Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
10 Nov	Mon	20	Exam Review			LAB 7	EXAM 2
11 Nov	Tue						
12 Nov	Wed	21	Boolean Algebra	13.2 – 13.3			
13 Nov	Thu						
14 Nov	Fri		Recitation				
15 Nov	Sat						
16 Nov	Sun						
17 Nov	Mon	22	Combinational Logic	13.3 – 13.5		LAB 10	
18 Nov	Tue						

Ask

Alma 5:26

26 And now behold, I say unto you, my brethren, if ye have experienced a change of heart, and if ye have felt to sing the song of redeeming love, **I would ask**, can ye feel so now?

Lecture 20 – Exam 2 Review

Chapters 4 – 6, 8

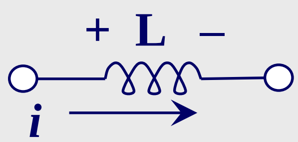
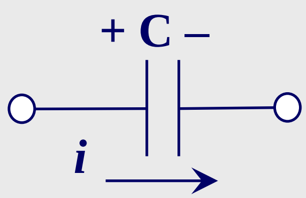
Exam 2

- 12 – 16 November (Monday – Friday)
- Chapters 4 – 6 and 8
- 15 questions
  - 12 multiple choice (answer on bubble sheet!)
    - 1 point each
  - 3 long answer (show your work!)
    - 4 or 5 points each
- Closed book!
  - One 3x5 card allowed
- Calculators allowed
- No time limit
- Study lecture slides and homework

Exam 2 Review…Overview

1. Capacitors and Inductors
2. Measuring Signal Strength
3. Phasors
4. Impedance
5. AC RLC Circuits
6. AC Equivalent Circuits
7. DC Transient Response
8. Frequency Response
9. Basic Filters
10. Op-Amps

Capacitors & Inductors

	Inductors	Capacitors
Passive sign convention		
Voltage	$v(t) = L\dfrac{di(t)}{dt}$	$v(t) = \dfrac{1}{C}\int_{t_0}^{t} i_C(\tau)d\tau + v(t_0)$
Current	$i(t) = \dfrac{1}{L}\int_{t_0}^{t} v_L(\tau)d\tau + i(t_0)$	$i(t) = C\dfrac{dv(t)}{dt}$
Power	$P_L(t) = Li(t)\dfrac{di(t)}{dt}$	$P_C(t) = Cv(t)\dfrac{dv(t)}{dt}$

Capacitors & Inductors

	Inductors	Capacitors
Energy	$W_L(t) = \frac{1}{2} L i(t)^2$	$W_C(t) = \frac{1}{2} C v(t)^2$
An **instantaneous** change is **not permitted** in:	Current	Voltage
Will permit an **instantaneous** change in:	Voltage	Current
With **DC source** element acts as a:	Short Circuit	Open Circuit

Capacitors & Inductors

1. What is the difference between the voltage and current behaviour of capacitors and inductors?

Capacitors & Inductors

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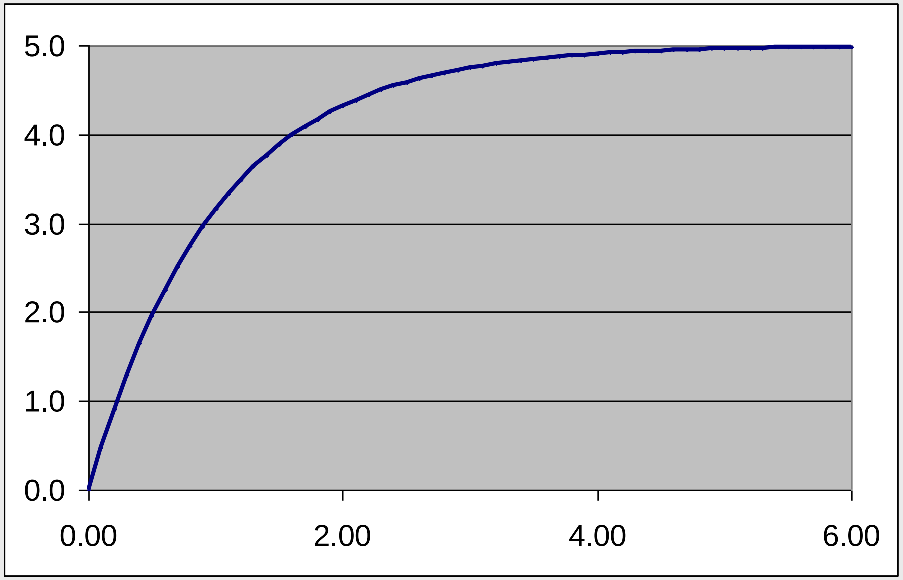
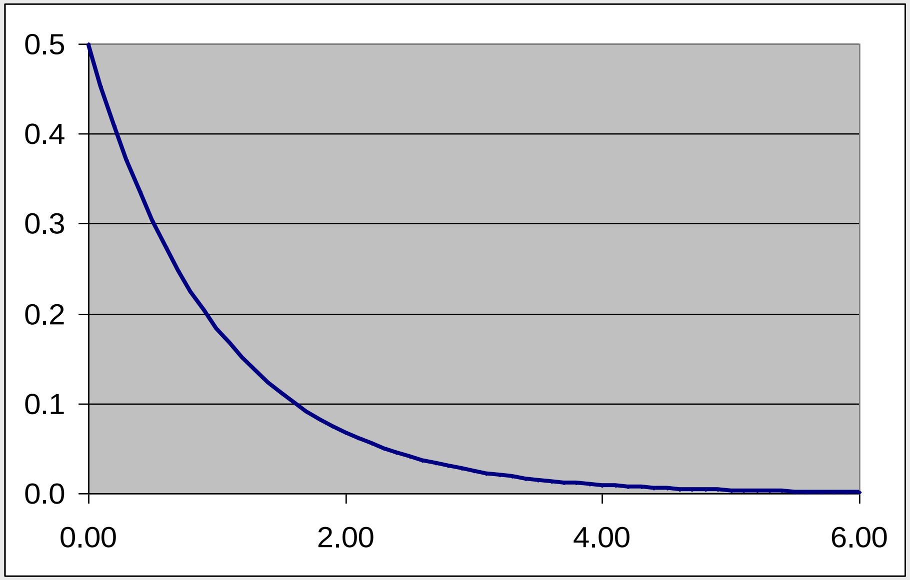


