Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
3 Nov	Mon	18	Operational Amplifiers	8.4		LAB 6	
4 Nov	Tue						
5 Nov	Wed	19	Binary Numbers	13.1 – 13.2			
6 Nov	Thu						
7 Nov	Fri		Recitation		HW 8		
8 Nov	Sat						
9 Nov	Sun						
10 Nov	Mon	20	Exam Review			LAB 7	EXAM 2
12 Nov	Tue						

Give to Receive

Alma 34:28

28 And now behold, my beloved brethren, I say unto you, do not suppose that this is all; for after ye have done all these things, if ye turn away the needy, and the naked, and visit not the sick and afflicted, and impart of your substance, if ye have, to those who stand in need—I say unto you, if ye do not any of these things, behold, **your prayer is vain**, and availeth you nothing, and ye are as hypocrites who do deny the faith.

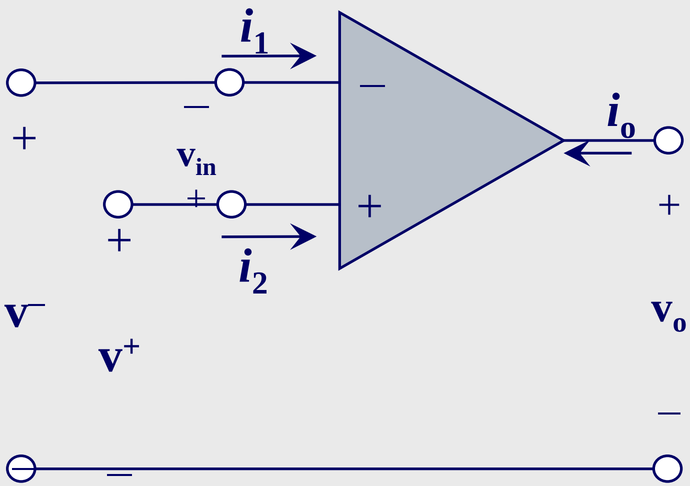
Lecture 18 – Operational Amplifiers

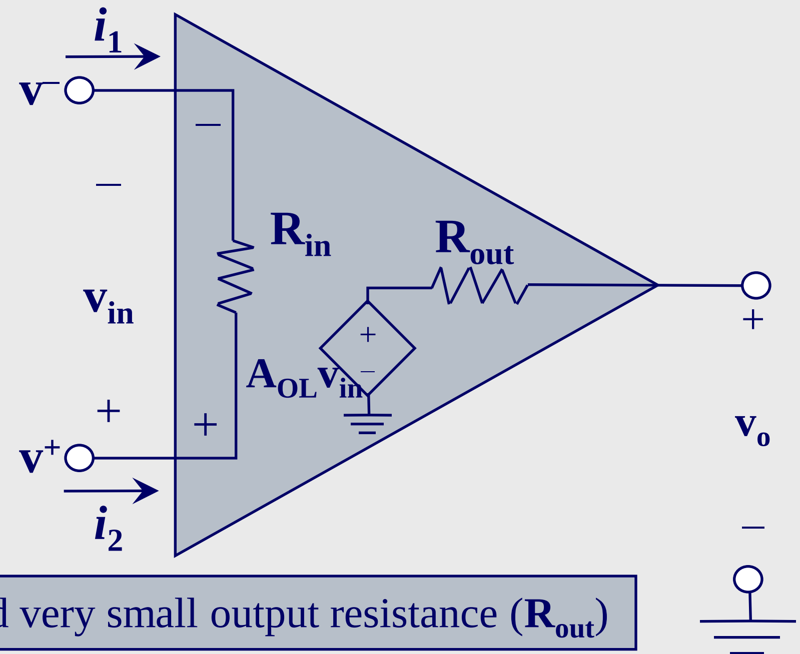
Answer questions from last lecture

Continue with Different OpAmp configurations

Op-Amps – Open-Loop Model

1. **How can $v^- \approx v^+$ when v_o is amplifying $(v^+ - v^-)$?**
2. How can an opAmp form a closed circuit when ($i_1 = i_2 = 0$) ?





NB: op-amps have near-infinite input resistance (**R_{in}**) and very small output resistance (**R_{out}**)

$$v_o \approx A_{OL} v_{in} = A_{OL}(v^+ - v^-)$$

A_{OL} – open-loop voltage gain

Op-Amps – Open-Loop Model

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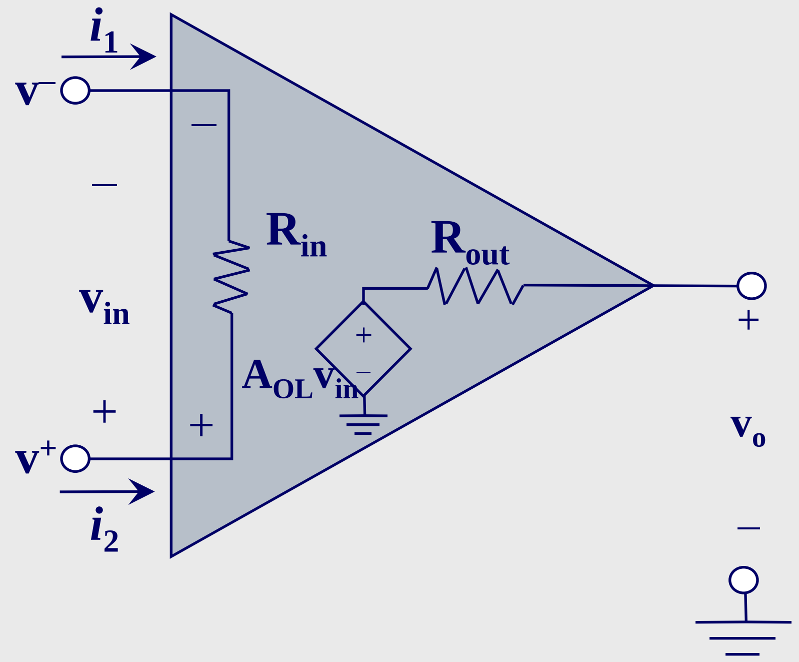


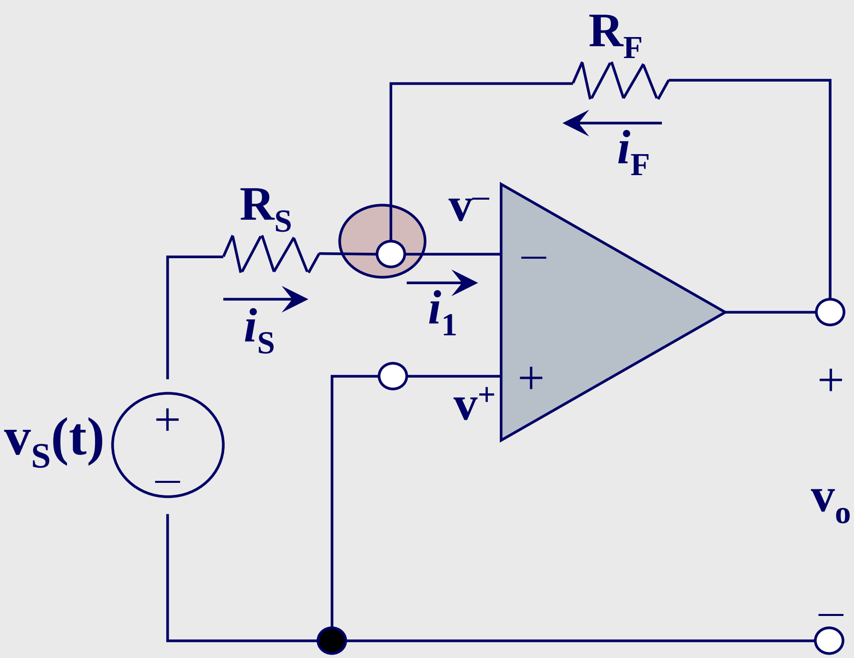
Ideally $i_1 = i_2 = 0$
(since $R_{in} \to \infty$)

$$v_o \approx A_{OL} v_{in}$$
$$= A_{OL}(v^+ - v^-)$$
$$v^+ = \frac{v_o}{A_{OL}} + v^-$$

What happens as $A_{OL} \to \infty$?
$\to v^- \approx v^+$

Op-Amps – Closed-Loop Mode

1. **How can $v^- \approx v^+$ when v_o is amplifying $(v^+ - v^-)$?**
2. How can an opAmp form a closed circuit when ($i_1 = i_2 = 0$) ?



$$i_S = -i_F$$

$$\frac{v_S - v^-}{R_S} = -\frac{v_o - v^-}{R_F}$$

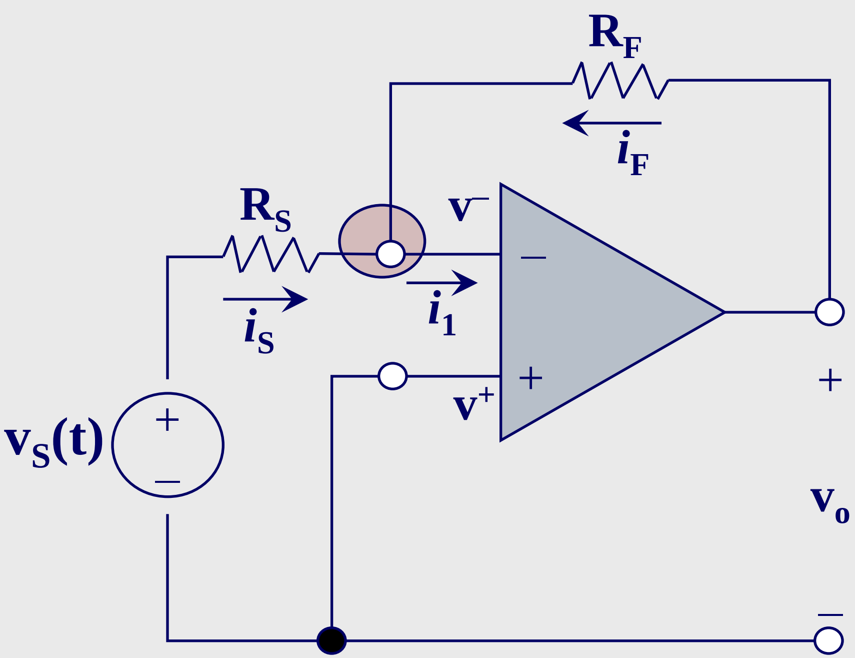
$$\frac{v_S}{R_S} - \left[\frac{(-v_o / A_{OL})}{R_S}\right] = -\frac{v_o}{R_F} + \left[\frac{(-v_o / A_{OL})}{R_F}\right]$$

$$\frac{v_S}{R_S} = -\frac{v_S}{R_S} - \frac{v_o}{A_{OL}R_F} - \frac{v_o}{A_{OL}R_S}$$

$$v_S = -v_o\left(\frac{1}{R_F / R_S} + \frac{1}{A_{OL}R_F / R_S} + \frac{1}{A_{OL}}\right)$$

