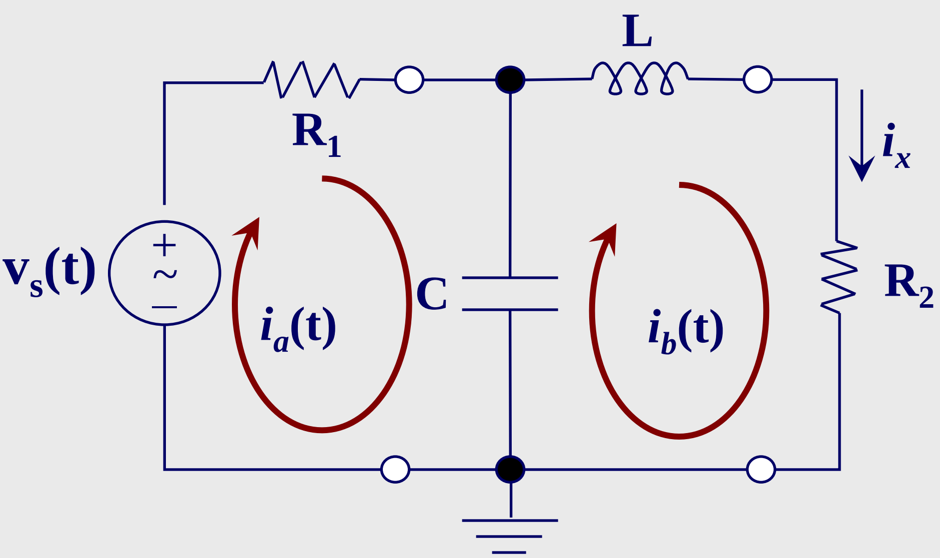
Lecture 14 – AC Circuit Analysis

Phasors allow the use of familiar network analysis

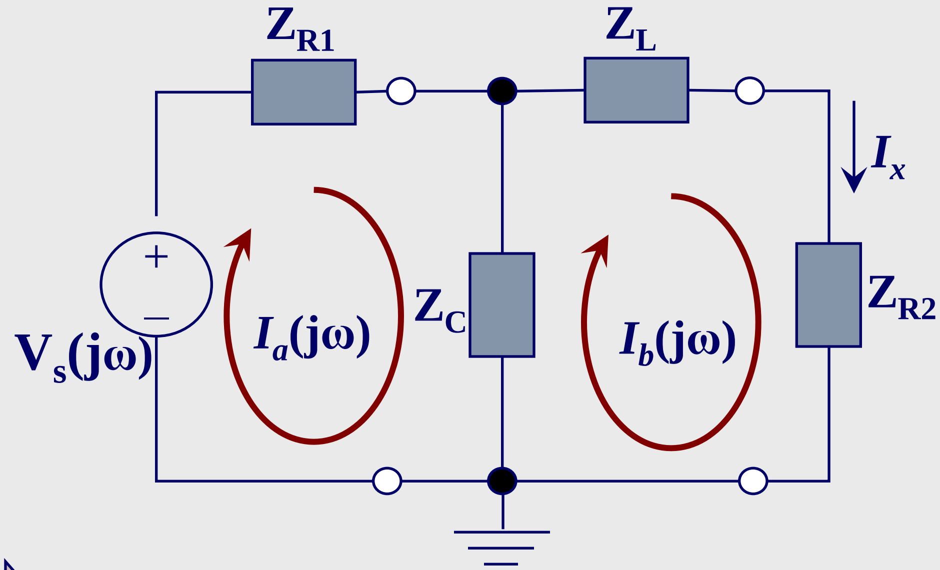
RLC Circuits

<u>Linear passive circuit elements</u>: resistors (**R**), inductors (**L**), and capacitors (**C**) (a.k.a. RLC circuits)

- Assume **RLC** circuit sources are **sinusoidal**



Time domain

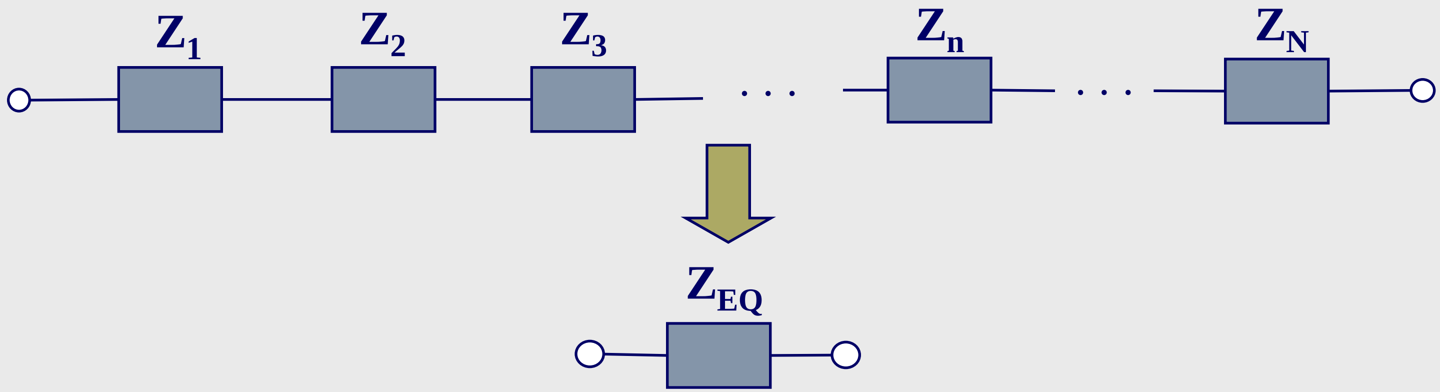


Frequency (phasor) domain

RLC Circuits - Series Impedances

- **<u>Series Rule</u>**: two or more circuit elements are said to be **in series** if the current from one element *exclusively* flows into the next element.
  - Impedances in **series** add the same way resistors in **series** add

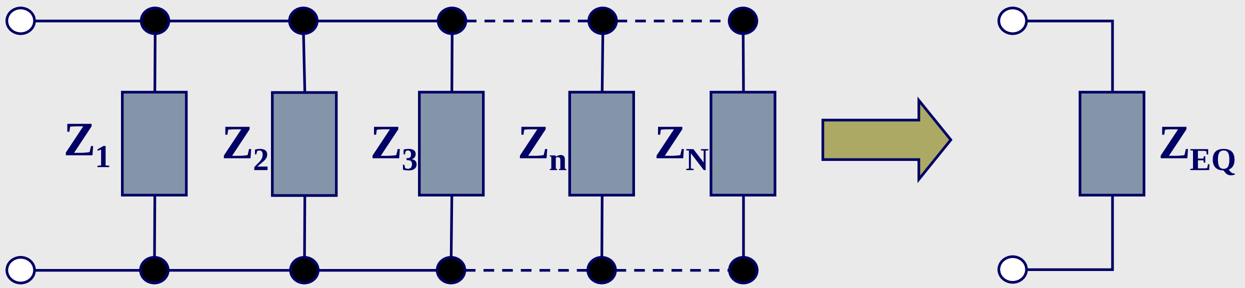
$$Z_{EQ} = \sum_{n=1}^{N} Z_n$$



RLC Circuits - Parallel Impedances

- **Parallel Rule**: two or more circuit elements are said to be **in parallel** if the elements share the *same* terminal**s**
  - Impedances in **parallel** add the same way resistors in **parallel** add

$$\frac{1}{Z_{EQ}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots \frac{1}{Z_N} \qquad Z_{EQ} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \cdots + \frac{1}{Z_N}}$$



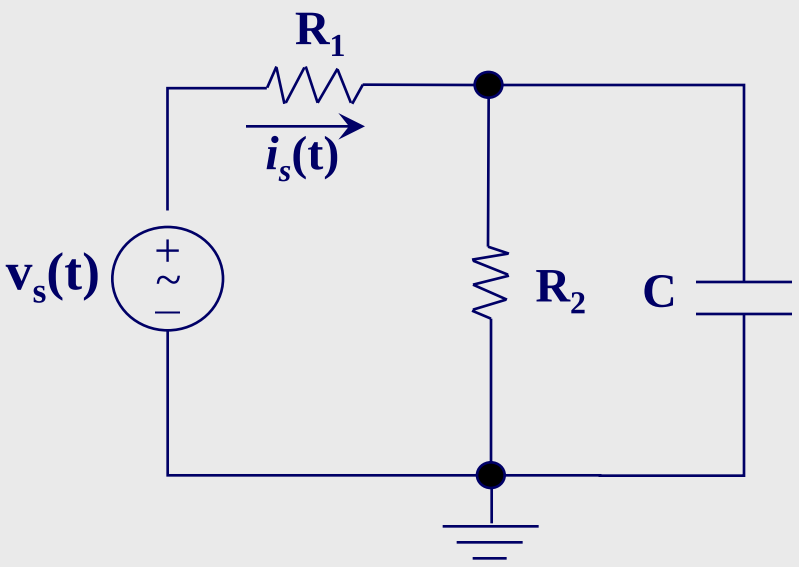
RLC Circuits

AC Circuit Analysis

1. Identify the AC sources and note the excitation frequency (**ω**)
2. Convert all sources to the phasor domain
3. Represent each circuit element by its impedance
4. Solve the resulting phasor circuit using network analysis methods
5. Convert from the phasor domain back to the time domain

RLC Circuits

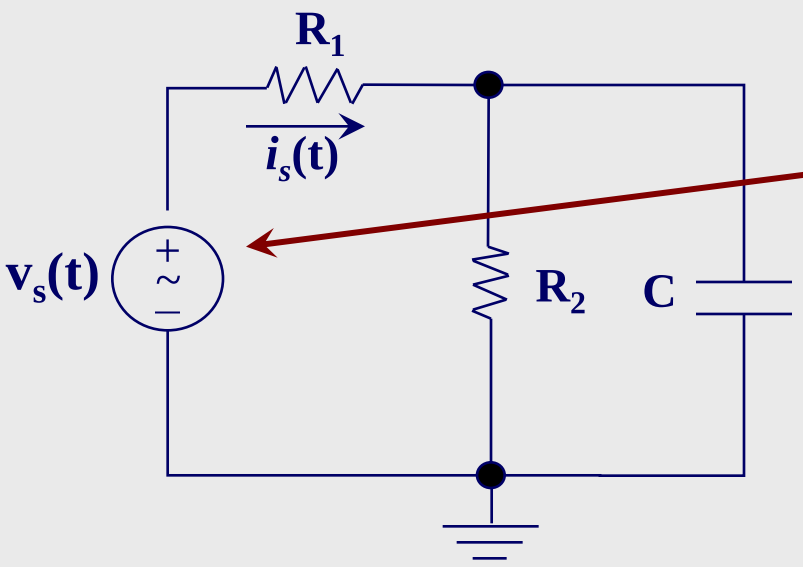
- **<u>Example1</u>**: find $\boldsymbol{i_s}$**(t)**
  - **v_s(t) = 10cos(ωt)**, ω = 377 rad/s **R_1** = 50Ω, **R_2** = 200Ω, C = 100uF



RLC Circuits

- **Example1**: find $\boldsymbol{i_s}$**(t)**
  - $\mathbf{v_s(t) = 10\cos(\omega t)}$, $\omega = 377$ rad/s $\mathbf{R_1} = 50\Omega$, $\mathbf{R_2} = 200\Omega$, $C = 100uF$

