Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
27 Oct	Mon	16	Sinusoidal Frequency Response	6.1		LAB 5	
28 Oct	Tue						
29 Oct	Wed	17	Operational Amplifiers	8.1 – 8.2			
30 Oct	Thu						
31 Oct	Fri		Recitation		HW 7		
1 Nov	Sat						
2 Nov	Sun						
3 Nov	Mon	18	Operational Amplifiers	8.3 – 8		LAB 6	
4 Nov	Tue						Exam 1

Easier to Maintain Than to Retake

Alma 59:9

9 And now as Moroni had supposed that there should be men sent to the city of Nephihah, to the assistance of the people to maintain that city, and knowing that it was **easier to keep the city from falling into the hands of the Lamanites than to retake it from them**, he supposed that they would easily maintain that city.

Lecture 16 – Frequency Response

Back to **AC** Circuits and **Phasors**

- Frequency Response
- Filters

Frequency Response

<u>Frequency Response H(jω)</u>: a measure of how the voltage/current/impedance of a load responds to the voltage/current of a source

$$H_V(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)}$$

$$H_I(j\omega) = \frac{I_L(j\omega)}{I_S(j\omega)}$$

$$H_Z(j\omega) = \frac{V_L(j\omega)}{I_S(j\omega)}$$

Frequency Response

$V_L(j\omega)$ is a phase-shifted and amplitude-scaled version of **$V_S(j\omega)$**

$$\frac{V_L(j\omega)}{V_S(j\omega)} = H_V(j\omega)$$

$$V_L(j\omega) = H_V(j\omega)V_S(j\omega)$$

Frequency Response

$V_L(j\omega)$ is a phase-shifted and amplitude-scaled version of **$V_S(j\omega)$**

$$\frac{V_L(j\omega)}{V_S(j\omega)} = H_V(j\omega)$$

$$V_L(j\omega) = H_V(j\omega) V_S(j\omega)$$

$$V_L e^{j\phi_L} = |H_V| e^{j\angle H_V} |V_S| e^{j\angle V_S}$$

$$V_L e^{j\phi_L} = |H_V||V_S| e^{j(\angle H_V + \angle V_S)}$$

NB : any complex number A can be expressed as

$$A = |A| e^{j\angle A}$$

$$A = |A| \angle A$$

$$V_L = |H_V||V_S|$$

$$\phi_L = \angle H_V + \angle V_S$$

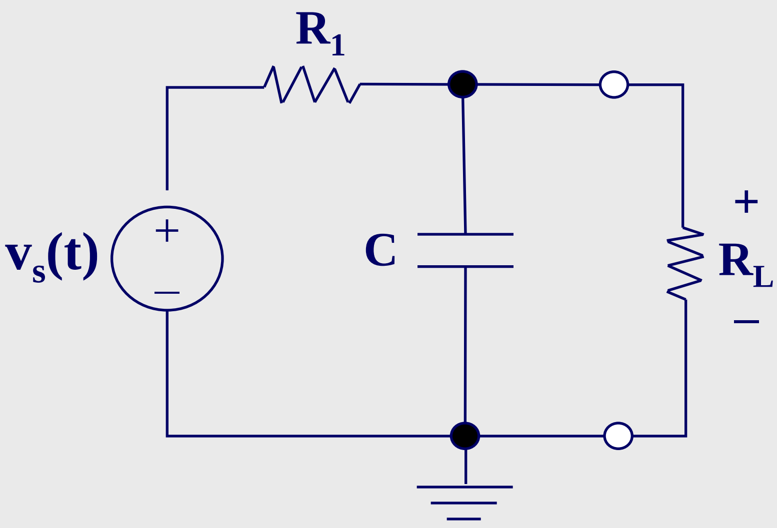
Amplitude scaled

Phase shifted

Frequency Response

<u>Example1</u>: compute the frequency response $\mathbf{H_V(j\omega)}$

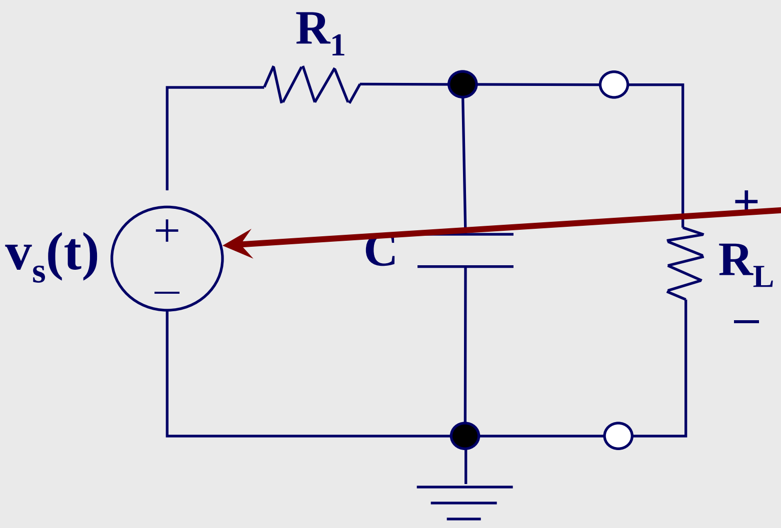
$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



Frequency Response

<u>Example1</u>: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1k\Omega$, $\mathbf{R_L} = 10k\Omega$, $\mathbf{C} = 10uF$



1. Note frequencies of AC sources

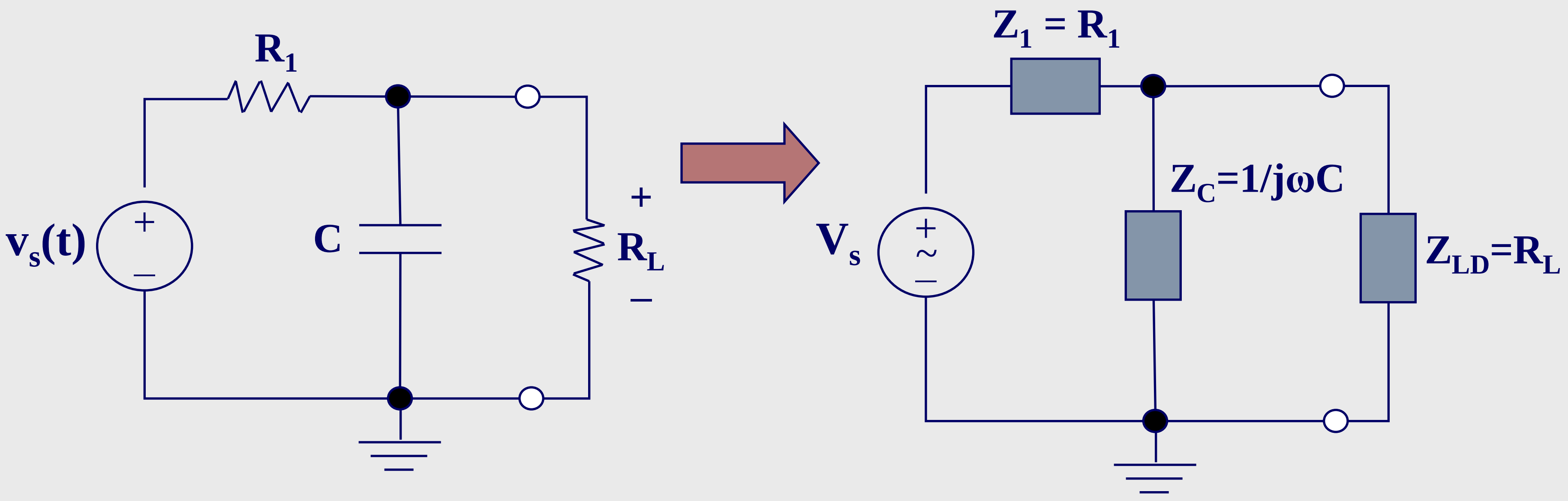
Only one AC source so frequency response $\mathbf{H_V(j\omega)}$ will be the function of a single frequency

Frequency Response

Example1: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$

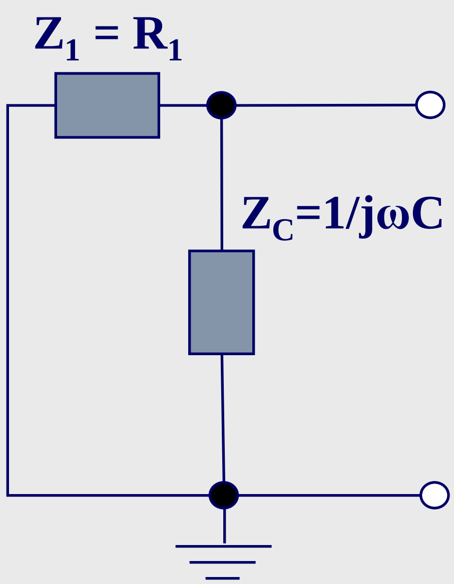
1. Note frequencies of AC sources
2. Convert to phasor domain



Frequency Response

<u>Example1</u>: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1}$ = 1kΩ, $\mathbf{R_L}$ = 10kΩ, **C** = 10uF



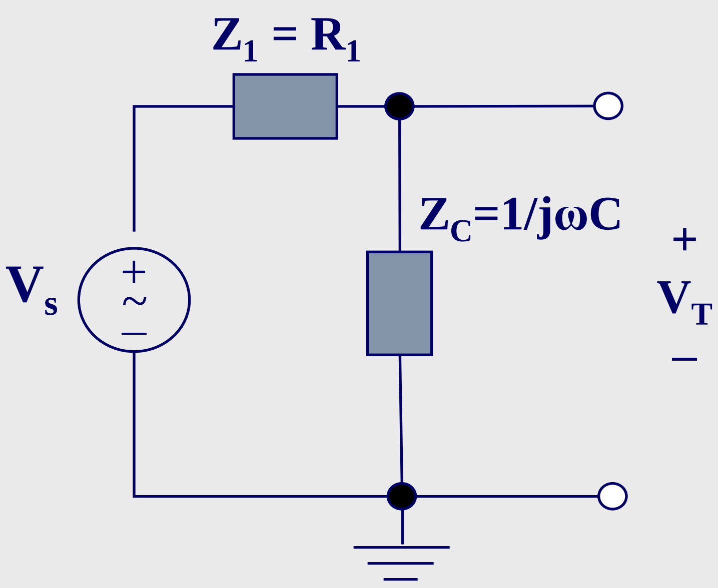
1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Thévenin equivalent

$$Z_T = Z_1 \parallel Z_C$$

Frequency Response

Example1: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1}$ = 1kΩ, $\mathbf{R_L}$ = 10kΩ, **C** = 10uF



