Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
29 Oct	Wed	17	Operational Amplifiers	8.1 – 8.2			
30 Oct	Thu						
31 Oct	Fri		Recitation		HW 7		
1 Nov	Sat						
2 Nov	Sun						
3 Nov	Mon	18	Operational Amplifiers	8.3 – 8.4		LAB 6	
4 Nov	Tue						
5 Nov	Wed	19	Binary Numbers	13.1 – 13.2			

Increase

<u>Helaman 12:2</u>

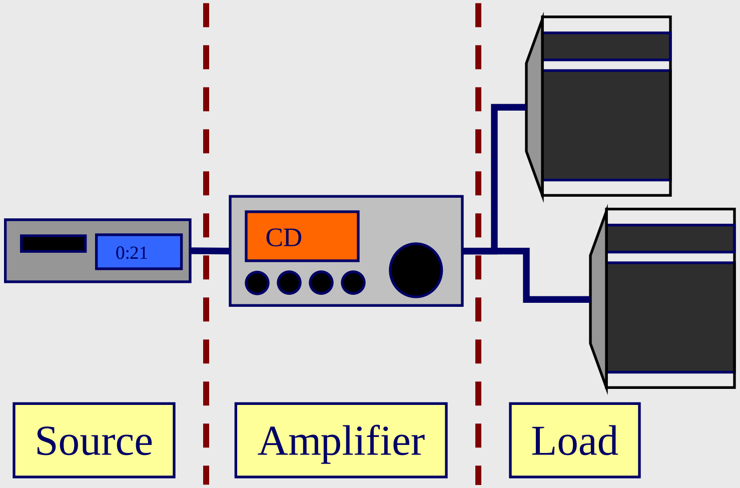
2 Yea, and we may see at the very time when he doth prosper his people, yea, in the **increase** of their fields, their flocks and their herds, and in gold, and in silver, and in all manner of precious things of every kind and art; sparing their lives, and delivering them out of the hands of their enemies; softening the hearts of their enemies that they should not declare wars against them; yea, and in fine, doing all things for the welfare and happiness of his people; yea, then is the time that they do harden their hearts, and do forget the Lord their God, and do trample under their feet the Holy One—yea, and this because of their ease, and their exceedingly great prosperity.

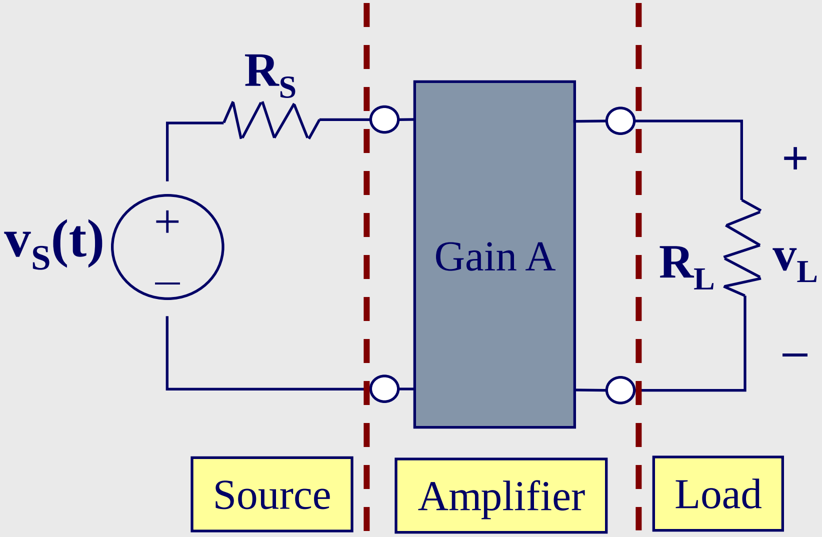
Lecture 17 – Operational Amplifiers

Ideal Amplifiers

<u>Amplifier</u>: a device for **increasing** the power of a signal.

- Power increase is called **gain** (**A**)



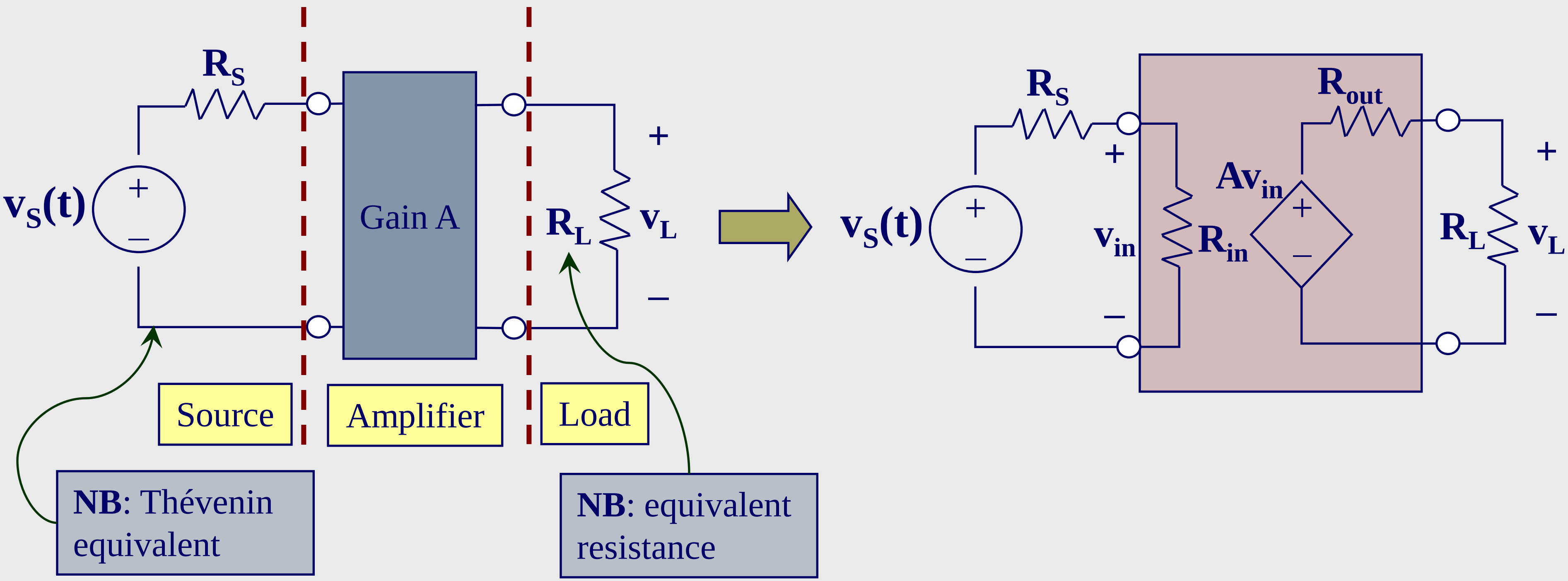


$$v_L(t) = Av_S(t)$$

Ideal Amplifiers

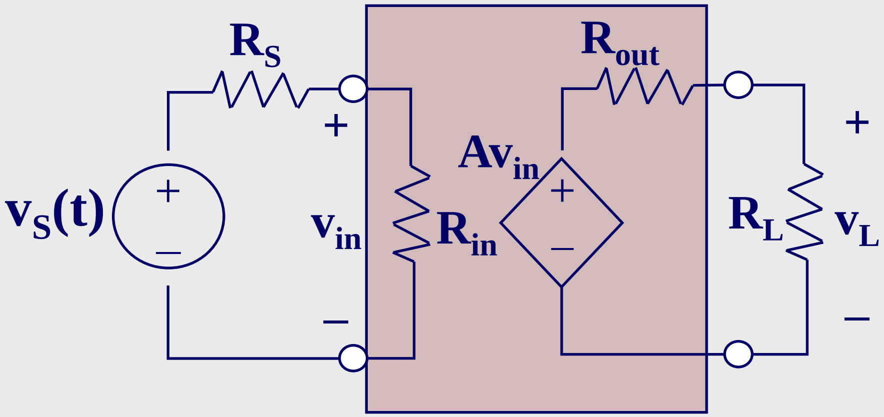
Simplified Amplifier Model:

- The source “sees” and equivalent **load** (**R_{in}**)
- The load “sees” and equivalent **source** (**Av_{in}**)



Ideal Amplifiers

The **gain** is **dependent on the source and load** (i.e. is different for different sources and loads)



$$v_{in} = v_S \frac{R_{in}}{R_S + R_{in}}$$

$$v_L = A v_{in} \frac{R_L}{R_{out} + R_L}$$

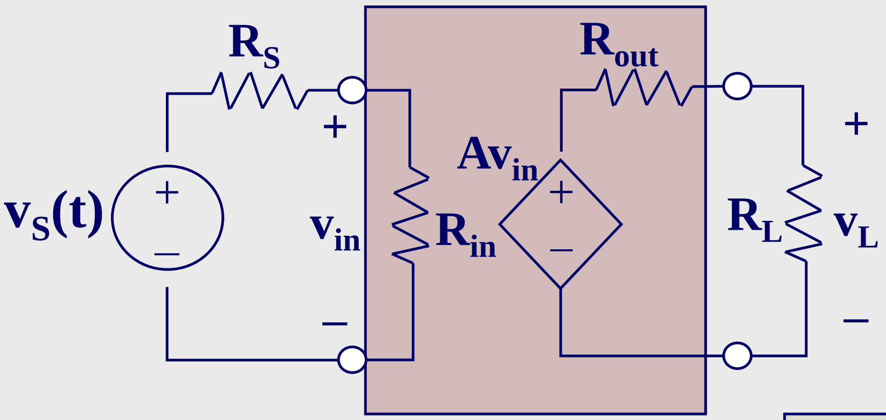
$$v_L = \left(A \frac{R_{in}}{R_S + R_{in}} \frac{R_L}{R_{out} + R_L} \right) v_S$$