1. Note frequencies of AC sources

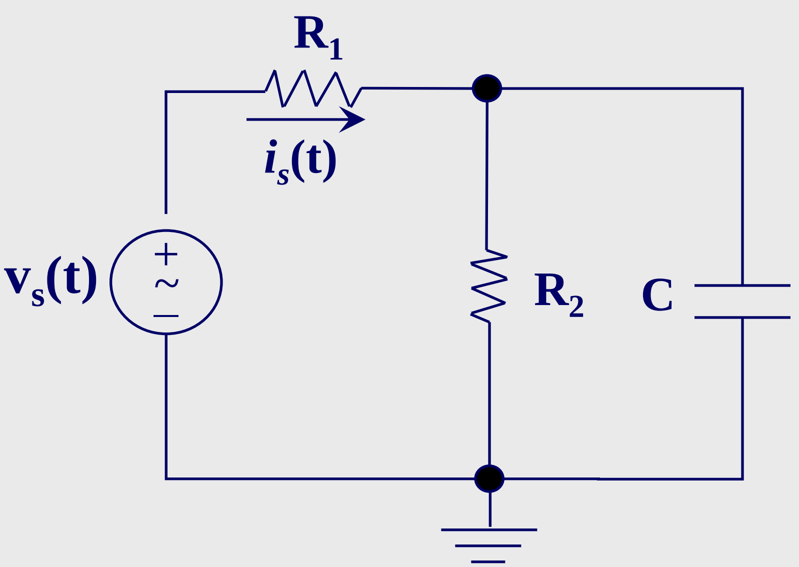


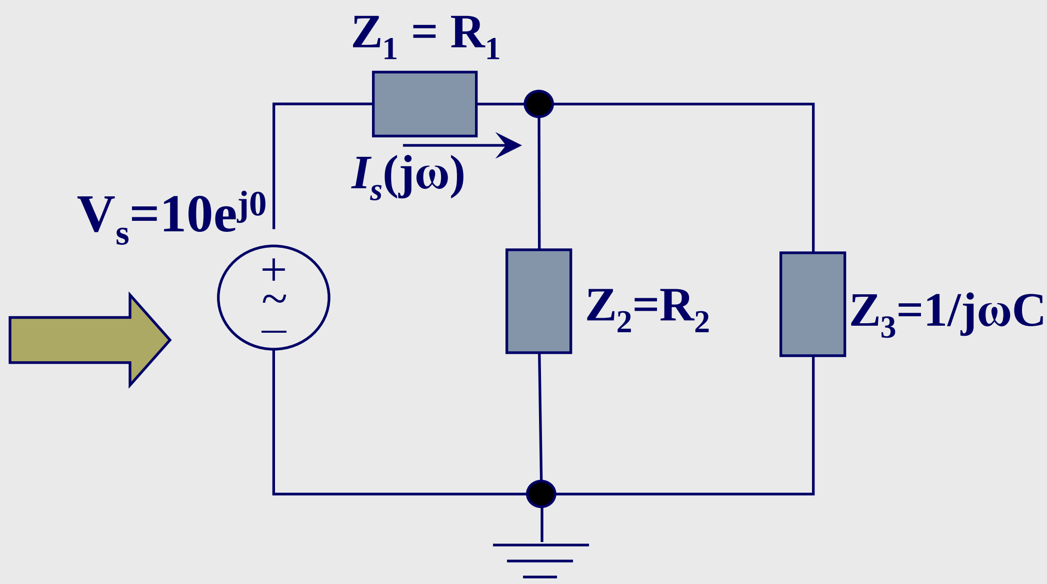
Only one AC source - **ω** = 377 rad/s

RLC Circuits

- **Example1**: find $i_s(t)$
  - $\mathbf{v_s(t) = 10\cos(\omega t)}$, ω = 377 rad/s $\mathbf{R_1}$ = 50Ω, $\mathbf{R_2}$ = 200Ω, C = 100uF

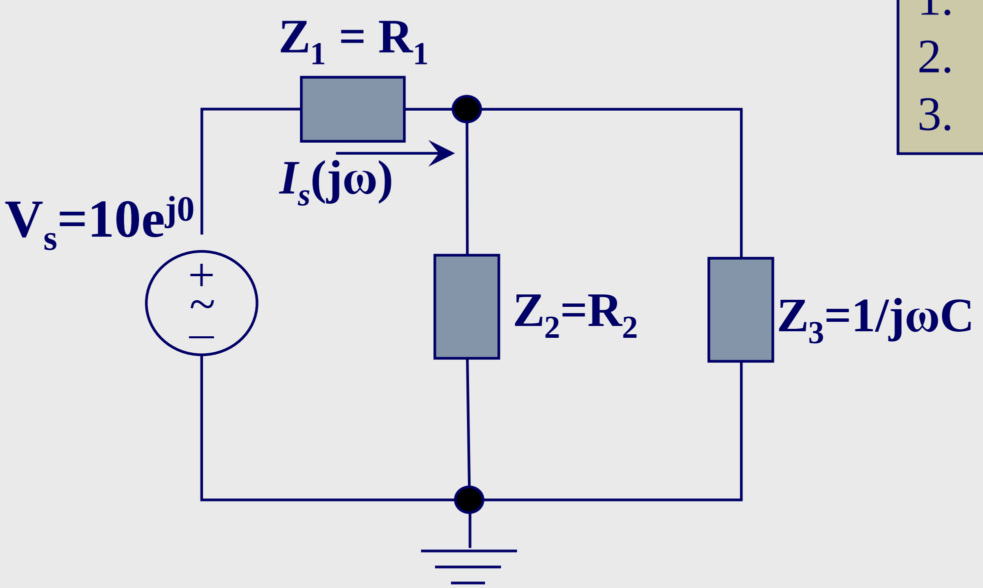
1. Note frequencies of AC sources
2. Convert to phasor domain





RLC Circuits

- **Example1**: find $\boldsymbol{i_s}$**(t)**
  - $v_s(t) = 10\cos(\omega t)$, $\omega = 377$ rad/s $R_1 = 50\Omega$, $R_2 = 200\Omega$, C = 100uF



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Represent each element by its impedance

$$v_s(t) = 10\cos(\omega t)$$

$$V_s(j\omega) = 10\angle 0 = 10e^{j0}$$

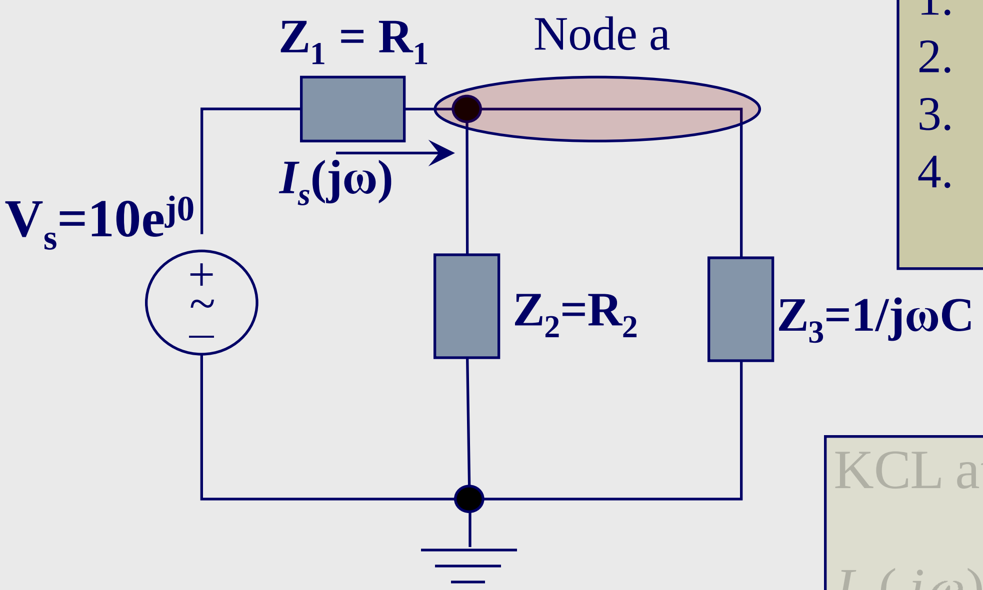
$$Z_1 = R_1$$

$$Z_2 = R_2$$

$$Z_3 = \frac{1}{j\omega C}$$

RLC Circuits

- **Example1**: find $\boldsymbol{i_s}$**(t)**
  - $v_s(t) = 10\cos(\omega t)$, ω = 377 rad/s R_1 = 50Ω, R_2 = 200Ω, C = 100uF



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Represent each element by its impedance
4. Solve using network analysis
   - Use node voltage and Ohm's law

KCL at Node a:

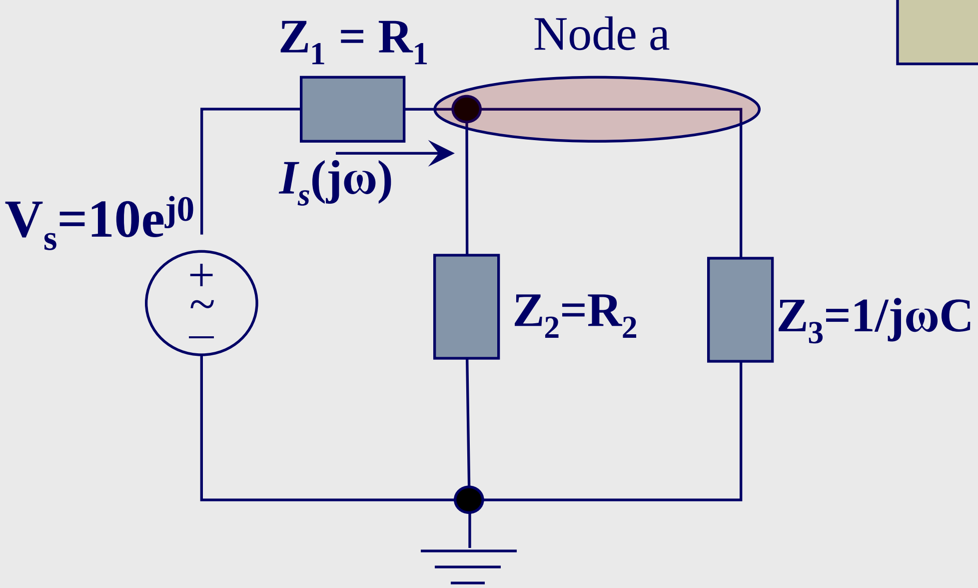
$$I_s(j\omega) = \frac{V_a(j\omega)}{Z_2 \parallel Z_3}$$

Ohm's Law:

$$I_s(j\omega) = \frac{V_1(j\omega)}{Z_1} = \frac{V_s(j\omega) - V_a(j\omega)}{Z_1}$$

RLC Circuits

- **Example1**: find $i_s(t)$
  - $\mathbf{v_s(t) = 10\cos(\omega t)}$, ω = 377 rad/s $\mathbf{R_1}$ = 50Ω, $\mathbf{R_2}$ = 200Ω, C = 100uF



4. Solve using network analysis
   - Use node voltage and Ohm's law

$$\frac{V_s - V_a}{Z_1} = \frac{V_a}{Z_2 \parallel Z_3}$$

$$\frac{V_s}{Z_1} = V_a\left(\frac{1}{Z_2 \parallel Z_3} + \frac{1}{Z_1}\right)$$