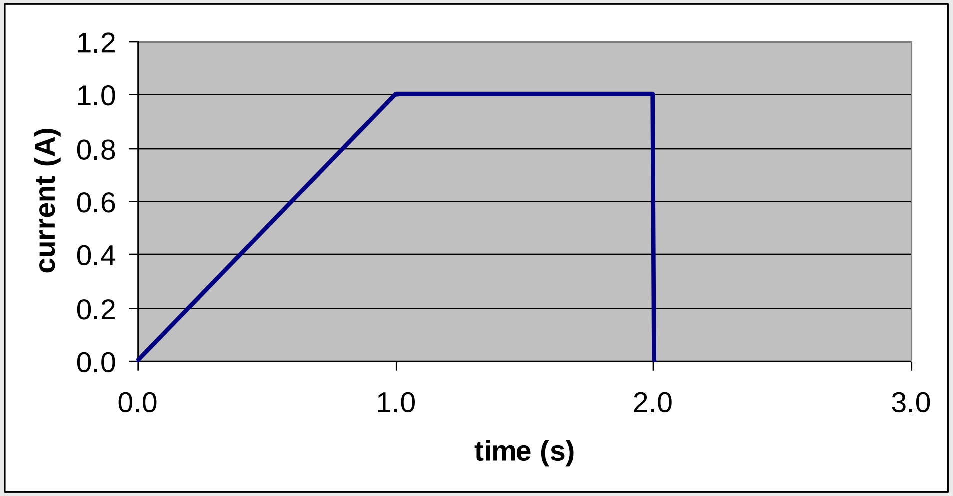


Capacitor voltage $\mathbf{v_C(t)}$
Inductor current $\boldsymbol{i_L}\mathbf{(t)}$
NB: neither can change instantaneously

Capacitor current $\boldsymbol{i_C}\mathbf{(t)}$
Inductor voltage $\mathbf{v_L(t)}$
NB: both can change instantaneously

Capacitors & Inductors

2. find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0



Capacitors & Inductors

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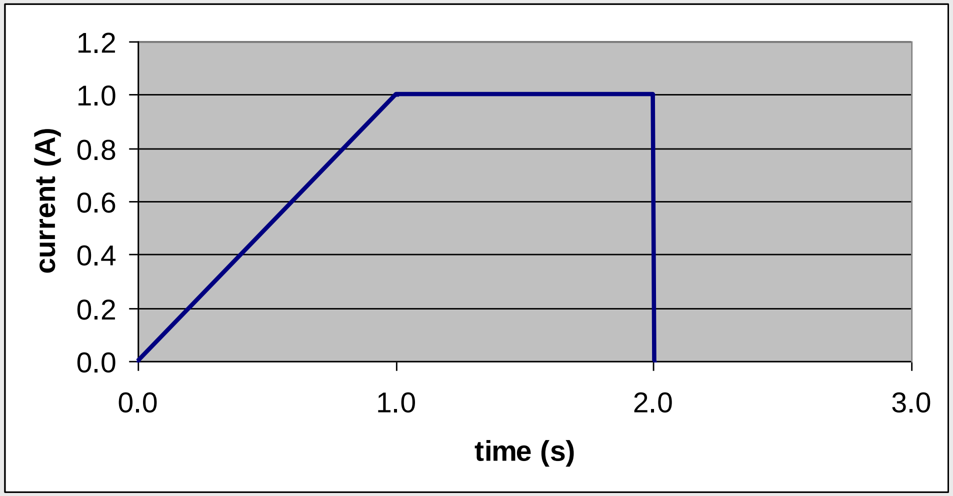


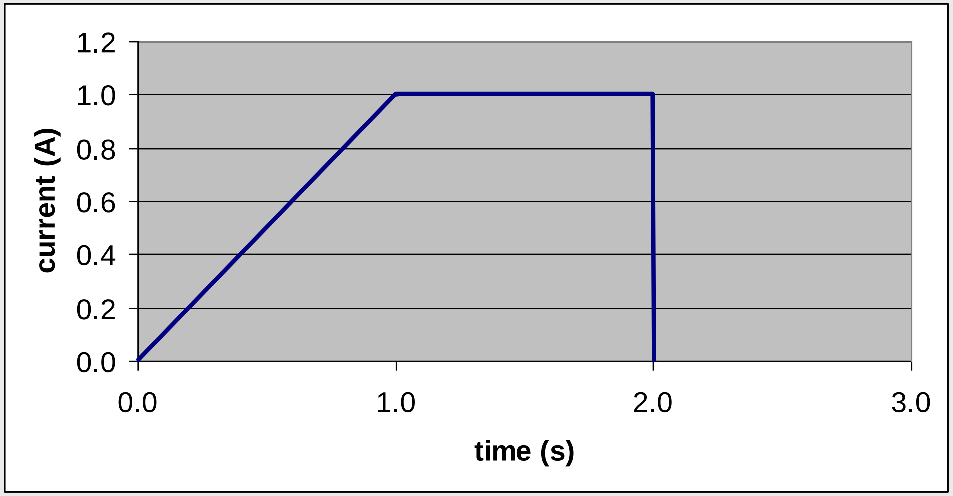
current i(t) in 4 intervals :

$$
\begin{aligned}
i &= 0 & t &\le 0 \\
&= t & 0 < t &\le 1 \\
&= 1 & 1 < t &\le 2 \\
&= 0 & 2 &< t
\end{aligned}
$$

$$v(t) = \frac{1}{C}\int_0^t i\,d\tau + v(0)$$

Capacitors & Inductors

2. find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0



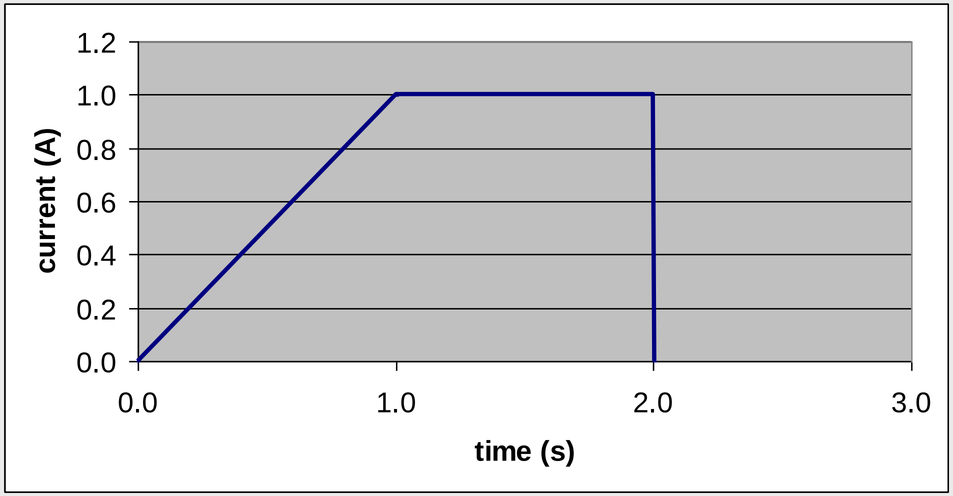
voltage v(t) in 4 intervals :