Op-Amps – Closed-Loop Mode

1. **How can $v^- \approx v^+$ when v_o is amplifying $(v^+ - v^-)$?**
2. How can an opAmp form a closed circuit when ($i_1 = i_2 = 0$) ?



$$v_S = -v_o\left(\frac{1}{R_F/R_S} + \frac{1}{A_{OL}R_F/R_S} + \frac{1}{A_{OL}}\right)$$

NB: if $\mathbf{A_{OL}}$ is very large these terms → 0

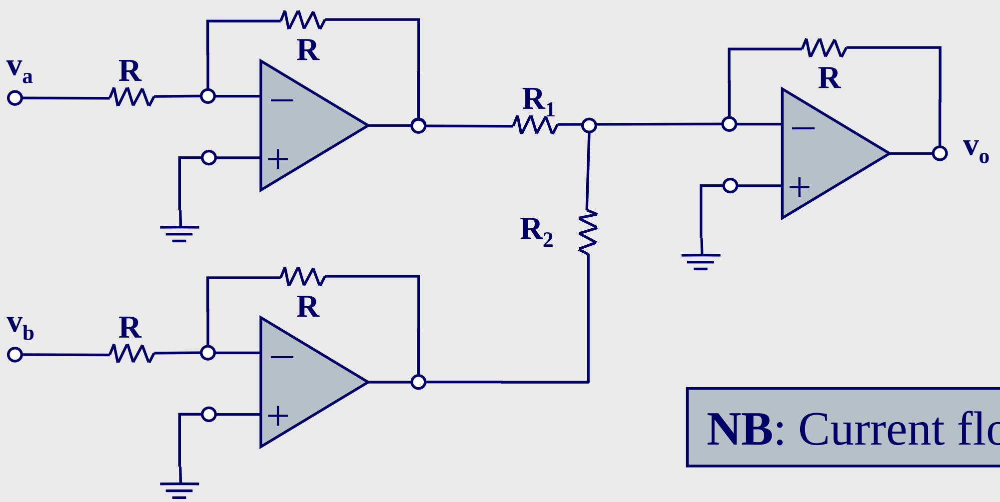
Closed - Loop Gain :

$$A_{CL} = \frac{v_o}{v_S} \approx -\frac{R_F}{R_S}$$

NB: if $\mathbf{A_{OL}}$ is NOT the same thing as $\mathbf{A_{CL}}$

Op-Amps – Closed-Loop Mode

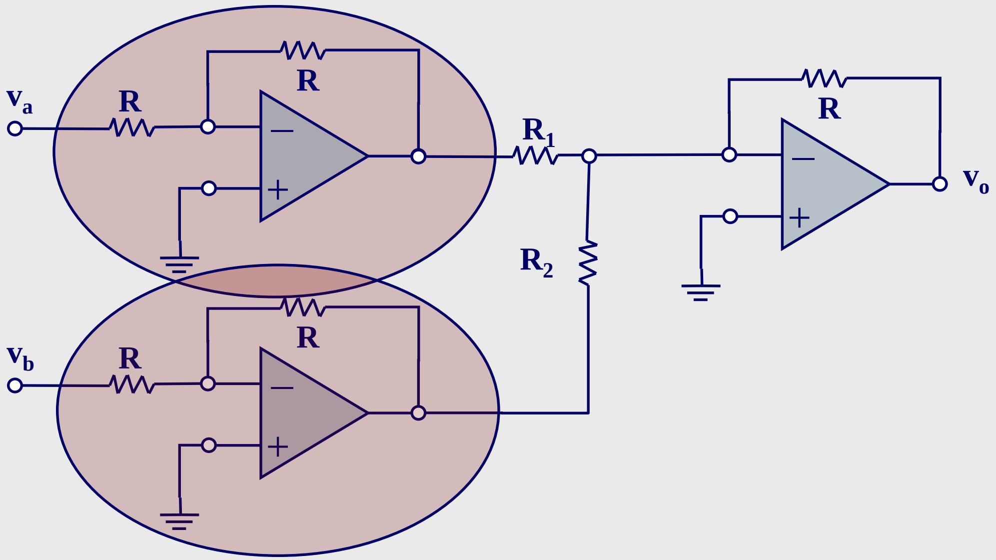
1. How can $\mathbf{v}^- \approx \mathbf{v}^+$ when $\mathbf{v_o}$ is amplifying ($\mathbf{v}^+$ - $\mathbf{v}^-$) ?
2. **How can an opAmp form a closed circuit when ($i_1 = i_2 = 0$) ?**



NB: Current flows through $\mathbf{R_1}$ and $\mathbf{R_2}$

Op-Amps – Closed-Loop Mode

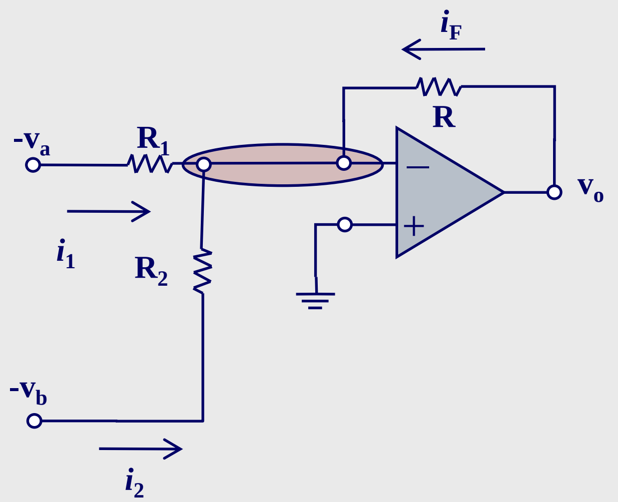
1. How can $\mathbf{v^-} \approx \mathbf{v^+}$ when $\mathbf{v_o}$ is amplifying ($\mathbf{v^+}$ - $\mathbf{v^-}$) ?
2. **How can an opAmp form a closed circuit when ($i_1 = i_2 = 0$) ?**



NB: Inverting amplifiers and ($\mathbf{R_S}$ = $\mathbf{R_F}$) → $\mathbf{v_o} = -\mathbf{v_i}$

Op-Amps – Closed-Loop Mode

1. How can $\mathbf{v}^{-} \approx \mathbf{v}^{+}$ when $\mathbf{v_o}$ is amplifying ($\mathbf{v}^{+}$ - $\mathbf{v}^{-}$) ?
2. **How can an opAmp form a closed circuit when ($i_1 = i_2 = 0$) ?**



$$i_1 + i_2 = -i_F$$

$$\frac{-v_a - v^-}{R_1} + \frac{-v_b - v^-}{R_2} = -\frac{v_o - v^-}{R}$$

$$\frac{-v_a - 0}{R_1} + \frac{-v_b - 0}{R_2} = -\frac{v_o - 0}{R}$$

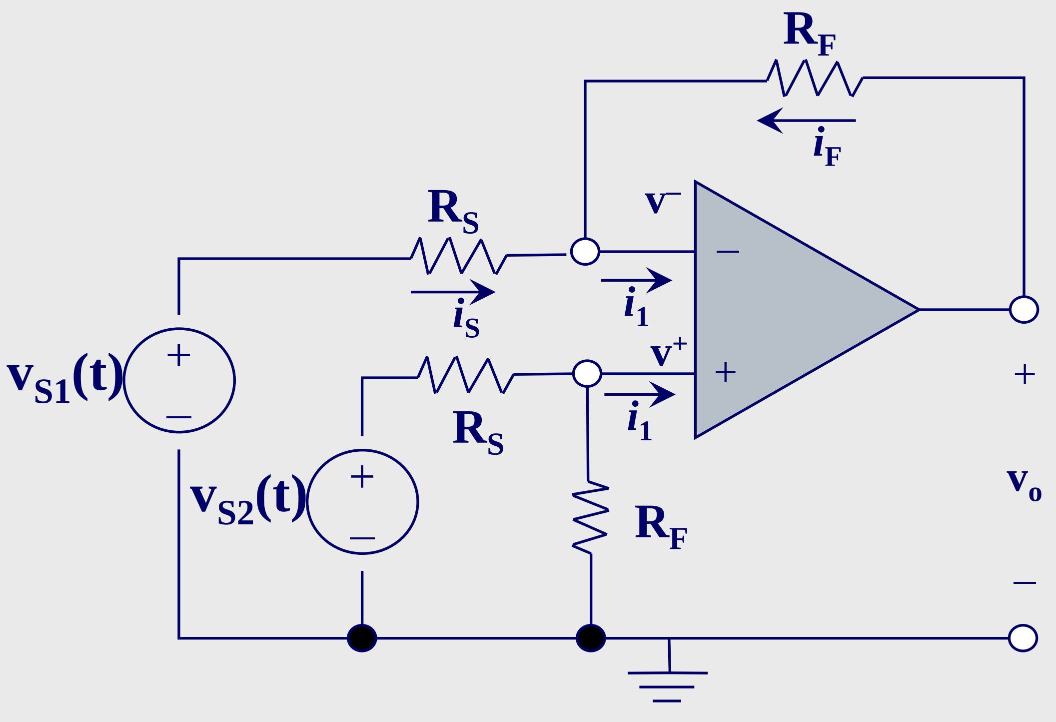
$$\frac{v_a}{R_1} + \frac{v_b}{R_2} = \frac{v_o}{R}$$

$$v_o = R\left(\frac{v_a}{R_1} + \frac{v_b}{R_2}\right)$$

More OpAmp Configurations

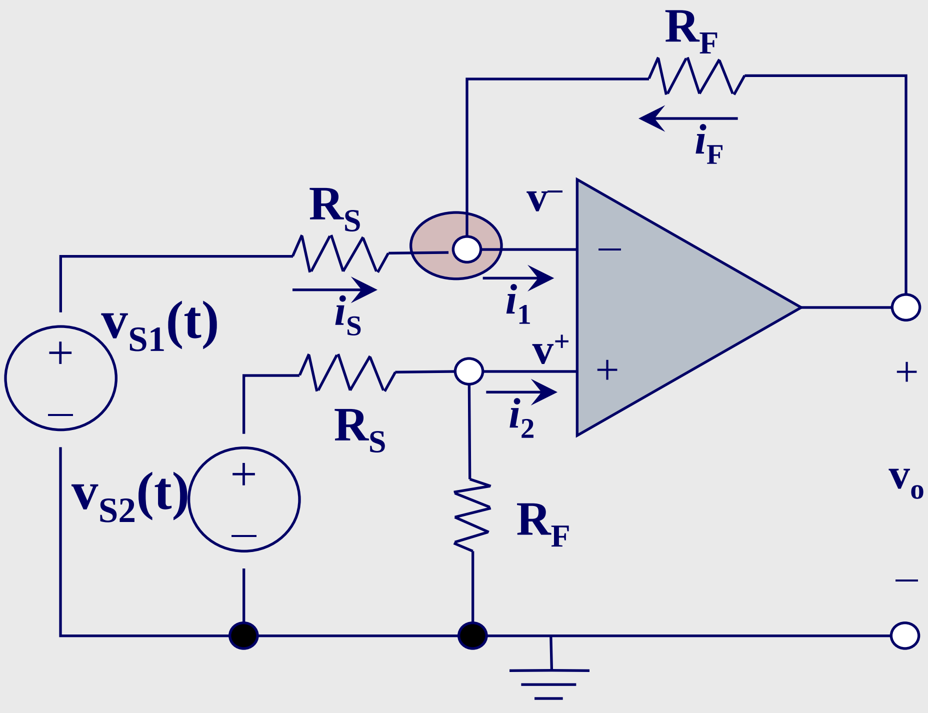
Op-Amps – Closed-Loop Mode

<u>The Differential Amplifier</u>: the signal to be amplified is the difference of two signals