NB: expression for $\mathbf{v_L}$ depends on the source ($\mathbf{R_S}$) and the load ($\mathbf{R_L}$)

Ideal Amplifiers

The gain can be made to be **almost** independent of the source (**R_S**) and the load (**R_L**)

- Let **R_{in}** $\rightarrow \infty$
- Let **R_{out}** $\rightarrow 0$



if $R_{in} \rightarrow \infty$:

$$v_{in} = v_S \frac{R_{in}}{R_S + R_{in}} \approx v_S$$

if $R_{out} \rightarrow 0$

$$v_L = Av_{in} \frac{R_L}{R_{out} + R_L} \approx Av_{in}$$

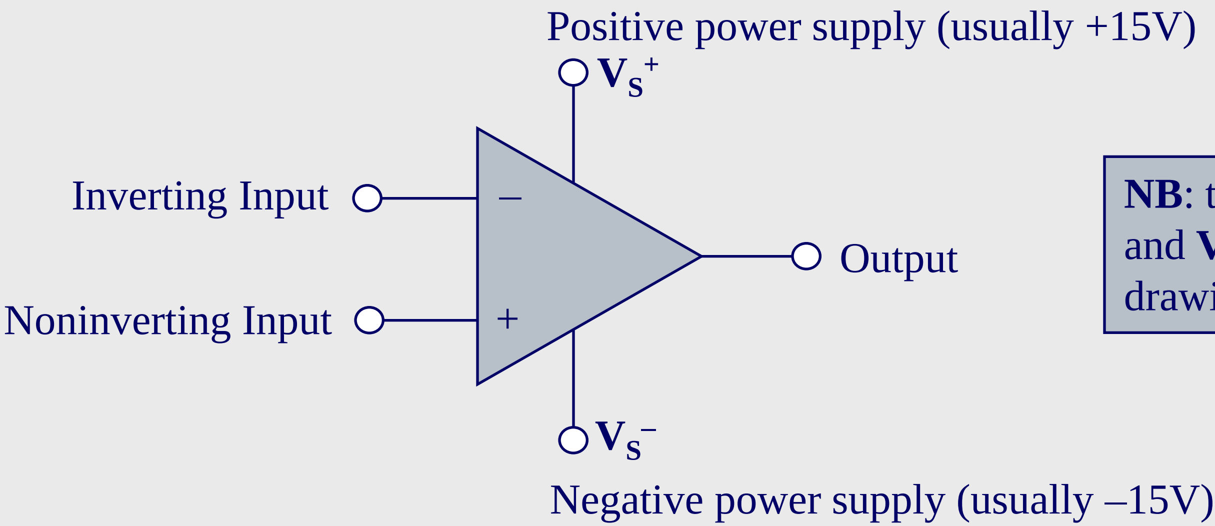
$$v_L \approx Av_S$$

NB: it is desirable for an amplifier to have:
- a very large input impedance
- a very small output impedance

Op-Amps

<u>Operational Amplifier</u>: originally designed (late 1960's) to perform mathematical operations (analog computer)

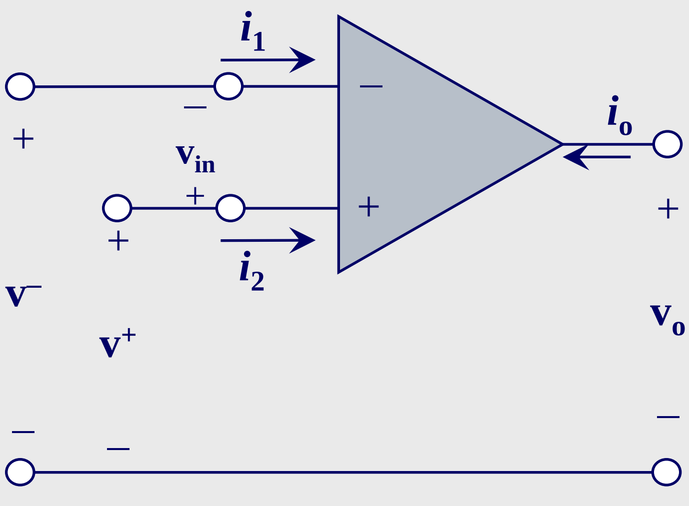
- Addition
- Subtraction
- Integration
- differentiation

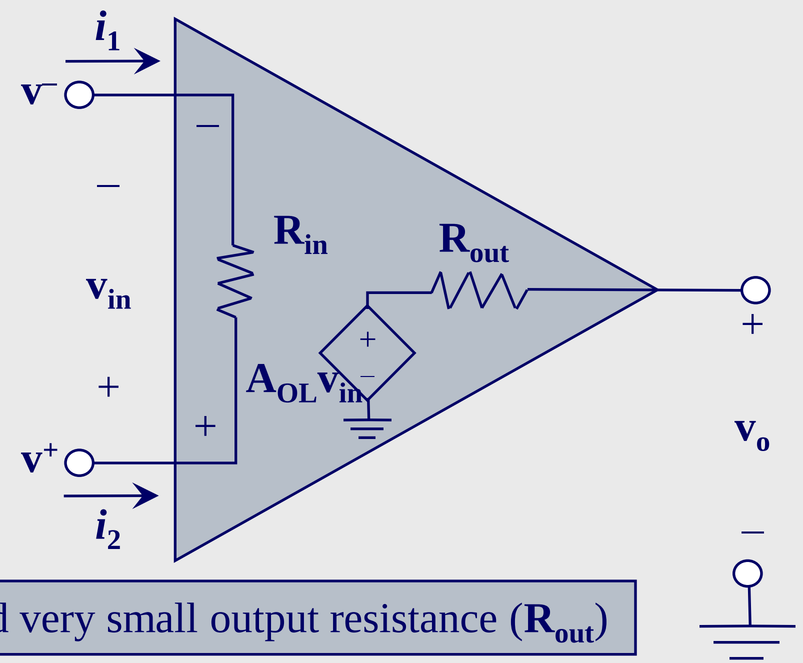


NB: the power supplies (**V_S^+** and **V_S^-**) are often omitted in drawings – they are implicit

Op-Amps – Open-Loop Mode

Open-Loop Model: an ideal op-amp acts like a **difference amplifier** (a device that amplifies the difference between two input voltages)





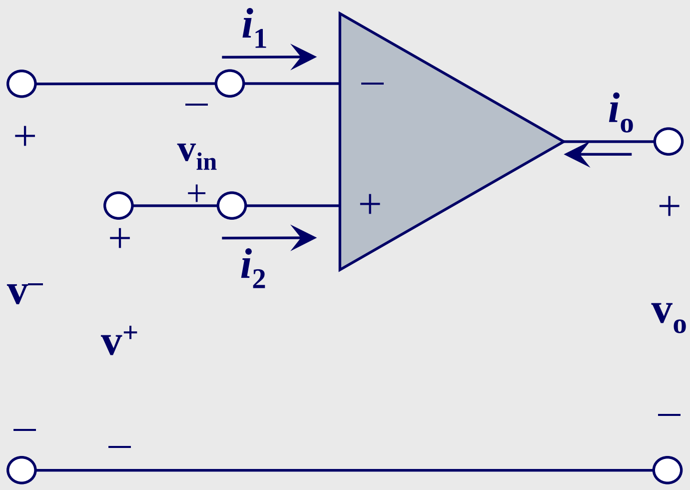
NB: op-amps have near-infinite input resistance (**R_{in}**) and very small output resistance (**R_{out}**)

$$v_o \approx A_{OL}v_{in} = A_{OL}(v^+ - v^-)$$

A_{OL} – open-loop voltage gain

Op-Amps – Open-Loop Mode

<u>Open-Loop Model</u>: an ideal op-amp acts like a **difference amplifier** (a device that amplifies the difference between two input voltages)