$$
\begin{aligned}
v &= 0 & t \le 0 \\
&= 2\int_0^t \tau d\tau + 0 & 0 < t \le 1 \\
&= 2\int_1^t (1) d\tau + v(1) & 1 < t \le 2 \\
&= 0 + v(2) & 2 < t
\end{aligned}
$$

$$v(t) = \frac{1}{C}\int_0^t i\,d\tau + v(0)$$

Capacitors & Inductors

2. find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0

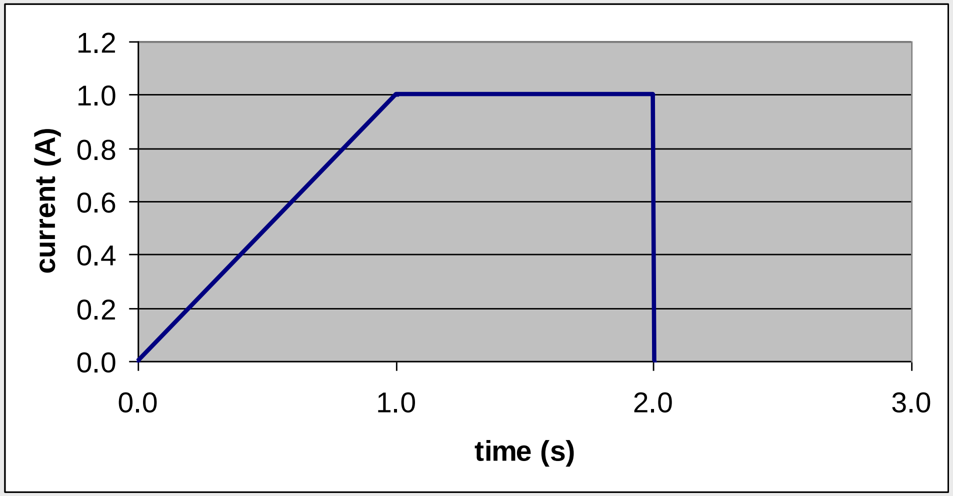


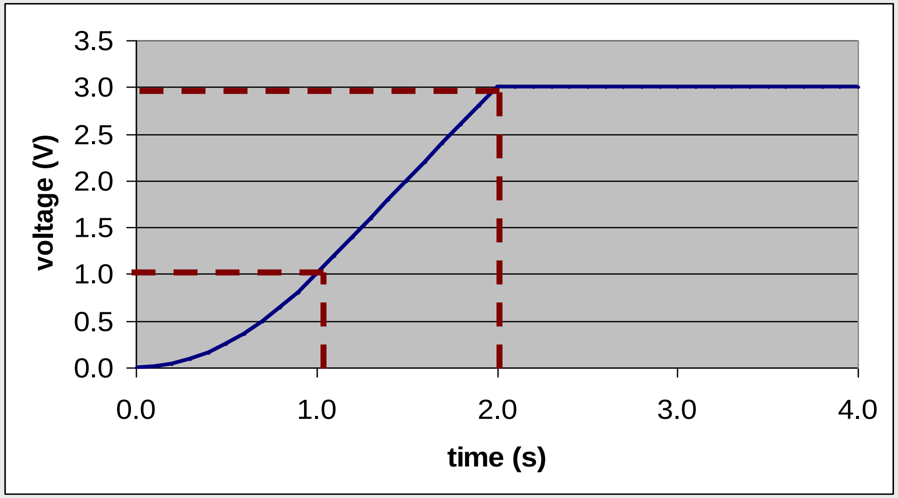
voltage v(t) in 4 intervals :

$$
\begin{aligned}
v &= 0 & t &\le 0 \\
&= t^2 & 0 < t &\le 1 \\
&= 2t - 1 & 1 < t &\le 2 \\
&= 3 & 2 &< t
\end{aligned}
$$

Capacitors & Inductors

2. find the voltage v(t) for a capacitor C = 0.5F with the current as shown and v(0) = 0





NB: The final value of the capacitor voltage after the current source has stopped charging the capacitor depends on two things:

1. The initial capacitor voltage
2. The history of the capacitor current

voltage v(t) in 4 intervals :

$$
\begin{aligned}
v &= 0 & t &\le 0 \\
&= t^2 & 0 &< t \le 1 \\
&= 2t-1 & 1 &< t \le 2 \\
&= 3 & 2 &< t
\end{aligned}
$$

Measuring Signal Strength

3. Compute the rms value of the sinusoidal current
   $i(t) = I \cos(\omega t)$

Measuring Signal Strength

3. Compute the rms value of the sinusoidal current **i(t) = I cos(ωt)**

$$\cos^2(t) = \frac{\cos(2t)+1}{2}$$

Integrating a sinusoidal waveform over 2 periods equals zero

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\cos^2(\omega\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\left[\frac{1}{2}+\frac{1}{2}\cos(2\omega\tau)\right]d\tau}$$

$$= \sqrt{\frac{1}{2}I^2 + \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\frac{I^2}{2}\cos(2\omega\tau)d\tau}$$

$$= \sqrt{\frac{1}{2}I^2 + 0}$$

$$= \frac{I}{\sqrt{2}}$$

Measuring Signal Strength

3. Compute the rms value of the sinusoidal current **i(t) = I cos(ωt)**

$$i_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\cos^2(2\omega\tau)d\tau}$$

$$= \sqrt{\frac{\omega}{2\pi}\int_0^{2\pi/\omega} I^2\left[\frac{1}{2}+\frac{1}{2}\cos(2\omega\tau)\right]d\tau}$$