Op-Amps – Closed-Loop Mode

<u>The Differential Amplifier</u>: the signal to be amplified is the difference of two signals



NB: an ideal op-amp with negative feedback has the properties

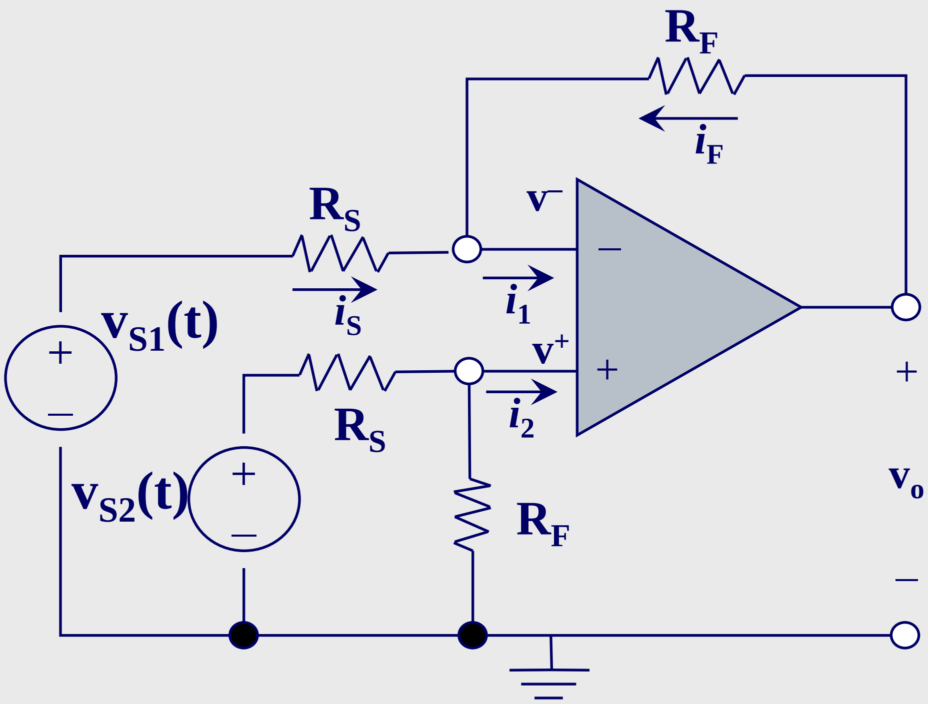
$$v^- = v^+$$

$$i_1 = i_2 = 0$$

$$\therefore i_F = -i_S$$

Op-Amps – Closed-Loop Mode

The Differential Amplifier: the signal to be amplified is the difference of two signals



Voltage Divider :

$$v^+ = v_{s2}\frac{R_F}{R_F + R_S}$$

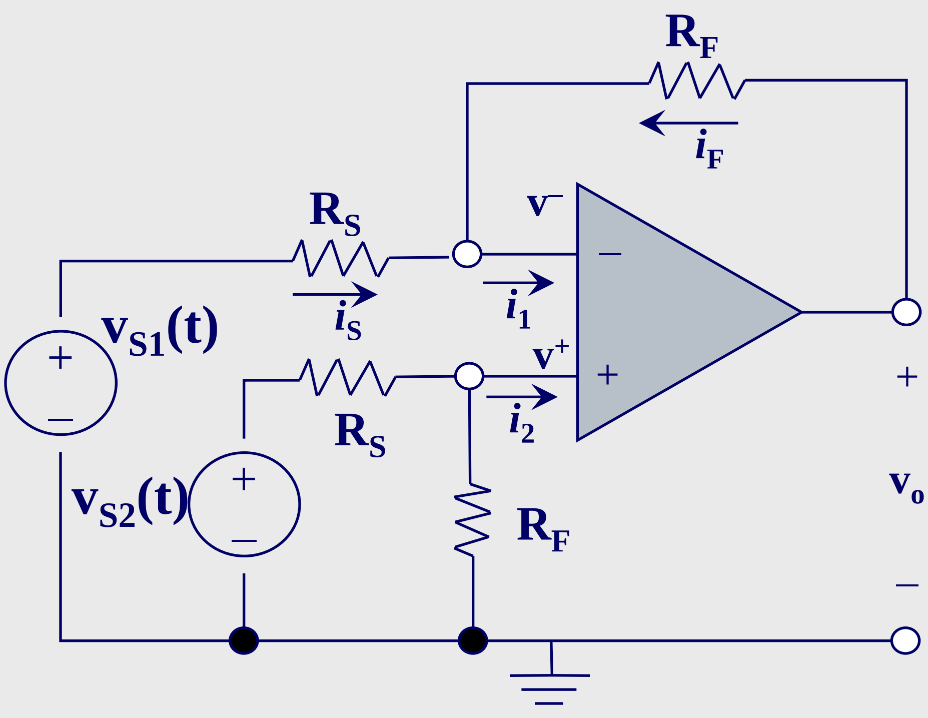
$$v^- = v^+ = v_{S1} + i_S R_S$$

$$i_S = \frac{v_{S1} - v^+}{R_S}$$

$$i_F = \frac{v_o - v^+}{R_F}$$

Op-Amps – Closed-Loop Mode

The Differential Amplifier: the signal to be amplified is the difference of two signals

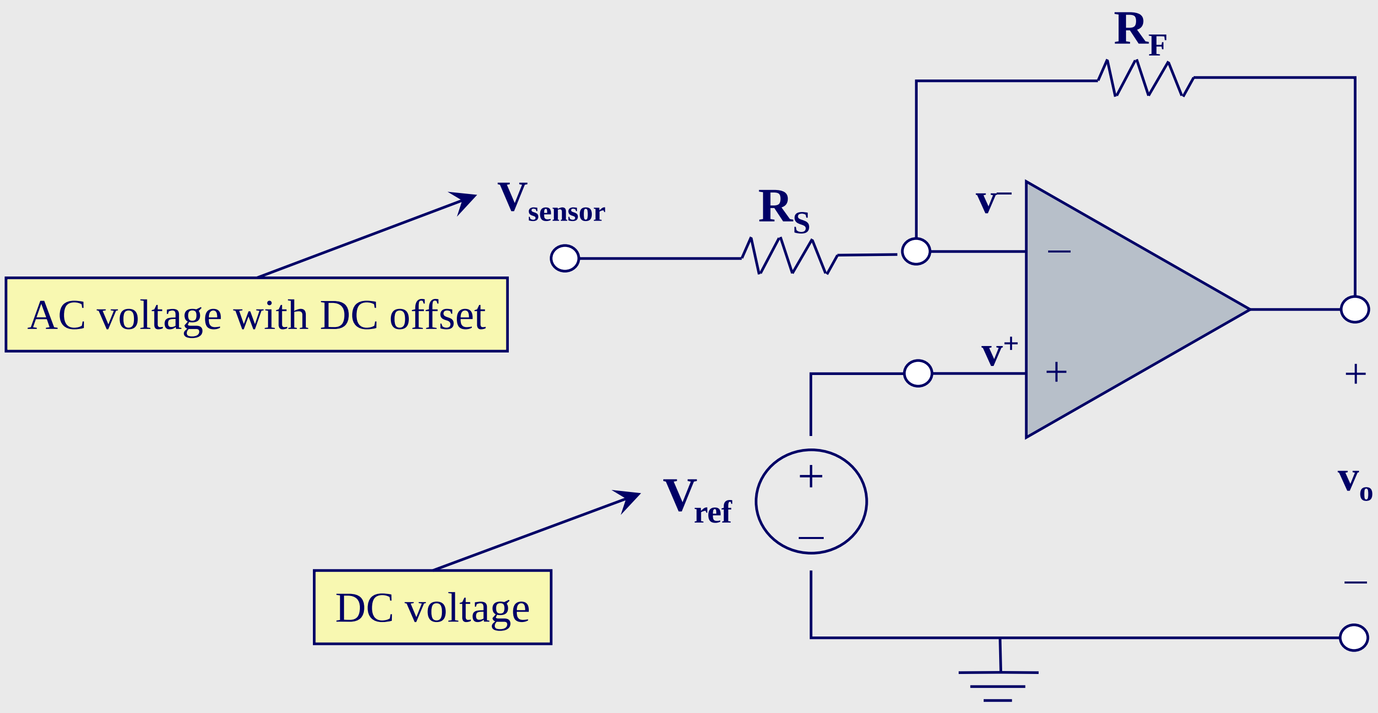


$$v_o = i_F R_F + v^+$$
$$= -i_S R_F + v^+$$

$$v_o = R_F \left[\frac{-v_{S1}}{R_S} + \frac{v_{S2}}{R_S + R_F} + \frac{v_{S2} R_F}{R_S (R_S + R_F)} \right]$$
$$= \frac{R_F}{R_S} (v_{S2} - v_{S1})$$