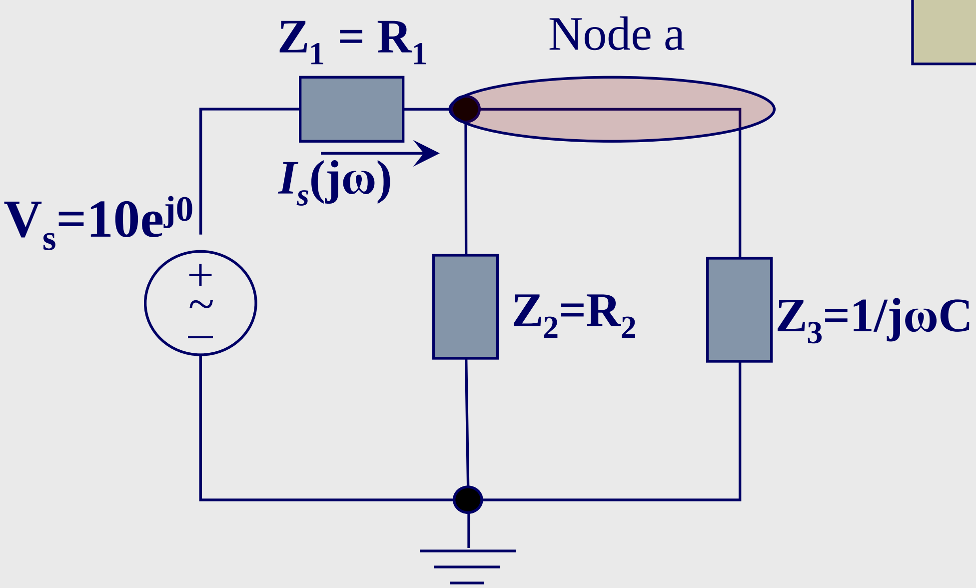
$$= V_a\left(\frac{R_2 + (1/j\omega C)}{R_2 \cdot (1/j\omega C)} + \frac{1}{R_1}\right)$$

$$= V_a\left(\frac{j\omega C R_2 + 1}{R_2} + \frac{1}{R_1}\right)$$

$$= V_a\left(\frac{(j\omega C R_1 R_2 + R_1) + R_2}{R_1 R_2}\right)$$

RLC Circuits

- **<u>Example1</u>**: find $\boldsymbol{i_s}$**(t)**
  - $\mathbf{v_s(t) = 10\cos(\omega t)}$, $\omega = 377$ rad/s $\mathbf{R_1} = 50\Omega$, $\mathbf{R_2} = 200\Omega$, C = 100uF



4. Solve using network analysis
   - Use node voltage and Ohm's law

$$\frac{V_s}{Z_1} = V_a\left(\frac{(j\omega C R_1 R_2 + R_1) + R_2}{R_1 R_2}\right)$$

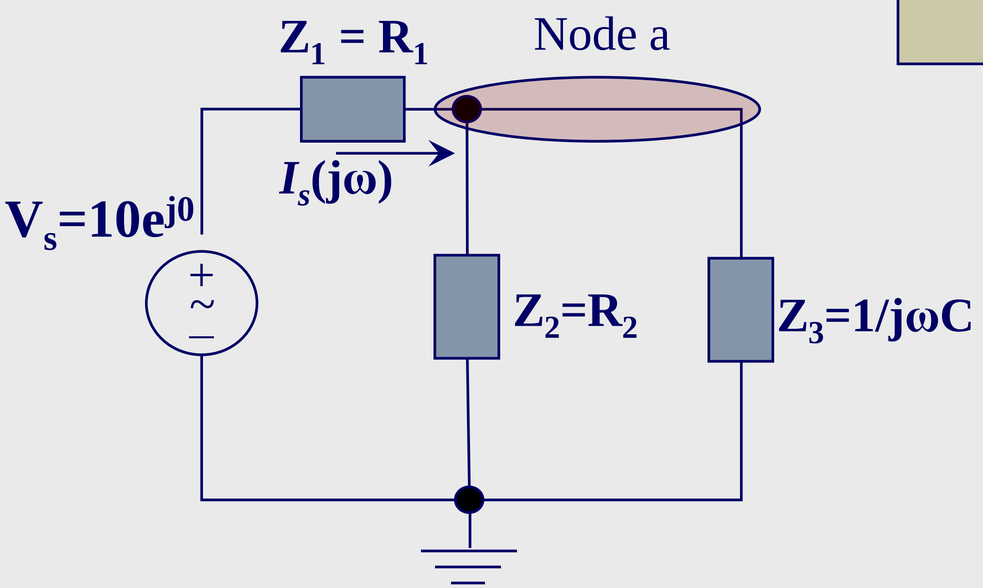
$$V_a = \frac{V_s}{R_1}\left(\frac{R_1 R_2}{(j\omega C R_1 R_2 + R_1) + R_2}\right)$$

$$= \frac{10\angle 0}{50}\left[\frac{(50)(200)}{j(377)(10^{-4})(50)(200) + (50) + (200)}\right]$$

$$= 4.421\angle(-0.9852)$$

RLC Circuits

- **Example1**: find $\boldsymbol{i_s(t)}$
  - $\mathbf{v_s(t) = 10cos(\omega t)}$, $\omega = 377$ rad/s $\mathbf{R_1} = 50\Omega$, $\mathbf{R_2} = 200\Omega$, $C = 100uF$



4. Solve using network analysis
   - Use node voltage and Ohm's law

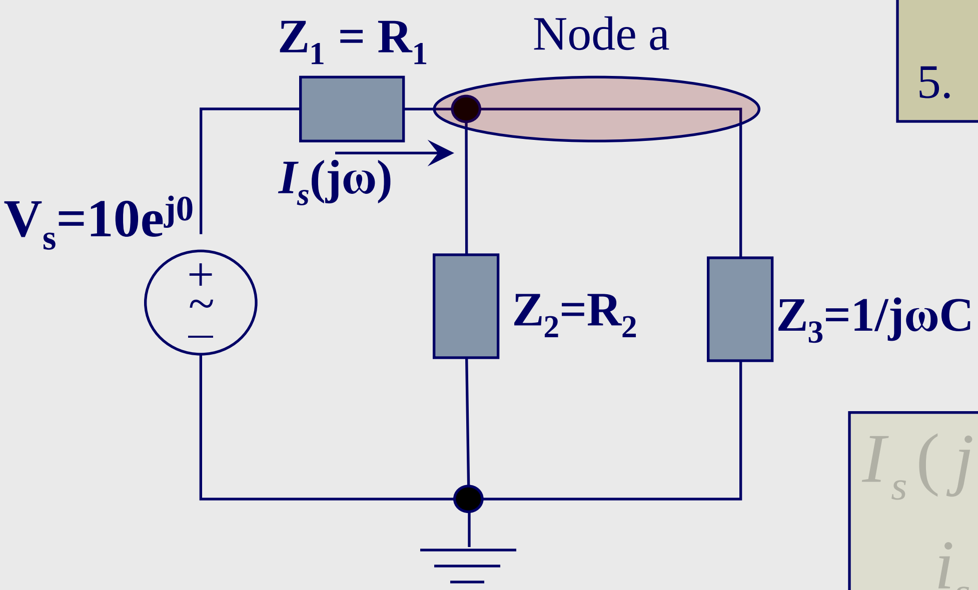
$$I_s(j\omega) = \frac{V_s(j\omega) - V_a(j\omega)}{Z_1}$$

$$= \frac{10\angle 0 - 4.421\angle(-.9852)}{50}$$

$$= 0.1681\angle 0.4537$$

RLC Circuits

- **Example1**: find $i_s(t)$
  - $v_s(t) = 10\cos(\omega t)$, ω = 377 rad/s R_1 = 50Ω, R_2 = 200Ω, C = 100uF



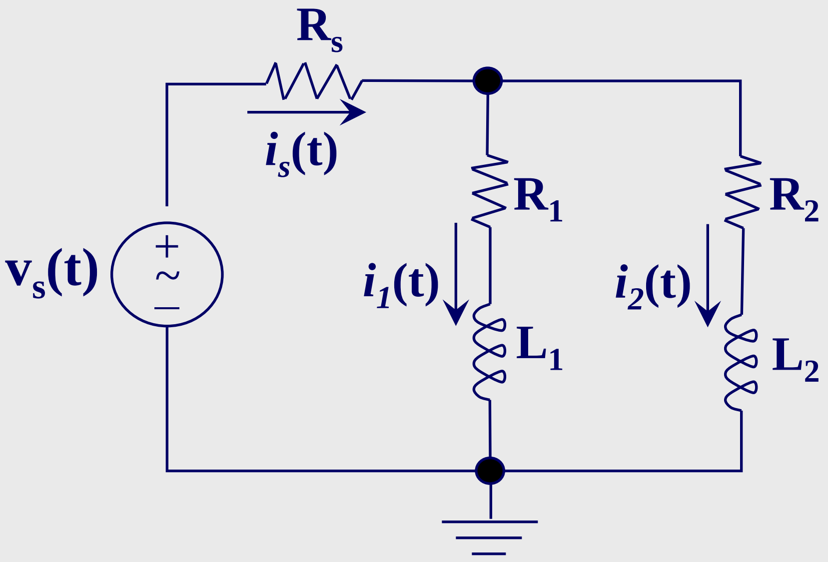
4. Solve using network analysis
   - Use node voltage and Ohm's law
5. Convert to time domain

$$I_s(j\omega) = 0.1681\angle 0.4537$$

$$i_s(t) = 0.1681\cos(377t + 0.4537)$$

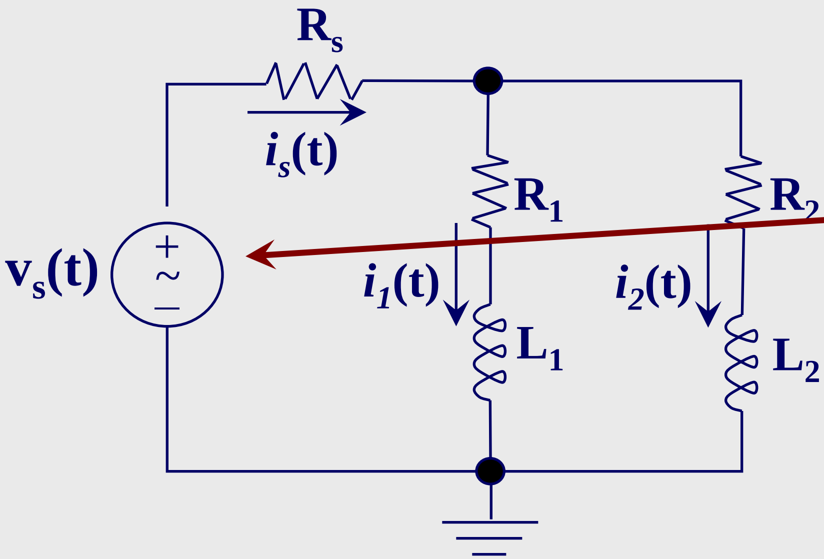
RLC Circuits

- **Example2**: find i_1 and i_2
  - $\mathbf{v_s(t)}$ = 155cos(ωt)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH



RLC Circuits

- **<u>Example2</u>**: find $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$
  - $\mathbf{v_s(t)}$ = 155cos(**ω**t)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH



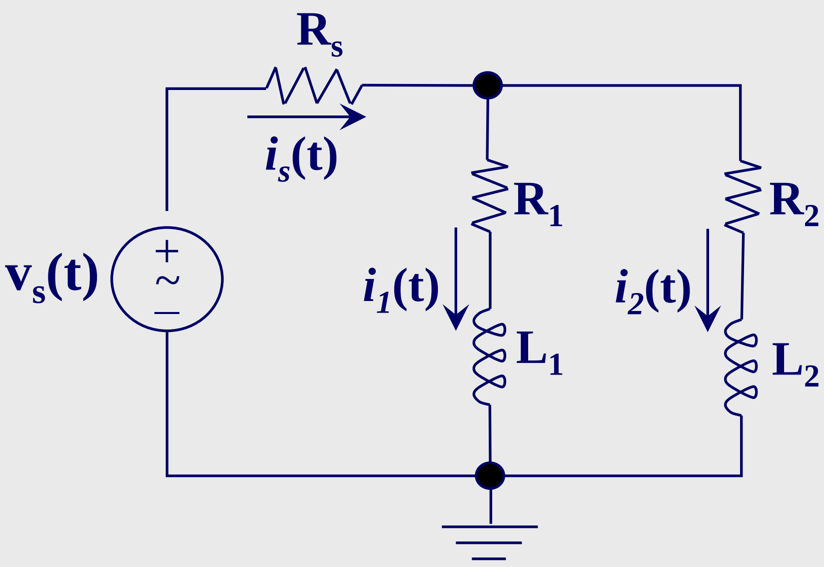
1. Note frequencies of AC sources

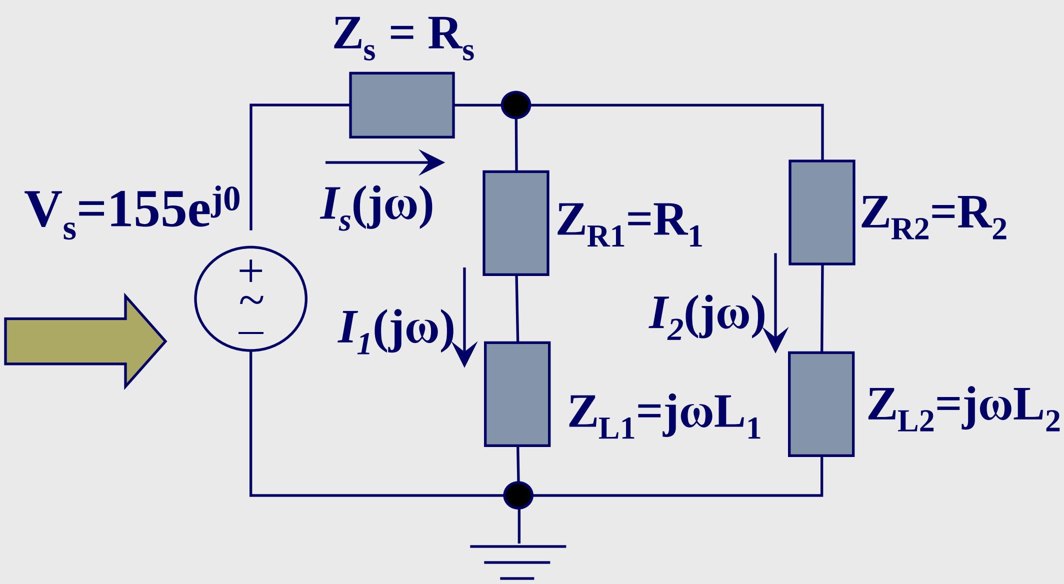
Only one AC source - **ω** = 377 rad/s

RLC Circuits

- **<u>Example2</u>**: find i_1 and i_2
  - $\mathbf{v_s(t)}$ = 155cos(ωt)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH

1. Note frequencies of AC sources
2. Convert to phasor domain

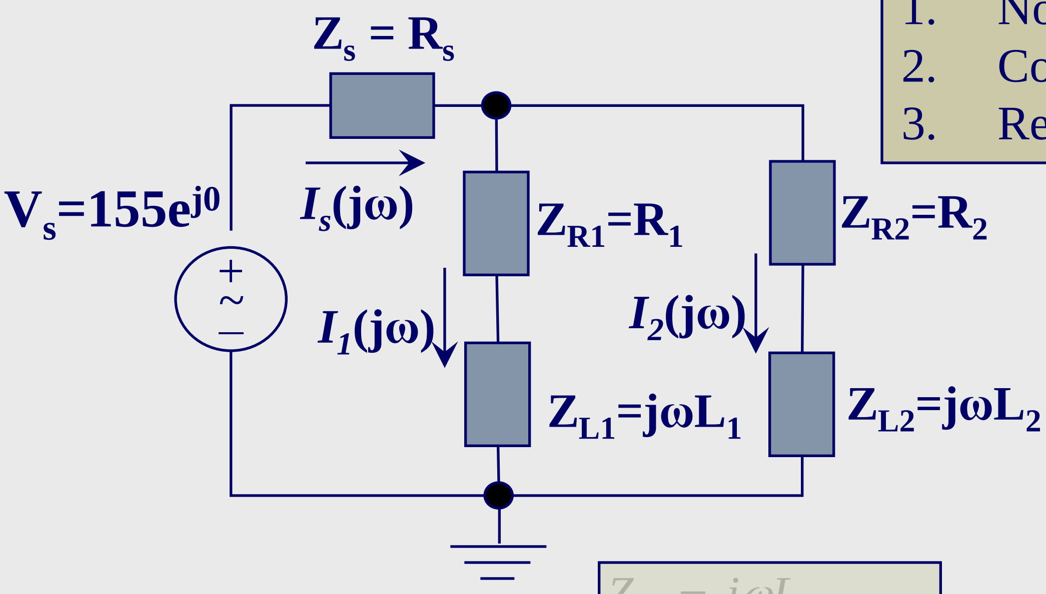




RLC Circuits

- **Example2**: find $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$
  - $\mathbf{v_s(t)}$ = 155cos(ωt)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH

1. Note frequencies of AC sources
2. Convert to phasor domain
3. Represent each element by its impedance



$$v_s(t) = 155\cos(\omega t)$$

$$V_s(j\omega) = 155\angle 0 = 155 e^{j0}$$

$$Z_{R1} = R_1 = 2\Omega$$

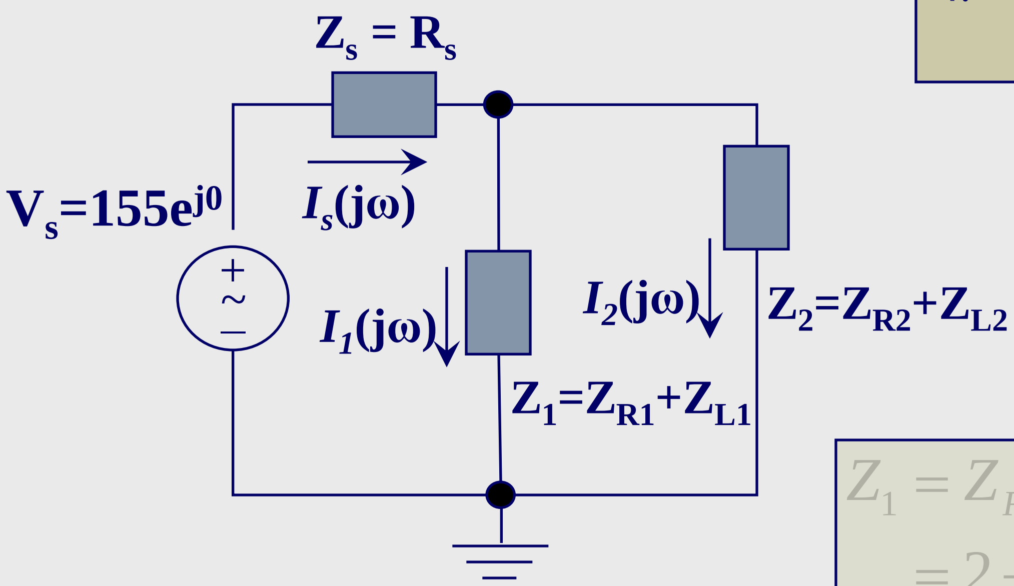
$$Z_{L1} = j\omega L_1 = j(377)(0.1) = j37.7\Omega$$

$$Z_{R2} = R_2 = 0.2\Omega$$

$$Z_{L2} = j\omega L_2 = j(377)(0.02) = j7.54\Omega$$

RLC Circuits

- **<u>Example2</u>**: find $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$
  - $\mathbf{v_s(t)}$ = 155cos(ωt)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH



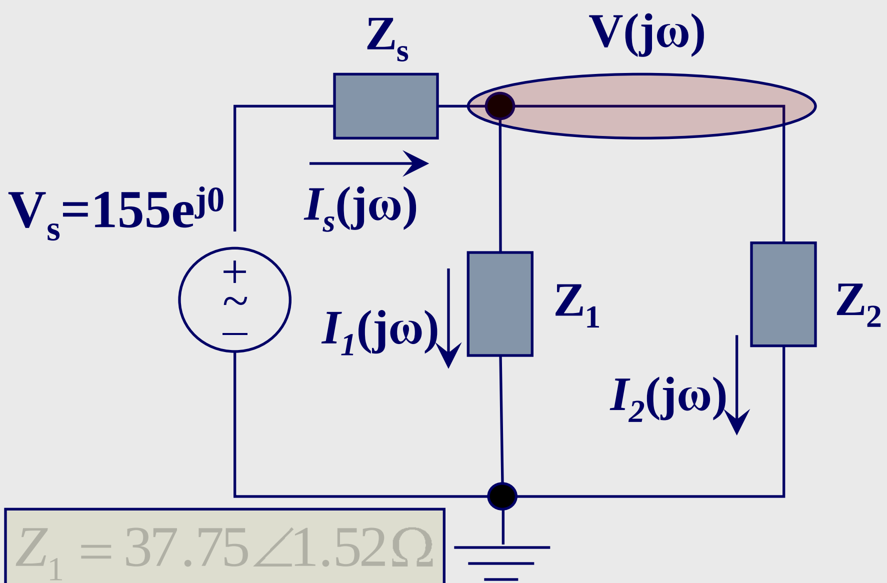
4. Solve using network analysis
   - Ohm's law

$$Z_1 = Z_{R1} + Z_{L1} = 2 + j37.7 = 37.75\angle 1.52\Omega$$

$$Z_2 = Z_{R2} + Z_{L2} = 0.2 + j7.54 = 7.54\angle 1.54\Omega$$

RLC Circuits

- **Example2**: find $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$
  - $\mathbf{v_s(t)}$ = 155cos(**ω**t)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH



4. Solve using network analysis
   - KCL

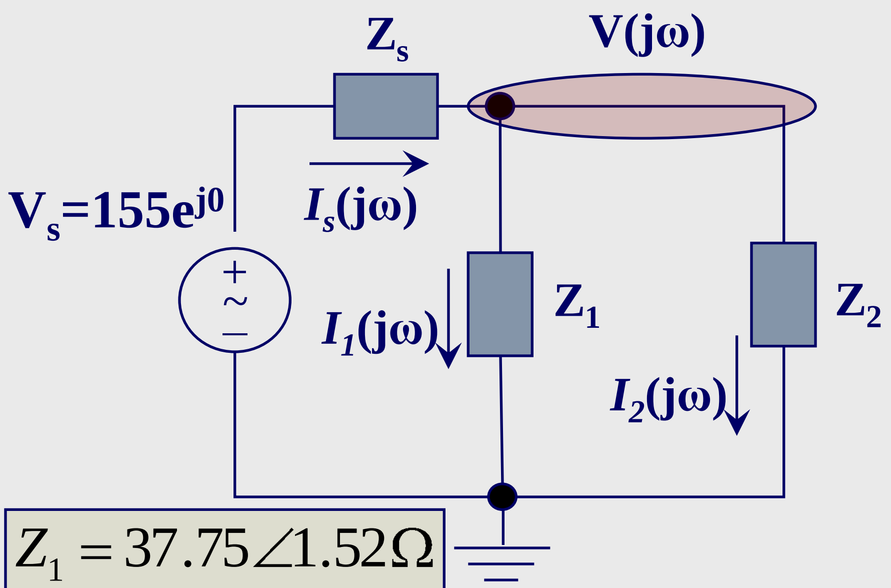
$$Z_1 = 37.75\angle 1.52\Omega$$
$$Z_2 = 7.54\angle 1.54\Omega$$
$$Z_s = 0.5\Omega$$

$$I_s(j\omega) - I_1(j\omega) - I_2(j\omega) = 0$$
$$\frac{V(j\omega)}{Z_1} + \frac{V(j\omega)}{Z_2} = \frac{V_s(j\omega) - V(j\omega)}{Z_s}$$
$$V(j\omega)\left(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_s}\right) = \frac{V_s(j\omega)}{Z_s}$$
$$V(j\omega) = 154.1\angle 0.079\, V$$

RLC Circuits

- **Example2**: find $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$
  - $\mathbf{v_s(t)}$ = 155cos(**ω**t)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH

4. Solve using network analysis
   - Ohm's Law



$$Z_1 = 37.75\angle 1.52\Omega$$
$$Z_2 = 7.54\angle 1.54\Omega$$
$$Z_s = 0.5\Omega$$