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$$= \sqrt{\frac{1}{2}I^2 + 0}$$

$$= \frac{I}{\sqrt{2}}$$

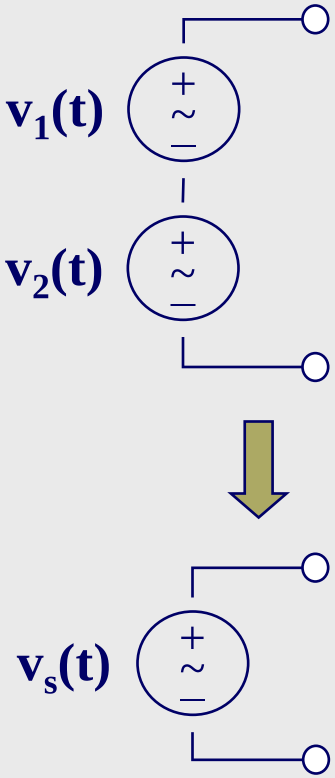
The RMS value of any sinusoid signal is always equal to 0.707 times the peak value (regardless of phase or frequency)

Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

$\mathbf{v_1(t) = 15cos(377t+\pi/4)}$

$\mathbf{v_2(t) = 15cos(377t+\pi/12)}$



Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

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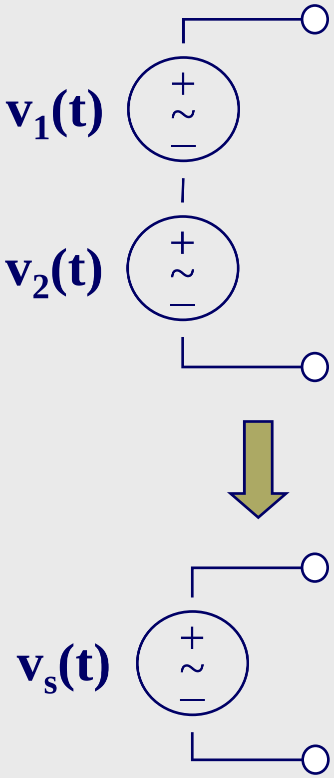

1. Write voltages in phasor notation

$$V_1(j\omega) = 15e^{j\pi/4} = 15\angle\frac{\pi}{4}\ V$$

$$V_2(j\omega) = 15e^{j\pi/12} = 15\angle\frac{\pi}{12}\ V$$

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Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

$v_1(t) = 15\cos(377t+\pi/4)$

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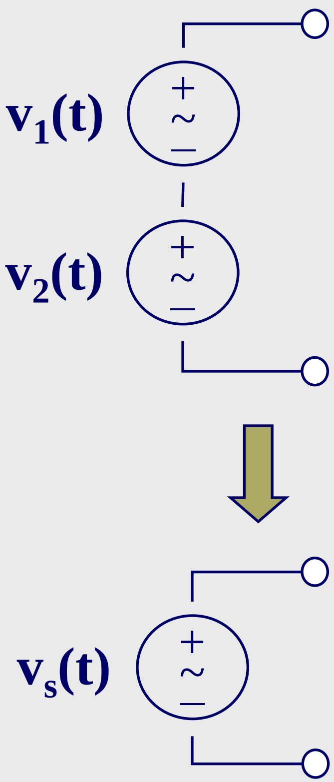


1. Write voltages in phasor notation
2. Convert phasor voltages from polar to rectangular form (see Appendix A)

$$V_1(j\omega) = 15\angle\frac{\pi}{4}\ V$$

Convert to rectangular:

$$V_1(j\omega) = 15\cos\left(\frac{\pi}{4}\right) + j15\sin\left(\frac{\pi}{4}\right)$$

$$= 10.61 + j10.61\ V$$

$$V_2(j\omega) = 15\angle\frac{\pi}{12}\ V$$

Convert to rectangular:

$$V_2(j\omega) = 15\cos\left(\frac{\pi}{12}\right) + j15\sin\left(\frac{\pi}{12}\right)$$

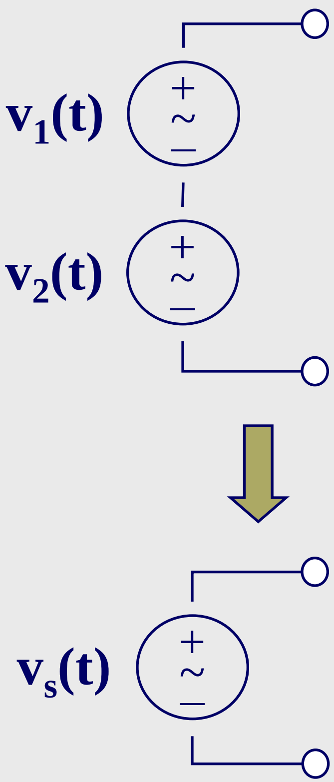
$$= 14.49 + j3.88\ V$$

Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

$v_1(t) = 15\cos(377t+\pi/4)$

$v_2(t) = 15\cos(377t+\pi/12)$



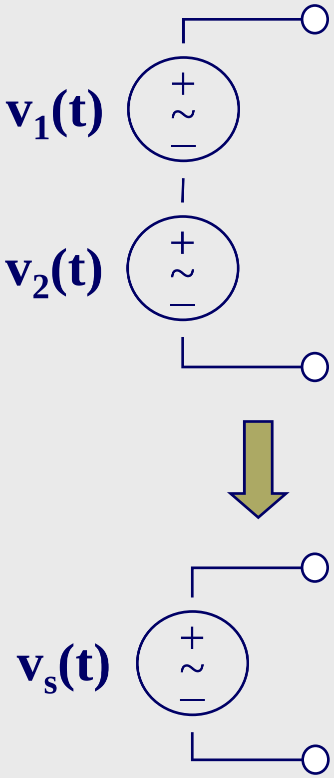
1. Write voltages in phasor notation
2. Convert phasor voltages from polar to rectangular form (see Appendix A)
3. Combine voltages

$$V_S(j\omega) = V_1(j\omega) + V_2(j\omega)$$
$$= 25.10 + j14.49$$

Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

$v_1(t) = 15\cos(377t+\pi/4)$
$v_2(t) = 15\cos(377t+\pi/12)$



1. Write voltages in phasor notation
2. Convert phasor voltages from polar to rectangular form (see Appendix A)
3. Combine voltages
4. Convert rectangular back to polar

$$V_S(j\omega) = 25.10 + j14.49$$

Convert to polar :