Op-Amps – Level Shifter

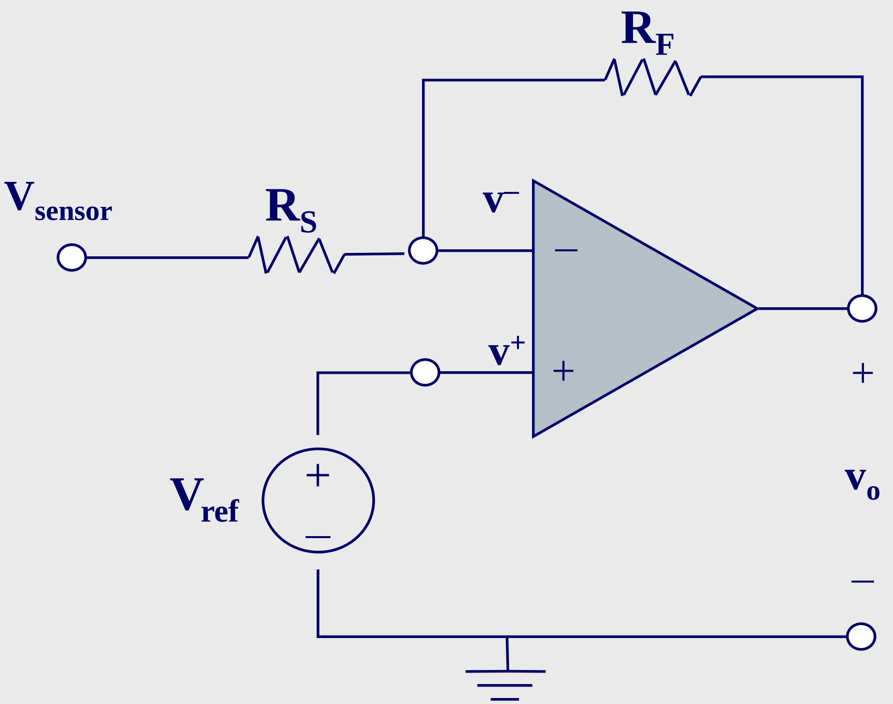
<u>Level Shifter</u>: can add or subtract a DC offset from a signal based on the values of $\mathbf{R_S}$ and/or $\mathbf{V_{ref}}$



Op-Amps – Level Shifter

Example1: design a level shifter such that it can remove a 1.8V DC offset from the sensor signal (Find $\mathbf{V_{ref}}$)

$\mathbf{R_S}$ = 10kΩ, $\mathbf{R_F}$ = 220kΩ, $\mathbf{v_s(t)}$ = 1.8+0.1cos(ωt)



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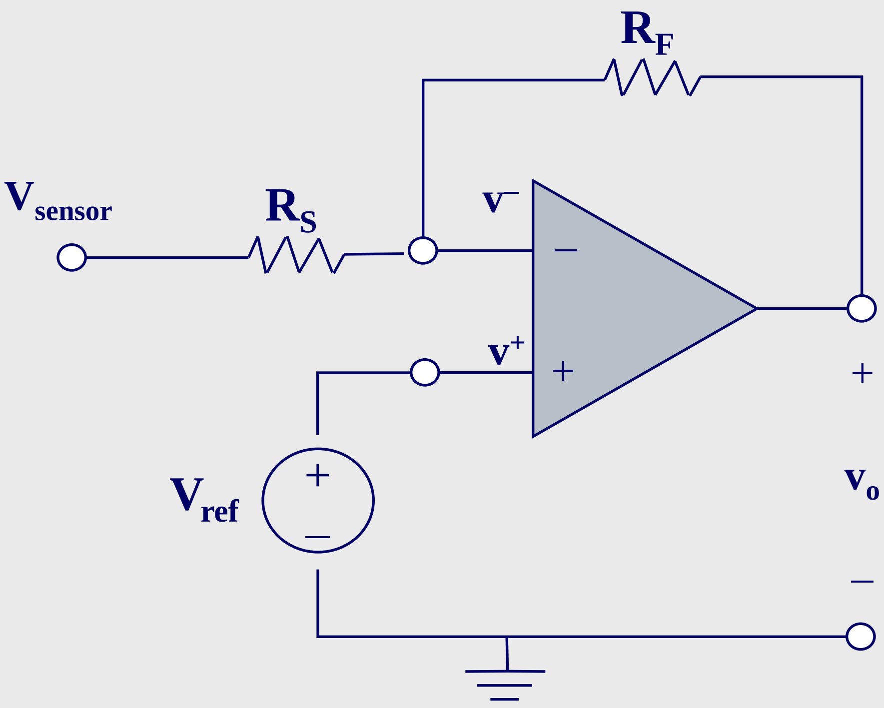

Find the **Closed-Loop** voltage gain by using the principle of **superposition** on each of the DC voltages

Op-Amps – Level Shifter

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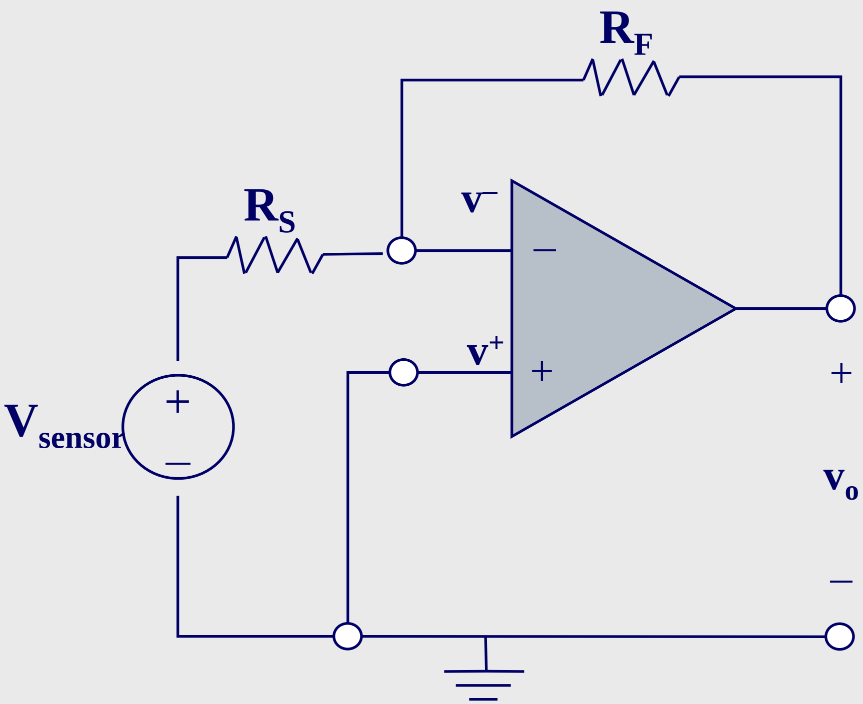

DC from sensor:
Inverting amplifier

$$v_{osen} = A_{CL} v_{senDC}$$
$$= -\frac{R_F}{R_S} v_{senDC}$$

Op-Amps – Level Shifter

Example1: design a level shifter such that it can remove a 1.8V DC offset from the sensor signal (Find $\mathbf{V_{ref}}$)

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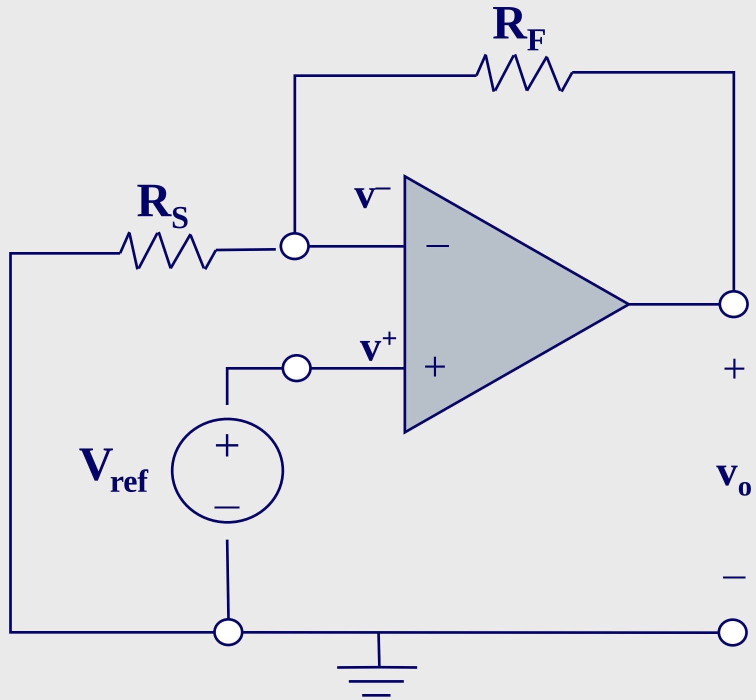

DC from reference:
Noninverting amplifier

$$v_{oref} = A_{CL} v_{ref}$$

$$= \left(1 + \frac{R_F}{R_S}\right) v_{ref}$$

Op-Amps – Level Shifter

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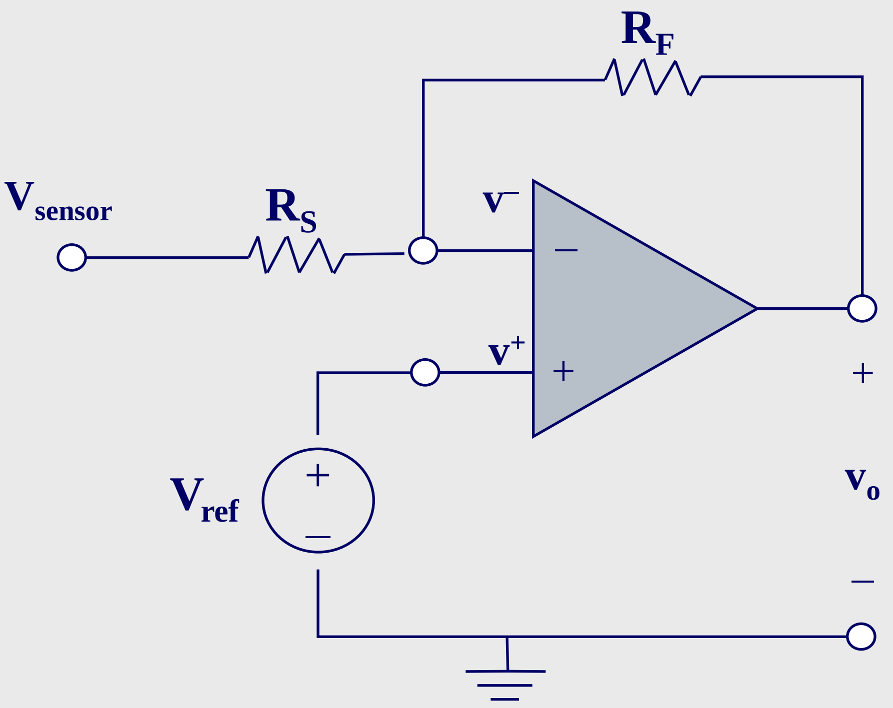
$\mathbf{R_S}$ = 10kΩ, $\mathbf{R_F}$ = 220kΩ, $\mathbf{v_s(t)}$ = 1.8+0.1cos(ωt)