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Thévenin equivalent

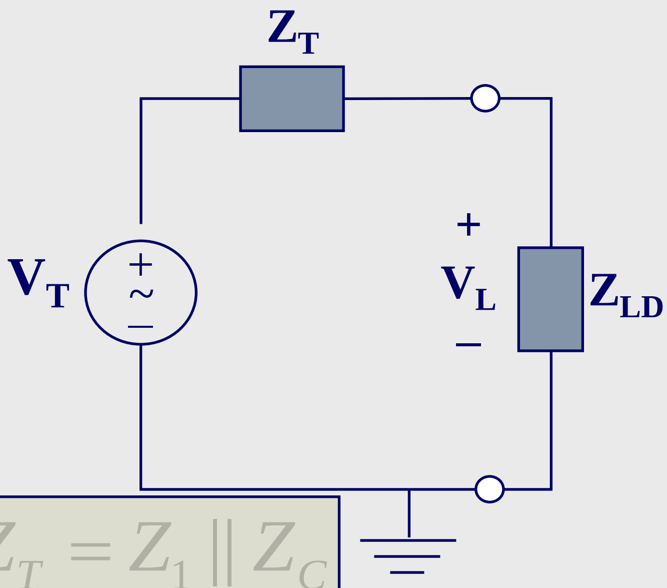
$$V_T(j\omega) = V_S(j\omega)\frac{Z_C}{Z_1 + Z_C}$$

$$Z_T = Z_1 \parallel Z_C$$

Frequency Response

<u>Example1</u>: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Thévenin equivalent
4. Find an expression for the load voltage

$$Z_T = Z_1 \| Z_C$$

$$V_T(j\omega) = V_S(j\omega)\frac{Z_C}{Z_1 + Z_C}$$

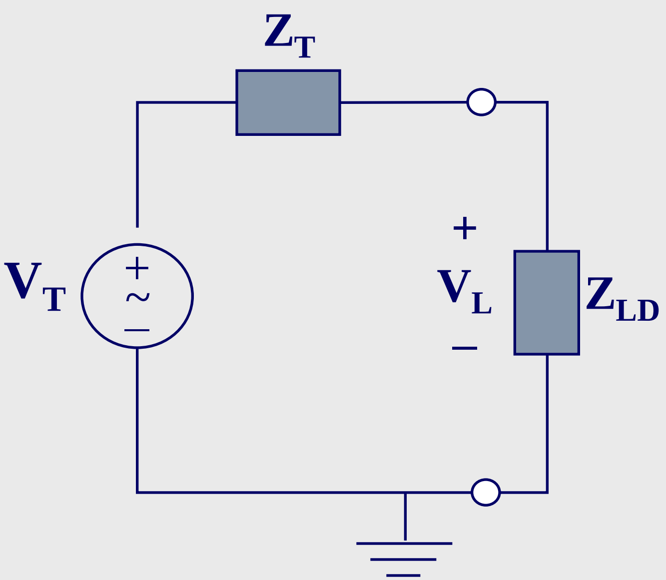
$$V_L(j\omega) = V_T(j\omega)\frac{Z_{LD}}{Z_T + Z_{LD}}$$

$$= \left(V_S(j\omega)\frac{Z_C}{Z_1 + Z_C}\right)\frac{Z_{LD}}{(Z_1 \| Z_C) + Z_{LD}}$$

Frequency Response

Example1: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1k\Omega$, $\mathbf{R_L} = 10k\Omega$, $\mathbf{C} = 10uF$



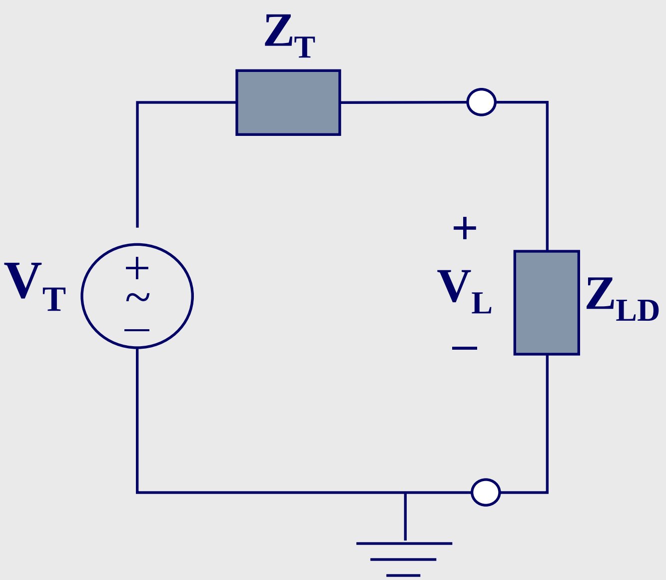
5. Find an expression for the frequency response

$$H_V(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)}$$
$$= \frac{Z_C}{Z_1 + Z_C} \cdot \frac{Z_{LD}}{(Z_1 \| Z_C) + Z_{LD}}$$

Frequency Response

Example1: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



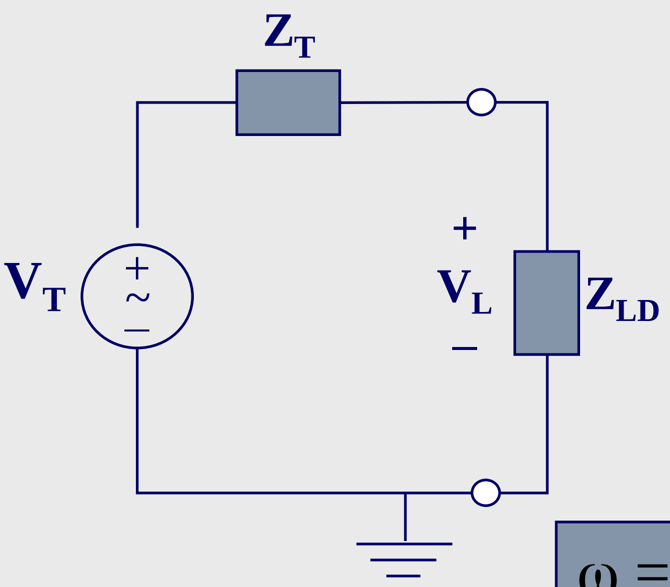
5. Find an expression for the frequency response

$$
\begin{aligned}
H_V(j\omega) &= \frac{Z_C}{Z_1 + Z_C} \cdot \frac{Z_{LD}}{\left(Z_1 \| Z_C\right) + Z_{LD}} \\
&= \frac{Z_{LD} Z_C}{Z_{LD}\left(Z_1 + Z_C\right) + Z_1 Z_C} \\
&= \frac{10^4/(j\omega \times 10^{-5})}{10^4\left[10^3 + 1/(j\omega \times 10^{-5})\right] + 10^3/(j\omega \times 10^{-5})} \\
&= \frac{100}{110 + j\omega} \\
&= \frac{100}{\sqrt{110^2 + \omega^2}} \angle -\arctan\left(\frac{\omega}{110}\right)
\end{aligned}
$$

Frequency Response

Example1: compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



5. Find an expression for the frequency response
   - Look at response for low frequencies (ω = 10) and high frequencies (ω = 10000)

$$H_V(j\omega) = \frac{100}{\sqrt{110^2 + \omega^2}} \angle -\arctan\left(\frac{\omega}{110}\right)$$

ω = 10

$$H_V(j\omega) = 0.905 \angle -0.0907$$

ω = 10000

$$H_V(j\omega) = 0.010 \angle -\frac{\pi}{2}$$

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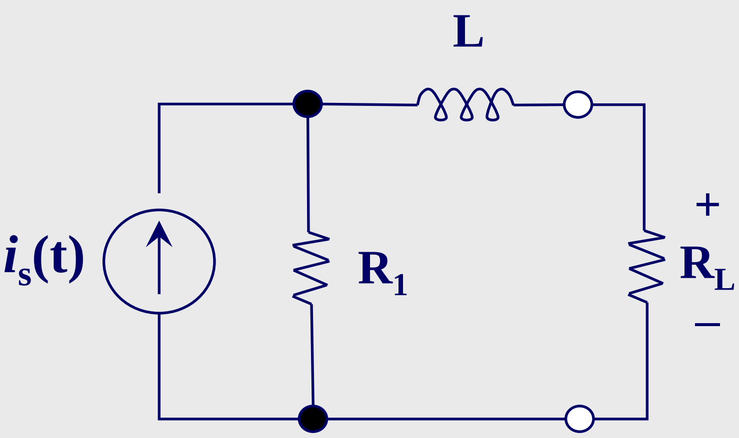
Frequency Response

<u>Example2</u>: compute the frequency response $\mathbf{H_Z(j\omega)}$

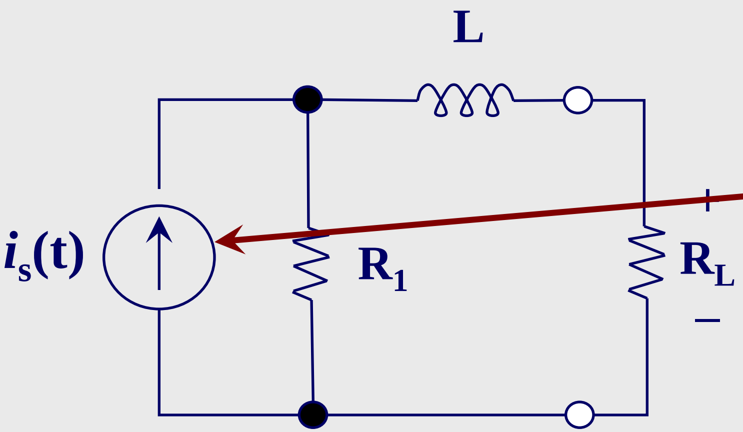
$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 4\text{k}\Omega$, $\mathbf{L} = 2\text{mH}$



Frequency Response

<u>Example2</u>: compute the frequency response $\mathbf{H_Z(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 4\text{k}\Omega$, $\mathbf{L} = 2\text{mH}$



1. Note frequencies of AC sources

Only one AC source so frequency response $\mathbf{H_Z(j\omega)}$ will be the function of a single frequency

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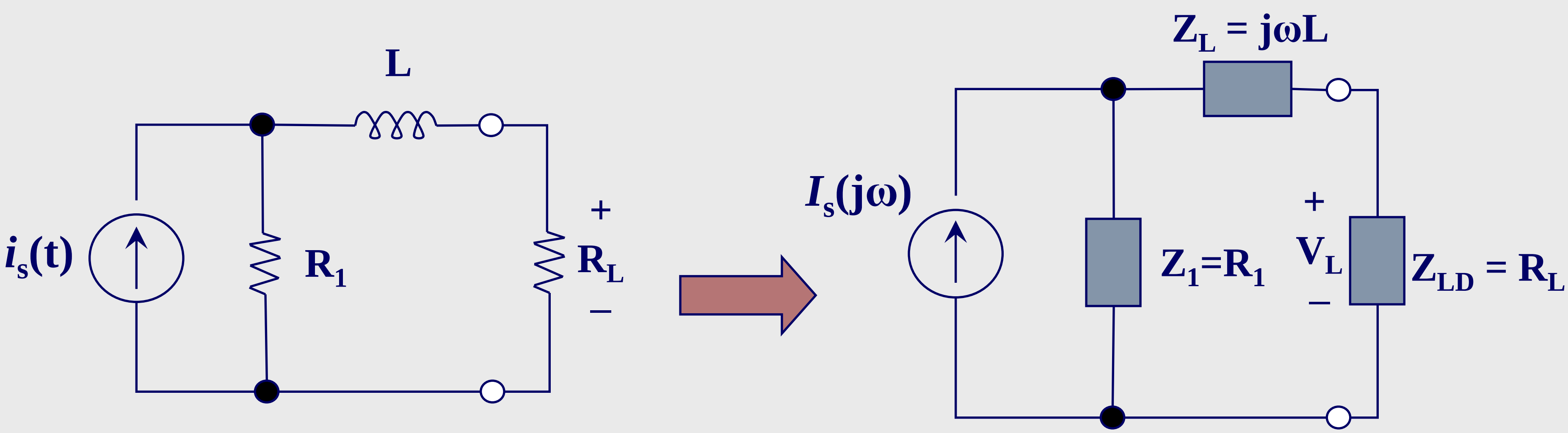
Frequency Response

<u>Example2</u>: compute the frequency response $\mathbf{H_Z(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 4\text{k}\Omega$, $\mathbf{L} = 2\text{mH}$

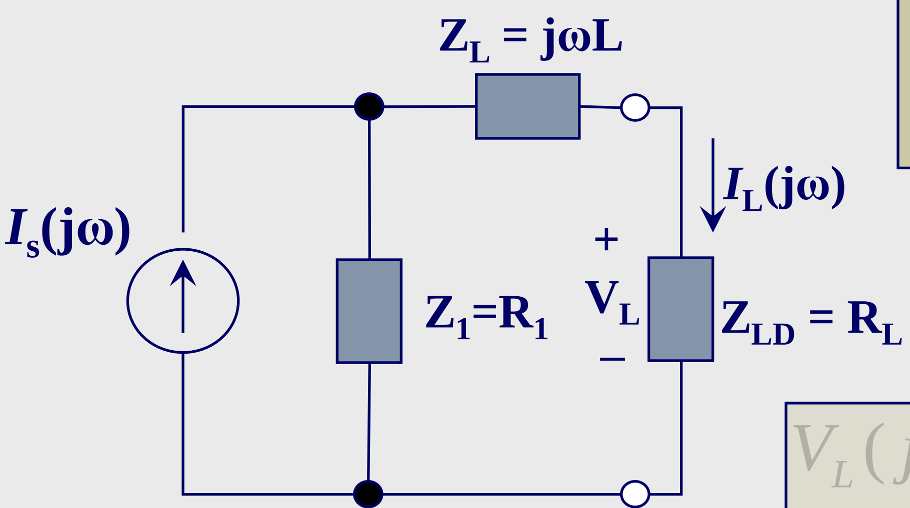
1. Note frequencies of AC sources
2. Convert to phasor domain



