### Schedule...

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
22 Oct	Wed	15	Transient Response 1 <sup>st</sup> Order Circuits	5.3 - 5.4			
23 Oct	Thu						
24 Oct	Fri		Recitation				
25 Oct	Sat						
26 Oct	Sun						
27 Oct	Mon	16	Sinusoidal Frequency Response	6.1		LAB 5	
28 Oct	Tue						
29 Oct	Wed	17	Operational Amplifiers	8.1 - 8.2	(	1	1



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## Good, Better, Best

### Elder Oaks (October 2007):

As we consider various choices, we should remember that it is not enough that something is **good**. Other choices are **better**, and still others are **best**. Even though a particular choice is more costly, its far greater value may make it the **best** choice of all.

I have never known of a man who looked back on his working life and said, "I just didn't spend enough time with my job."



# Lecture 15 – Transient Response of 1<sup>st</sup> Order Circuits

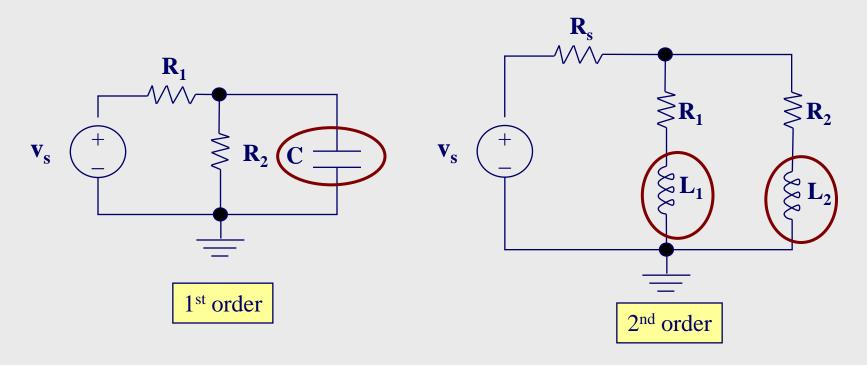
DC Steady-State Transient Response



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## 1<sup>st</sup> Order Circuits

Electric circuit 1<sup>st</sup> order system: any circuit containing a single energy storing element (either a capacitor or an inductor) and any number of sources and resistors



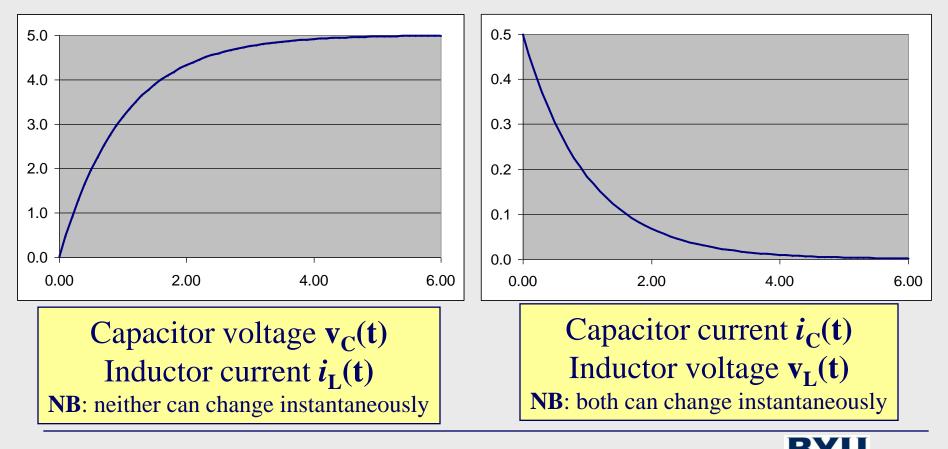


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Discussion  $#15 - 1^{st}$  Order Transient Response

### Capacitor/Inductor Voltages/Currents

Review of capacitor/inductor currents and voltages



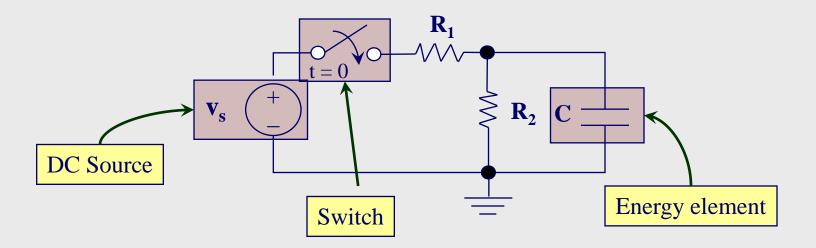
Discussion  $#15 - 1^{st}$  Order Transient Response

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## **Transient Response**

### **Transient response of a circuit consists of 3 parts**:

- Steady-state response prior to the switching on/off of a DC source
- 2. Transient response the circuit **adjusts** to the **DC source**
- 3. Steady-state response following the transient response





### 1. DC Steady State

### 1<sup>st</sup> and 3<sup>rd</sup> Step in Transient Response



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<u>**DC steady-state</u>**: the **stable** voltages and currents in a circuit connected to a DC source</u>

$$i_C(t) = C \frac{dv_C(t)}{dt}$$
 capacitor current  
 $i_C(t) \to 0$  as  $t \to \infty$  steady state current

Capacitors act like **open circuits** at DC steady-state

$$v_L(t) = L \frac{di_L(t)}{dt}$$
 inudctor v oltage  
 $v_L(t) \to 0$  as  $t \to \infty$  steady state voltage