$$v_{oDC} = v_{osen} + v_{oref}$$

$$= -\frac{R_F}{R_S} v_{senDC} + \left(1 + \frac{R_F}{R_S}\right) v_{ref}$$

Op-Amps – Level Shifter

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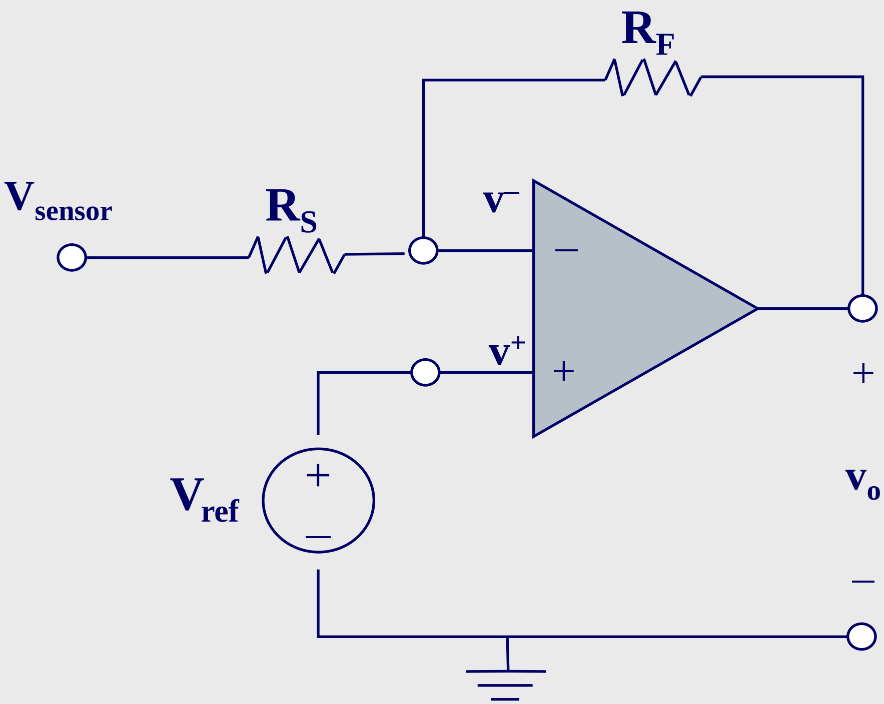


Since the desire is to remove all DC from the output we require:

$$0 = -\frac{R_F}{R_S}v_{sDC} + \left(1+\frac{R_F}{R_S}\right)v_{ref}$$

$$v_{ref} = v_{sDC}\frac{R_F/R_S}{1+R_F/R_S}$$

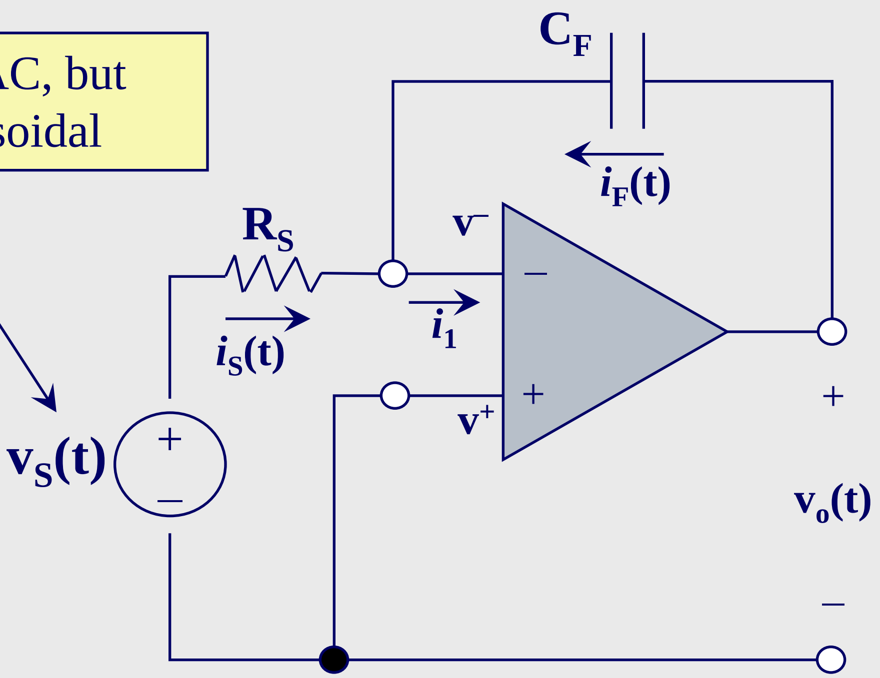
$$= (1.8)\frac{220/10}{1+220/10}$$

$$= 1.714\ V$$

Op-Amps – Ideal Integrator

The Ideal Integrator: the output signal is the integral of the input signal (over a period of time)

The input signal is AC, but not necessarily sinusoidal



NB: Inverting amplifier setup with $\mathbf{R_F}$ replaced with a capacitor

Op-Amps – Ideal Integrator

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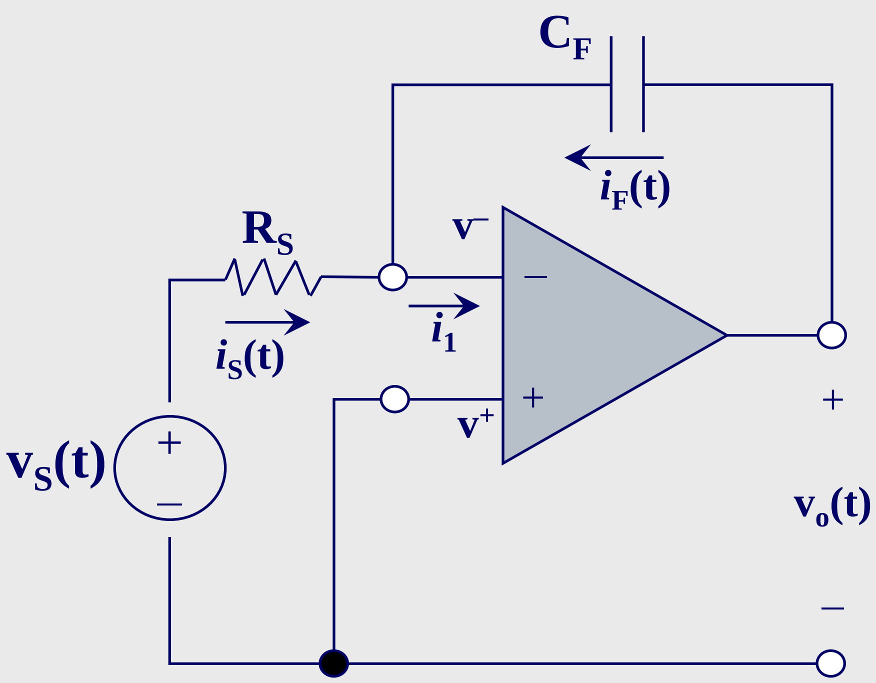


$$i_S(t) = -i_F(t)$$

$$\frac{v_S(t)}{R_S} = -C_F \frac{d\left[v_o(t) - v^-(t)\right]}{dt}$$

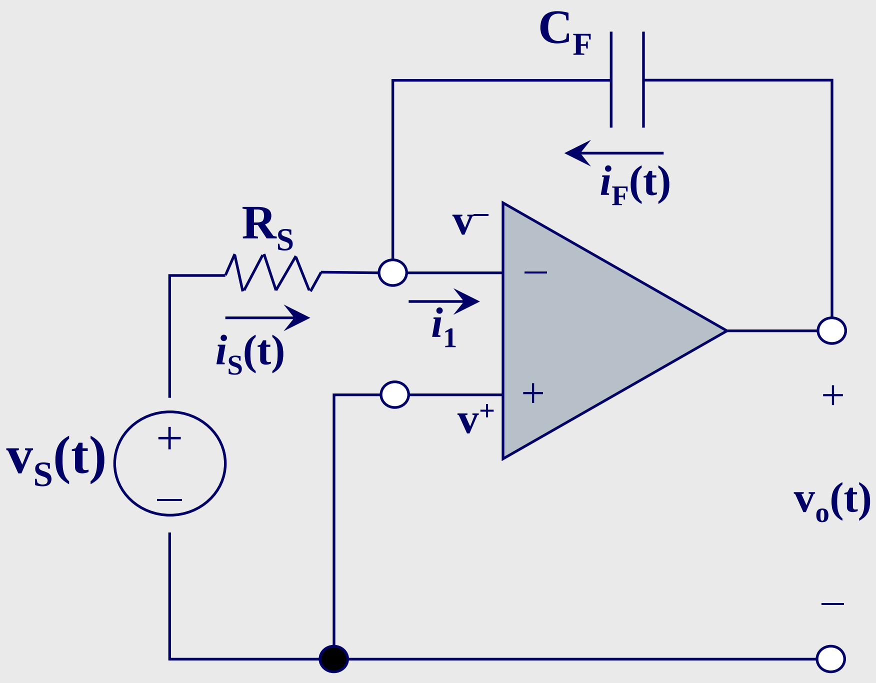
$$\frac{dv_o(t)}{dt} = -\frac{v_S(t)}{R_S C_F}$$

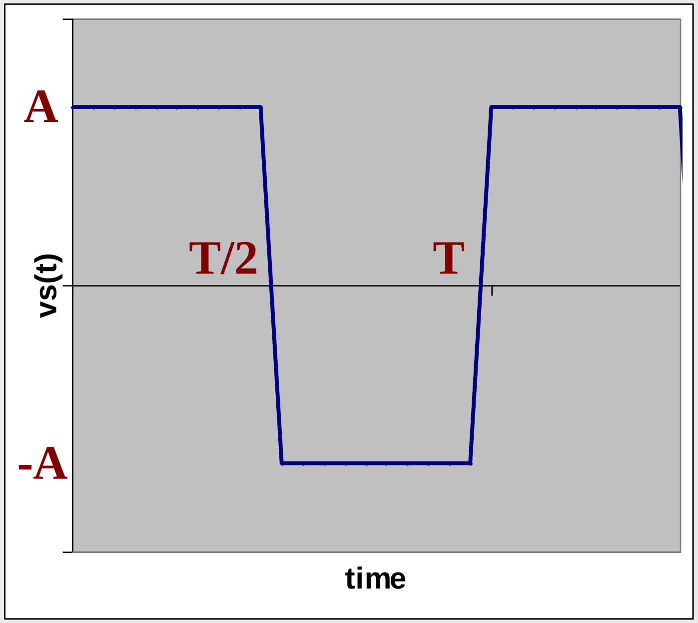
$$v_o(t) = -\frac{1}{R_S C_F}\int_{-\infty}^{t} v_S(\tau)\,d\tau$$

Op-Amps – Ideal Integrator

Example2: find the output voltage if the input is a square wave of amplitude +/–**A** with period **T**