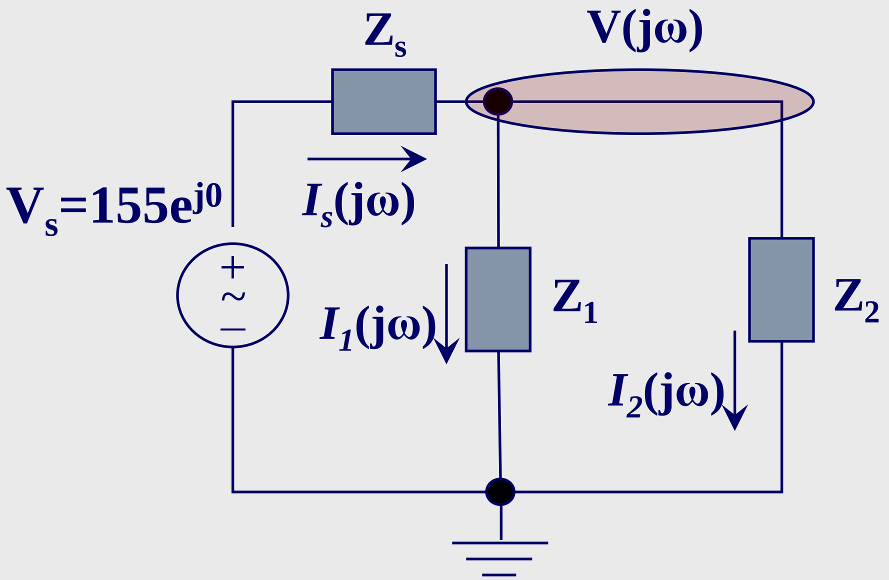
$$I_1(j\omega) = \frac{V(j\omega)}{Z_1} = 4.083\angle -1.439$$

$$I_2(j\omega) = \frac{V(j\omega)}{Z_2} = 20.44\angle -1.465$$

RLC Circuits

- **Example2**: find $\boldsymbol{i_1}$ and $\boldsymbol{i_2}$
  - $\mathbf{v_s(t)}$ = 155cos(ωt)V, **ω** = 377 rads/s, $\mathbf{R_s}$ = 0.5Ω, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 0.2Ω, $\mathbf{L_1}$ = 0.1H, $\mathbf{L_2}$ = 20mH

5. Convert to Time domain

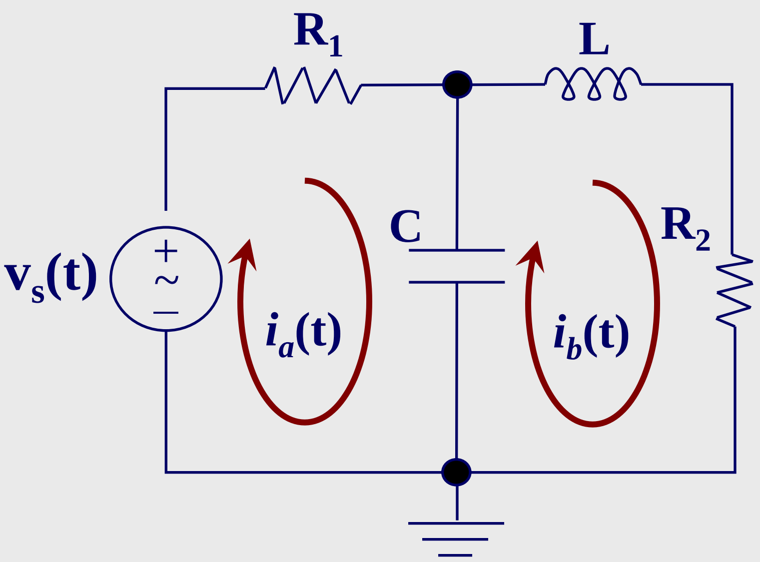


$$I_1(j\omega) = 4.083\angle -1.439$$
$$i_1(t) = 4.083\cos(377t - 1.439)\ A$$

$$I_2(j\omega) = 20.44\angle -1.465$$
$$i_2(t) = 20.44\cos(377t - 1.465)\ A$$

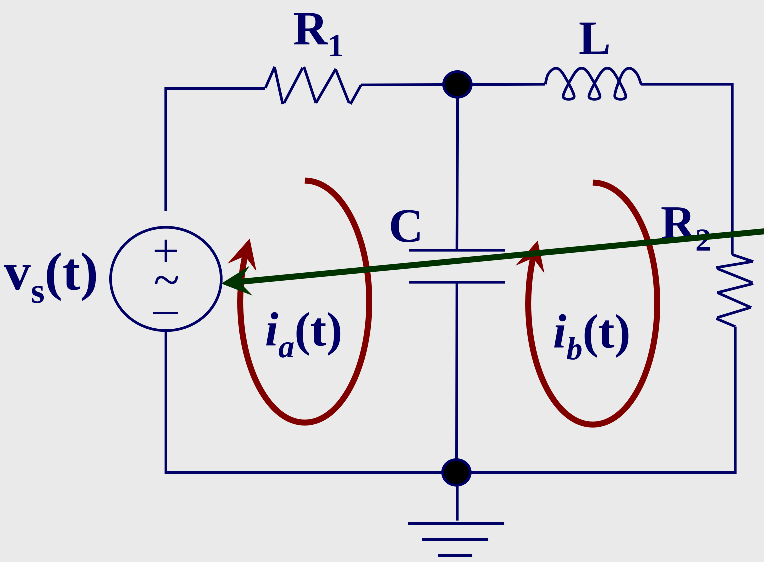
RLC Circuits

- **Example3**: find $\boldsymbol{i_a}$**(t)** and $\boldsymbol{i_b}$**(t)**
  - $\mathbf{v_s(t)}$ = 15cos(**1500**t)V, $\mathbf{R_1}$ = 100Ω, $\mathbf{R_2}$ = 75Ω, **L** = 0.5H, **C** = 1uF



RLC Circuits

- **Example3**: find $\boldsymbol{i_a}$**(t)** and $\boldsymbol{i_b}$**(t)**
  - $\mathbf{v_s(t)}$ = 15cos(**1500**t)V, $\mathbf{R_1}$ = 100Ω, $\mathbf{R_2}$ = 75Ω, **L** = 0.5H, **C** = 1uF

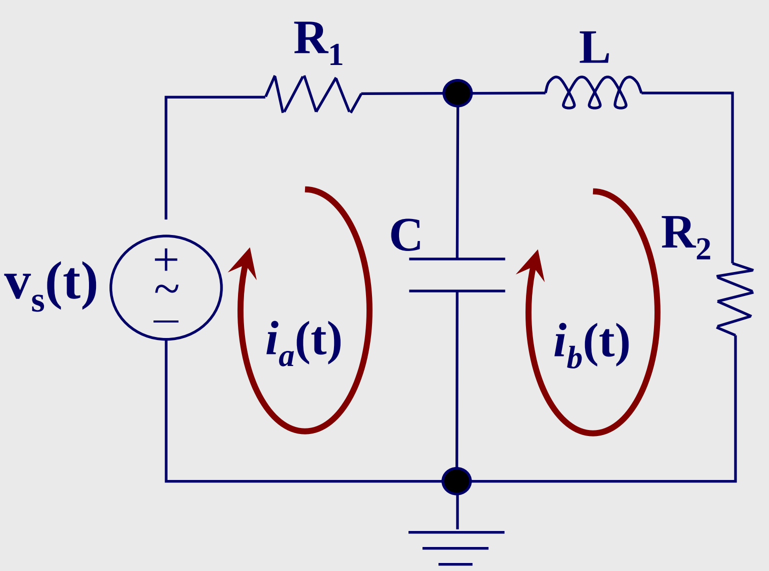


1. Note frequencies of AC sources

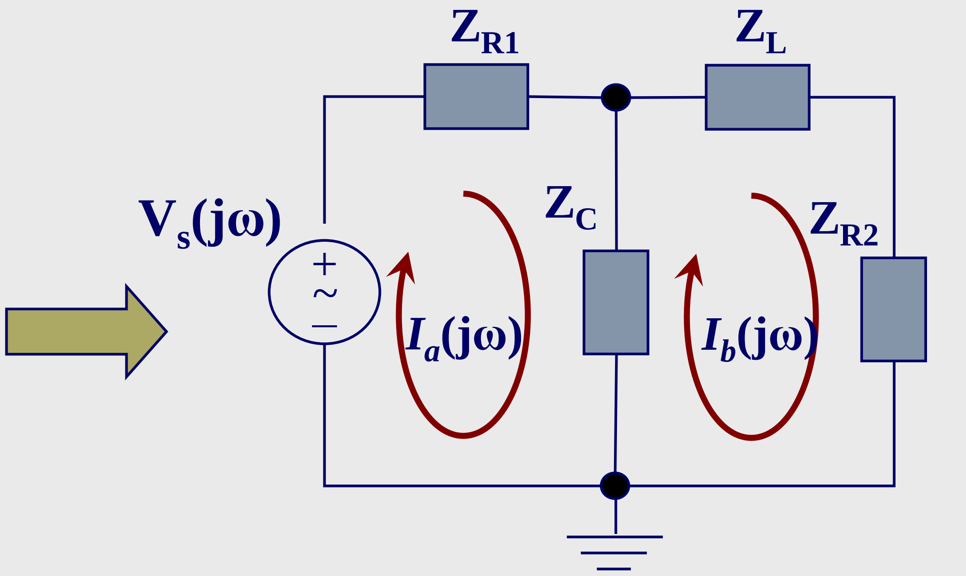
Only one AC source - **ω** = 1500 rad/s

RLC Circuits

- **<u>Example3</u>**: find $\boldsymbol{i_a(t)}$ and $\boldsymbol{i_b(t)}$
  - $\mathbf{v_s(t)}$ = 15cos(**1500**t)V, $\mathbf{R_1}$ = 100Ω, $\mathbf{R_2}$ = 75Ω, **L** = 0.5H, **C** = 1uF

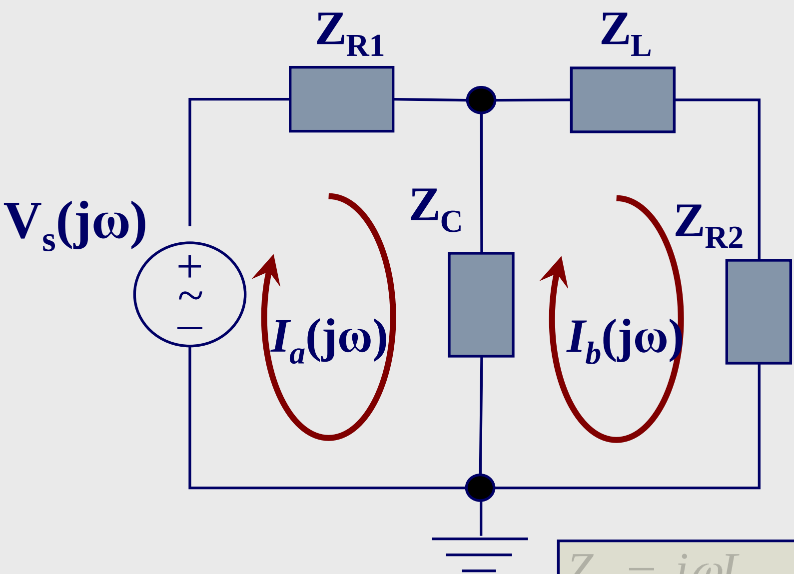


1. Note frequencies of AC sources
2. Convert to phasor domain



RLC Circuits

- **Example3**: find ***i**$_a$***(t)** and ***i**$_b$***(t)**
  - **v$_s$(t)** = 15cos(**1500**t)V, **R$_1$** = 100Ω, **R$_2$** = 75Ω, **L** = 0.5H, **C** = 1uF