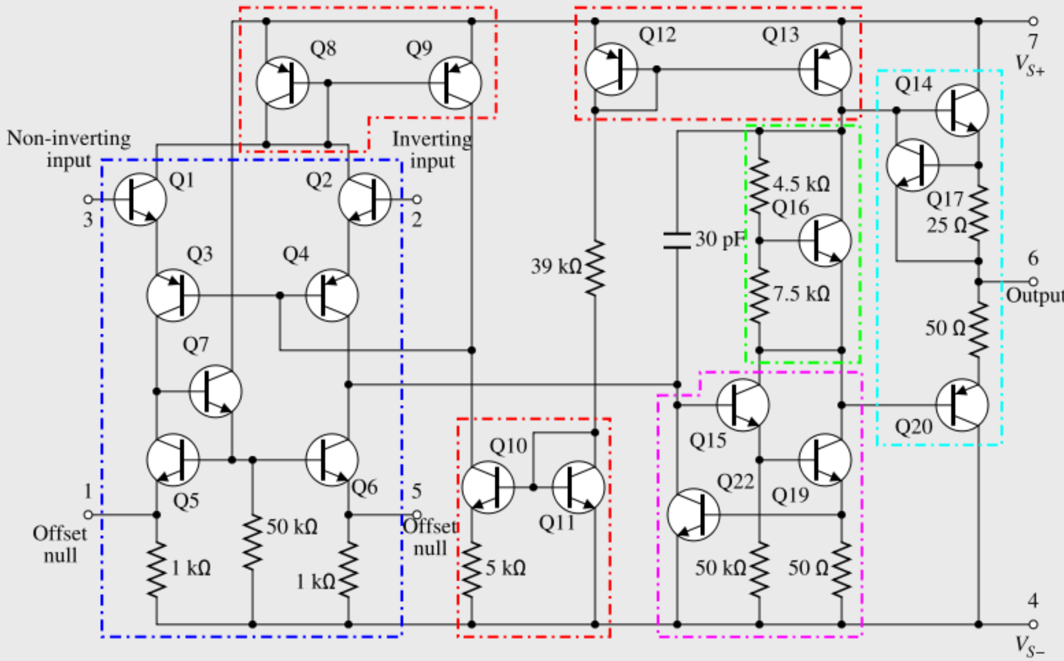


Ideally $i_1 = i_2 = 0$
(since $\mathbf{R_{in}} \rightarrow \infty$)

$$i_1 = i_2 = 0$$

Op-Amps – Open-Loop Mode

<u>Open-Loop Model</u>: an ideal op-amp acts like a **difference amplifier** (a device that amplifies the difference between two input voltages)

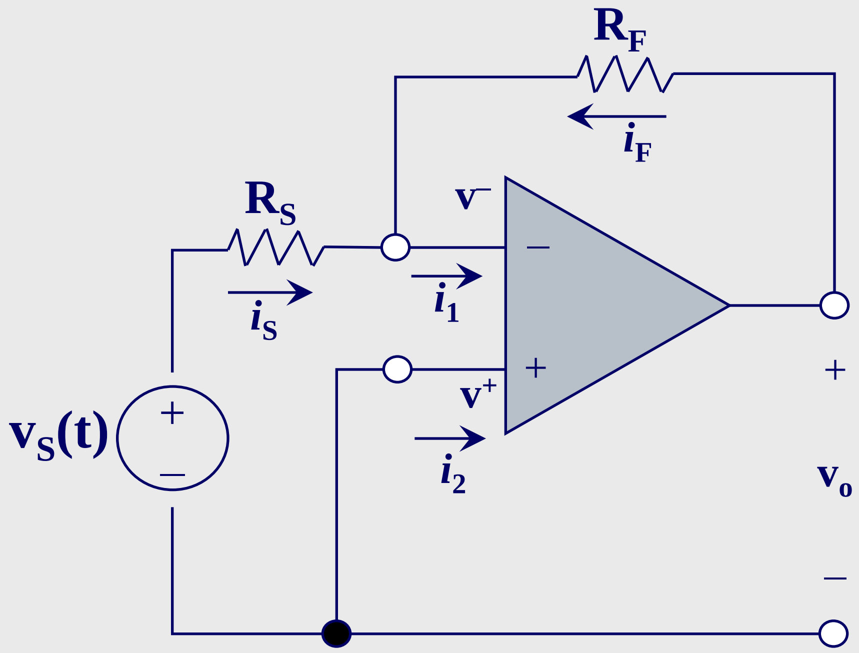


$$i_1 = i_2 = 0$$

What? No input current??
In reality there is a **small** current

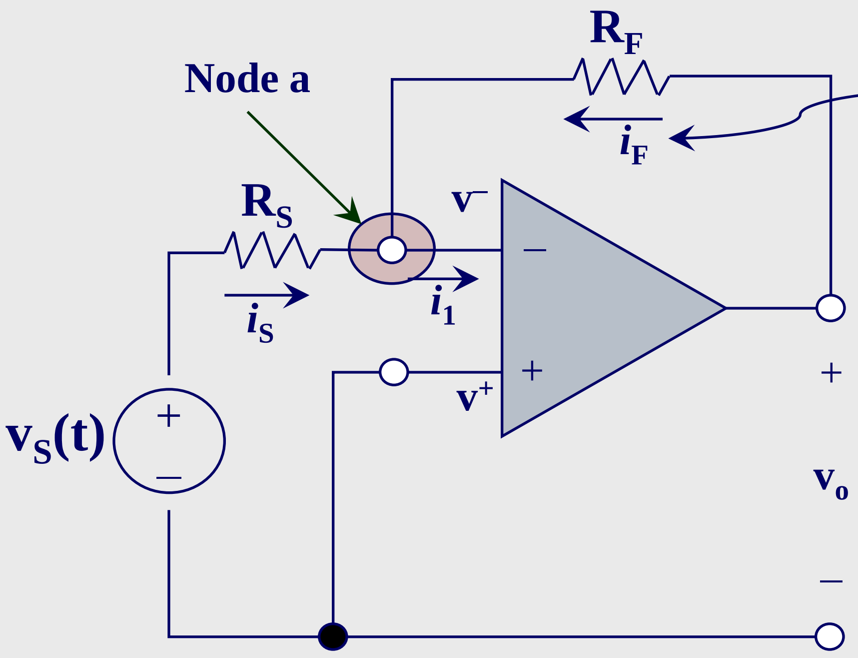
Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal



Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal



<u>Feedback current</u>: current from the output is fed back into the input of the op-amp

$$i_S = \frac{v_S - v^-}{R_S}$$

$$i_F = \frac{v_o - v^-}{R_F}$$

$$i_1 = 0$$

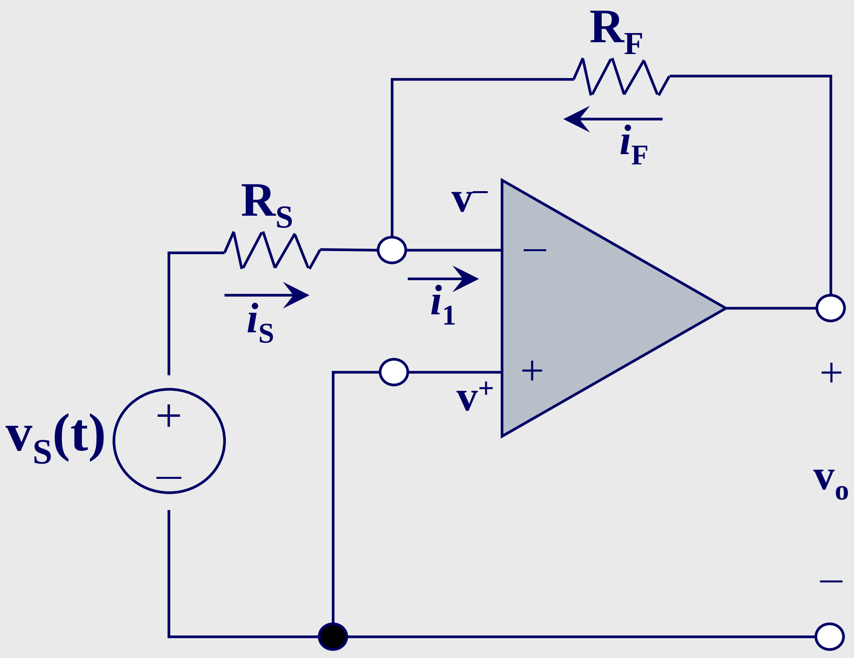
KCL at Node a :

$$i_S + i_F - i_1 = 0$$

$$i_S = -i_F$$

Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal

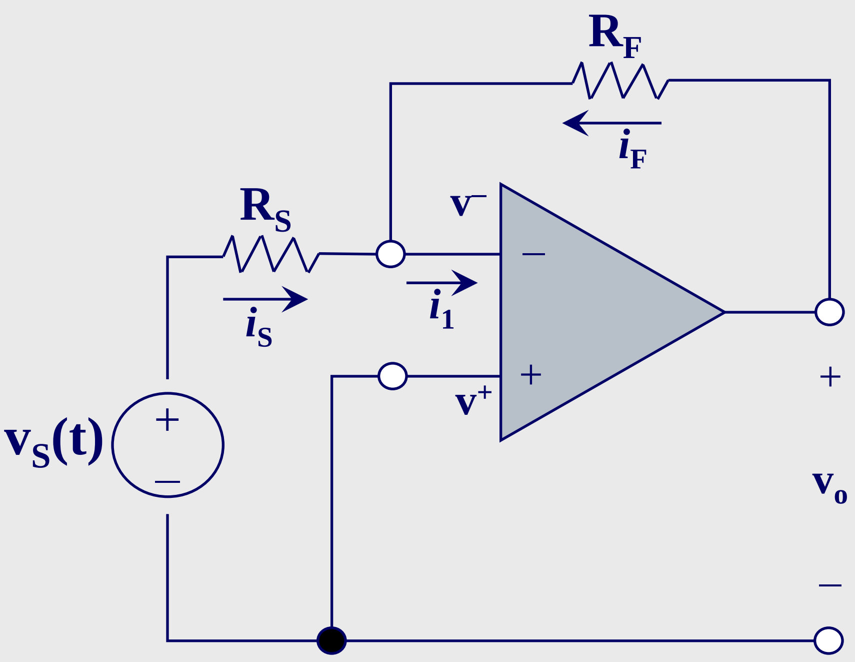


From Open-Loop Model:

$$v_o = A_{OL}(v^+ - v^-)$$
$$= A_{OL}(0 - v^-)$$
$$= -A_{OL}v^-$$
$$v^- = -\frac{v_o}{A_{OL}}$$

Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal



$$i_S = -i_F$$

$$\frac{v_S - v^-}{R_S} = -\frac{v_o - v^-}{R_F}$$

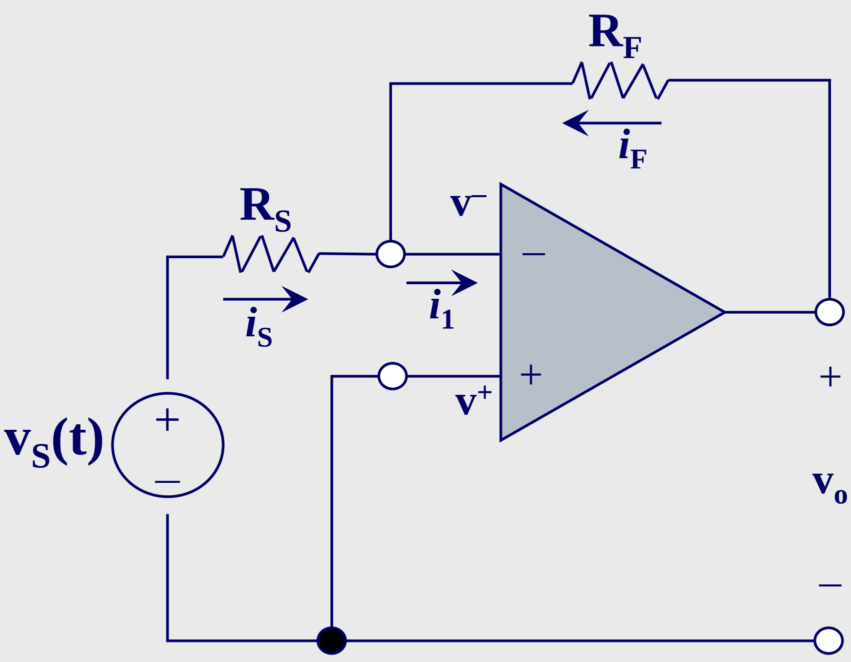
$$\frac{v_S}{R_S} - \left[\frac{(-v_o / A_{OL})}{R_S}\right] = -\frac{v_o}{R_F} + \left[\frac{(-v_o / A_{OL})}{R_F}\right]$$

$$\frac{v_S}{R_S} = -\frac{v_S}{R_S} - \frac{v_o}{A_{OL} R_F} - \frac{v_o}{A_{OL} R_S}$$

$$v_S = -v_o \left(\frac{1}{R_F / R_S} + \frac{1}{A_{OL} R_F / R_S} + \frac{1}{A_{OL}} \right)$$

Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal



$$v_S = -v_o\left(\frac{1}{R_F/R_S} + \frac{1}{A_{OL}R_F/R_S} + \frac{1}{A_{OL}}\right)$$

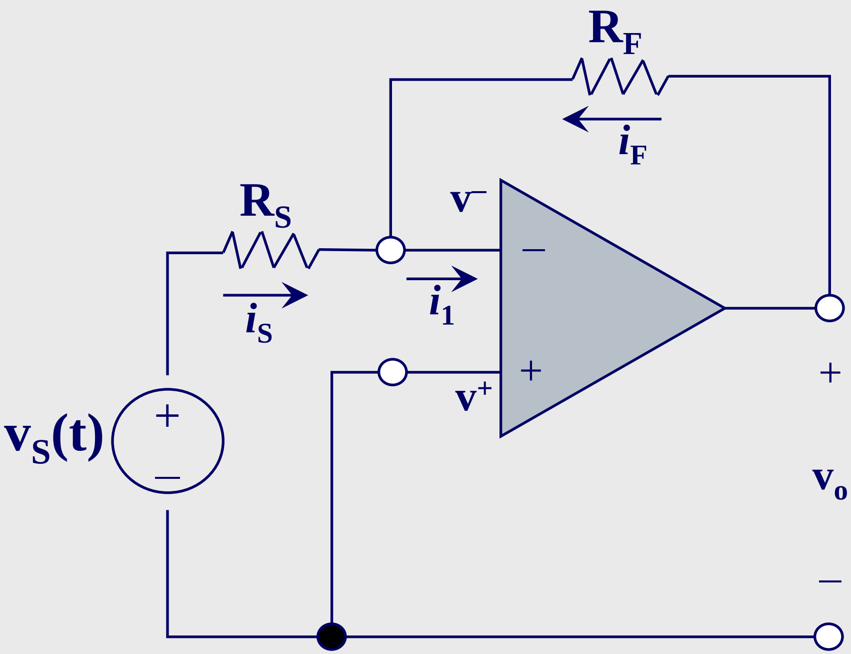
NB: if $\mathbf{A_{OL}}$ is very large these terms → 0

Closed-Loop Gain:

$$A_{CL} = \frac{v_o}{v_S} \approx -\frac{R_F}{R_S}$$

Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal



From Open - Loop Model :

$$v^- = -\frac{v_{out}}{A_{OL}}$$

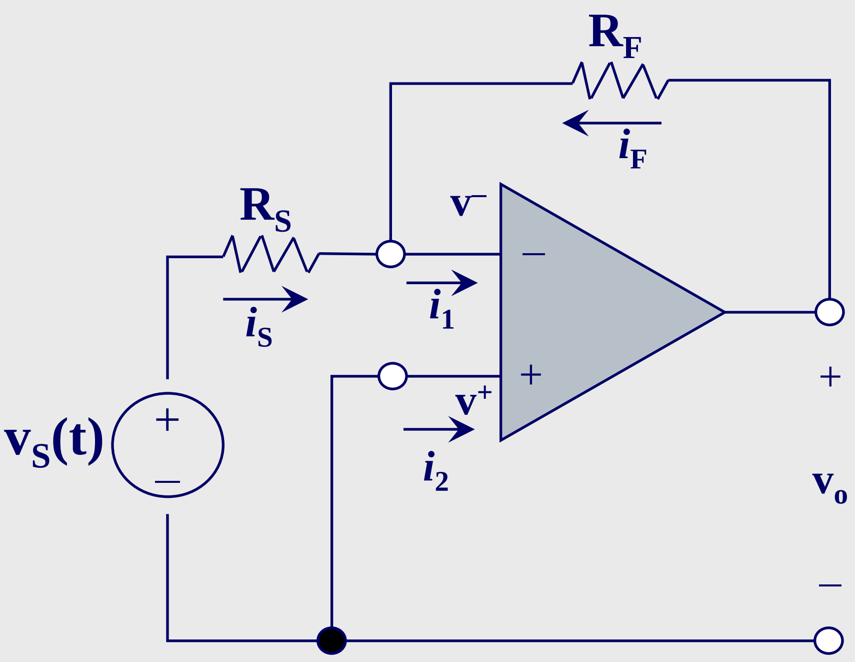
NB: as $\mathbf{A_{OL}} \rightarrow \infty$ $\mathbf{v^-} \rightarrow 0$

As $A_{OL} \rightarrow \infty$

$$v^- \approx v^+$$

Op-Amps – Closed-Loop Mode

<u>The Inverting Amplifier</u>: the signal to be amplified is connected to the inverting terminal