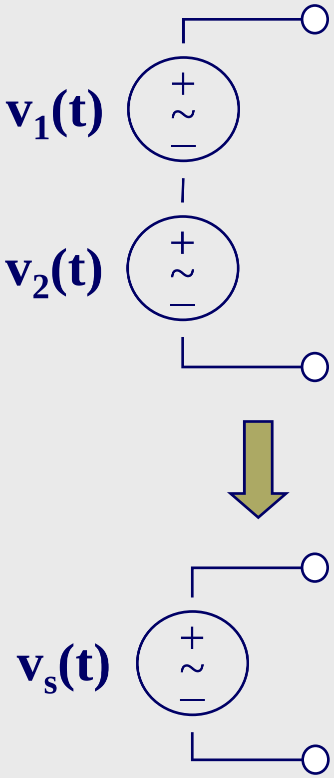
$$r = \sqrt{(25.10)^2 + (14.49)^2}$$
$$= 28.98$$
$$\theta = \tan^{-1}\left(\frac{14.49}{25.10}\right)$$
$$= \frac{\pi}{6}$$
$$V_S(j\omega) = 28.98\angle\frac{\pi}{6}$$

Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

$\mathbf{v_1(t) = 15cos(377t+\pi/4)}$

$\mathbf{v_2(t) = 15cos(377t+\pi/12)}$



1. Write voltages in phasor notation
2. Convert phasor voltages from polar to rectangular form (see Appendix A)
3. Combine voltages
4. Convert rectangular back to polar
5. Convert from phasor to time domain

$$V_S(j\omega) = 28.98\angle\frac{\pi}{6}$$

$$v_S(t) = 28.98\cos\left(377t + \frac{\pi}{6}\right)$$

Bring **ωt** back

NB: the answer is **NOT** simply the addition of the amplitudes of $\mathbf{v_1(t)}$ and $\mathbf{v_2(t)}$ (i.e. 15 + 15), and the addition of their phases (i.e. $\pi/4 + \pi/12$)

Phasors

4. compute the phasor voltage for the equivalent voltage $\mathbf{v_s(t)}$

   $\mathbf{v_1(t) = 15\cos(377t+\pi/4)}$

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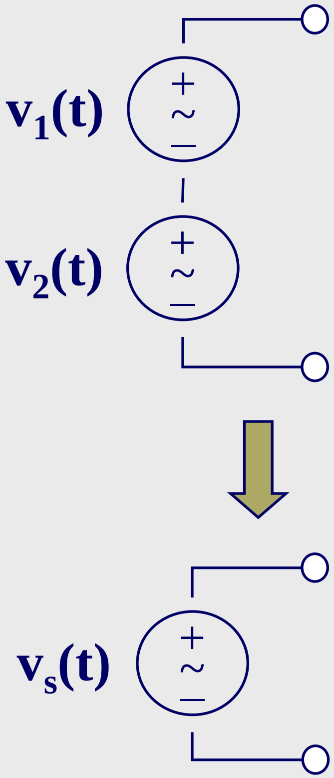

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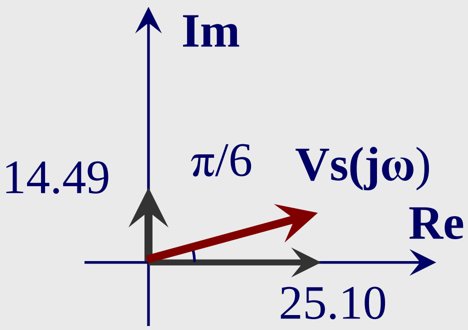
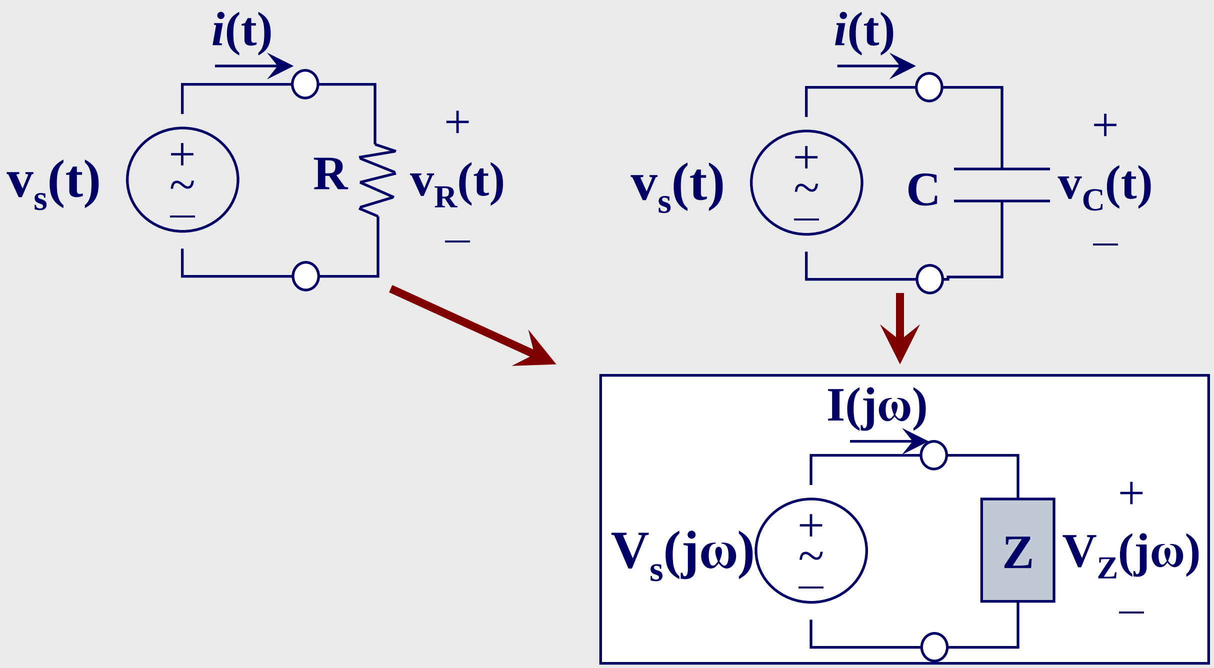