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Represent each element by its impedance

$$v_s(t) = 15\cos(\omega t)$$
$$V_s(j\omega) = 15\angle 0 = 15e^{j0}$$

$$Z_{R1} = R_1 = 100\Omega$$

$$Z_L = j\omega L = j(1500)(0.5) = j750\Omega$$

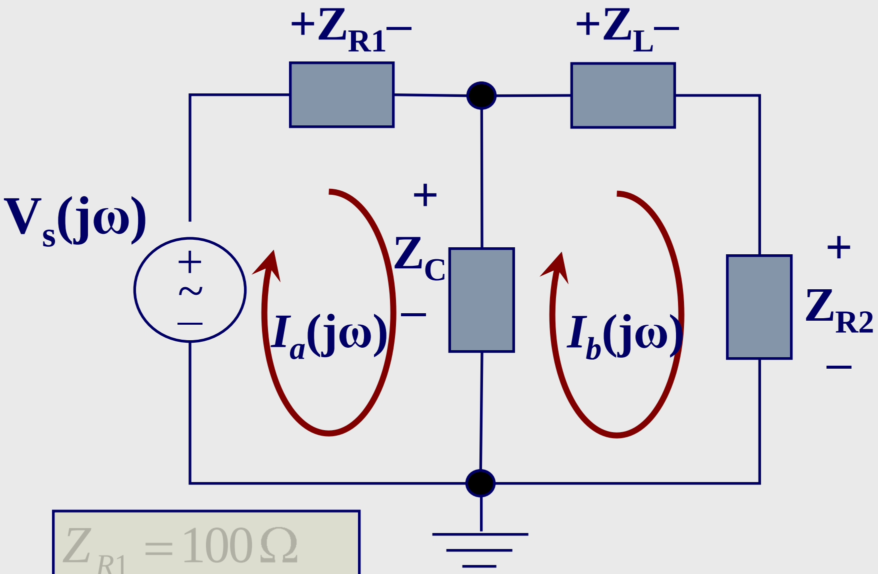
$$Z_{R2} = R_2 = 75\Omega$$

$$Z_C = 1/j\omega C = 1/j(1500)(10^{-6}) = -j667\Omega$$

BYU Electrical Engineering Computer Engineering

RLC Circuits

- **<u>Example3</u>**: find $\boldsymbol{i_a(t)}$ and $\boldsymbol{i_b(t)}$
  - $\mathbf{v_s(t)} = 15\cos(\mathbf{1500}t)$V, $\mathbf{R_1} = 100\Omega$, $\mathbf{R_2} = 75\Omega$, $\mathbf{L} = 0.5$H, $\mathbf{C} = 1$uF



$$Z_{R1} = 100\,\Omega$$
$$Z_{R2} = 75\Omega$$
$$Z_L = j750\,\Omega$$
$$Z_C = -j667\,\Omega$$

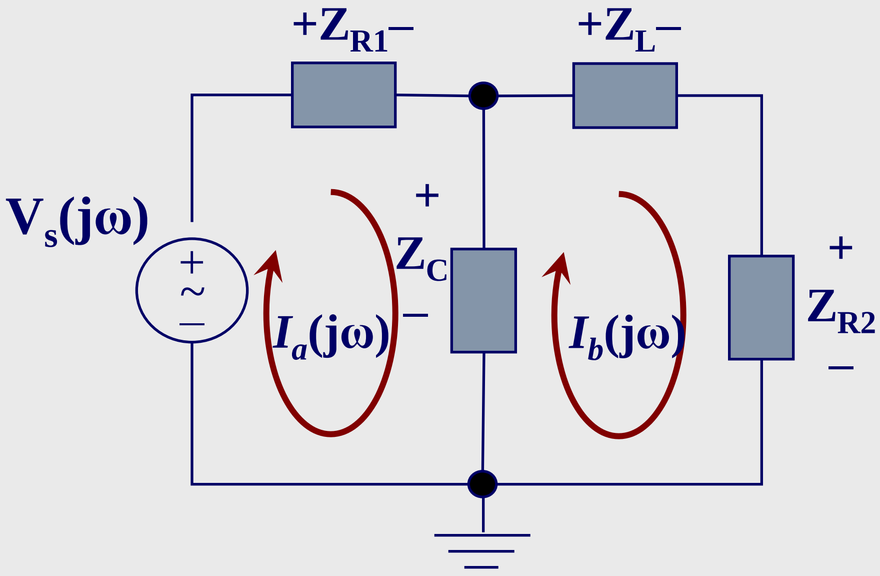
4. Solve using network analysis
   - Mesh current

$$\text{KVL at a}:$$
$$-V_s(j\omega) + V_{R1}(j\omega) + V_C(j\omega) = 0$$
$$I_a Z_{R1} + (I_a - I_b)Z_C = V_s$$
$$I_a(Z_{R1} + Z_C) - I_b Z_C = V_s$$

$$\text{KVL at b}:$$
$$-V_C(j\omega) + V_L(j\omega) + V_{R2}(j\omega) = 0$$
$$-(I_a - I_b)Z_C + I_b Z_L + I_b Z_{R2} = 0$$
$$-I_a Z_C + I_b(Z_C + Z_L + Z_{R2}) = 0$$

RLC Circuits

- **<u>Example3</u>**: find $\boldsymbol{i_a(t)}$ and $\boldsymbol{i_b(t)}$
  - $\mathbf{v_s(t)} = 15\cos(\mathbf{1500}t)\text{V}$, $\mathbf{R_1} = 100\Omega$, $\mathbf{R_2} = 75\Omega$, $\mathbf{L} = 0.5\text{H}$, $\mathbf{C} = 1\text{uF}$



4. Solve using network analysis
   - Mesh current

$$I_a(100 - j667) + I_b(j667) = 15$$

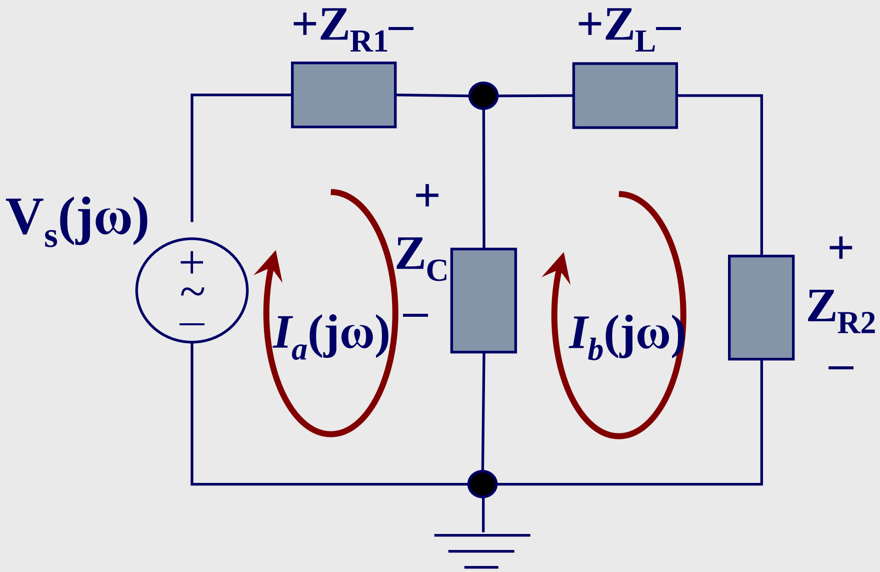
$$I_a(j667) + I_b(75 + j83) = 0$$

$$I_1 = 0.0032\angle 0.917\ A$$

$$I_2 = 0.019\angle -1.49\ A$$

RLC Circuits

- **Example3**: find $\boldsymbol{i_a}$**(t)** and $\boldsymbol{i_b}$**(t)**
  - $\mathbf{v_s(t)}$ = 15cos(**1500**t)V, $\mathbf{R_1}$ = 100Ω, $\mathbf{R_2}$ = 75Ω, **L** = 0.5H, **C** = 1uF