NB: two important results for an ideal op-amp with negative feedback

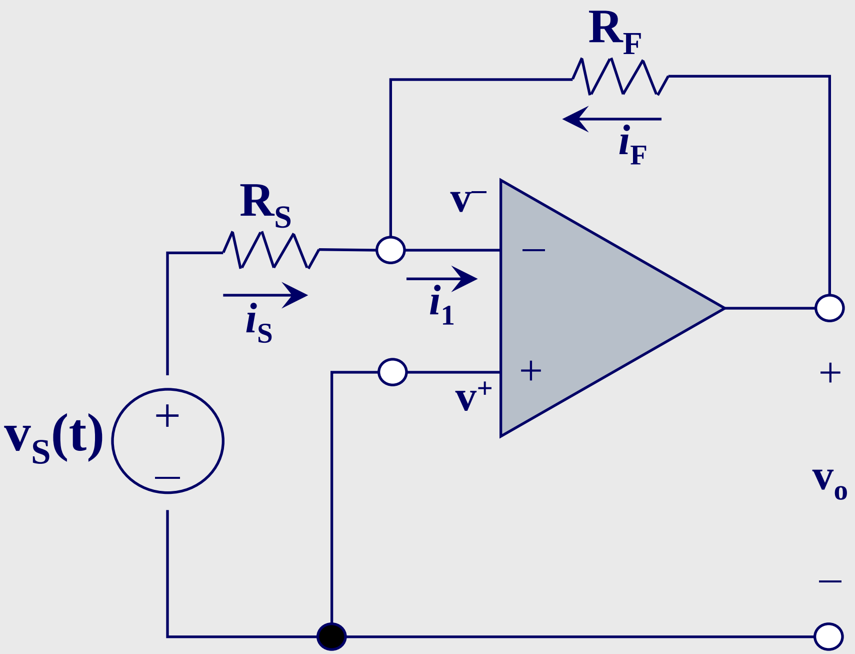
$$i_1 = i_2 = 0$$

$$v^- = v^+$$

Op-Amps – Closed-Loop Mode

<u>Example1</u>: determine $\mathbf{A_{OL}}$ and $\mathbf{v_o}$

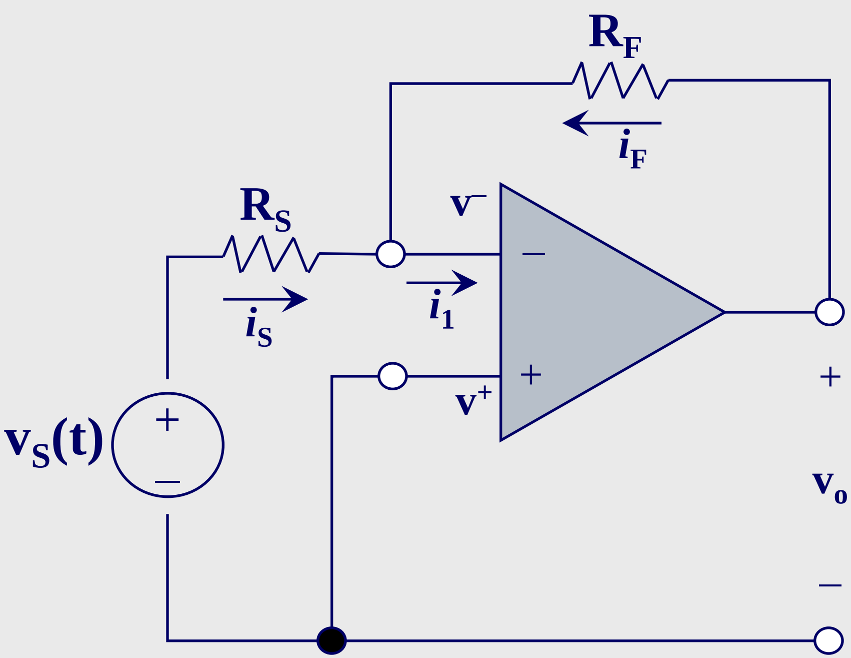
$\mathbf{R_S}$ = 1kΩ, $\mathbf{R_F}$ = 10kΩ, $\mathbf{v_s(t)}$ = Acos(ωt), A=0.015, ω = 50 rads/s



Op-Amps – Closed-Loop Mode

<u>Example1</u>: determine $\mathbf{A_{OL}}$ and $\mathbf{v_o}$

$\mathbf{R_S} = 1\text{k}\Omega$, $\mathbf{R_F} = 10\text{k}\Omega$, $\mathbf{v_s(t)} = A\cos(\omega t)$, $A=0.015$, $\omega = 50$ rads/s



$$A_{CL} = \frac{v_o}{v_S} = -\frac{R_F}{R_S}$$

$$v_o = -\frac{R_F}{R_S}v_S$$

$$= -\frac{10^4}{10^3}\left[0.015\cos(\omega t)\right]$$

$$= -0.15\cos(\omega t)$$

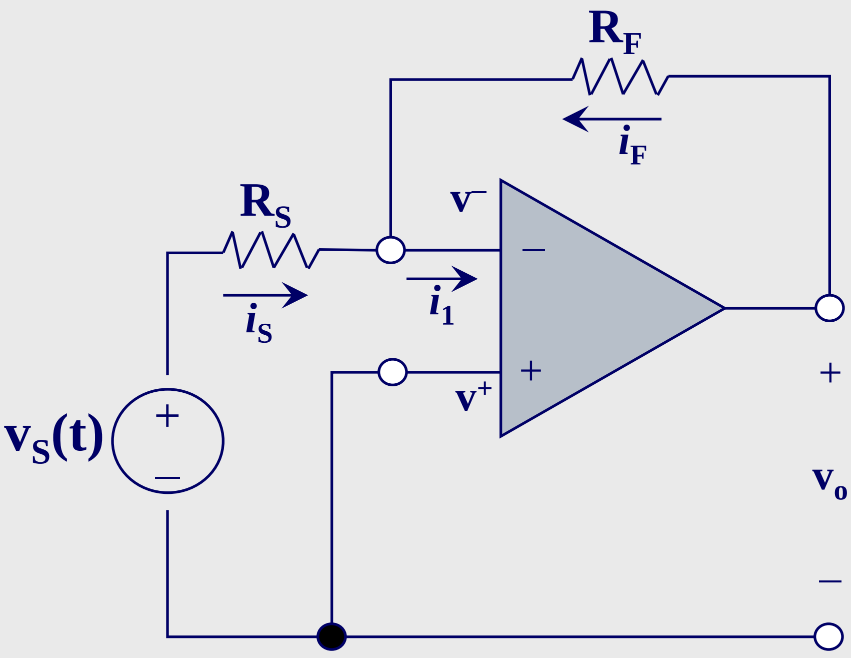
$$A_{CL} = -\frac{R_F}{R_S}$$

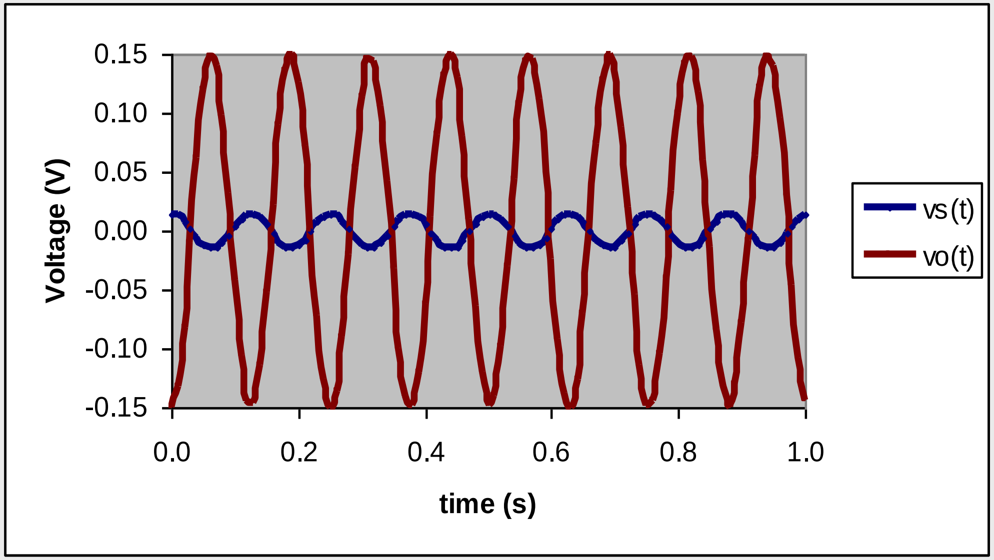
$$= -10$$

Op-Amps – Closed-Loop Mode

Example1: determine $\mathbf{A_{OL}}$ and $\mathbf{v_o}$

$\mathbf{R_S}$ = 1kΩ, $\mathbf{R_F}$ = 10kΩ, $\mathbf{v_s(t)}$ = Acos(ωt), A=0.015, ω = 50 rads/s

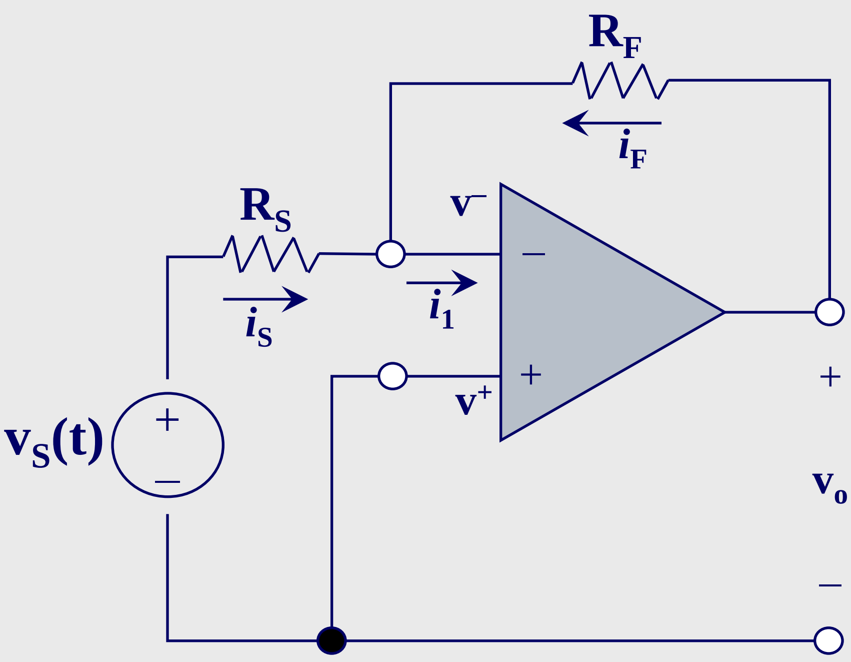




Op-Amps – Closed-Loop Mode

Example2: What is the min/max gain, and gain uncertainty if 5% tolerance resistors are used?