Inductors act like **short circuits** at DC steady-state



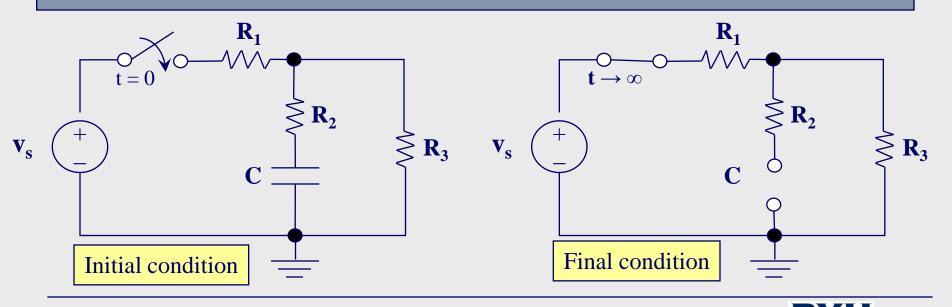
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**Initial condition x(0)**: DC steady state **before** a switch is first activated

 $\land$  **x**(**0**<sup>-</sup>): right before the switch is closed

 $\land$  **x**(**0**<sup>+</sup>): right after the switch is closed

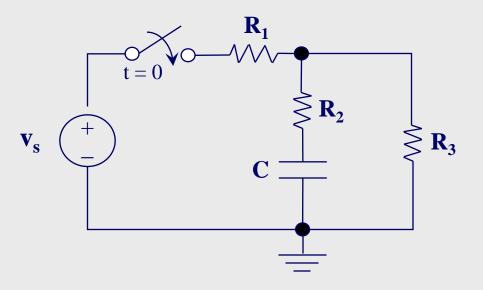
**Final condition**  $x(\infty)$ : DC steady state a long time **after** a switch is activated



Discussion  $#15 - 1^{st}$  Order Transient Response

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• Example1: determine the final condition capacitor voltage  $\mathbf{v_s} = 12V, \mathbf{R_1} = 100\Omega, \mathbf{R_2} = 75\Omega, \mathbf{R_3} = 250\Omega, \mathbf{C} = 1$ uF

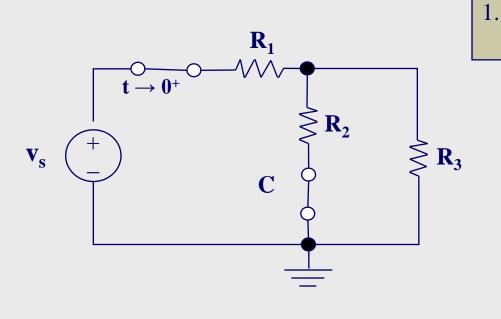




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**Example1**: determine the final condition capacitor voltage  $\mathbf{v_s} = 12V, \mathbf{R_1} = 100\Omega, \mathbf{R_2} = 75\Omega, \mathbf{R_3} = 250\Omega, \mathbf{C} = 1$ uF



. Close the switch and find **initial** conditions to the capacitor

**NB: Initially**  $(t = 0^+)$  current across the capacitor changes **instantly** but voltage cannot change instantly thus it acts as a **short circuit** 

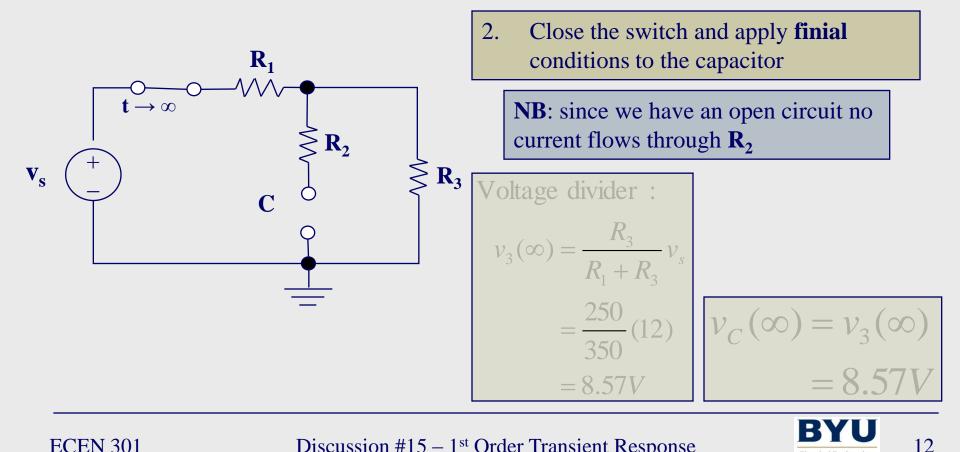
$$v_C(0^+) = v_C(0^-)$$
$$= 0V$$



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Discussion  $#15 - 1^{st}$  Order Transient Response

**Example1**: determine the final condition capacitor voltage  $v_s = 12V, R_1 = 100\Omega, R_2 = 75\Omega, R_3 = 250\Omega, C = 1 \mu F$ 



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Discussion  $#15 - 1^{st}$  Order Transient Response

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**Remember** – capacitor voltages and inductor currents cannot change instantaneously

Capacitor voltages and inductor currents don't change right before closing and right after closing a switch

$$v_C(0^+) = v_C(0^-)$$
  
 $i_L(0^+) = i_L(0^-)$ 

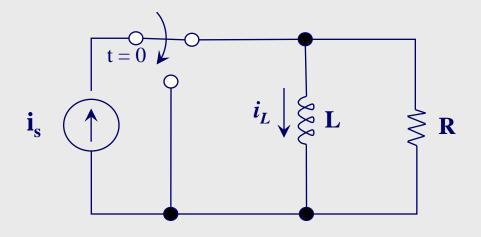


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### Example2: find the initial and final current conditions at the inductor