Impedance

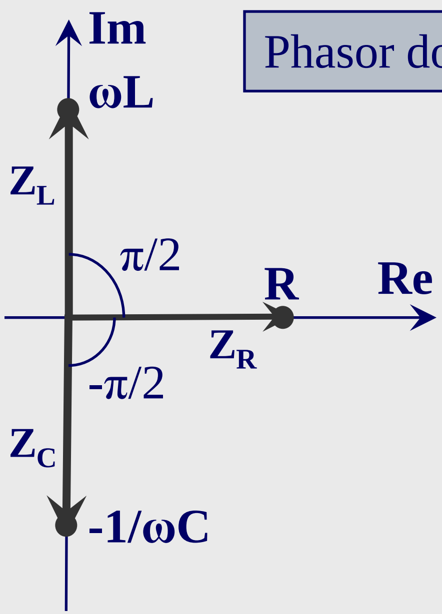
Impedance: complex resistance (has no physical significance)

- will allow us to use network analysis methods such as node voltage, mesh current, etc.
- Capacitors and inductors act as **frequency-dependent** resistors

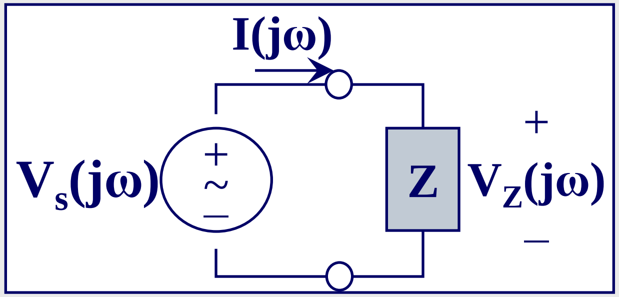


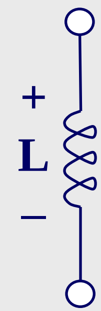
Impedance

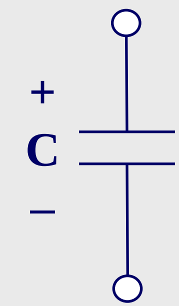
Impedance of resistors, inductors, and capacitors



Phasor domain







$$Z_R(j\omega) = R$$

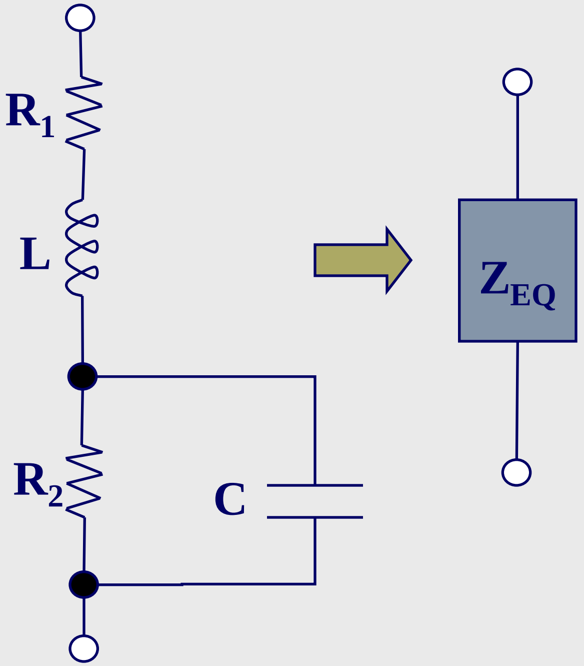
$$Z_L(j\omega) = j\omega L$$

$$Z_C(j\omega) = \frac{1}{j\omega C}$$

Impedance

5. find the equivalent impedance ($\mathbf{Z_{EQ}}$)

$\boldsymbol{\omega}$ = 10^4 rads/s, **C** = 10uF, $\mathbf{R_1}$ = 100Ω, $\mathbf{R_2}$ = 50Ω, **L** = 10mH



Impedance

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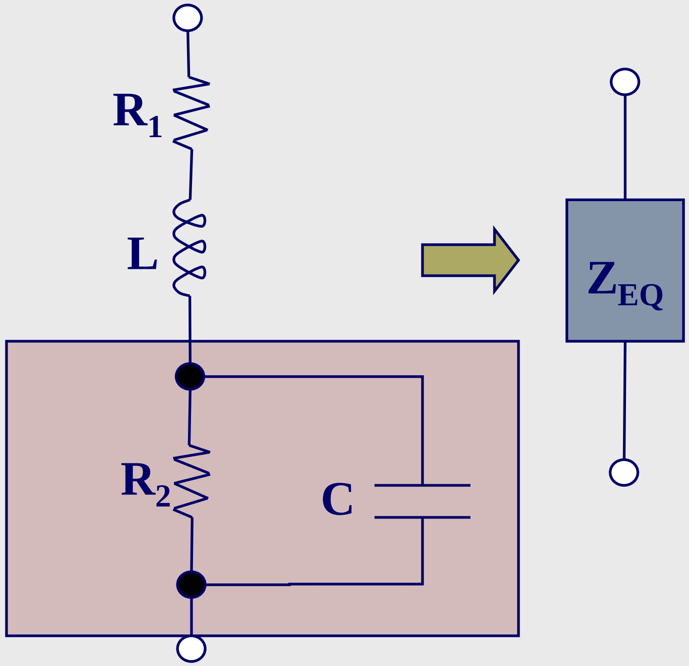
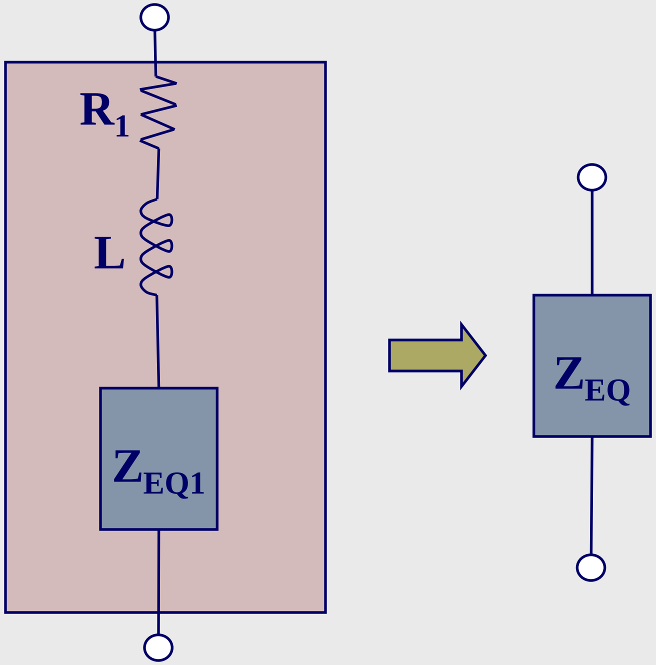


$$
\begin{aligned}
Z_{EQ1} &= Z_{R2} \parallel Z_C \\
&= \frac{R_2(1/j\omega C)}{R_2 + (1/j\omega C)} \\
&= \frac{R_2}{1 + j\omega C R_2} \\
&= \frac{50}{1 + j(10^4)(10\times10^{-6})(50)} \\
&= \frac{50}{1+j5} \\
&= 1.92 - j9.62 \\
&= 9.81\angle(-1.37)\ \Omega
\end{aligned}
$$