Frequency Response

<u>Example2</u>: compute the frequency response $\mathbf{H_Z(j\omega)}$

$\mathbf{R_1} = 1k\Omega$, $\mathbf{R_L} = 4k\Omega$, $\mathbf{L} = 2mH$



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Current divider & Ohm's Law

$$I_L(j\omega) = I_S(j\omega)\frac{Z_1 \| (Z_L + Z_{LD})}{(Z_L + Z_{LD})}$$

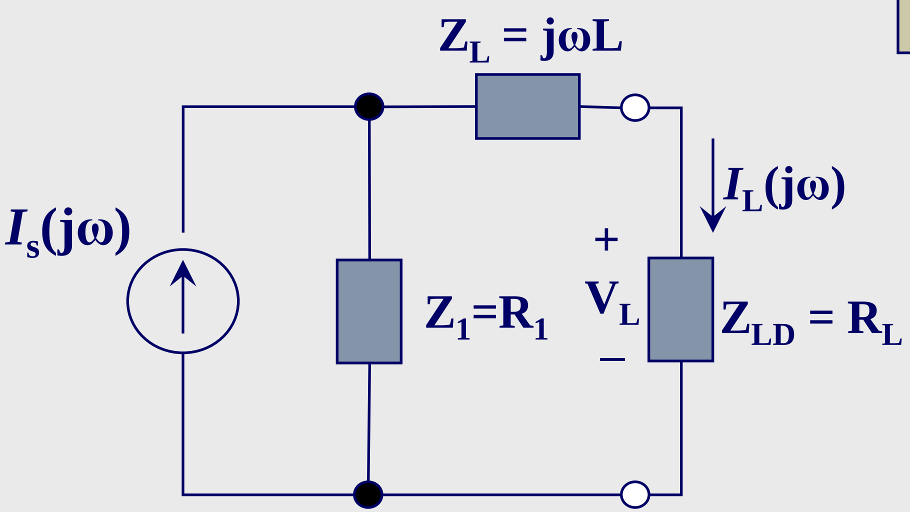
$$V_L(j\omega) = I_L(j\omega) Z_{LD}$$

$$= \left(I_S(j\omega)\frac{Z_1 \| (Z_L + Z_{LD})}{(Z_L + Z_{LD})} \right) Z_{LD}$$

Frequency Response

<u>Example2</u>: compute the frequency response $\mathbf{H_Z(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 4\text{k}\Omega$, $\mathbf{L} = 2\text{mH}$



4. Find an expression for the frequency response

$$H_Z(j\omega) = \frac{V_L(j\omega)}{I_S(j\omega)}$$

$$= \frac{Z_1 \| (Z_L + Z_{LD})}{(Z_L + Z_{LD})} Z_{LD}$$

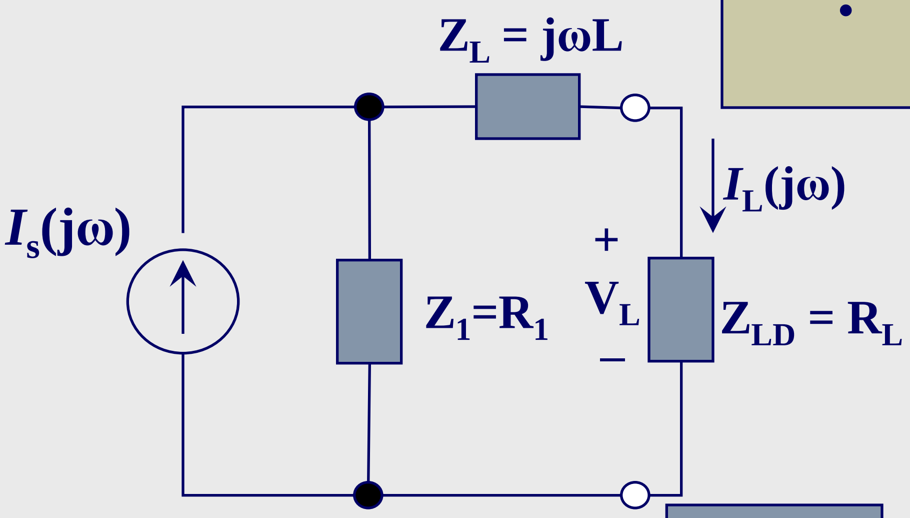
$$= \frac{R_L}{1 + R_L / R_1 + j\omega L / R_1}$$

$$= \frac{800}{1 + j\omega 0.4 \times 10^{-6}}$$

Frequency Response

Example2: compute the frequency response $\mathbf{H_Z(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 4\text{k}\Omega$, $\mathbf{L} = 2\text{mH}$



4. Find an expression for the frequency response
   - Look at response for low frequencies (ω = 10) and high frequencies (ω = 10000)

$$H_Z(j\omega) = \frac{800}{1 + j\omega 0.4 \times 10^{-6}}$$

ω = 10

$$H_Z(j\omega) = 800\angle -4.0$$

ω = 10000

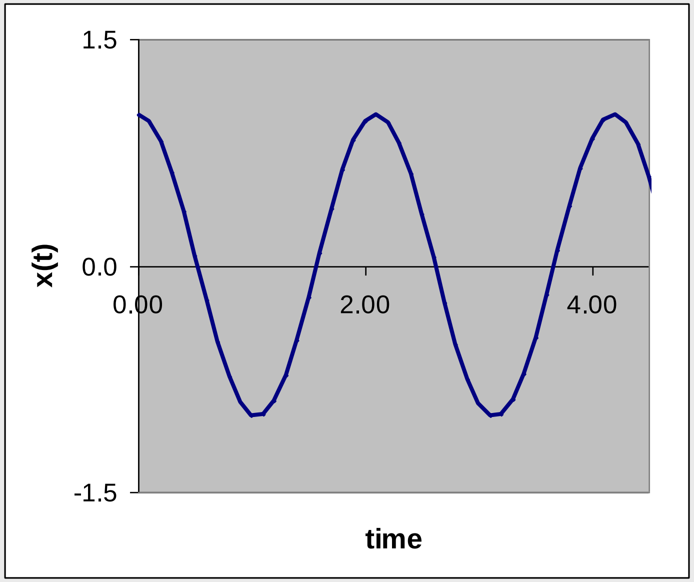
$$H_Z(j\omega) = 800\angle -0.0040$$

1^{st} and 2^{nd} Order RLC Filters

Graphing in Frequency Domain
Filter Orders
Resonant Frequencies
Basic Filters

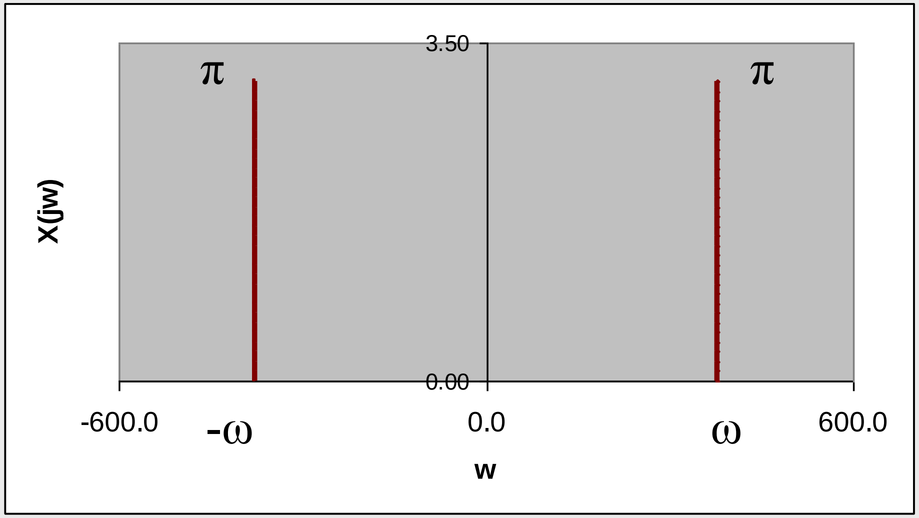
Frequency Domain

<u>Graphing in the frequency domain</u>: helpful in order to understand filters



$\cos(\omega_0 t)$

Time domain

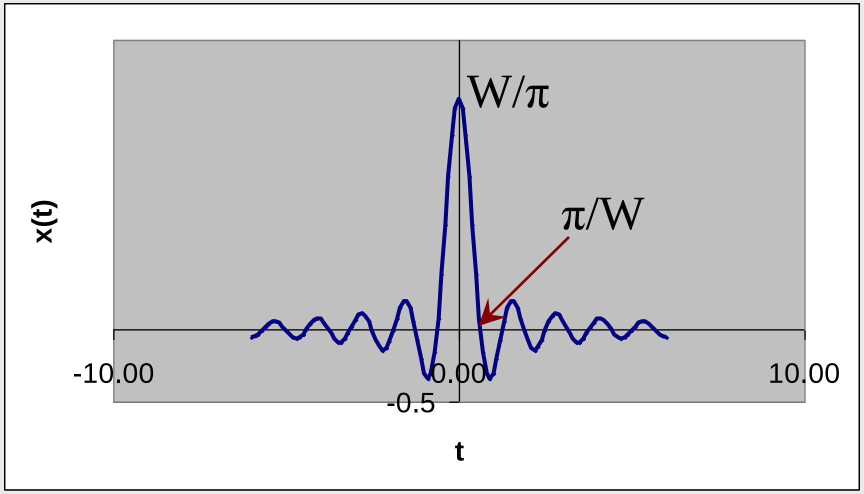


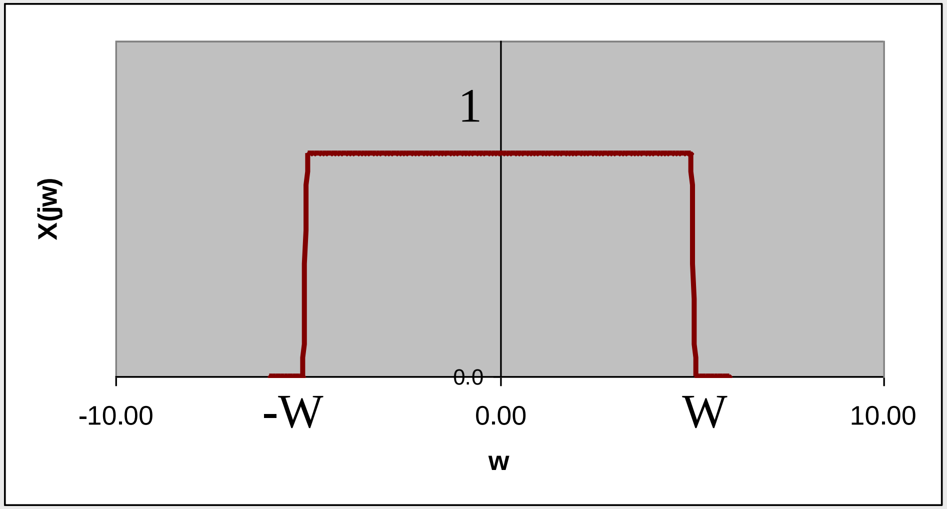
$\pi[\delta(\omega - \omega_0) + \delta(\omega - \omega_0)]$