5. Convert to Time domain

$$I_1 = 0.0032\angle 0.917\ A$$

$$i_1(t) = 3.2\cos(1500t + 0.917)\ mA$$

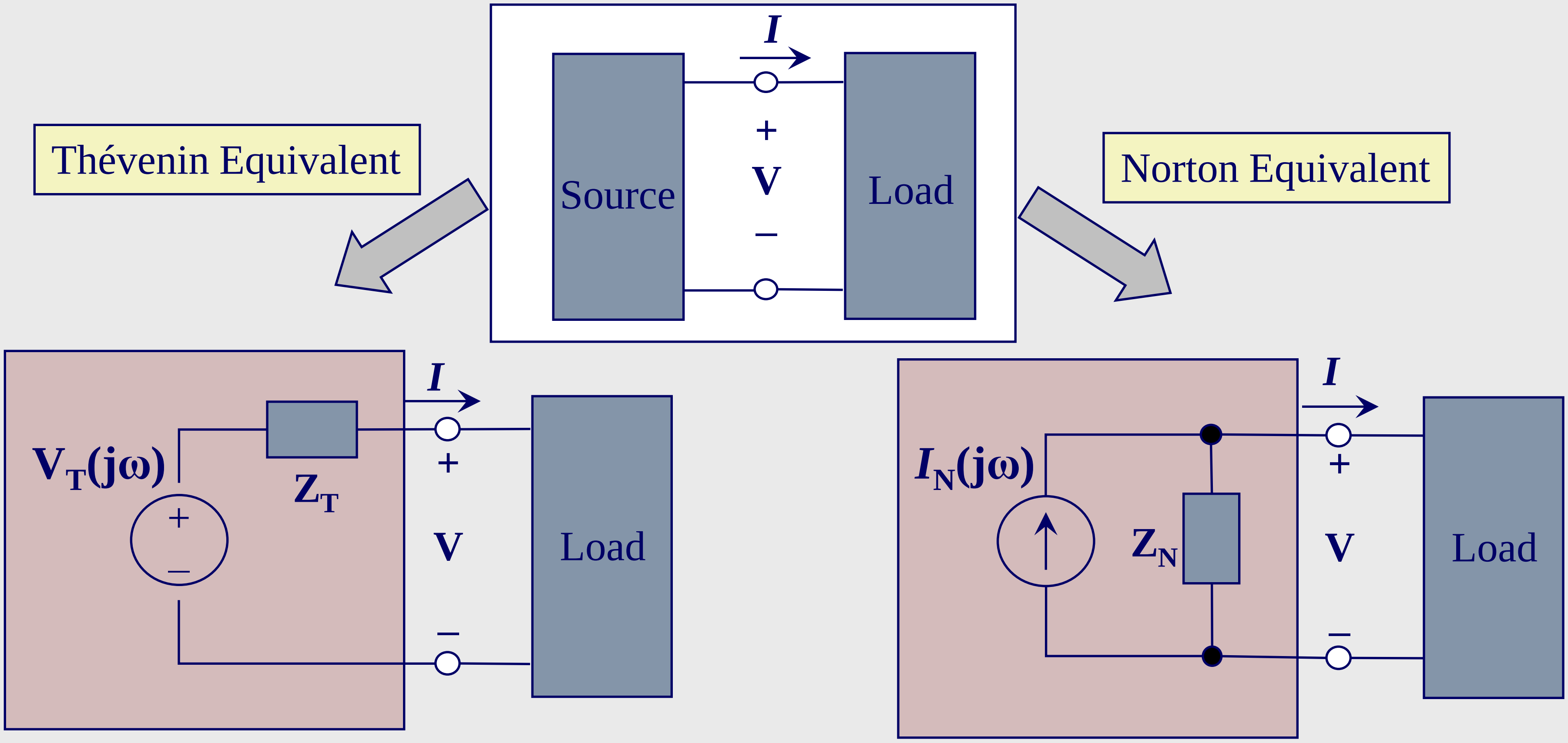
$$I_2 = 0.019\angle -1.49\ A$$

$$i_2(t) = 19\cos(1500t - 1.49)\ mA$$

AC Equivalent Circuits

Thévenin and **Norton** equivalent circuits apply in AC analysis

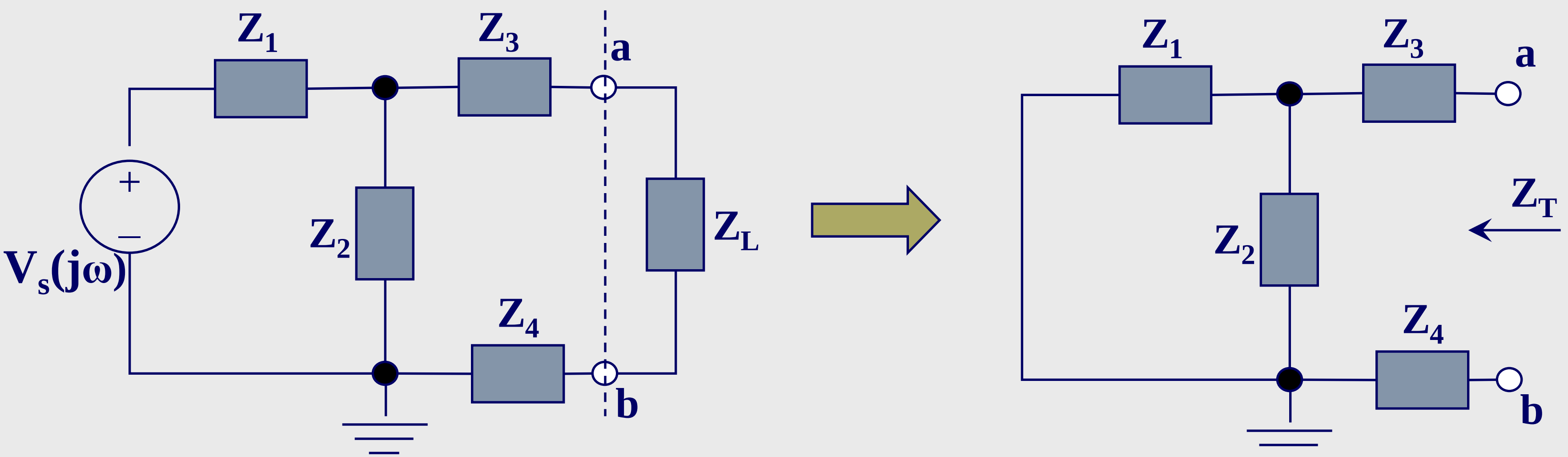
- Equivalent voltage/current will be **complex** and **frequency dependent**



AC Equivalent Circuits

Computation of Thévenin and Norton Impedances:

1. Remove the load (open circuit at load terminal)
2. Zero all independent sources
   - Voltage sources ⟶ short circuit (**v** = 0)
   - Current sources ⟶ open circuit (***i*** = 0)
3. Compute equivalent impedance **across load terminals** (with load removed)

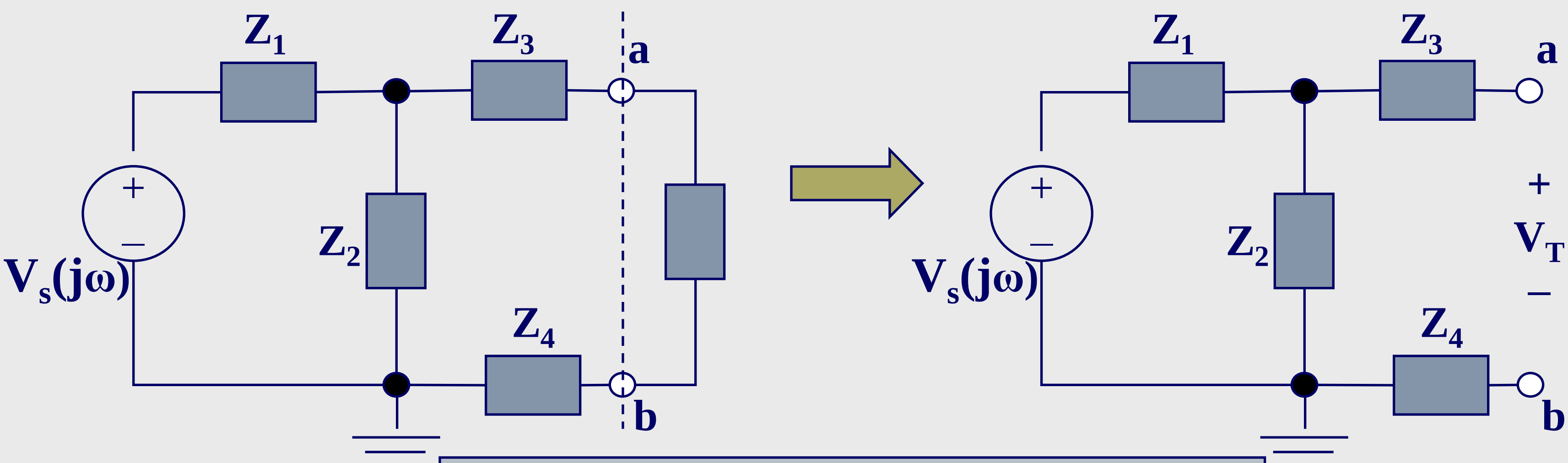


NB: same procedure as equivalent resistance

AC Equivalent Circuits

Computing Thévenin voltage:

1. Remove the load (open circuit at load terminals)
2. Define the open-circuit voltage (**V_{oc}**) across the load terminals
3. Chose a network analysis method to find **V_{oc}**
   - node, mesh, superposition, etc.
4. Thévenin voltage **V_T = V_{oc}**

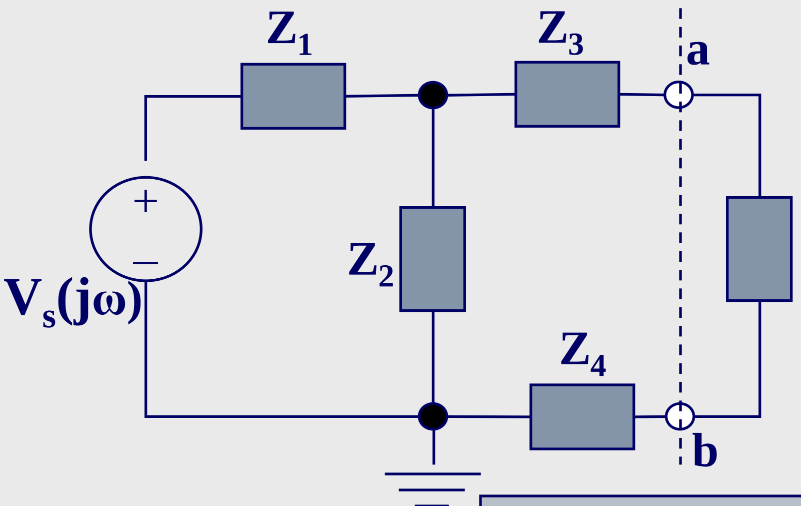


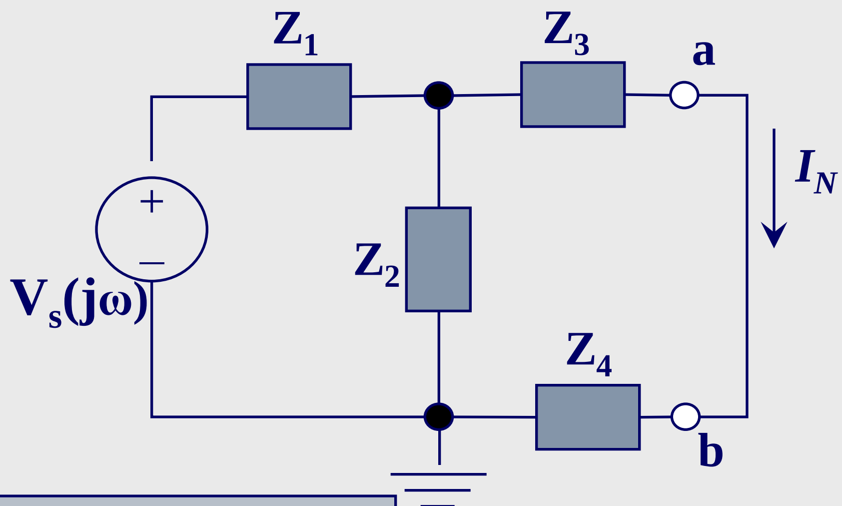
NB: same procedure as equivalent resistance

AC Equivalent Circuits

Computing Norton current:

1. Replace the load with a short circuit
2. Define the short-circuit current ($\boldsymbol{I_{sc}}$) across the load terminals
3. Chose a network analysis method to find $\boldsymbol{I_{sc}}$
   - node, mesh, superposition, etc.
4. Norton current $\boldsymbol{I_N} = \boldsymbol{I_{sc}}$

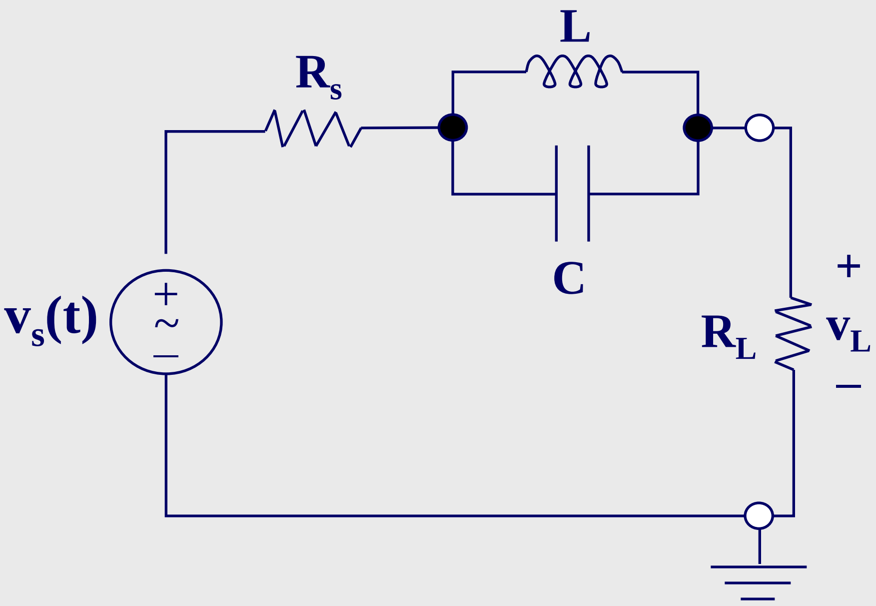




NB: same procedure as equivalent resistance

AC Equivalent Circuits

- **Example4**: find the Thévenin equivalent
  - $\boldsymbol{\omega} = 10^3$ rads/s, $\mathbf{R_s} = 50\Omega$, $\mathbf{R_L} = 50\Omega$, $\mathbf{L} = 10$mH, $\mathbf{C} = 0.1$uF

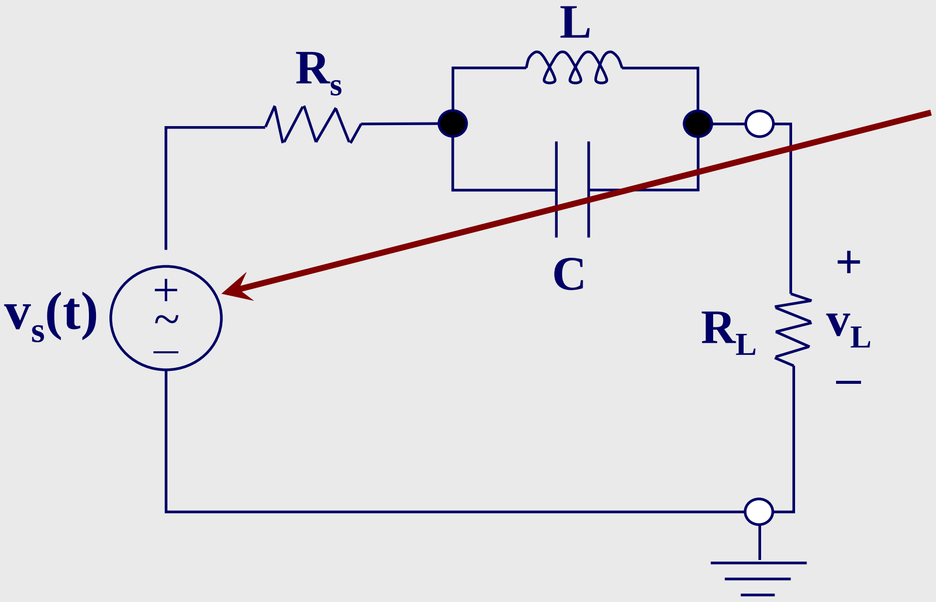


AC Equivalent Circuits

- **Example4**: find the Thévenin equivalent
  - **ω** = 10^3 rads/s, **R_s** = 50Ω, **R_L** = 50Ω, **L** = 10mH, **C** = 0.1uF