Impedance

5. find the equivalent impedance (**Z_{EQ}**)

ω = 10^4 rads/s, **C** = 10uF, **R_1** = 100Ω, **R_2** = 50Ω, **L** = 10mH



$$
\begin{aligned}
Z_{EQ} &= Z_{R1} + Z_L + Z_{EQ1} \\
&= R_1 + j\omega L + 9.81\angle(-1.37) \\
&= 100 + j(10^4)(10^{-2}) + 1.92 - j9.62 \\
&= 101.92 + j90.38 \\
&= 136.2\angle 0.723\ \Omega
\end{aligned}
$$

NB: at this frequency (**ω**) the circuit has an **inductive impedance** (reactance or phase is positive)

$$Z_{EQ1} = 9.81\angle(-1.37)\ \Omega$$

AC RLC Circuits

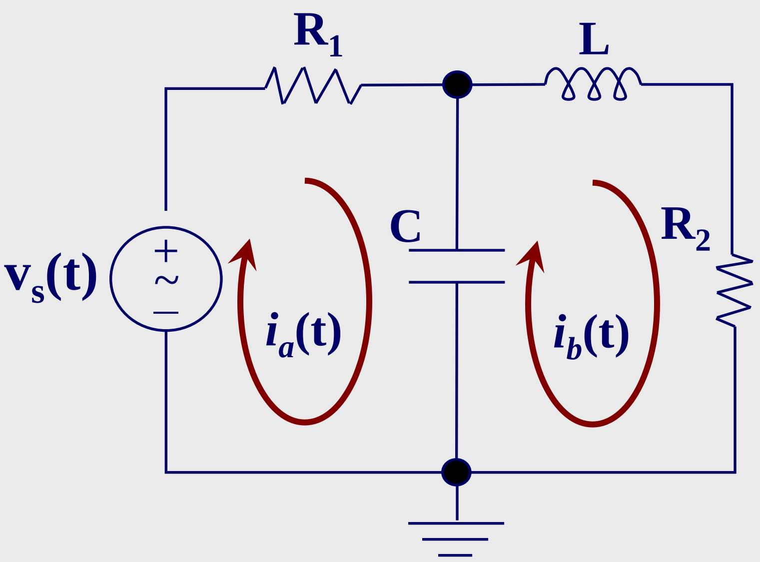
AC Circuit Analysis

1. Identify the AC sources and note the excitation frequency (ω)
2. Convert all sources to the phasor domain
3. Represent each circuit element by its impedance
4. Solve the resulting phasor circuit using network analysis methods
5. Convert from the phasor domain back to the time domain

AC RLC Circuits

6. find $i_a(t)$ and $i_b(t)$

$v_s(t) = 15\cos(1500t)$V, $R_1 = 100\Omega$, $R_2 = 75\Omega$, $L = 0.5$H, $C = 1$uF



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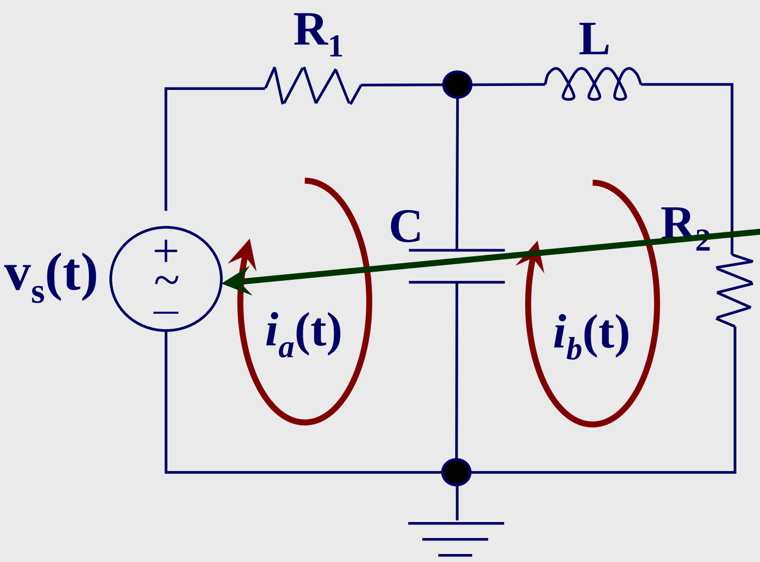