Frequency domain

Frequency Domain

<u>Graphing in the frequency domain</u>: helpful in order to understand filters





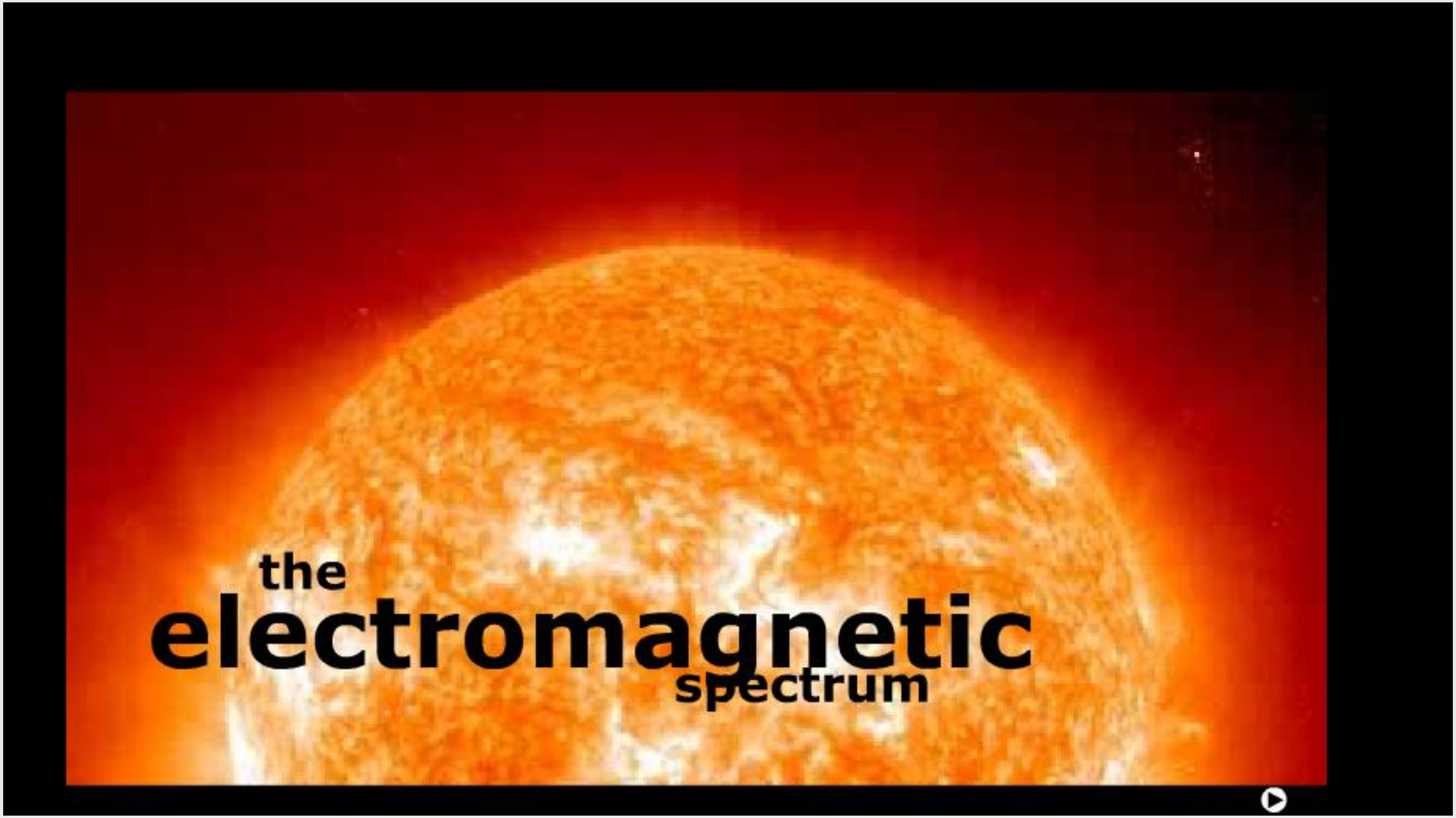
$\mathrm{sinc}(\omega_0 t)$

$$X(j\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$

Time domain

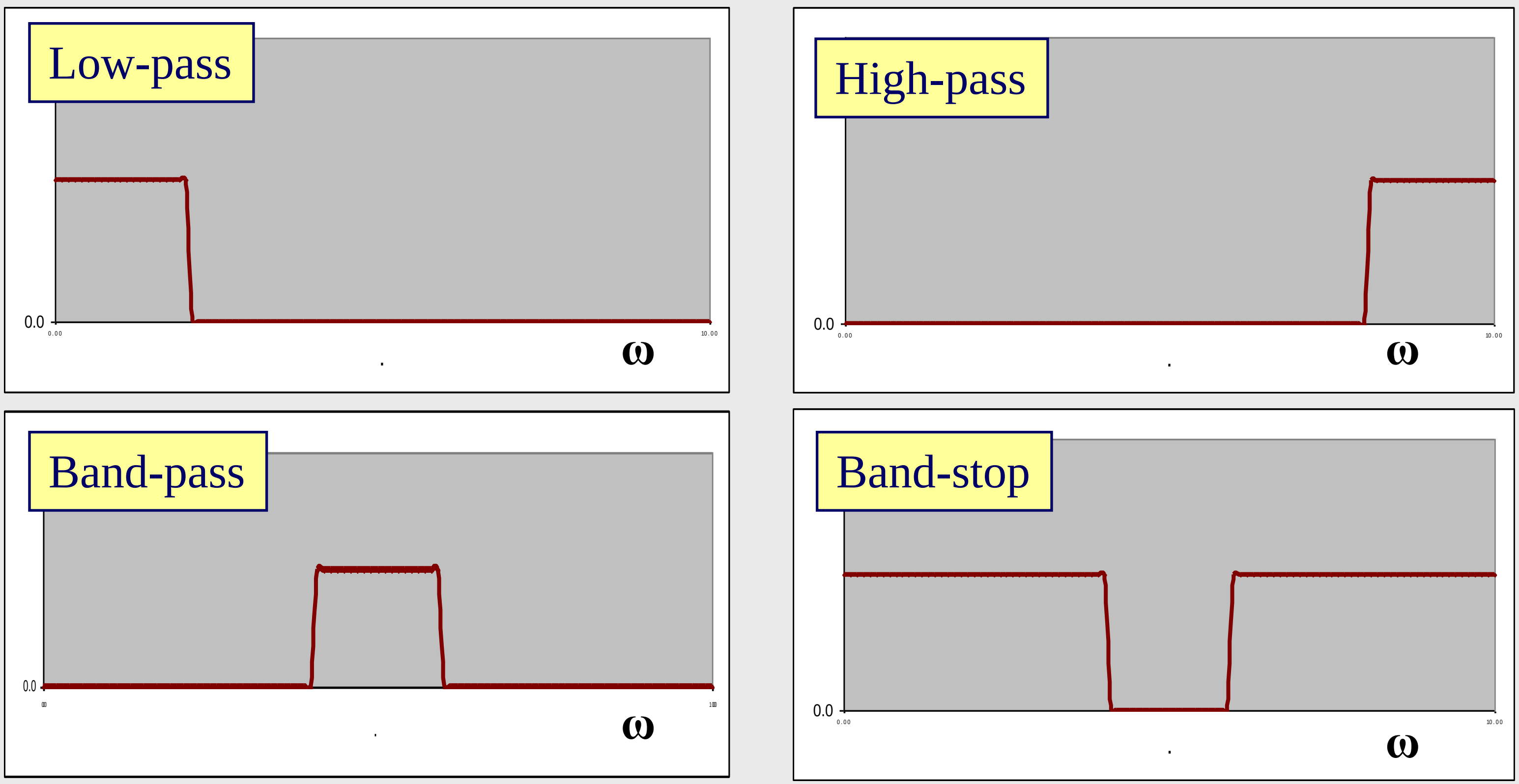
Frequency domain

Electromagnetic (Frequency) Spectrum



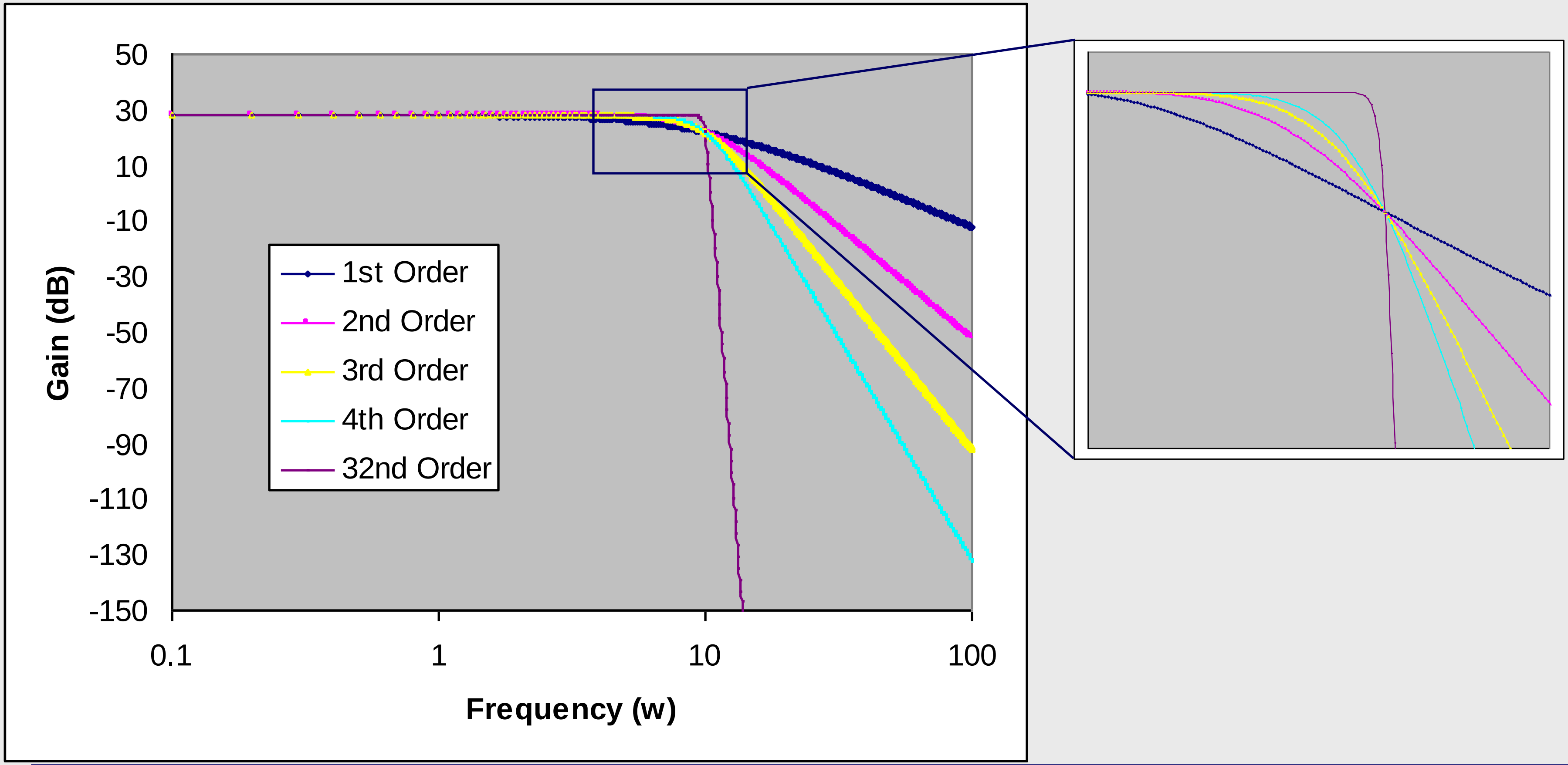
Basic Filters

<u>Electric circuit filter</u>: attenuates (reduces) or eliminates signals at unwanted frequencies