T = 10ms, $\mathbf{C_F}$ = 1uF, $\mathbf{R_S}$ = 10kΩ





Op-Amps – Ideal Integrator

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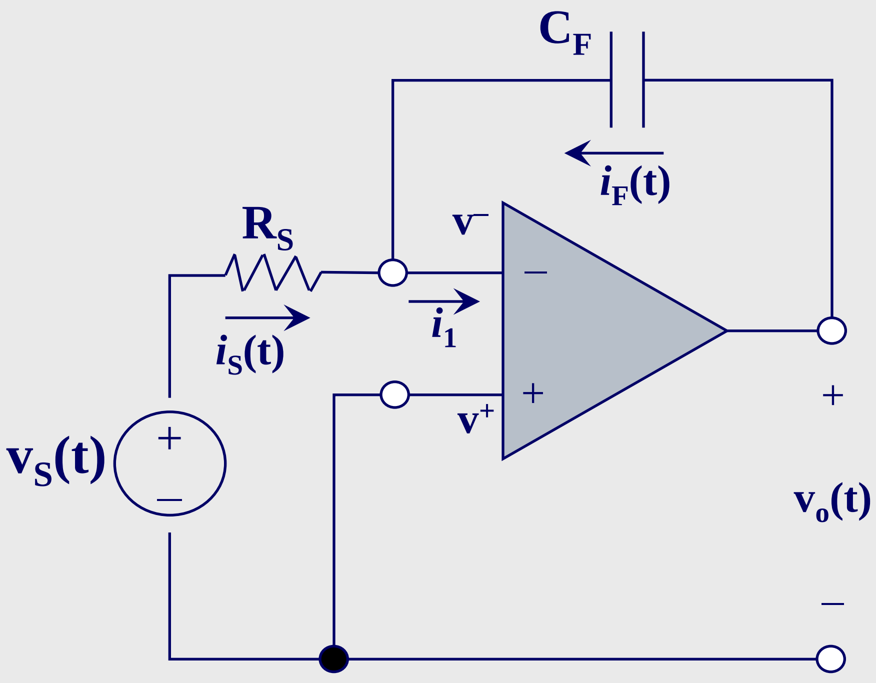

$$v_o(t) = -\frac{1}{R_S C_F}\int_{-\infty}^{t} v_S(\tau)d\tau$$

$$= -\frac{1}{R_S C_F}\left[\int_{-\infty}^{0} v_S(\tau)d\tau + \int_{0}^{t} v_S(\tau)d\tau\right]$$

$$= v_o(0) - \frac{1}{R_S C_F}\int_{0}^{t} v_S(\tau)d\tau$$

Op-Amps – Ideal Integrator

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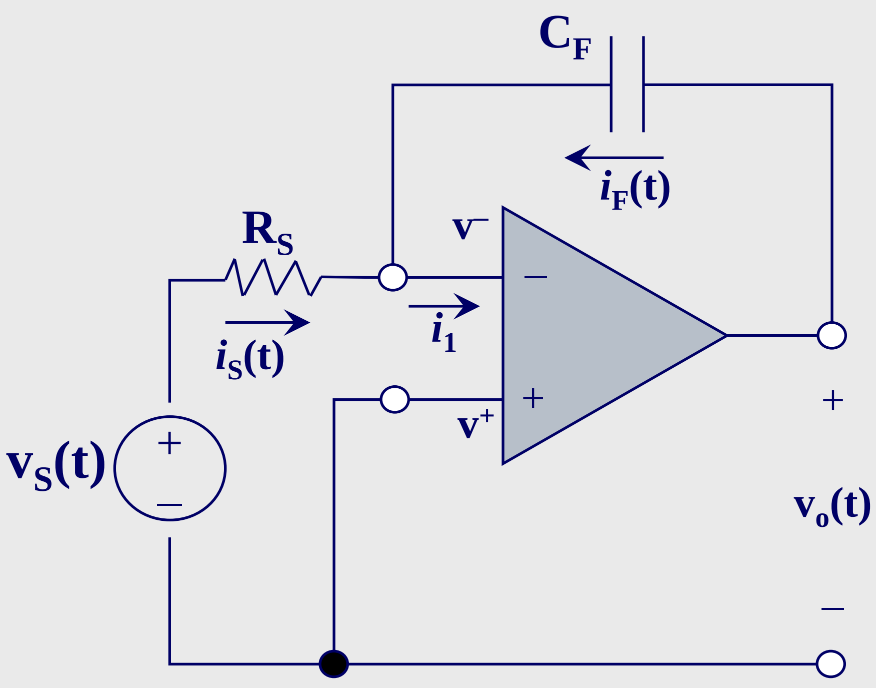

NB: since the $\mathbf{v_s(t)}$ is periodic, we can find $\mathbf{v_o(t)}$ over a single period – and repeat

$$0 < t \leq T/2$$

$$v_o(t) = v_o(0) - \frac{1}{R_S C_F}\int_0^t v_S(\tau)d\tau$$

$$= 0 - 100\int_0^t A d\tau$$

$$= -100 A\tau\Big|_0^t$$

$$= -100 At$$

Op-Amps – Ideal Integrator

<u>Example2</u>: find the output voltage if the input is a square wave of amplitude +/–**A** with period **T**

T = 10ms, $\mathbf{C_F}$ = 1uF, $\mathbf{R_S}$ = 10kΩ

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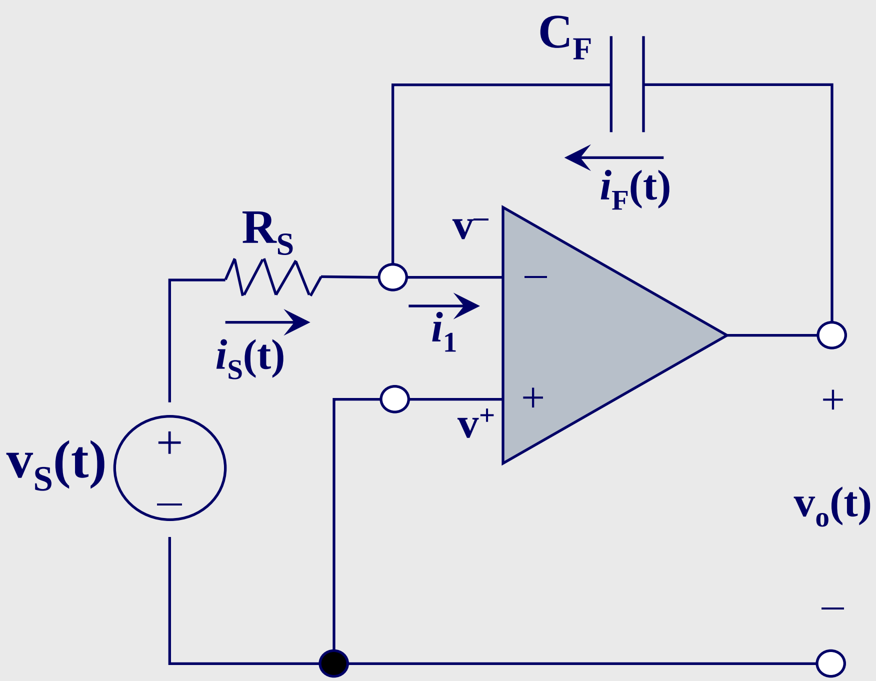

$$T/2 < t \le T$$

$$v_o(t) = v_o\left(\frac{T}{2}\right) - \frac{1}{R_S C_F}\left[\int_{T/2}^{t} v_S(\tau)d\tau\right]$$

$$= -100A\frac{T}{2} - 100\int_{T/2}^{t}(-A)d\tau$$

$$= -100A\frac{T}{2} + 100A\tau\Big|_{T/2}^{t}$$

$$= -100A(T-t)$$

Op-Amps – Ideal Integrator

<u>Example2</u>: find the output voltage if the input is a square wave of amplitude +/–**A** with period **T**

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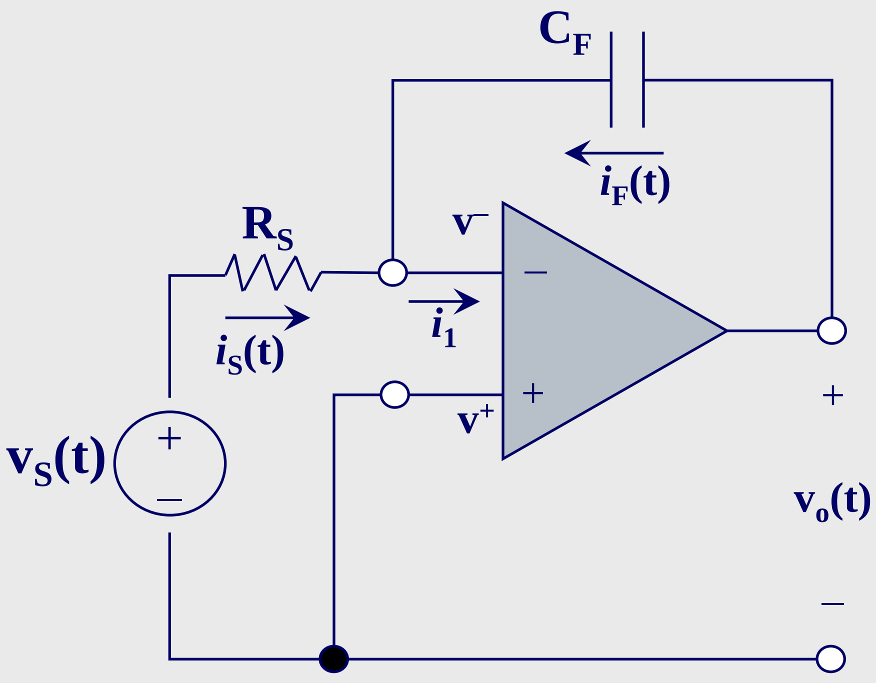
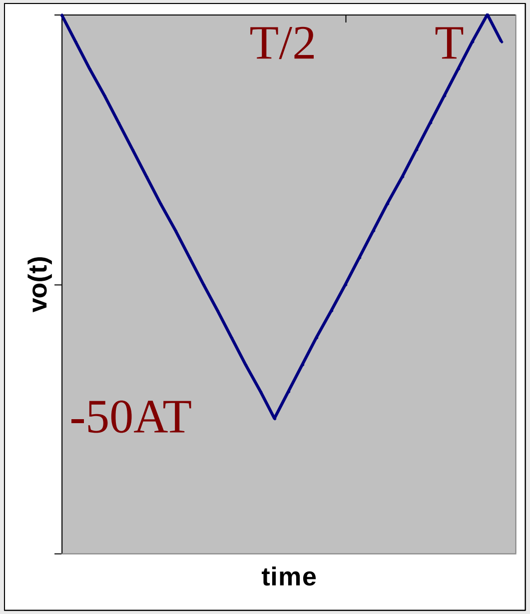
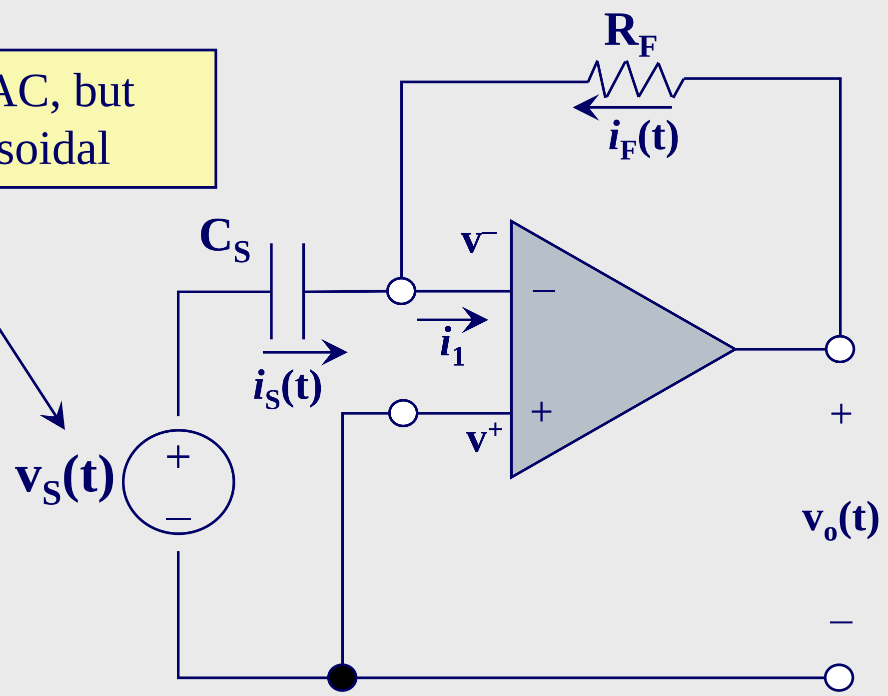