 $i_s = 10 \text{mA}$ 





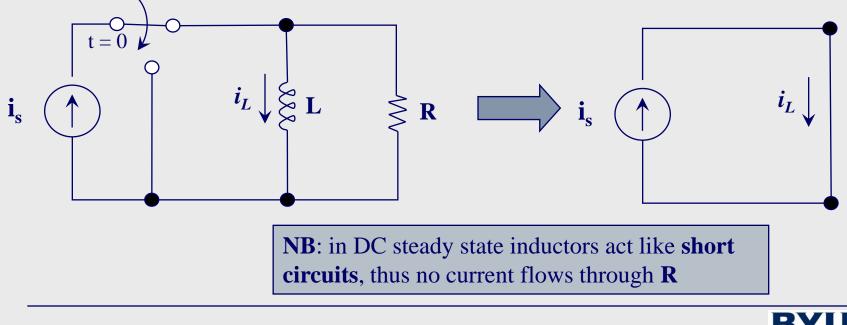
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### Example2: find the initial and final current conditions at the inductor

 $i_s = 10 \text{mA}$ 

1. Initial conditions – assume the current across the inductor is in steady-state.

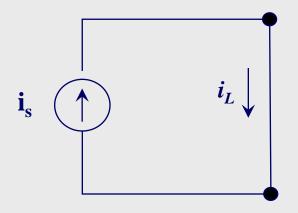


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### Example2: find the initial and final current conditions at the inductor

 $i_s = 10 \text{mA}$ 

1. Initial conditions – assume the current across the inductor is in steady-state.



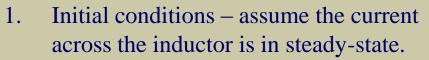
$$i_L(0^-) = i_s$$
$$= 10 mA$$



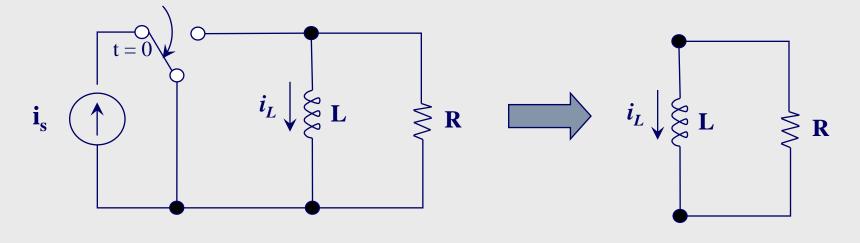
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### Example2: find the initial and final current conditions at the inductor

 $i_s = 10 \text{mA}$ 



2. Throw the switch



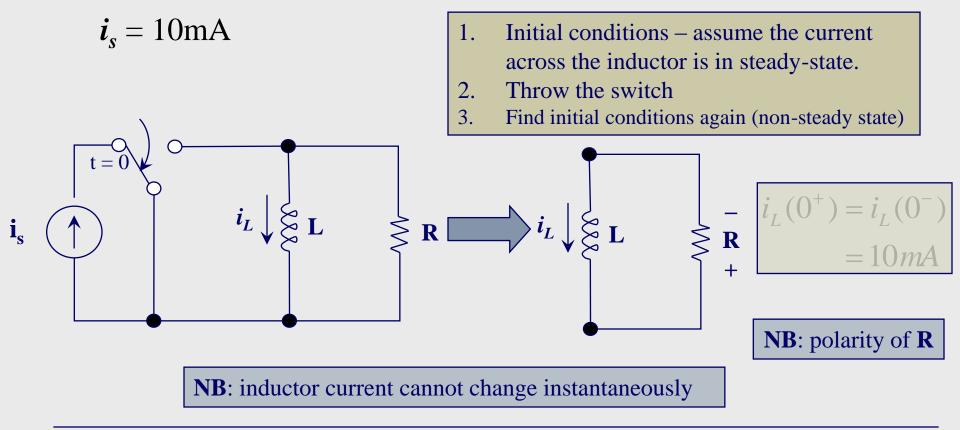
**NB**: inductor current cannot change instantaneously



Discussion #15 – 1<sup>st</sup> Order Transient Response

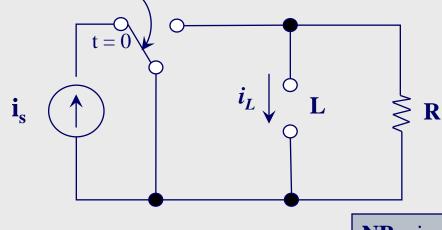
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### Example2: find the initial and final current conditions at the inductor



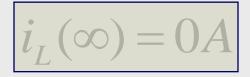
## Example2: find the initial and final current conditions at the inductor

 $i_s = 10 \text{mA}$ 



### 1. Initial conditions – assume the current across the inductor is in steady-state.

- 2. Throw the switch
- 3. Find initial conditions again (non-steady state)
- 4. Final conditions (steady-state)



**NB**: since there is **no source attached** to the inductor, its current is drained by the resistor **R** 



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## 2. Adjusting to Switch

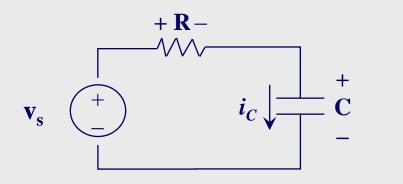
### 2<sup>nd</sup> Step in Transient Response

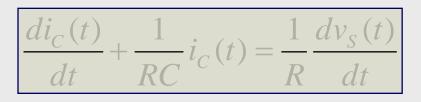


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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation





**NB**: Review lecture 11 for derivation of these equations

$$\frac{dv_C(t)}{dt} + \frac{1}{RC}v_C(t) = \frac{1}{RC}v_S(t)$$

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Discussion  $#15 - 1^{st}$  Order Transient Response

Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

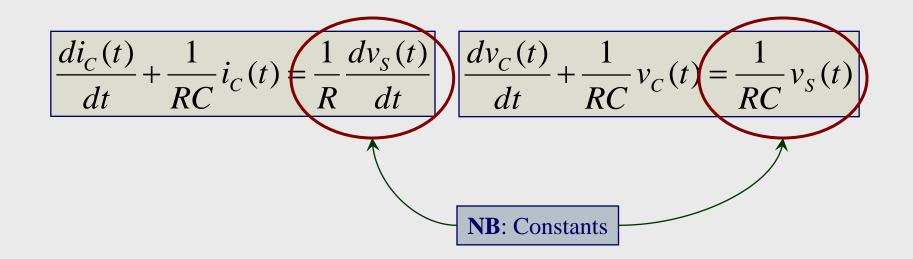
$$\frac{di_C(t)}{dt} + \frac{1}{RC}i_C(t) = \frac{1}{R}\frac{dv_S(t)}{dt} \qquad \frac{dv_C(t)}{dt} + \frac{1}{RC}v_C(t) = \frac{1}{RC}v_S(t)$$



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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

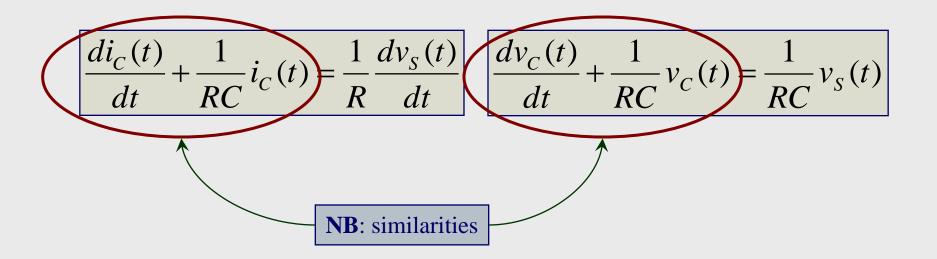




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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation





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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

In General :  

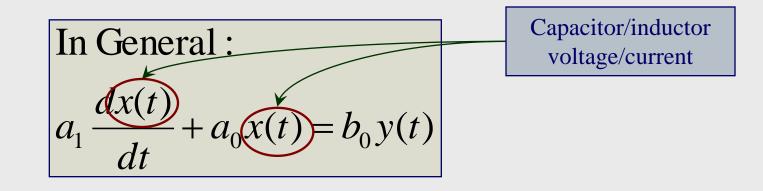
$$a_1 \frac{dx(t)}{dt} + a_0 x(t) = b_0 y(t)$$



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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

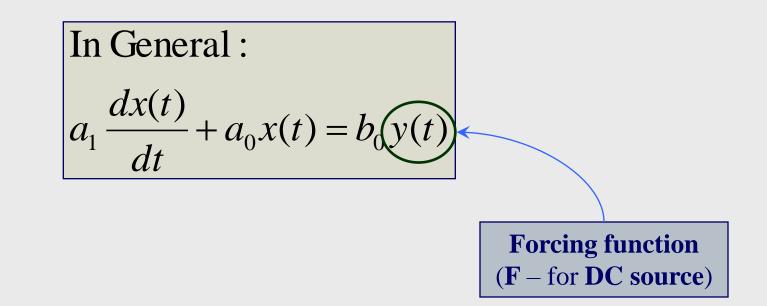




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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

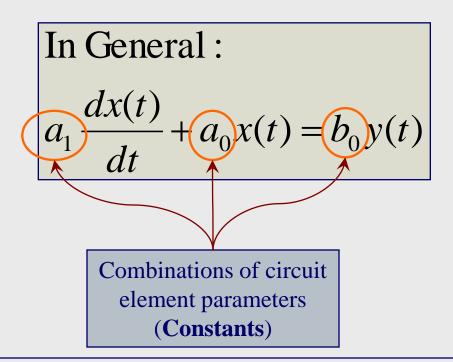




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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

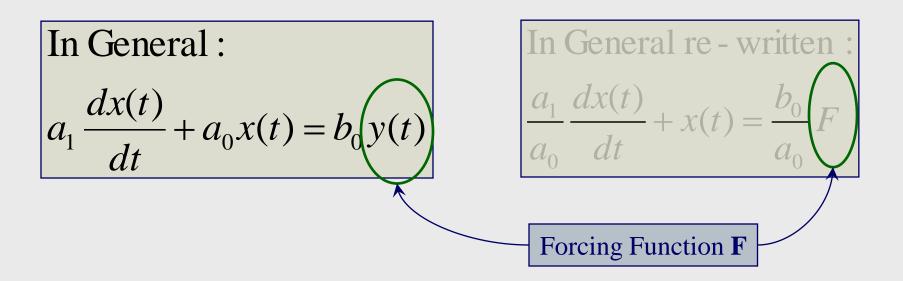




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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

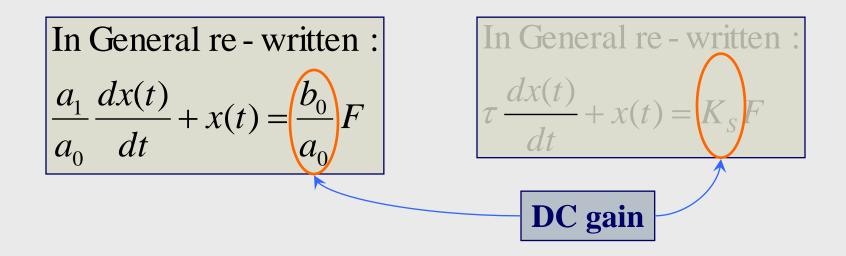




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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation

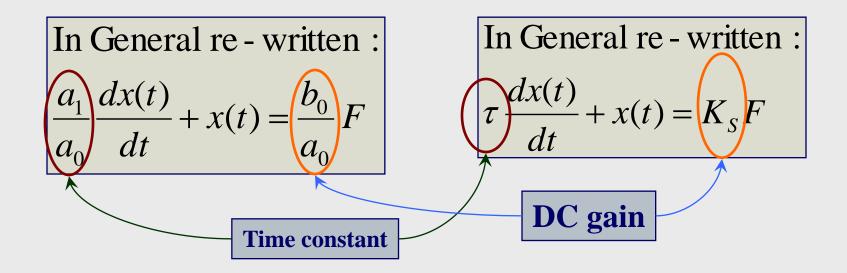




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Expressions for voltage and current of a 1<sup>st</sup> order circuit will be a 1<sup>st</sup> order differential equation





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The solution to this equation (the **complete response**) consists of two parts:

- ▲ Natural response (homogeneous solution)
  - Forcing function equal to zero

▲ Forced response (particular solution)

$$\tau \frac{dx(t)}{dt} + x(t) = K_s F$$

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Natural response (homogeneous solution)

▲ Forcing function equal to zero

$$\tau \frac{dx_N(t)}{dt} + x_N(t) = 0$$
$$\frac{dx_N(t)}{dt} = -\frac{x_N(t)}{\tau}$$

Has known solution of the form:

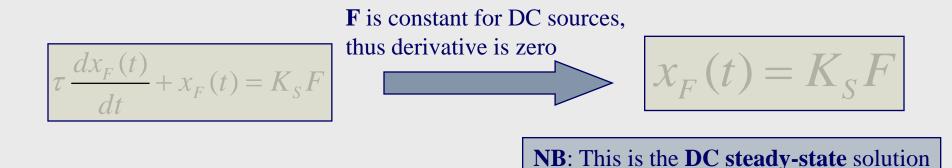


 $x_N(t) = \alpha e^{-t}$ 



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Forced response (particular solution)



### $x_F(t) = x(\infty) = K_S F$

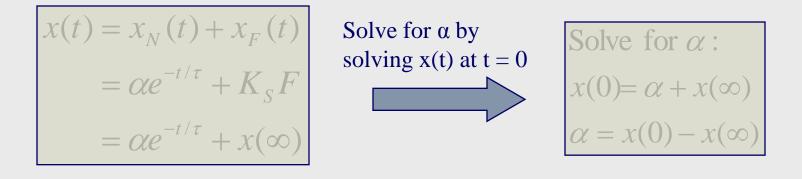


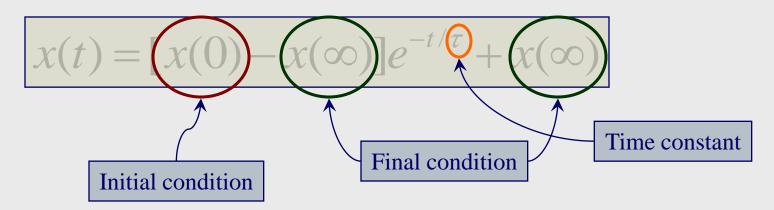
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### General Solution of 1st Order Circuits

### **Complete response** (natural + forced)



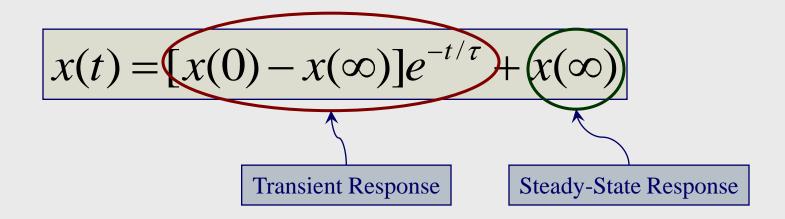




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**Complete response** (natural + forced)





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#### 3. DC Steady-State + Transient Response

#### Full Transient Response

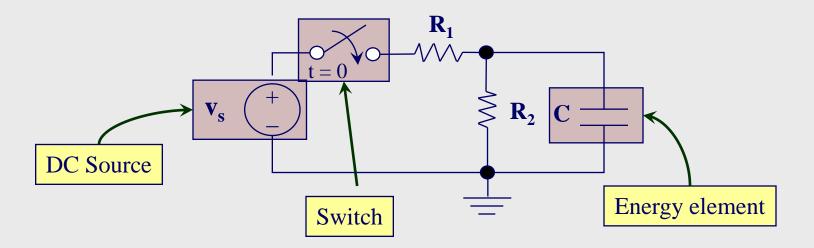


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#### **Transient response of a circuit consists of 3 parts**:

- Steady-state response prior to the switching on/off of a DC source
- 2. Transient response the circuit **adjusts** to the **DC source**
- 3. Steady-state response following the transient response





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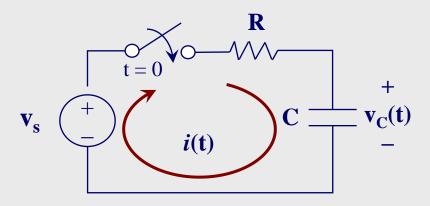
#### Solving 1<sup>st</sup> order transient response:

- 1. Solve the **DC** steady-state circuit:
  - ▲ Initial condition  $\mathbf{x}(\mathbf{0}^{-})$ : before switching (on/off)
  - Final condition  $\mathbf{x}(\infty)$ : After any transients have died out  $(t \to \infty)$
- 2. Identify  $x(0^+)$ : the circuit initial conditions
  - Capacitors:  $\mathbf{v}_{\mathbf{C}}(\mathbf{0}^+) = \mathbf{v}_{\mathbf{C}}(\mathbf{0}^-)$
  - A Inductors:  $i_{\rm L}(0^+) = i_{\rm L}(0^-)$
- 3. Write a differential equation for the circuit at time  $t = 0^+$ 
  - Reduce the circuit to its Thévenin or Norton equivalent
    - ▲ The energy storage element (capacitor or inductor) is the load
  - A The differential equation will be either in terms of  $\mathbf{v}_{\mathbf{C}}(\mathbf{t})$  or  $\mathbf{i}_{\mathbf{L}}(\mathbf{t})$
  - Reduce this equation to standard form
- 4. Solve for the **time constant** 
  - $\land \quad \text{Capacitive circuits: } \boldsymbol{\tau} = \mathbf{R}_{\mathbf{T}}\mathbf{C}$
  - $\land Inductive circuits: \tau = L/R_T$
- 5. Write the **complete response** in the form:

 $\mathbf{x}(t) = \mathbf{x}(\infty) + [\mathbf{x}(0) - \mathbf{x}(\infty)] e^{-t/\tau}$ 

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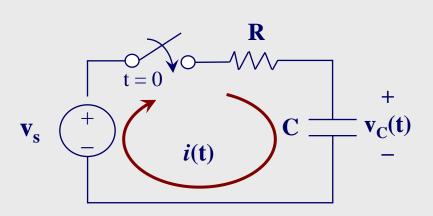
• <u>Example3</u>: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 



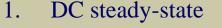


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**Example3**: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 



**NB**: as  $t \to \infty$  the capacitor acts like an open circuit thus  $\mathbf{v}_{\mathbf{C}}(\infty) = \mathbf{v}_{\mathbf{S}}$ 



- a) Initial condition:  $\mathbf{v}_{\mathbf{C}}(\mathbf{0})$
- b) Final condition:  $\mathbf{v}_{\mathbf{C}}(\infty)$

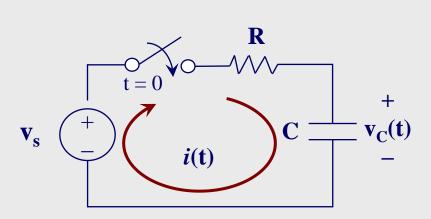
$$v_C(t < 0) = v_C(0^-) = 5V$$

$$v_C(\infty) = v_S$$
  
= 12V



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**Example3**: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 



2. Circuit initial conditions:  $v_{C}(0^{+})$ 

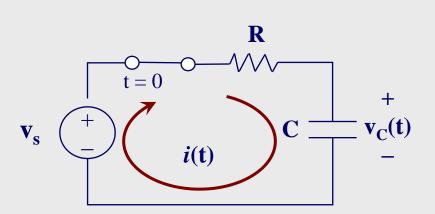
$$v_C(0^+) = v_C(0^-)$$
$$= 5V$$



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**Example3**: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 



3. Write differential equation (already in Thévenin equivalent) at t = 0

$$KVL:$$

$$-v_{S} + v_{R}(t) + v_{C}(t) = 0$$

$$i_{C}(t)R + v_{C}(t) = v_{S}$$

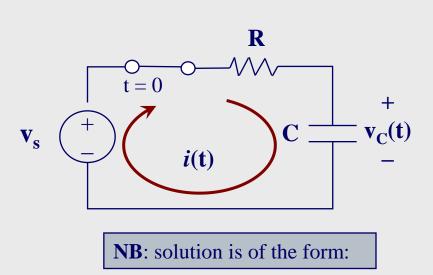
$$RC \frac{dv_{C}(t)}{dt} + v_{C}(t) = v_{S}$$



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**Example3**: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 



$$\tau \frac{dx(t)}{dt} + x(t) = K_s F$$

3. Write differential equation (already in Thévenin equivalent) at t = 0

$$KVL:$$

$$-v_{S} + v_{R}(t) + v_{C}(t) = 0$$

$$i_{C}(t)R + v_{C}(t) = v_{S}$$

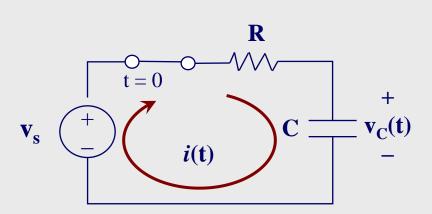
$$RC \frac{dv_{C}(t)}{dt} + v_{C}(t) = v_{S}$$



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**Example3**: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 



#### 4. Find the time constant $\tau$

$$\tau = RC$$
  
= (1000)(470×10<sup>-6</sup>)  
= 0.47

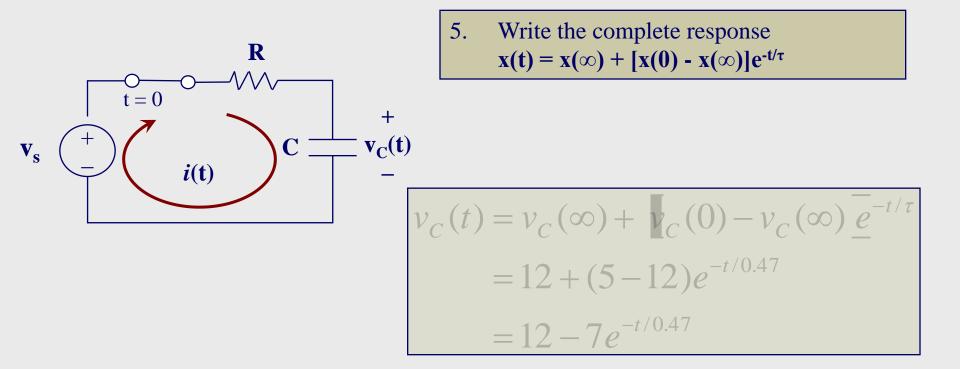
$$K_{S} = 1 \qquad F = v_{S} = 12$$



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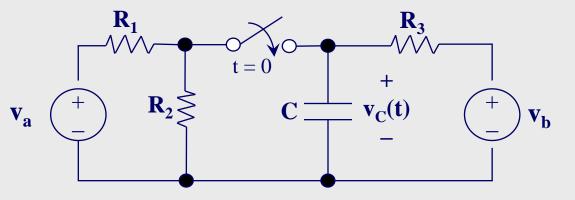
**Example3**: find  $\mathbf{v}_{c}(\mathbf{t})$  for all t  $\mathbf{v}_{s} = 12V, \mathbf{v}_{C}(\mathbf{0}) = 5V, \mathbf{R} = 1000\Omega, \mathbf{C} = 470 \mathrm{uF}$ 