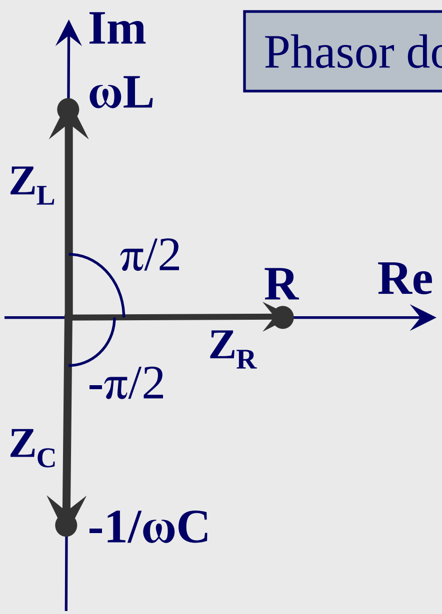
Filter Orders

Higher filter orders provide a higher quality filter

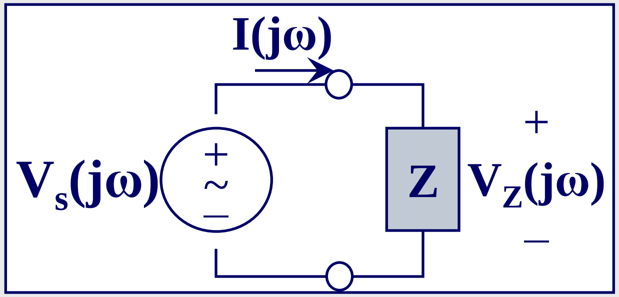


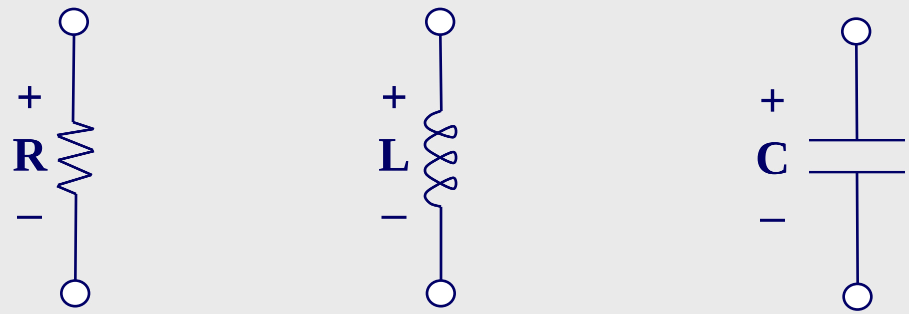
Impedance

Impedance of resistors, inductors, and capacitors



Phasor domain





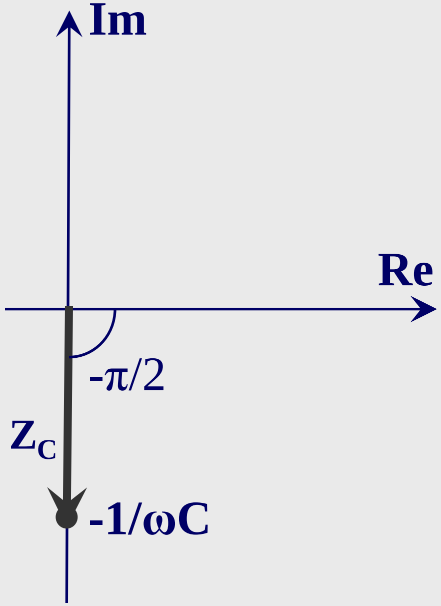
$$Z_R(j\omega) = R$$

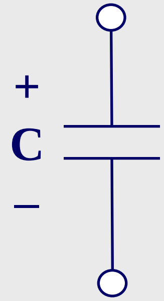
$$Z_L(j\omega) = j\omega L$$

$$Z_C(j\omega) = \frac{1}{j\omega C}$$

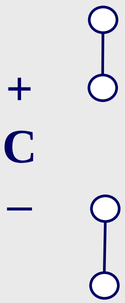
Impedance

Impedance of capacitors

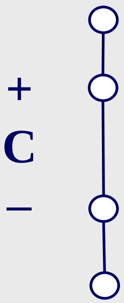




$$Z_C(j\omega) = \frac{1}{j\omega C}$$



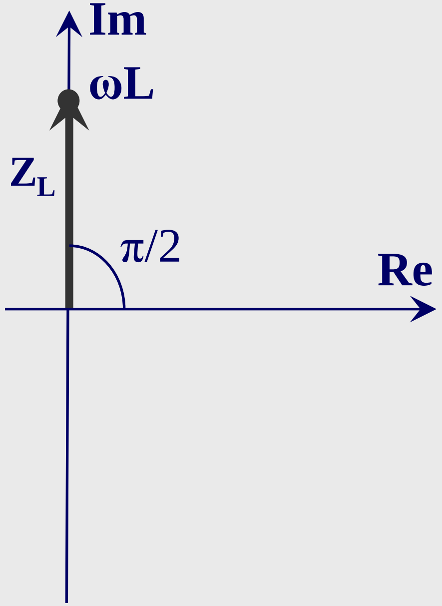
$$\text{as } \omega \to 0$$
$$Z_C(j\omega) \to \frac{1}{0}$$
$$= \infty$$

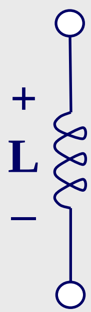


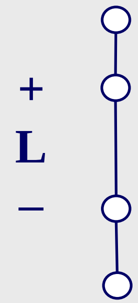
$$\text{as } \omega \to \infty$$
$$Z_C(j\omega) \to \frac{1}{\infty}$$
$$= 0$$

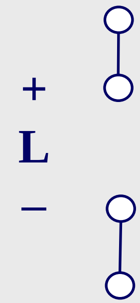
Impedance

Impedance of inductors









$$Z_L(j\omega) = j\omega L$$

as $\omega \to 0$

$$Z_C(j\omega) \to 0$$

as $\omega \to \infty$

$$Z_C(j\omega) \to \infty$$

Resonant Frequency

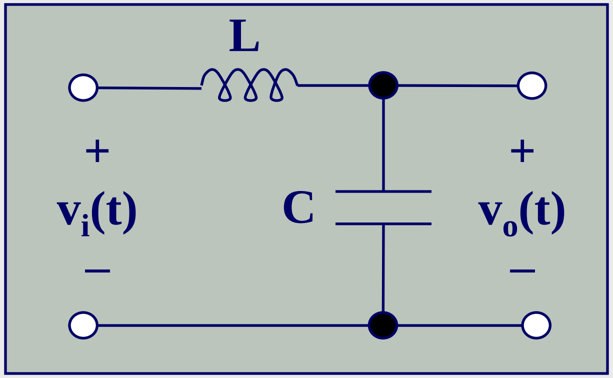
<u>Resonant Frequency (ω_n)</u>: the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

$$Z_L(j\omega) = -Z_C(j\omega)$$

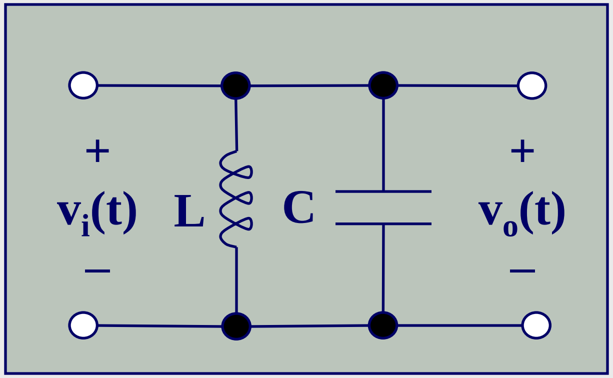
$$j\omega L = -\frac{1}{j\omega C}$$

$$\omega_n = \frac{1}{\sqrt{LC}}$$

Impedances in **series**



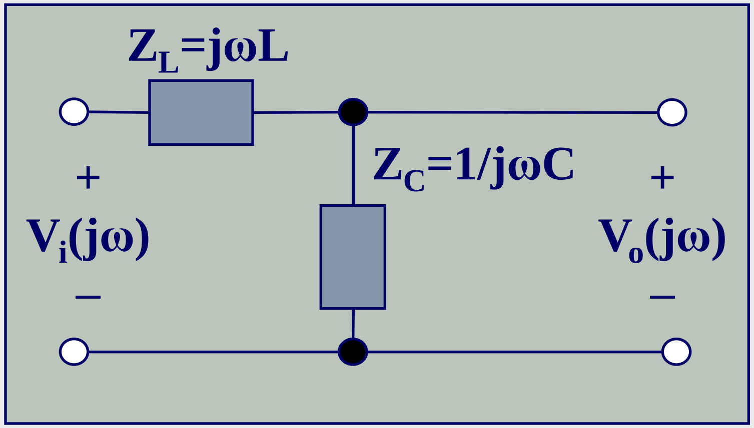
Impedances in **parallel**



Resonant Frequency

<u>Resonant Frequency (ω_n)</u>: the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

Impedances in **series**



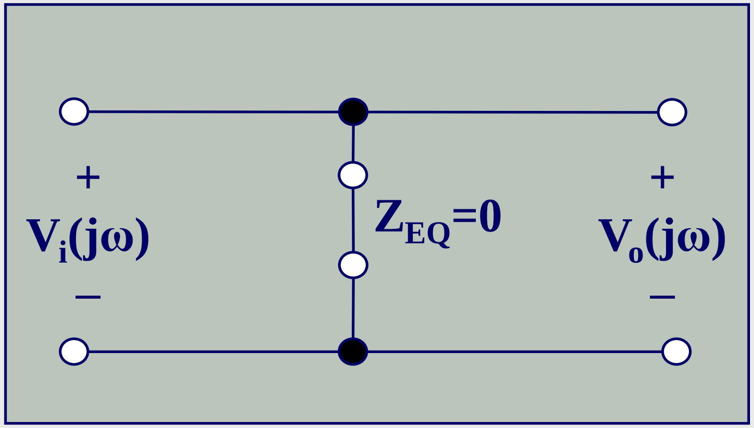
$$
\begin{aligned}
Z_L &= j\omega L \\
&= j\left(\frac{1}{\sqrt{LC}}\right)L \\
&= j\left(\frac{1}{\sqrt{LC}}\right)L \cdot \left(\frac{\sqrt{LC}}{\sqrt{LC}}\right) \\
&= jL\frac{\sqrt{LC}}{LC} \\
&= \frac{j\sqrt{LC}}{C}
\end{aligned}
$$

$$
\begin{aligned}
Z_C &= 1/j\omega C \\
&= 1/j\left(\frac{1}{\sqrt{LC}}\right)C \\
&= \frac{\sqrt{LC}}{jC} \\
&= -\frac{j\sqrt{LC}}{C}
\end{aligned}
$$

Resonant Frequency

Resonant Frequency (ω_n): the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

Impedances in **series**



$$V_o(j\omega)=0$$

$$Z_L = \frac{j\sqrt{LC}}{C}$$

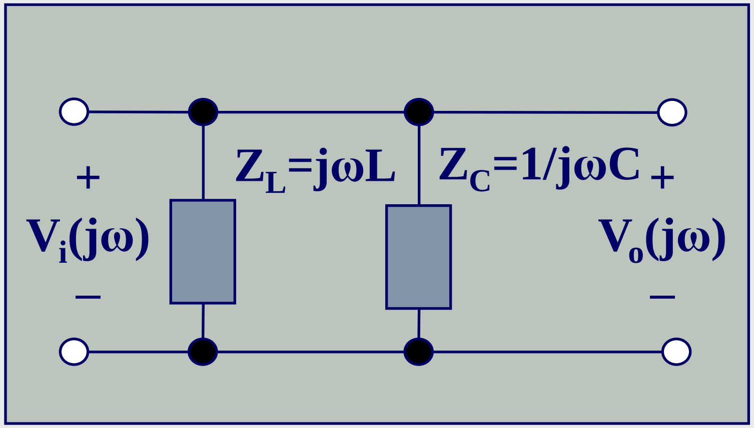
$$Z_C = -\frac{j\sqrt{LC}}{C}$$

$$Z_{EQ} = Z_L + Z_C$$
$$= 0$$

Resonant Frequency

<u>Resonant Frequency (ω_n)</u>: the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