R_S = 1kΩ, **R_F** = 10kΩ, **$v_s(t)$** = Acos(ωt), A=0.015, ω = 50 rads/s



$$A_{CL\,nom} = -10$$

NB: nominal gain

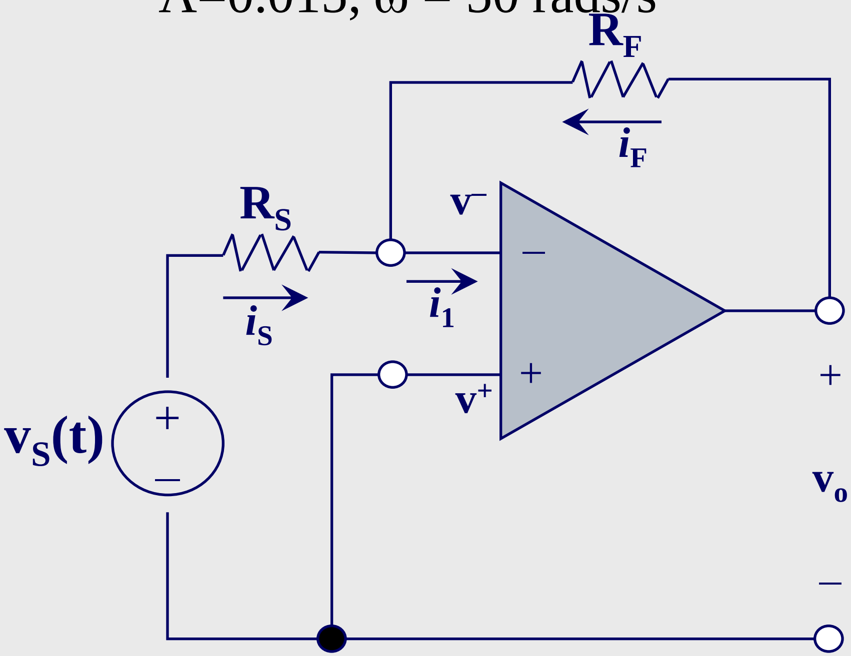
$$A_{CL\,min} = -\frac{R_{F\,min}}{R_{S\,max}} = -\frac{9500}{1050} = -9.05$$

$$A_{CL\,max} = -\frac{R_{F\,max}}{R_{S\,min}} = -\frac{10500}{950} = -11.05$$

Op-Amps – Closed-Loop Mode

Example2: What is the min/max gain, and gain uncertainty if 5% tolerance resistors are used?

$\mathbf{R_S} = 1\mathrm{k}\Omega$, $\mathbf{R_F} = 10\mathrm{k}\Omega$, $\mathbf{v_s(t)} = A\cos(\omega t)$,
$A = 0.015$, $\omega = 50$ rads/s



Max % Error :

$$100 \times \frac{A_{CL\,nom} - \left|A_{CL\,min}\right|}{A_{CL\,nom}}$$

$$= 100 \times \frac{10 - 9.05}{10}$$

$$= 9.5\%$$

Min % Error :

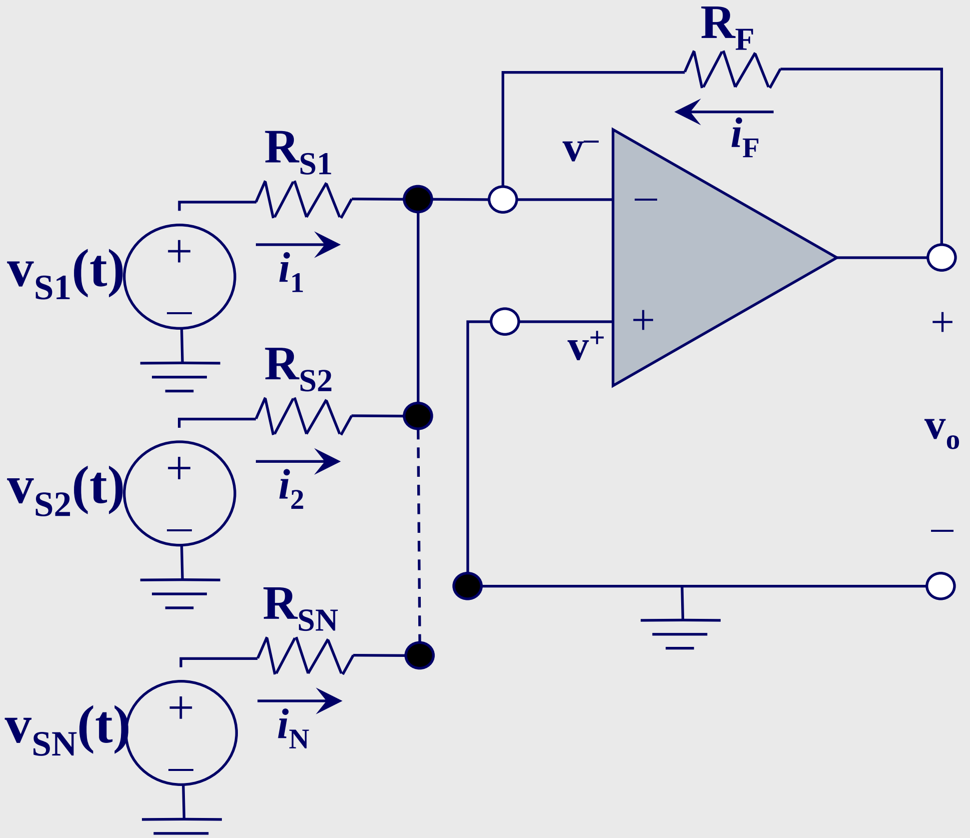
$$100 \times \frac{A_{CL\,nom} - \left|A_{CL\,max}\right|}{A_{CL\,nom}}$$

$$= 100 \times \frac{10 - 11.05}{10}$$

$$= -10.5\%$$

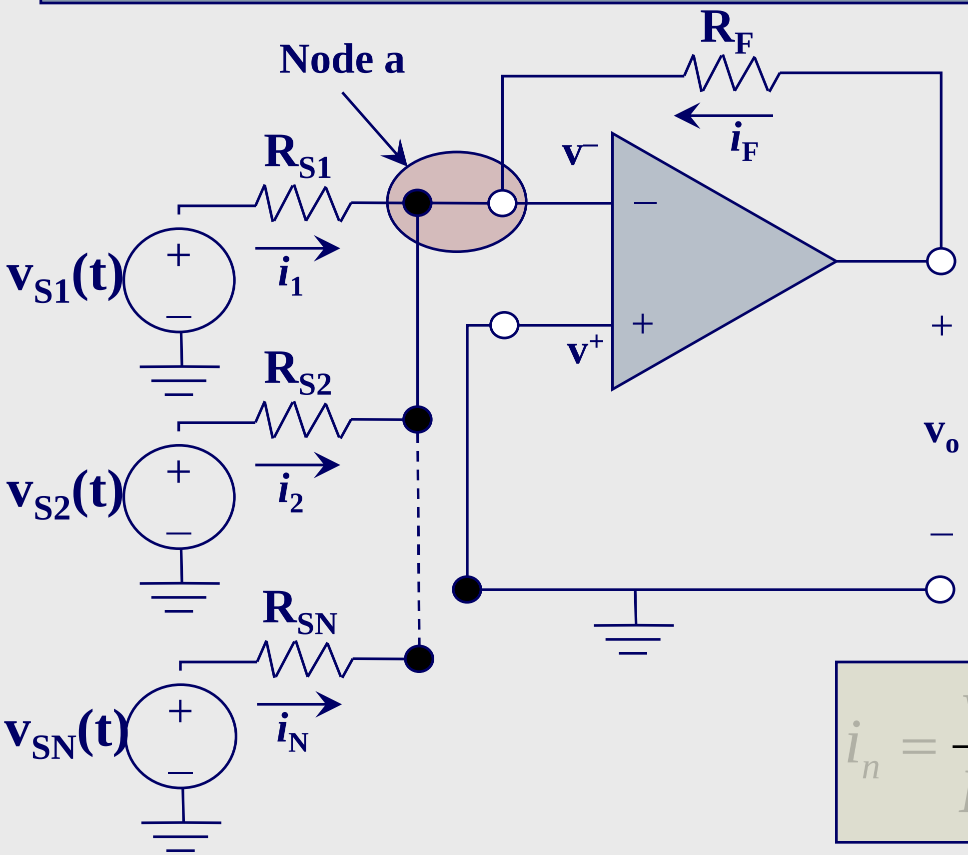
Op-Amps – Closed-Loop Mode

The Summing Amplifier: sources are summed together independently of load and source impedances



Op-Amps – Closed-Loop Mode

The Summing Amplifier: sources are summed together independently of load and source impedances