Op-Amps – Ideal Differentiator

The Ideal Differentiator: the output signal is the derivative of the input signal (over a period of time)

The input signal is AC, but not necessarily sinusoidal



NB: Inverting amplifier setup with $\mathbf{R_S}$ replaced with a capacitor

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Op-Amps – Ideal Differentiator

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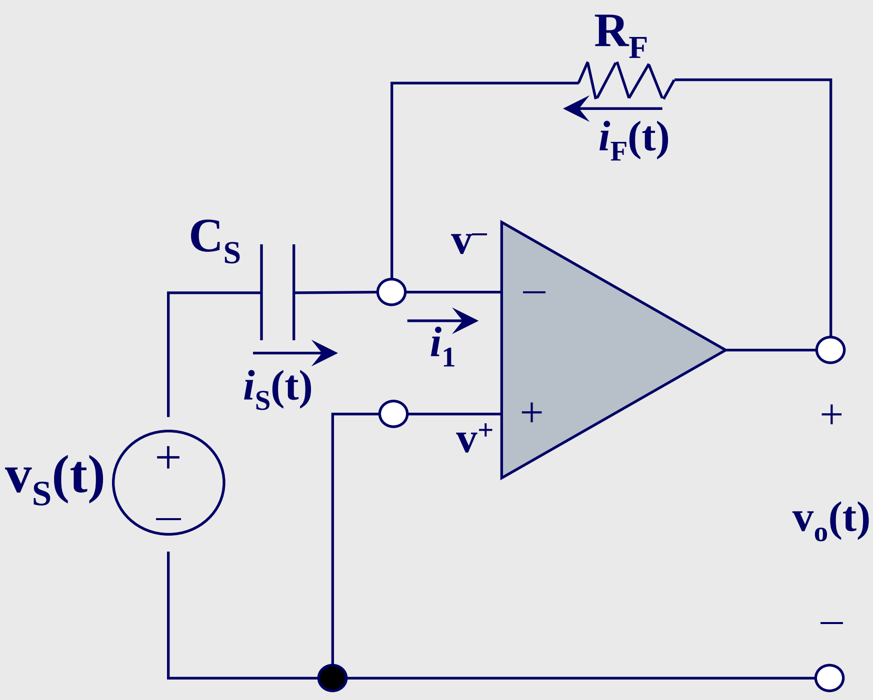


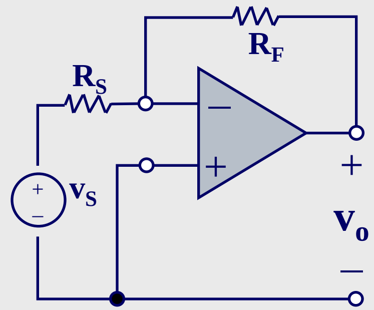
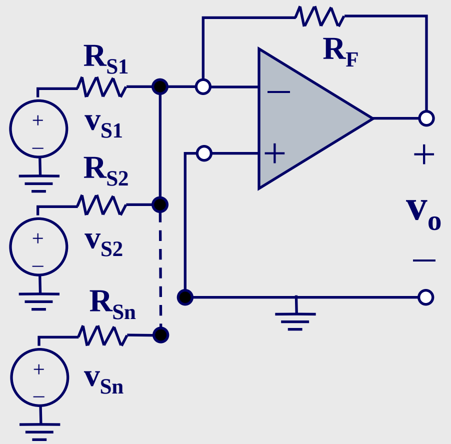
$$i_S(t) = -i_F(t)$$

$$C_S \frac{d\left(v_S(t) - v^-(t)\right)}{dt} = -\frac{v_o(t)}{R_F}$$

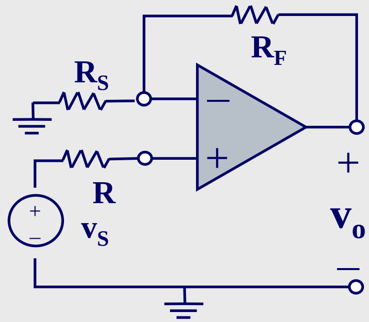
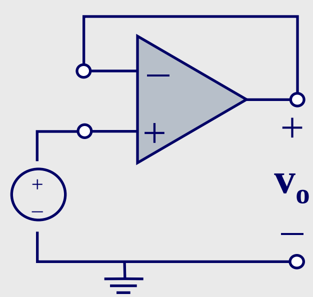
$$v_o(t) = -R_F C_S \frac{dv_S(t)}{dt}$$

NB: this type of differentiator is rarely used in practice since it amplifies noise

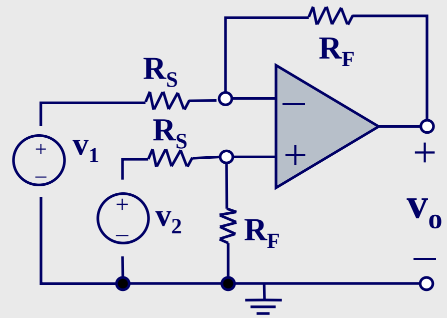
Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Inverting Amplifier		$v_o = A_{CL} v_S = -\frac{R_F}{R_S} v_S$
Summing Amplifier		$v_o = \sum_{n=1}^{N} A_{OLn} v_{Sn} = -\sum_{n=1}^{N} \frac{R_F}{R_n} v_{Sn}$

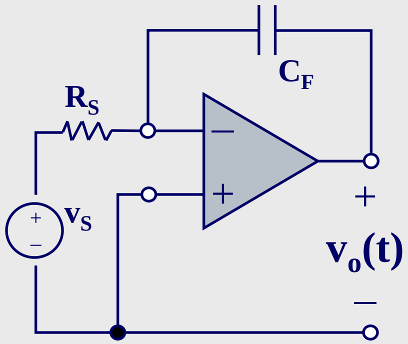
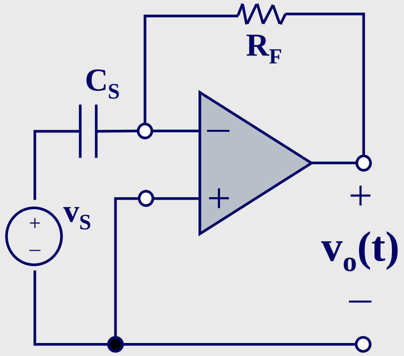
Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Noninverting Amplifier		$v_o = A_{CL} v_S = \left(1 + \frac{R_F}{R_S}\right) v_S$
Voltage Follower		$v_o = A_{CL} v_s = v_s$

Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Differential Amplifier		$v_o = \frac{R_F}{R_S}(v_{S2} - v_{S1})$

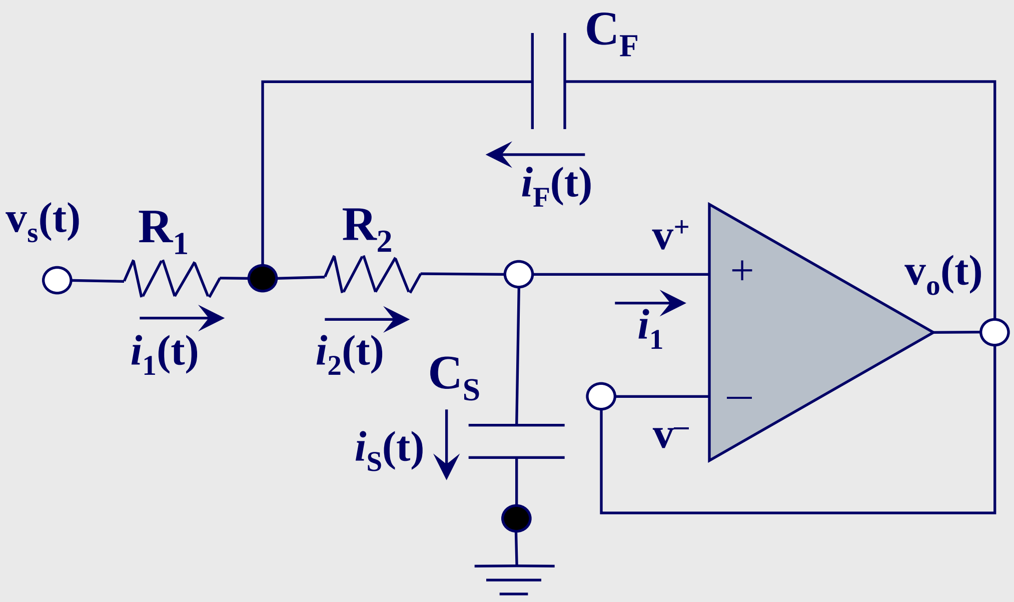
Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Ideal Integrator		$v_o = -\frac{1}{R_F C_S}\int_{-\infty}^{t} v_S(\tau)d\tau$
Ideal Differentiator		$v_o = -R_F C_S \frac{dv_S(t)}{dt}$

Op-Amps

<u>Example3</u>: find an expression for the gain if $\mathbf{v_s(t)}$ is sinusoidal