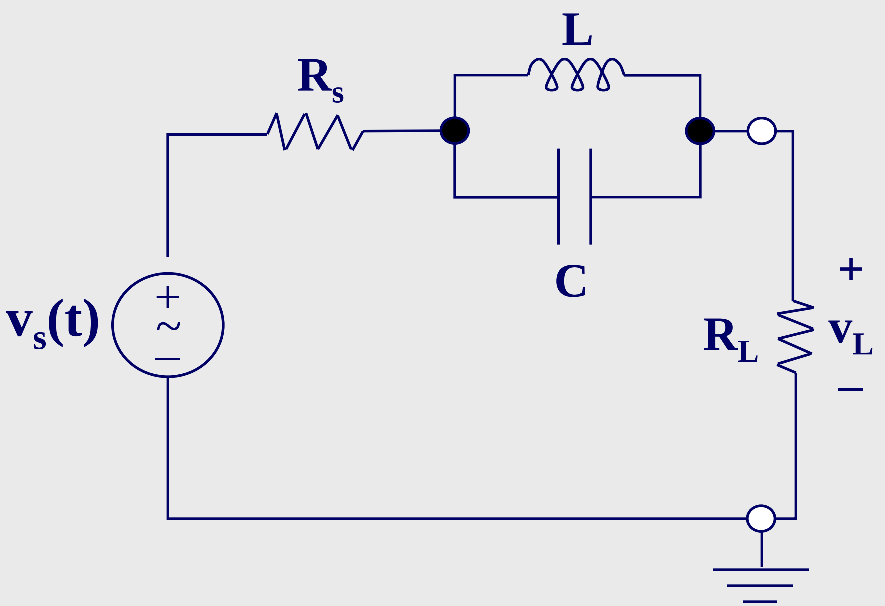
1. Note frequencies of AC sources

Only one AC source - **ω** = 10^3 rad/s

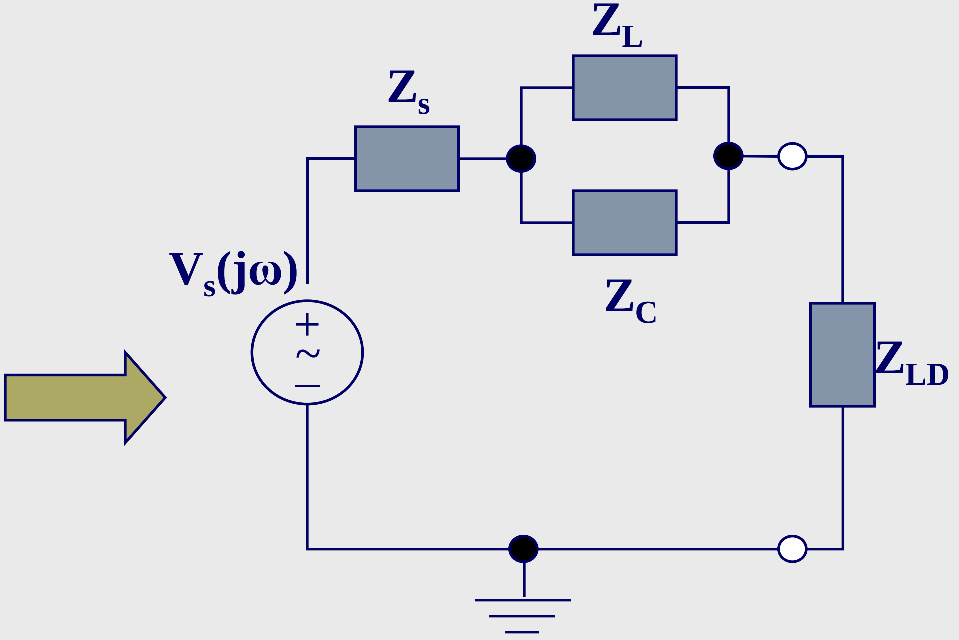


AC Equivalent Circuits

- **Example4**: find the Thévenin equivalent
  - $\omega = 10^3$ rads/s, $R_s = 50\Omega$, $R_L = 50\Omega$, $L = 10mH$, $C = 0.1uF$

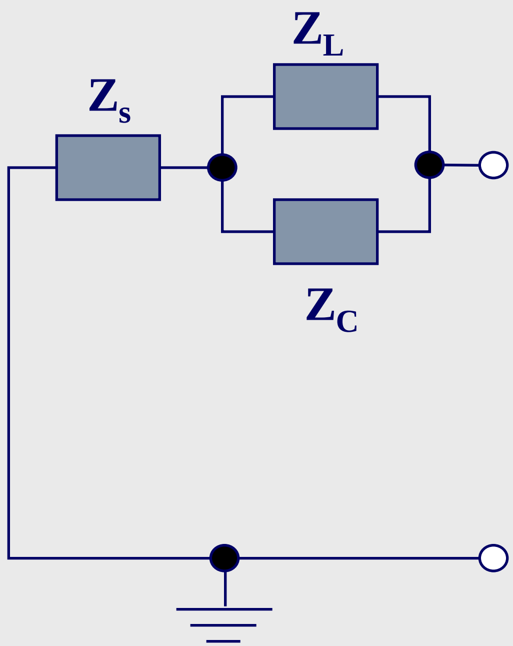


1. Note frequencies of AC sources
2. Convert to phasor domain



AC Equivalent Circuits

- **Example4**: find the Thévenin equivalent
  - **ω** = 10^3 rads/s, $\mathbf{R_s}$ = 50Ω, $\mathbf{R_L}$ = 50Ω, **L** = 10mH, **C** = 0.1uF

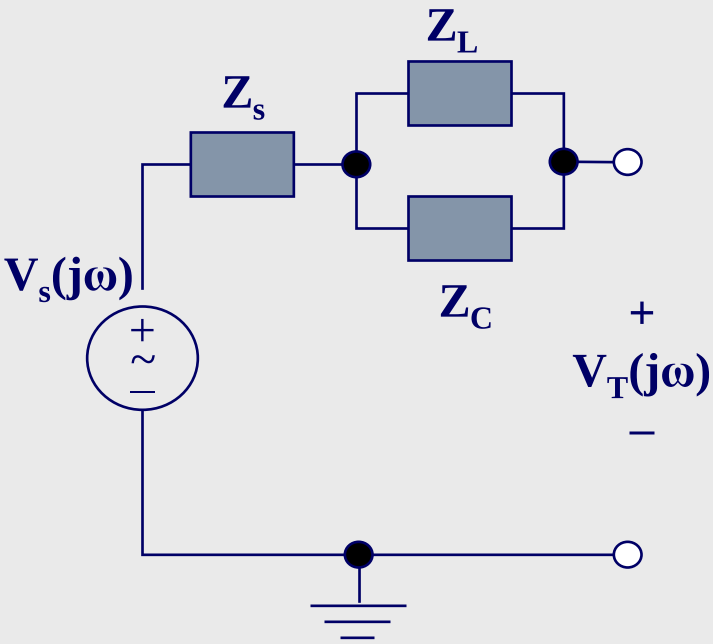


1. Note frequencies of AC sources
2. Convert to phasor domain
3. Find $\mathbf{Z_T}$
   - Remove load & zero sources

$$
\begin{aligned}
Z_T &= Z_S + Z_C \parallel Z_L \\
&= R_S + \frac{(j\omega L)(1/j\omega C)}{(j\omega L)+(1/j\omega C)} \\
&= R_S + j\frac{\omega L}{1-\omega^2 LC} \\
&= 50 + j65.414 \\
&= 82.33\angle 0.9182
\end{aligned}
$$

AC Equivalent Circuits

- **Example4**: find the Thévenin equivalent
  - **ω** = 10^3 rads/s, $\mathbf{R_s}$ = 50Ω, $\mathbf{R_L}$ = 50Ω, **L** = 10mH, **C** = 0.1uF



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Find $\mathbf{Z_T}$
   - Remove load & zero sources
4. Find $\mathbf{V_T(j\omega)}$
   - Remove load

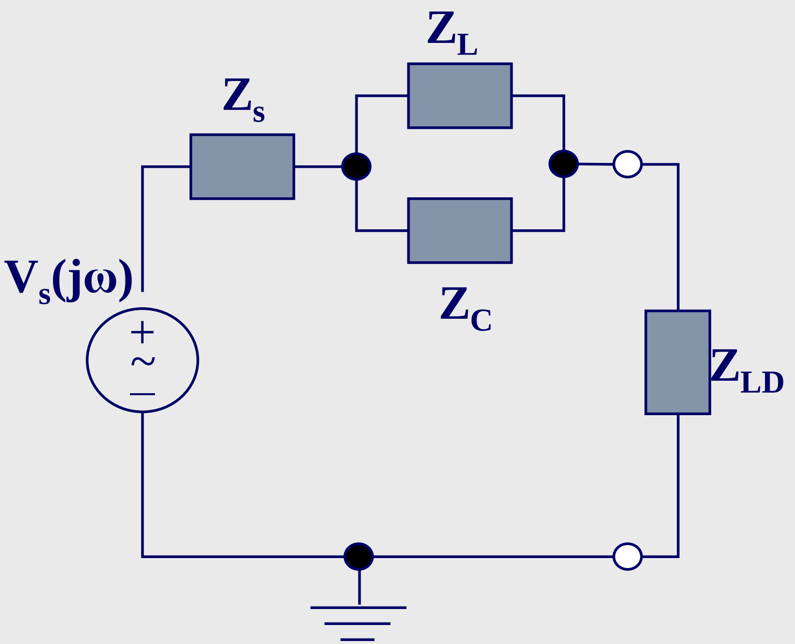
NB: Since no current flows in the circuit once the load is removed:

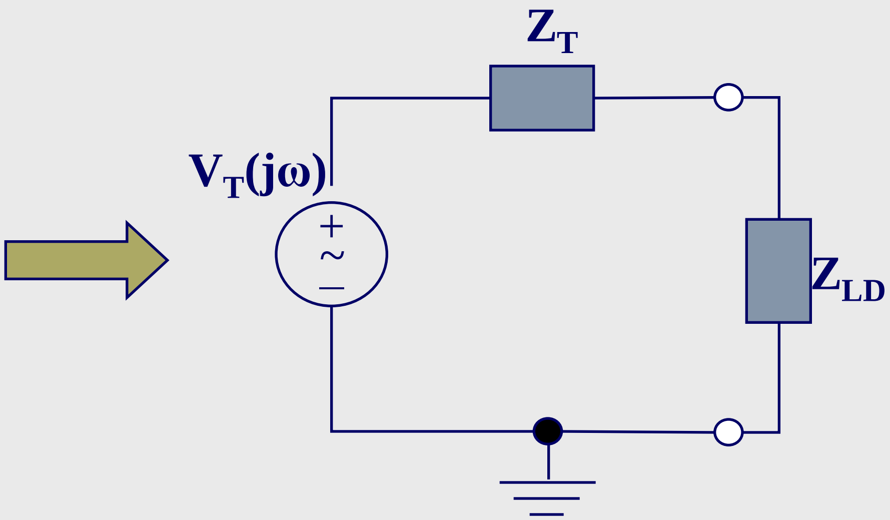
$$Z_T = 82.33\angle 0.9182$$

$$V_T = V_S$$

AC Equivalent Circuits

- **<u>Example4</u>**: find the Thévenin equivalent
  - **ω** = 10^3 rads/s, $\mathbf{R_s}$ = 50Ω, $\mathbf{R_L}$ = 50Ω, **L** = 10mH, **C** = 0.1uF





$$V_T = V_S$$

$$Z_T = 82.33\angle 0.9182$$