KCL at Node a :

$$i_1 + i_2 + \ldots + i_N = -i_F$$

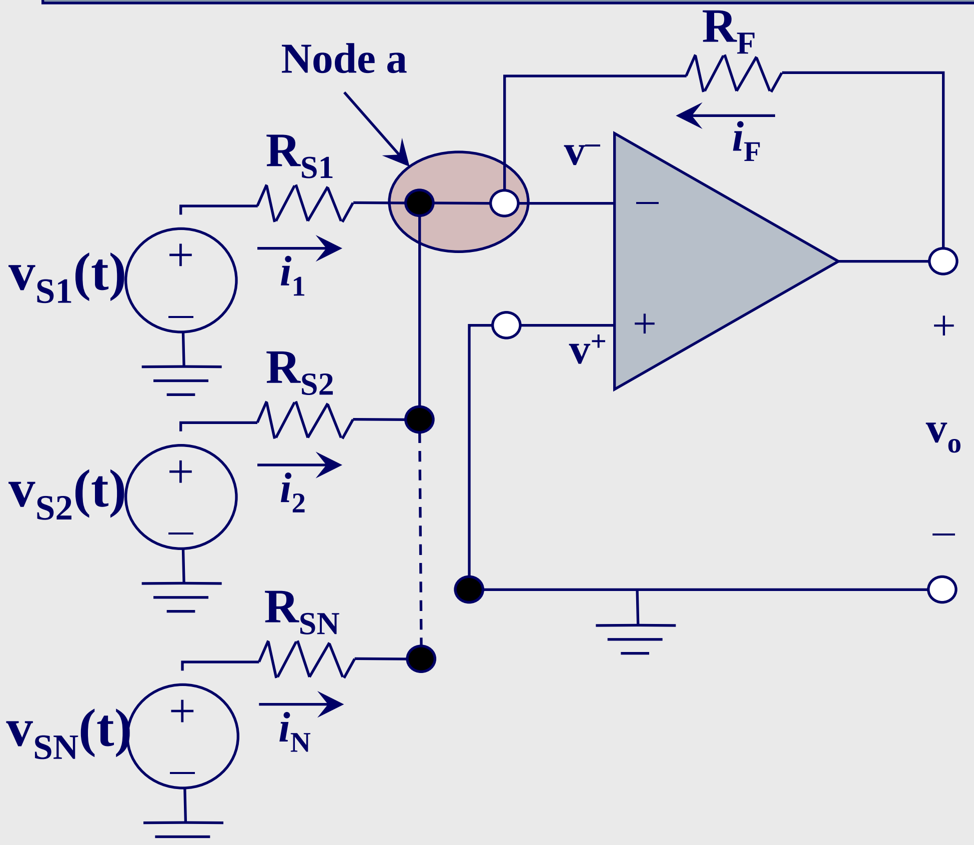
NB: $\mathbf{v}^- = \mathbf{v}^+ = 0$

$$i_n = \frac{v_{Sn}}{R_{Sn}} \qquad n = 1, 2, \ldots, \mathrm{N}$$

$$i_F = \frac{v_o}{R_F}$$

Op-Amps – Closed-Loop Mode

<u>The Summing Amplifier</u>: sources are summed together independently of load and source impedances

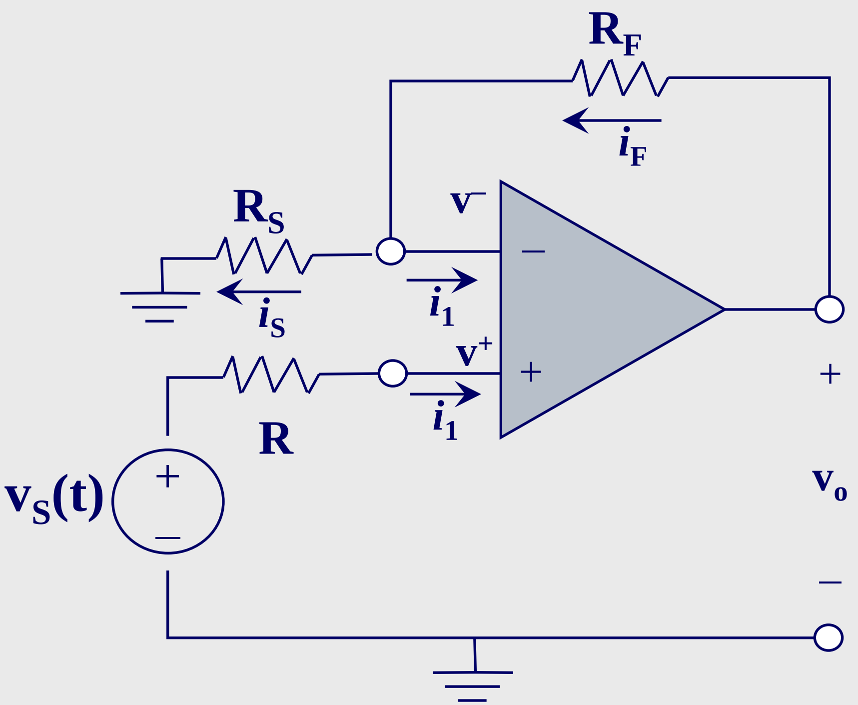


$$\sum_{n=1}^{N} \frac{v_{sn}}{R_{sn}} = -\frac{v_o}{R_F}$$

$$v_o = -\sum_{n=1}^{N} \frac{R_F}{R_{sn}} v_{sn}$$

Op-Amps – Closed-Loop Mode

<u>The Noninverting Amplifier</u>: the signal to be amplified is connected to the noninverting terminal



Op-Amps – Closed-Loop Mode

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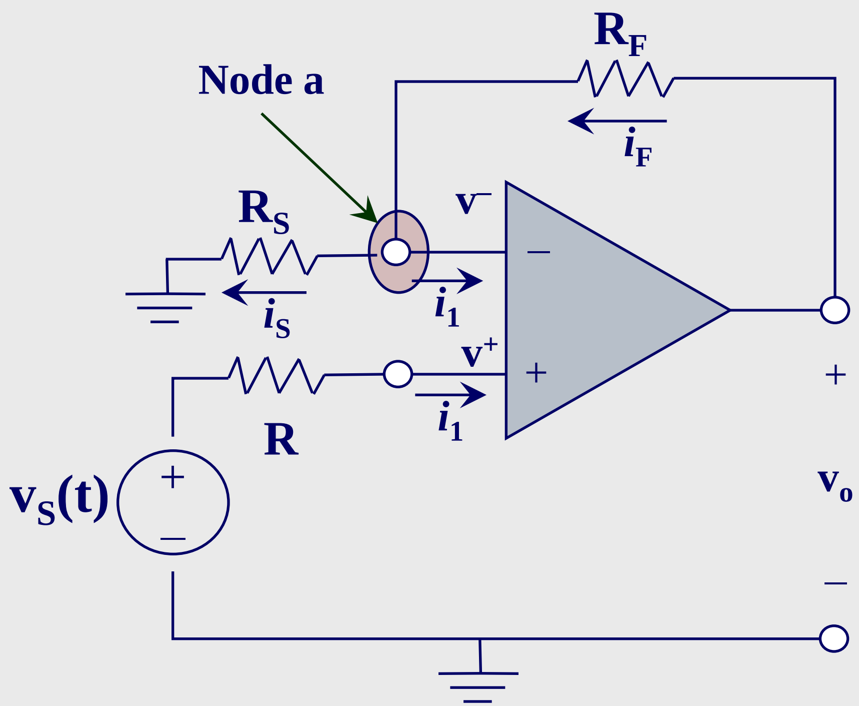


$$i_S = \frac{v_S}{R_S}$$

$$i_F = \frac{v_o - v^-}{R_F}$$

$$i_1 = 0$$

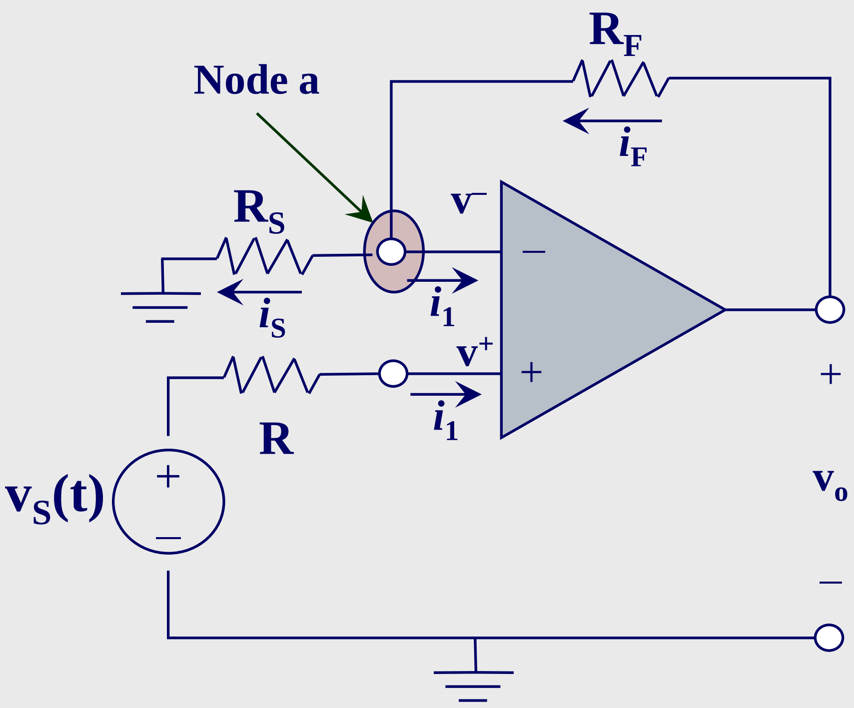
KCL at Node a :

$$-i_S + i_F - i_1 = 0$$

$$i_S = i_F$$

Op-Amps – Closed-Loop Mode

<u>The Noninverting Amplifier</u>: the signal to be amplified is connected to the noninverting terminal



Since $i_1 = 0$:

(i.e. no current through R ∴ no voltage drop)

$$v^+ = v_S = v^-$$

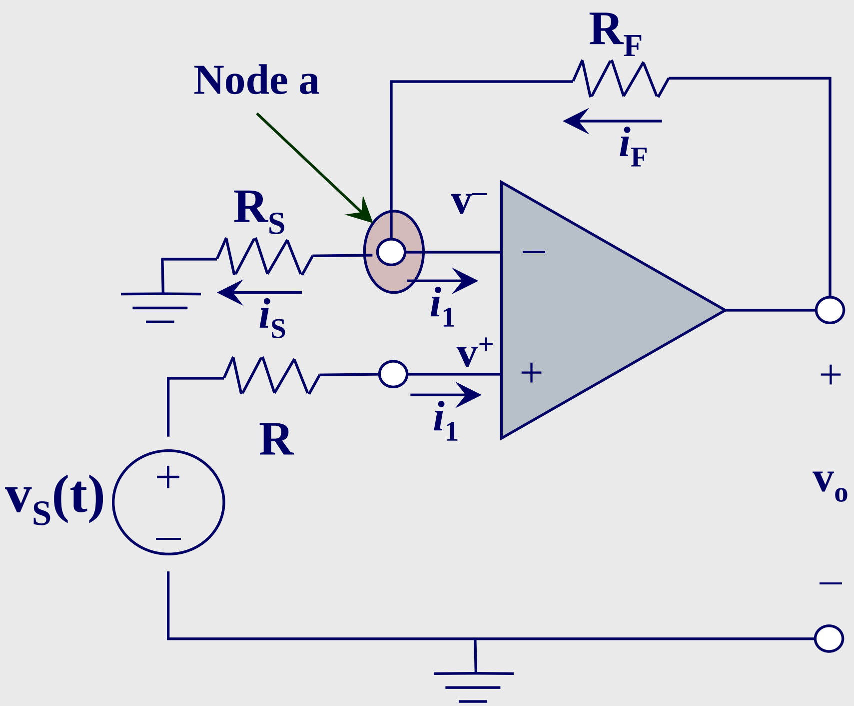
$$i_F = i_S$$

$$\frac{v_o - v_S}{R_F} = \frac{v_S}{R_S}$$

$$\frac{v_o}{v_S} = 1 + \frac{R_F}{R_S}$$

Op-Amps – Closed-Loop Mode

<u>The Noninverting Amplifier</u>: the signal to be amplified is connected to the noninverting terminal



Closed - Loop Gain :

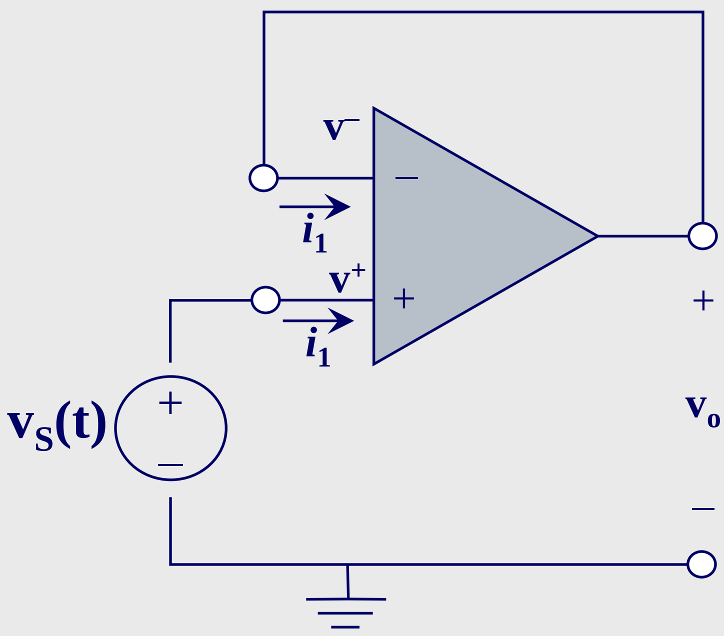
$$A_{CL} = \frac{v_o}{v_S}$$

$$= 1 + \frac{R_F}{R_S}$$

NB: always positive and greater than or equal to 1

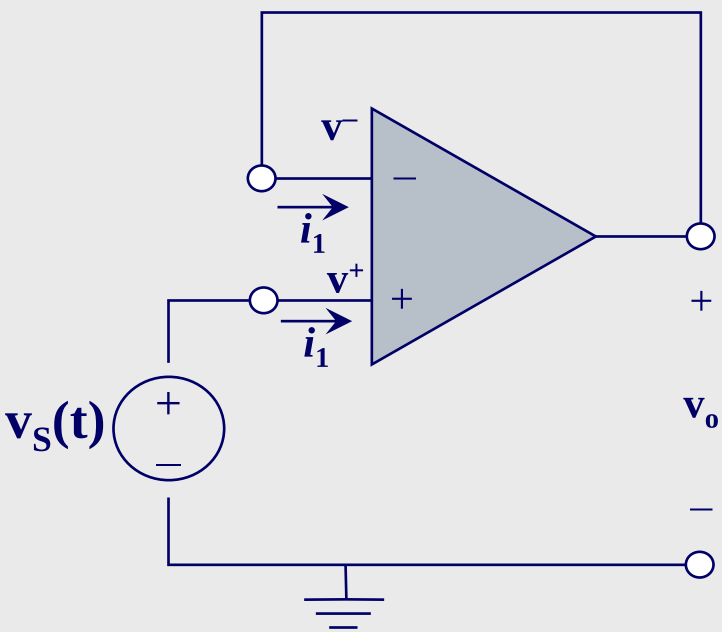
Op-Amps – Closed-Loop Mode

Voltage Follower: the voltage on the output of the op-amp is equal to the source voltage



Op-Amps – Closed-Loop Mode

<u>Voltage Follower</u>: the voltage on the output of the op-amp is equal to the source voltage



NB: an ideal op-amp with negative feedback has the property

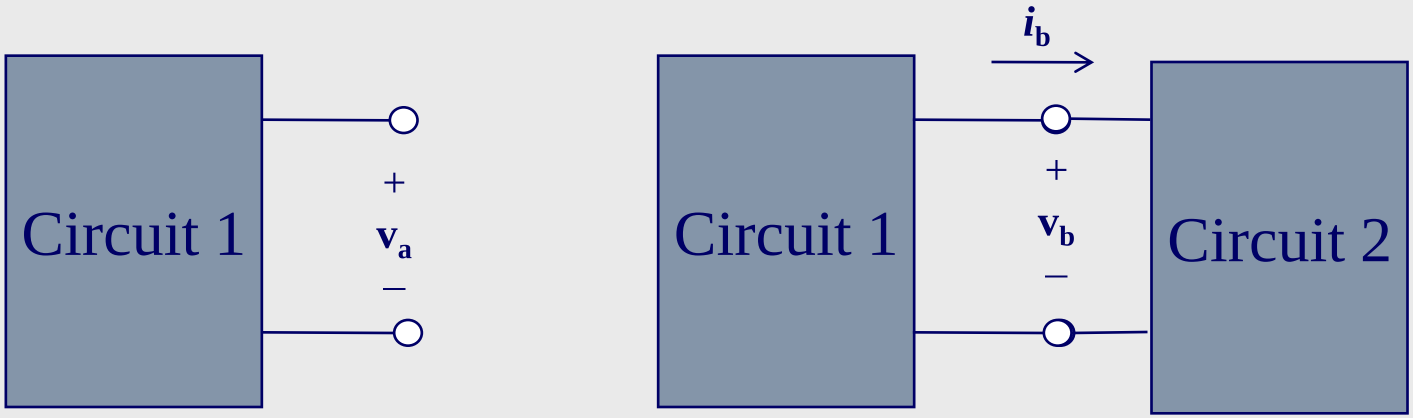
$$v^{-} = v^{+}$$

$$v^{+} = v_s \qquad v^{-} = v_o$$

$$v_o = v_S$$

Op-Amps – Closed-Loop Mode

Voltage Follower: the voltage on the output of the op-amp is equal to the source voltage



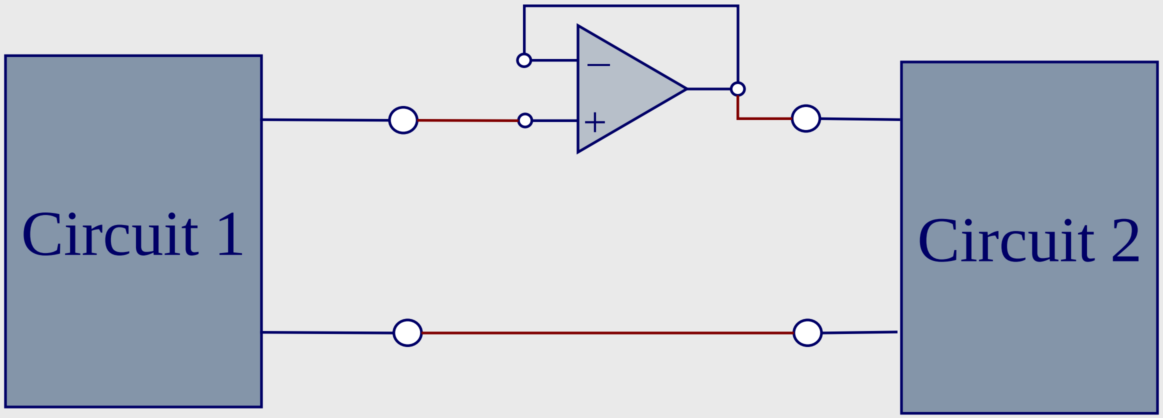
If $\mathbf{v_a} \neq \mathbf{v_b}$

$\boldsymbol{i_b}$ is the **load current**

Loading: changing the behaviour of one circuit by connecting another circuit to it

Op-Amps – Closed-Loop Mode

<u>Voltage Follower</u>: the voltage on the output of the op-amp is equal to the source voltage



Voltage follower can be used to prevent **loading** **(**$i_b = 0$**)**

<u>Loading</u>: changing the behaviour of one circuit by connecting another circuit to it