Impedances in **parallel**



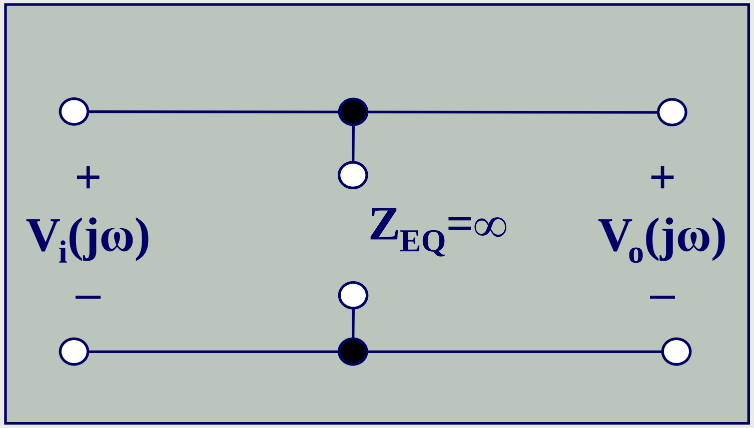
$$
\begin{aligned}
Z_L &= j\omega L \\
&= j\left(\frac{1}{\sqrt{LC}}\right)L \\
&= j\left(\frac{1}{\sqrt{LC}}\right)L \cdot \left(\frac{\sqrt{LC}}{\sqrt{LC}}\right) \\
&= jL\frac{\sqrt{LC}}{LC} \\
&= \frac{j\sqrt{LC}}{C}
\end{aligned}
$$

$$
\begin{aligned}
Z_C &= 1/\, j\omega C \\
&= 1/\, j\left(\frac{1}{\sqrt{LC}}\right)C \\
&= \frac{\sqrt{LC}}{jC} \\
&= -\frac{j\sqrt{LC}}{C}
\end{aligned}
$$

Resonant Frequency

<u>Resonant Frequency (ω_n)</u>: the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

Impedances in **parallel**



$$V_o(j\omega) = V_i(j\omega)$$

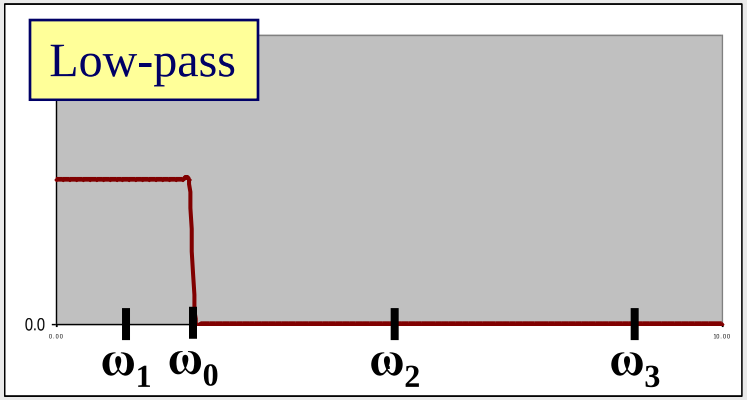
$$Z_L = \frac{j\sqrt{LC}}{C}$$

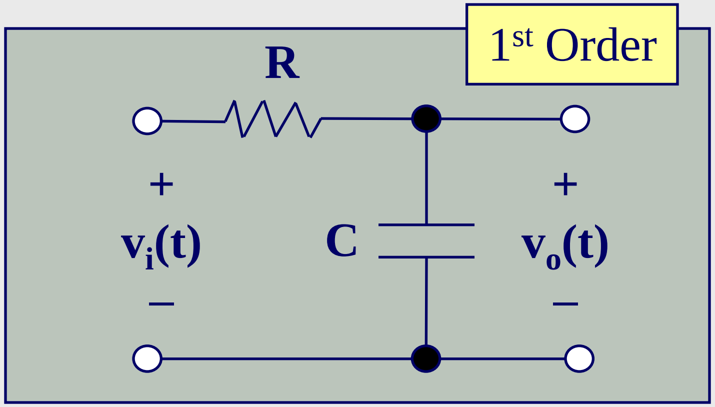
$$Z_C = -\frac{j\sqrt{LC}}{C}$$

$$\begin{aligned} Z_{EQ} &= Z_L \parallel Z_C \\ &= \frac{Z_L Z_C}{Z_L + Z_C} \\ &= \frac{L/C}{0} \\ &= \infty \end{aligned}$$

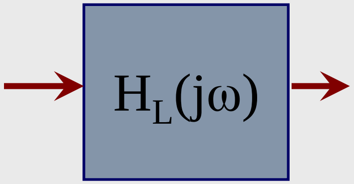
Low-Pass Filters

Low-pass Filters: only allow signals under the **cutoff frequency** (ω_0) to pass

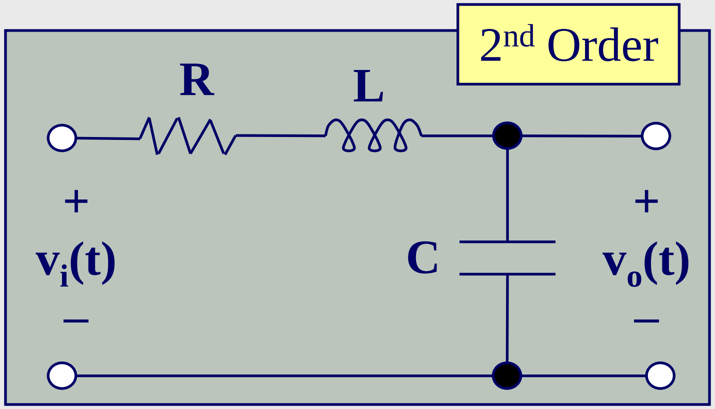




$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t)$$

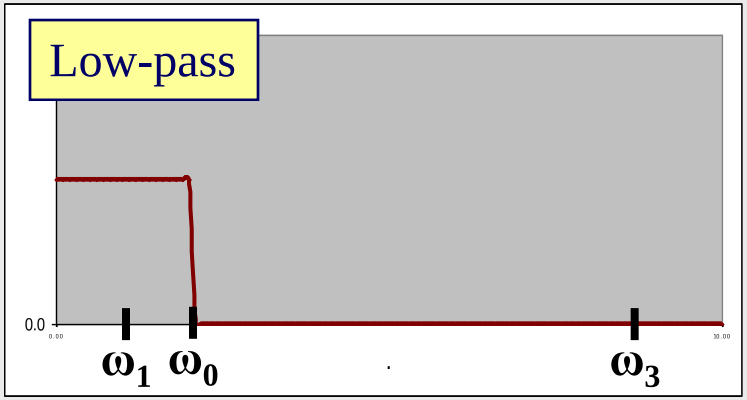


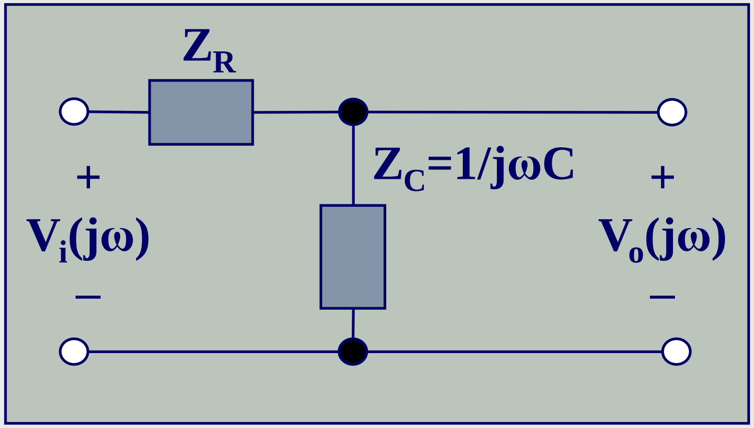
$$v_o(t) = \cos(\omega_1 t)$$



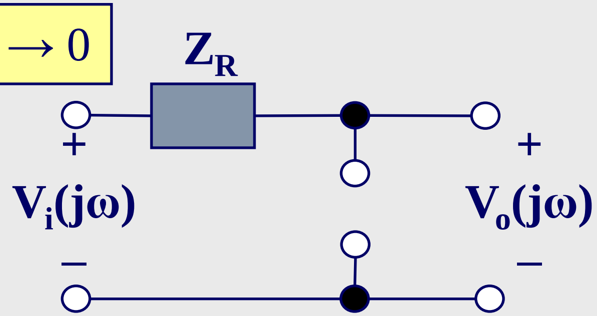
Low-Pass Filters

1st Order Low-pass Filters:



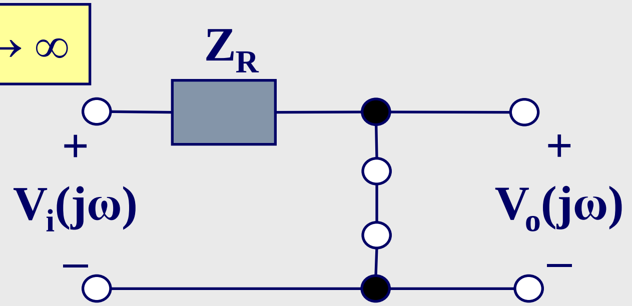


ω_1: $\omega \rightarrow 0$



$$v_o(t) = v_i(t)$$

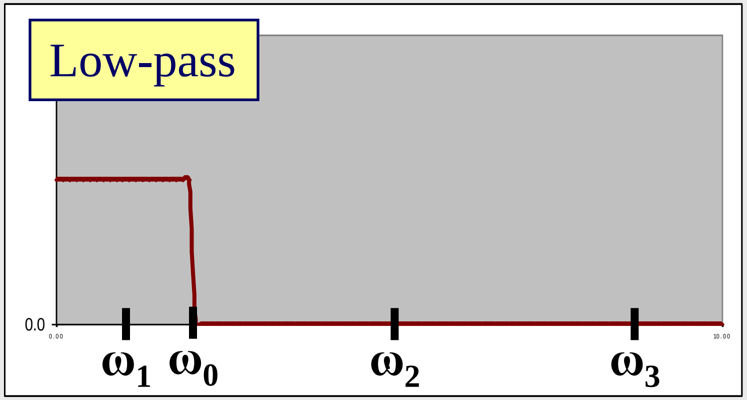
ω_3: $\omega \rightarrow \infty$

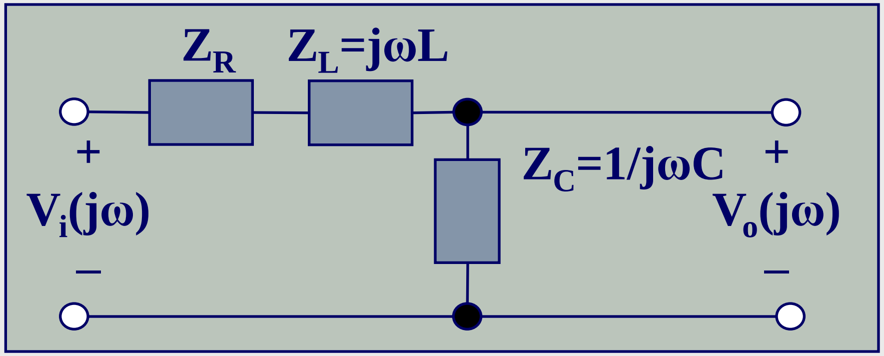


$$v_o(t) = 0$$

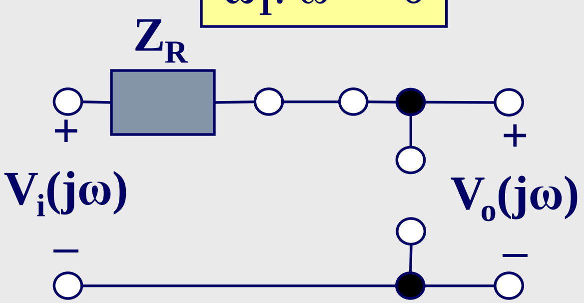
Low-Pass Filters

2nd Order Low-pass Filters:



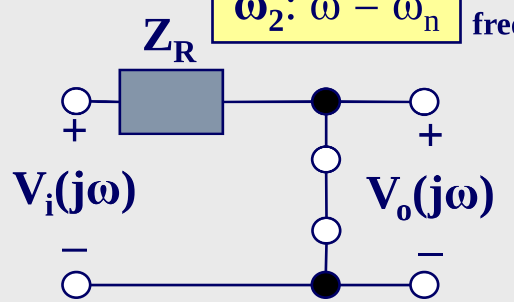


ω_1: $\omega \to 0$



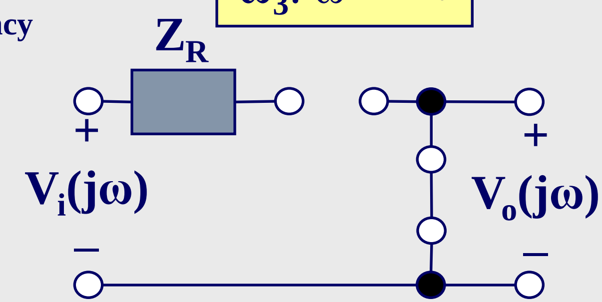
$$v_o(t) = v_i(t)$$

ω_2: $\omega = \omega_n$ Resonant frequency



$$v_o(t) = 0$$

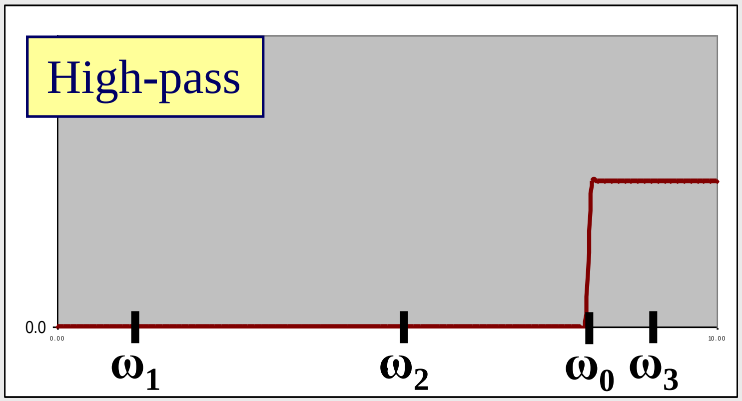
ω_3: $\omega \to \infty$

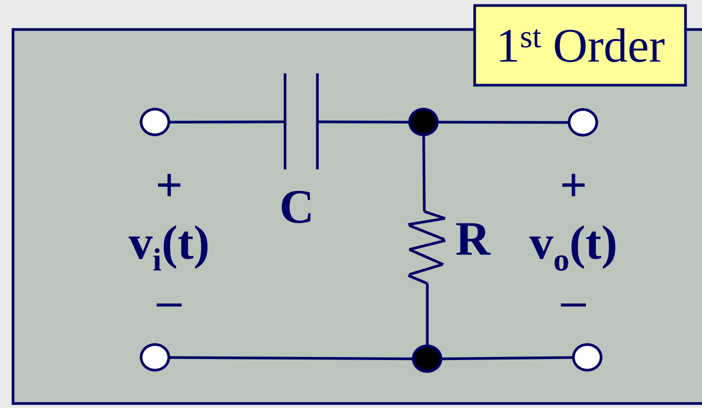


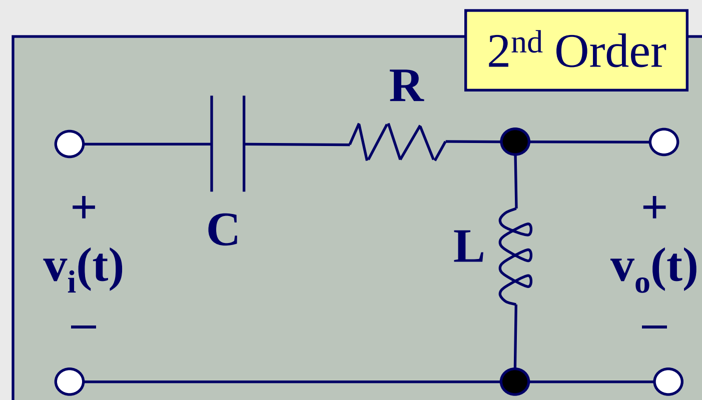
$$v_o(t) = 0$$

High-Pass Filters

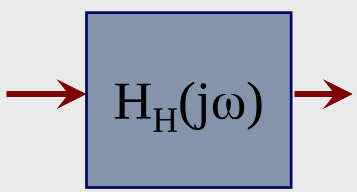
<u>High-pass Filters</u>: only allow signals above the **cutoff frequency** **(ω_0)** to pass







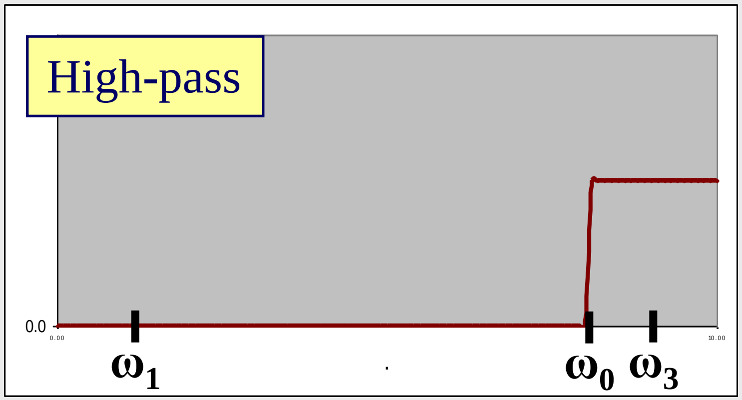
$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t)$$

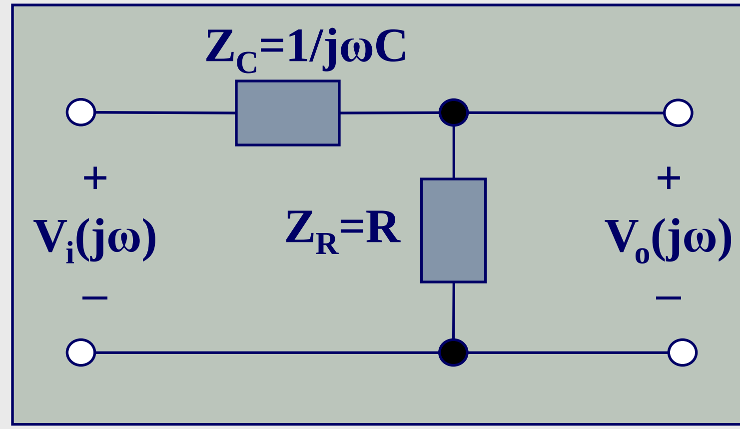


$$v_o(t) = \cos(\omega_3 t)$$

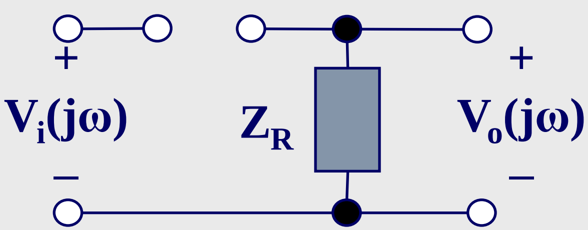
High-Pass Filters

1st Order High-pass Filters:



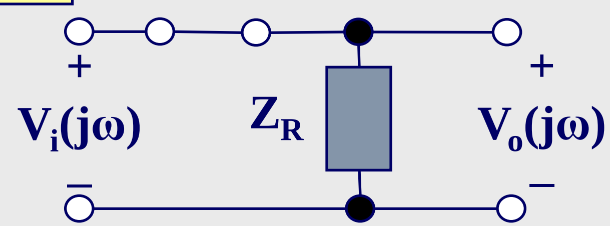


ω_1: $\omega \rightarrow 0$



$v_o(t) = 0$

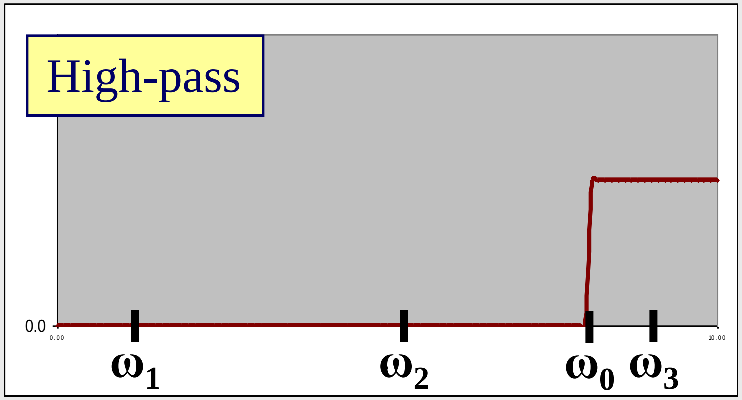
ω_3: $\omega \rightarrow \infty$

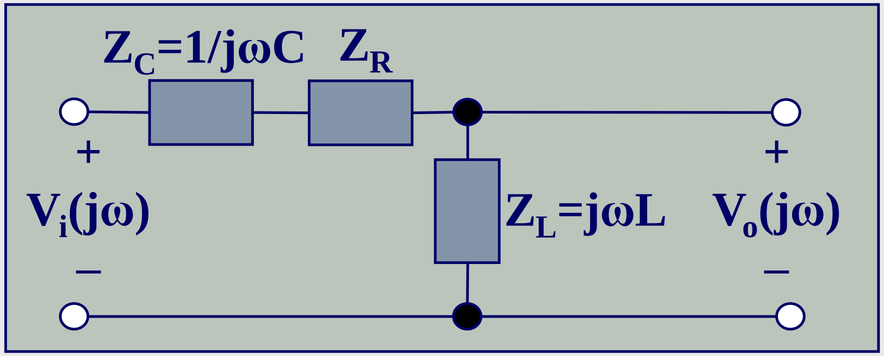


$v_o(t) = v_i(t)$

High-Pass Filters

2nd Order High-pass Filters:



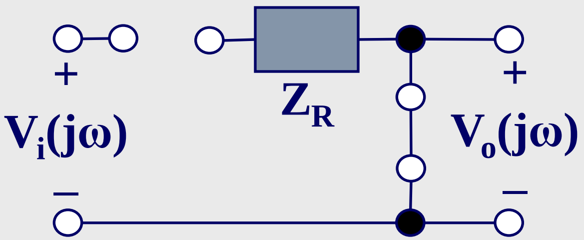


ω_1: $\omega \rightarrow 0$

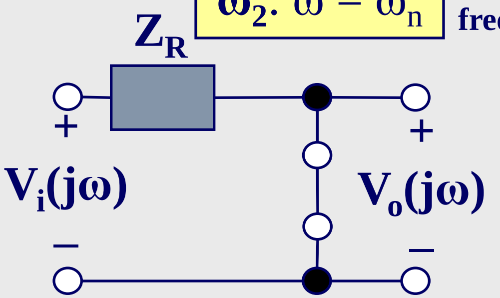
ω_2: $\omega = \omega_n$

Resonant frequency

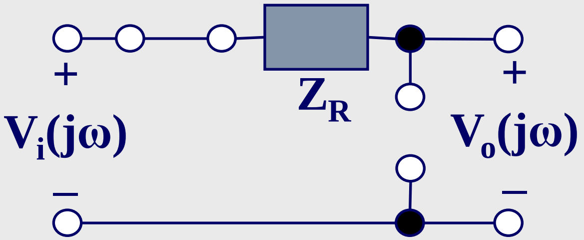
ω_3: $\omega \rightarrow \infty$



$v_o(t) = 0$



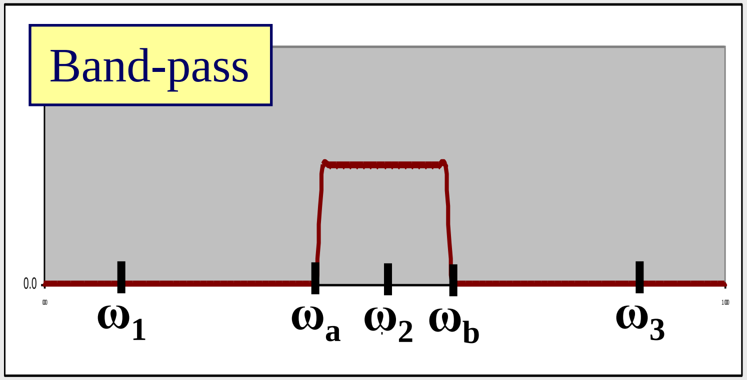
$v_o(t) = 0$

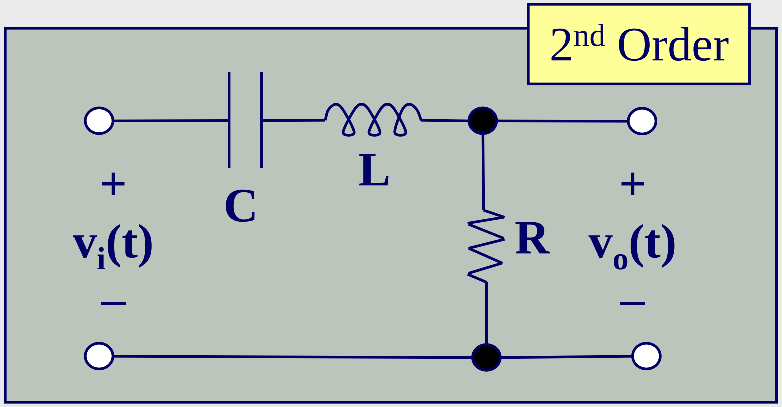


$v_o(t) = v_i(t)$

Band-Pass Filters

Band-pass Filters: only allow signals between the **passband** (ω_a to ω_b) to pass

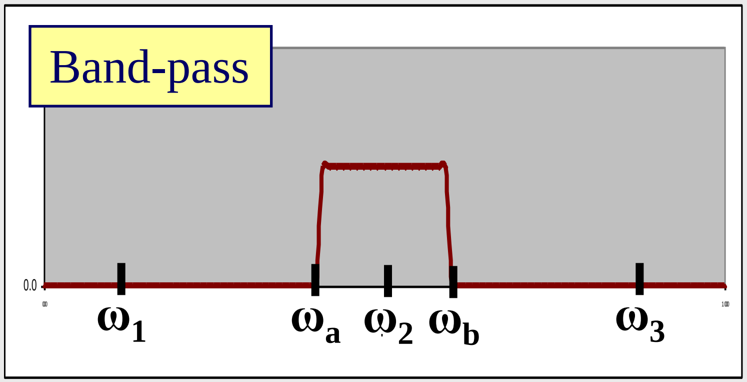


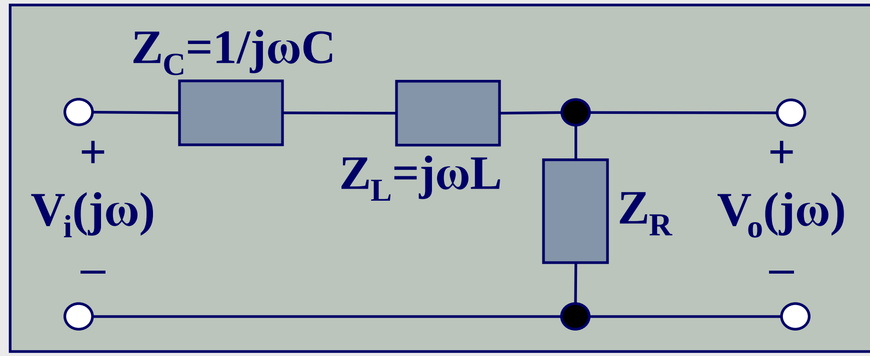


$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) \rightarrow \boxed{H_B(j\omega)} \rightarrow v_o(t) = \cos(\omega_2 t)$$

Band-Pass Filters

2nd Order Band-pass Filters:

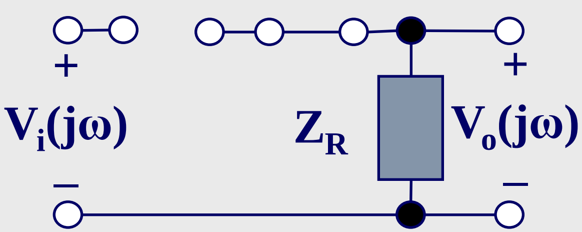


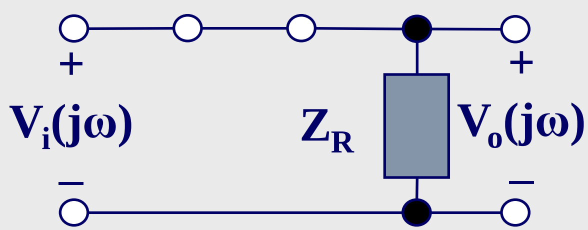


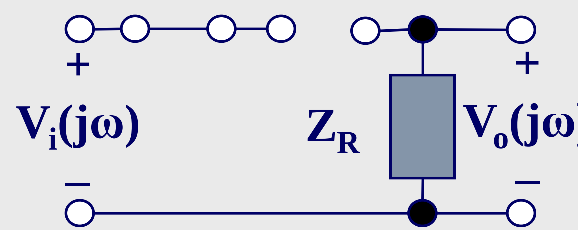
ω_1: $\omega \rightarrow 0$

ω_2: $\omega = \omega_n$ **Resonant frequency**

ω_3: $\omega \rightarrow \infty$







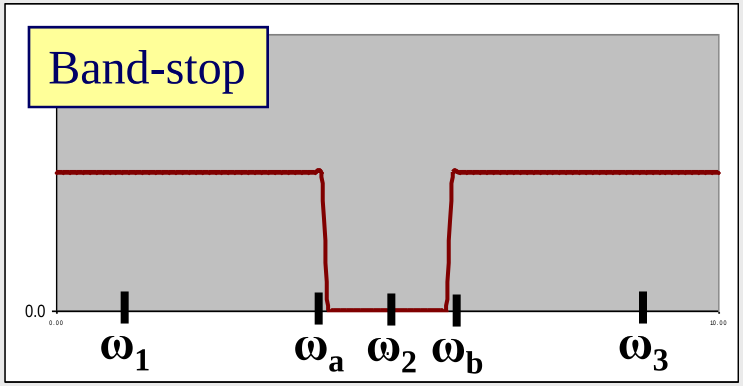
$$v_o(t) = 0$$

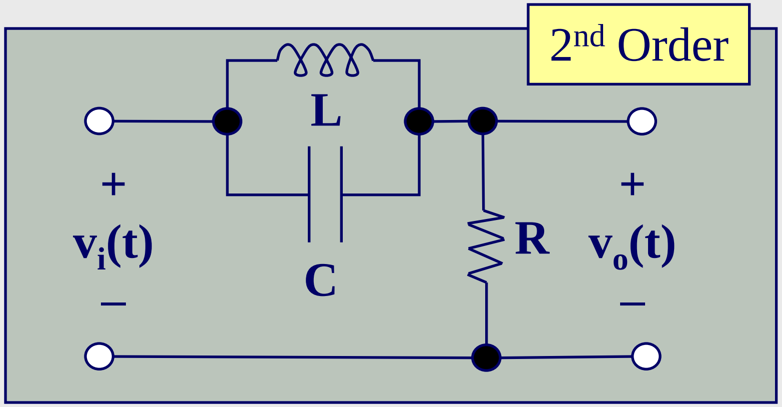
$$v_o(t) = v_i(t)$$

$$v_o(t) = 0$$

Band-Stop Filters

<u>Band-stop Filters</u>: allow signals except those between the **passband** (ω_a to ω_b) to pass





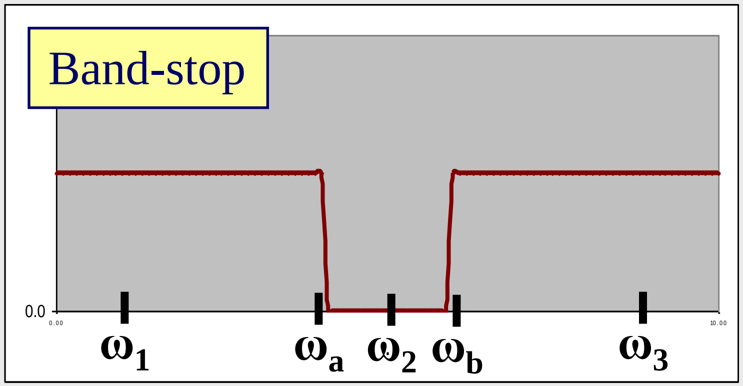
$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t)$$

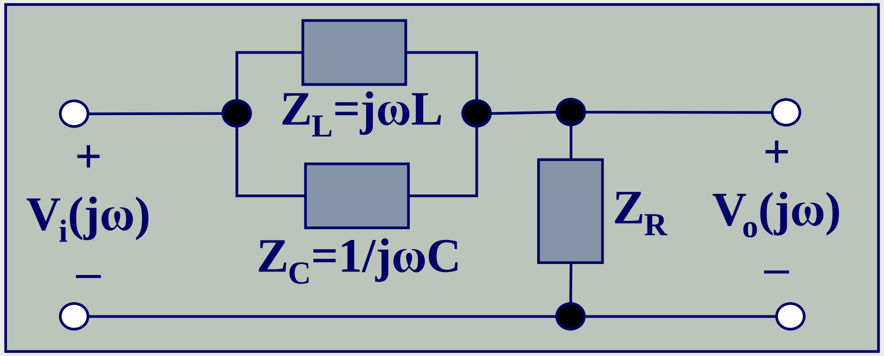
→ $H_N(j\omega)$ →

$$v_o(t) = \cos(\omega_1 t) + \cos(\omega_3 t)$$

Band-Stop Filters

2nd Order Band-stop Filters:

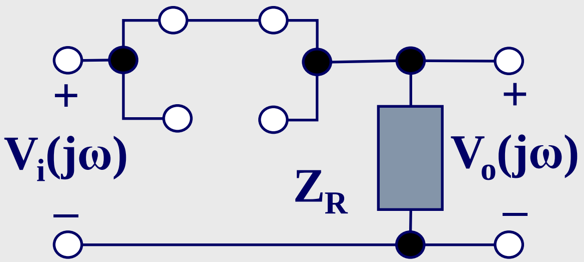


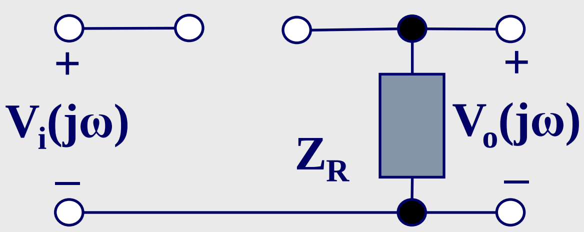


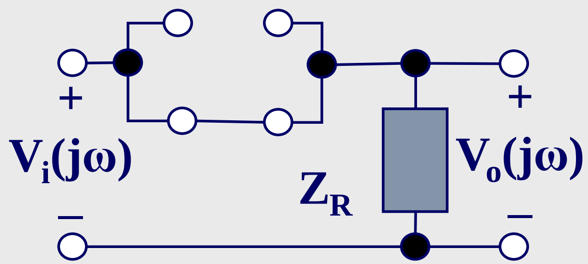
ω_1: $\omega \to 0$

ω_2: $\omega = \omega_n$ **Resonant frequency**

ω_3: $\omega \to \infty$







$v_o(t) = v_i(t)$

$v_o(t) = 0$

$v_o(t) = v_i(t)$