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• Example4: find 
$$\mathbf{v_c}(\mathbf{t})$$
 for all t  
 $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1uF$ 

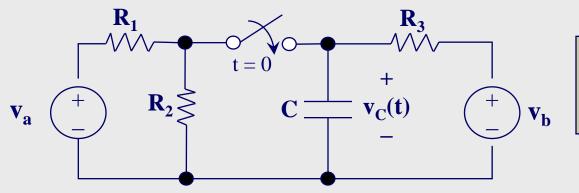




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• Example4: find 
$$\mathbf{v_c}(\mathbf{t})$$
 for all t  
 $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1$ uF



1. DC steady-state

- a) Initial condition:  $v_{C}(0)$
- b) Final condition:  $v_{C}(\infty)$

$$v_C(0^-) = v_b$$
$$= 5V$$

For  $t \to \infty \mathbf{v_c}(\infty)$  is not so easily determined – it will be equal to  $\mathbf{v_T}$  (the open circuited Thévenin equivalent)

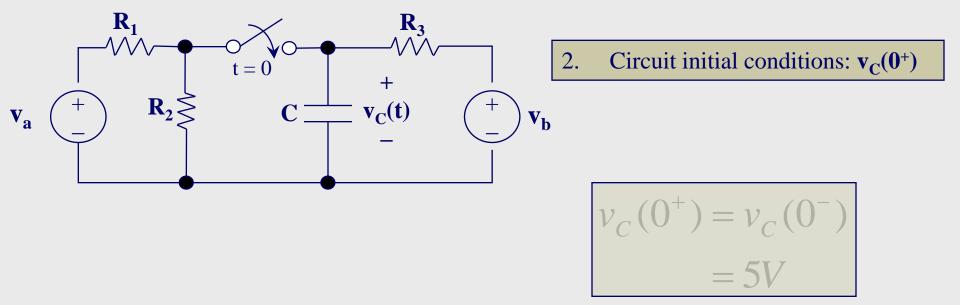
For t < 0 the capacitor has been charged by  $\mathbf{v}_{\mathbf{h}}$  thus  $\mathbf{v}_{\mathbf{C}}(\mathbf{0}) = \mathbf{v}_{\mathbf{h}}$ 

$$v_C(\infty) = v_T$$



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• Example4: find 
$$\mathbf{v}_{c}(\mathbf{t})$$
 for all t  
 $\mathbf{v}_{a} = 12V, \mathbf{v}_{b} = 5V, \mathbf{R}_{1} = 10\Omega, \mathbf{R}_{2} = 5\Omega, \mathbf{R}_{3} = 10\Omega, \mathbf{C} = 1$ uF



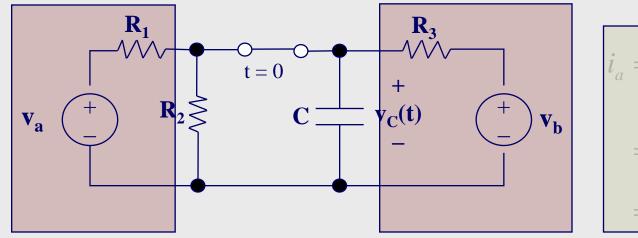


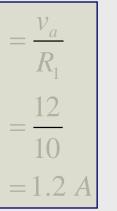
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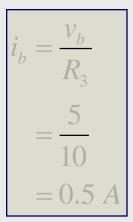
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• Example4: find  $\mathbf{v_c}(\mathbf{t})$  for all t  $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1$ uF

3. Write differential equation at t = 0a) Find Thévenin equivalent







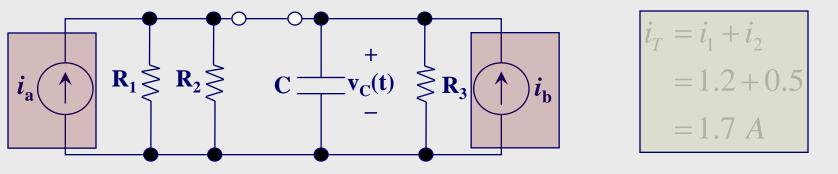


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**ECEN 301** 

• Example4: find  $\mathbf{v_c}(\mathbf{t})$  for all t  $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1$ uF

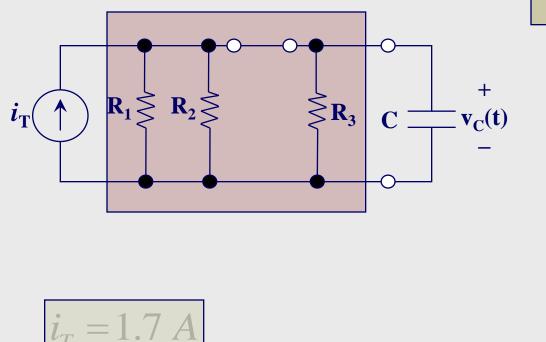
3. Write differential equation at t = 0a) Find Thévenin equivalent





ECEN 301

• Example4: find 
$$\mathbf{v_c}(\mathbf{t})$$
 for all t  
 $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1uF$ 