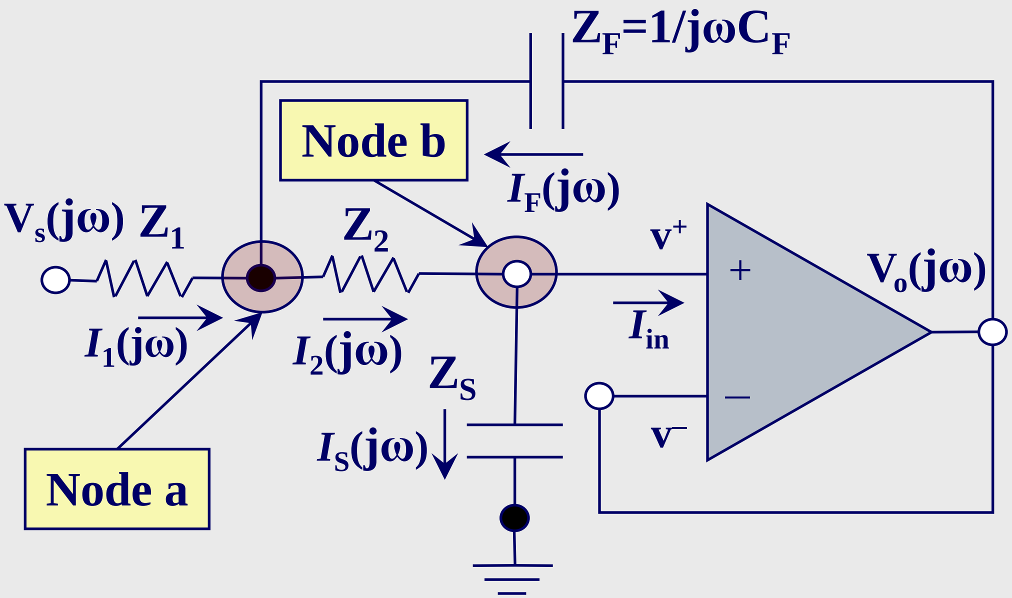
$\mathbf{C_F}$ = 1/6 F, $\mathbf{R_1}$ = 3Ω, $\mathbf{R_2}$ = 2Ω, $\mathbf{C_S}$ = 1/6 F



Op-Amps

Example3: find an expression for the gain

$C_F = 1/6$ F, $R_1 = 3\Omega$, $R_2 = 2\Omega$, $C_S = 1/6$ F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**

KCL at a :

$$I_1 + I_F - I_2 = 0$$

$$\frac{V_S - V_a}{Z_1} + \frac{V_o - V_a}{Z_F} - \frac{V_a - V^+}{Z_2} = 0$$

$$V_a\left(\frac{1}{Z_1} + \frac{1}{Z_F} + \frac{1}{Z_2}\right) - V_o\left(\frac{1}{Z_2} + \frac{1}{Z_F}\right) = V_S\left(\frac{1}{Z_1}\right)$$

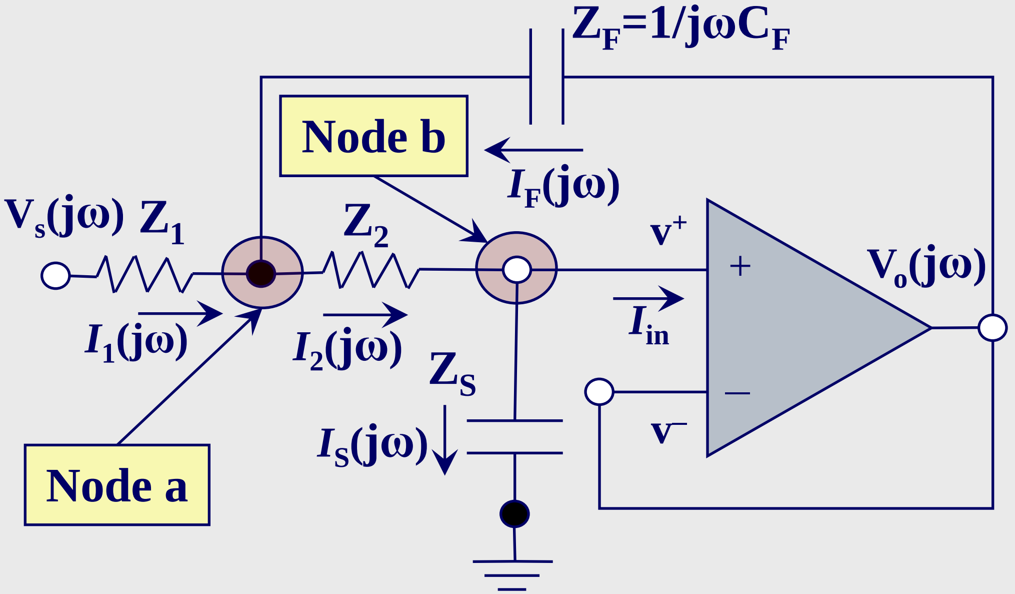
$$V_a\left(\frac{1}{3} + \frac{j\omega}{6} + \frac{1}{2}\right) - V_o\left(\frac{1}{2} + \frac{j\omega}{6}\right) = V_S\left(\frac{1}{3}\right)$$

NB: $v_+ = v^- = v_o$
and $I_{in} = 0$

Op-Amps

Example3: find an expression for the gain

$\mathbf{C_F}$ = 1/6 F, $\mathbf{R_1}$ = 3Ω, $\mathbf{R_2}$ = 2Ω, $\mathbf{C_S}$ = 1/6 F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**

KCL at b :

$$I_2 - I_S - I_{in} = 0$$

$$\frac{V_a - V_o}{Z_2} - \frac{V_o - 0}{Z_S} = 0$$

$$V_a\left(\frac{1}{Z_2}\right) - V_o\left(\frac{1}{Z_S} + \frac{1}{Z_2}\right) = 0$$

$$V_a\left(\frac{1}{2}\right) - V_o\left(\frac{j\omega}{6} + \frac{1}{2}\right) = 0$$

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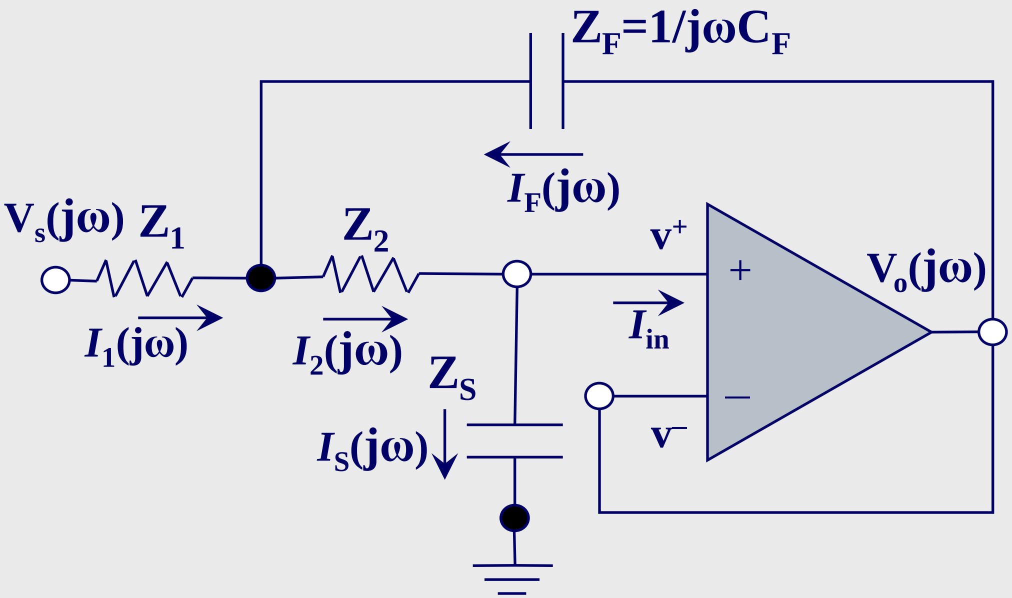
<u>Example3</u>: find an expression for the gain

C_F = 1/6 F, R_1 = 3Ω, R_2 = 2Ω, C_S = 1/6 F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**
3. Express $\mathbf{V_o}$ in terms of $\mathbf{V_s}$

$$V_a(5+j\omega) - V_o(3+j\omega) = 2V_S$$

$$3V_a - V_o(3+j\omega) = 0$$

$$V_o = -\frac{6V_S}{\omega^2 - j5\omega - 6}$$

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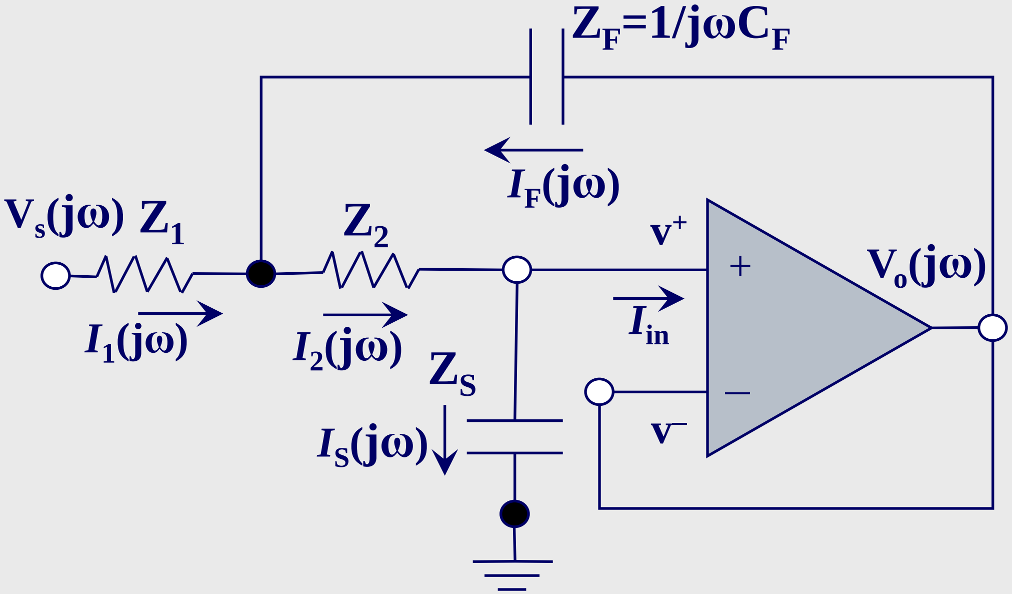
Example3: find an expression for the gain

C_F = 1/6 F, **R_1** = 3Ω, **R_2** = 2Ω, **C_S** = 1/6 F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**
3. Express **V_o** in terms of **V_S**
4. Find the gain (**V_o/V_S**)

$$\frac{V_o}{V_S} = -\frac{6}{\omega^2 - j5\omega - 6}$$