3. Write differential equation at t = 0a) Find Thévenin equivalent

$$R_{T} = R_{1} || R_{2} || R_{3}$$

$$= \frac{R_{1}R_{2}R_{3}}{R_{1}R_{2} + R_{1}R_{3} + R_{2}R_{3}}$$

$$= \frac{(10)(5)(10)}{(10)(5) + (10)(10) + (5)(10)}$$

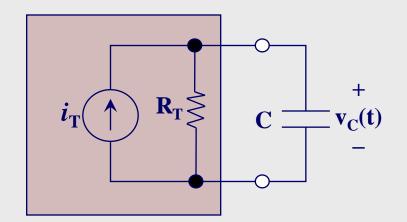
$$= \frac{500}{200}$$

$$= 2.5\Omega$$



#### **ECEN 301**

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$$v_T = i_T R_T$$
  
= (1.7)(2.5)  
= 4.25 V

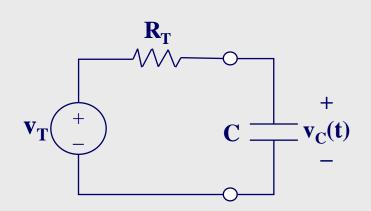


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**ECEN 301** 

 $l_T = 1.7 A$  $R_T = 2.50$ 

• Example4: find  $\mathbf{v_c}(\mathbf{t})$  for all t  $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1\mathrm{uF}$ 



- 3. Write differential equation at t = 0
  - a) Find Thévenin equivalent
  - b) Reduce equation to standard form

$$KVL:$$

$$-v_{T} + v_{RT}(t) + v_{C}(t) = 0$$

$$i_{C}(t)R_{T} + v_{C}(t) = v_{T}$$

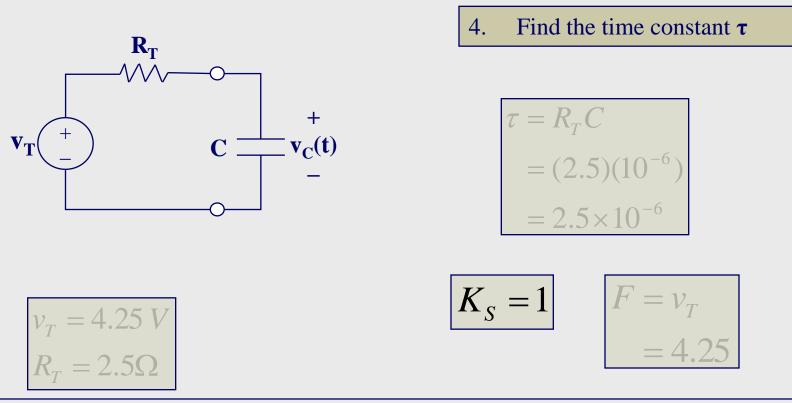
$$CR_{T} \frac{dv_{C}(t)}{dt} + v_{C}(t) = v_{T}$$



#### **ECEN 301**

 $v_T = 4.25 V$  $R_- = 2.50$ 

• Example4: find  $\mathbf{v_c}(\mathbf{t})$  for all t  $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1$ uF

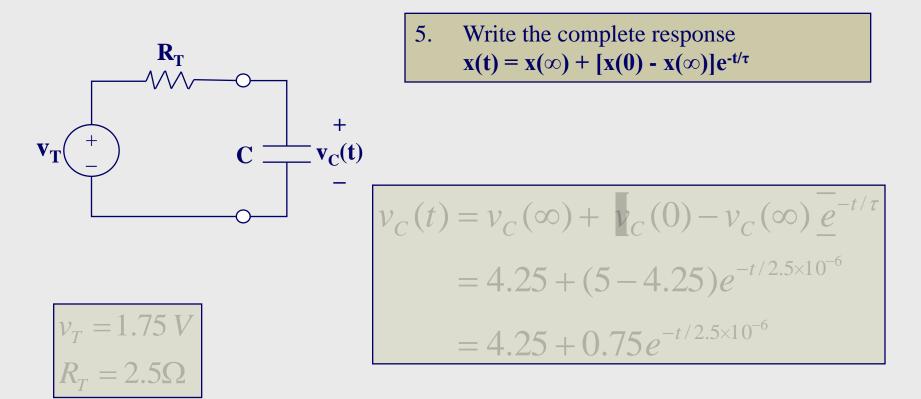




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#### **ECEN 301**

• Example4: find  $\mathbf{v_c}(\mathbf{t})$  for all t  $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1$ uF



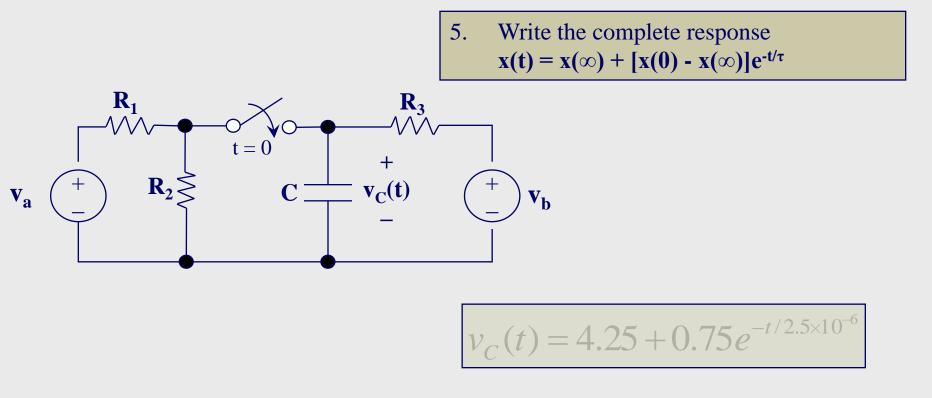
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Discussion #15 – 1<sup>st</sup> Order Transient Response



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• Example4: find  $\mathbf{v_c}(\mathbf{t})$  for all t  $\mathbf{v_a} = 12V, \mathbf{v_b} = 5V, \mathbf{R_1} = 10\Omega, \mathbf{R_2} = 5\Omega, \mathbf{R_3} = 10\Omega, \mathbf{C} = 1$ uF





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**ECEN 301**