1. Note frequencies of AC sources

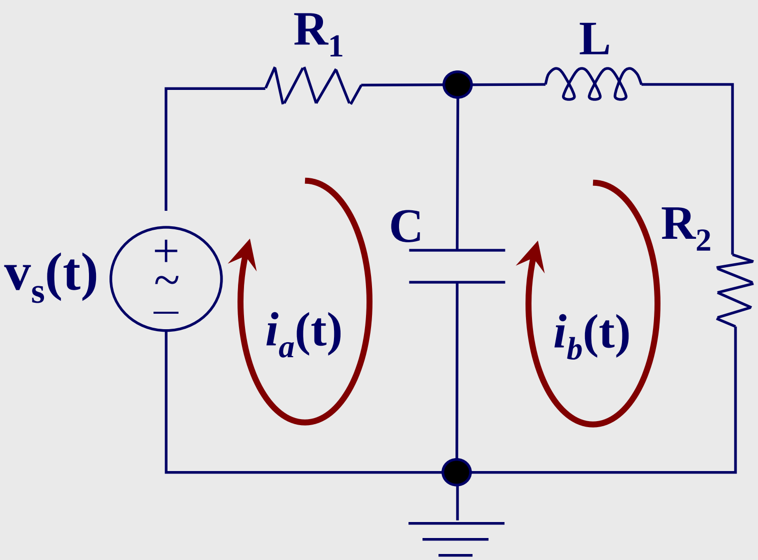
Only one AC source - ω = 1500 rad/s

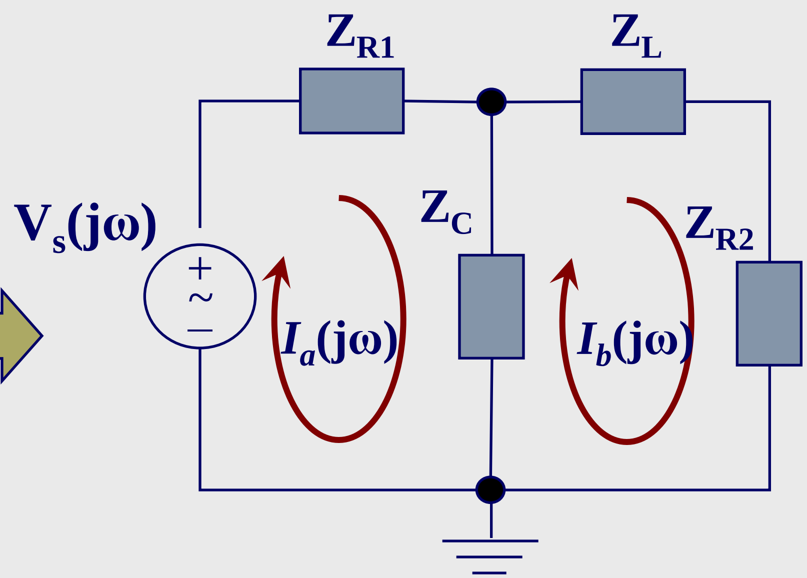
AC RLC Circuits

6. find $i_a(t)$ and $i_b(t)$

$\mathbf{v_s(t)}$ = 15cos(**1500**t)V, $\mathbf{R_1}$ = 100Ω, $\mathbf{R_2}$ = 75Ω, **L** = 0.5H, **C** = 1uF

1. Note frequencies of AC sources
2. Convert to phasor domain





AC RLC Circuits

6. find $i_a(t)$ and $i_b(t)$

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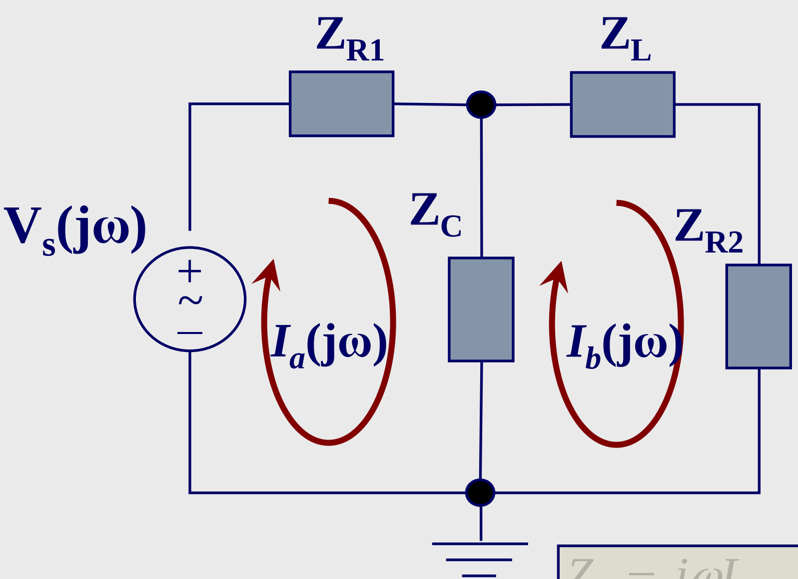


1. Note frequencies of AC sources
2. Convert to phasor domain
3. Represent each element by its impedance

$$v_s(t) = 15\cos(\omega t)$$
$$V_s(j\omega) = 15\angle 0 = 15e^{j0}$$

$$Z_{R1} = R_1 = 100\Omega$$

$$Z_L = j\omega L = j(1500)(0.5) = j750\Omega$$

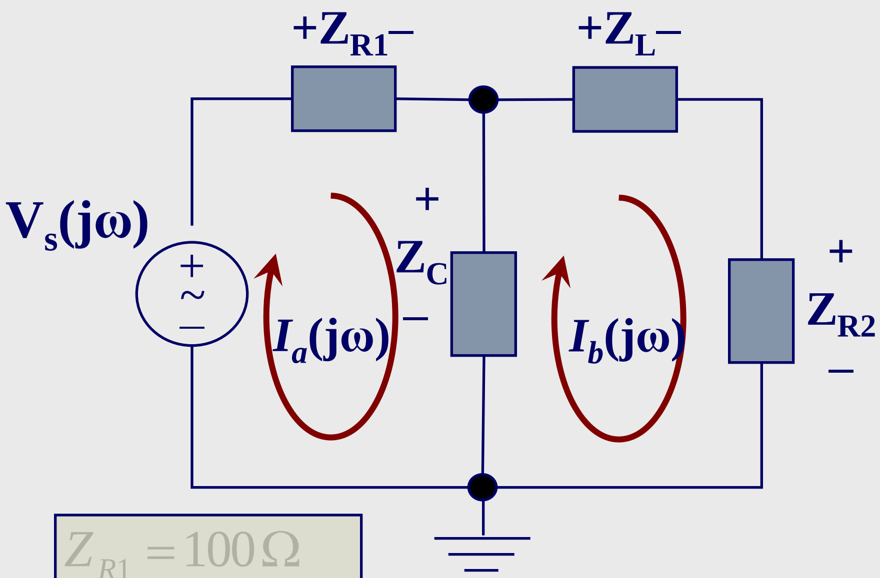
$$Z_{R2} = R_2 = 75\Omega$$

$$Z_C = 1/j\omega C = 1/j(1500)(10^{-6}) = -j667\Omega$$

AC RLC Circuits

6. find $\boldsymbol{i_a}$**(t)** and $\boldsymbol{i_b}$**(t)**

$\mathbf{v_s(t)} = 15\cos(\mathbf{1500}t)$V, $\mathbf{R_1} = 100\Omega$, $\mathbf{R_2} = 75\Omega$, $\mathbf{L} = 0.5$H, $\mathbf{C} = 1$uF



$$Z_{R1} = 100\,\Omega$$
$$Z_{R2} = 75\Omega$$
$$Z_L = j750\,\Omega$$
$$Z_C = -j667\,\Omega$$

4. Solve using network analysis
   - Mesh current

KVL at a :

$$-V_s(j\omega) + V_{R1}(j\omega) + V_C(j\omega) = 0$$
$$I_a Z_{R1} + (I_a - I_b) Z_C = V_s$$
$$I_a(Z_{R1} + Z_C) - I_b Z_C = V_s$$

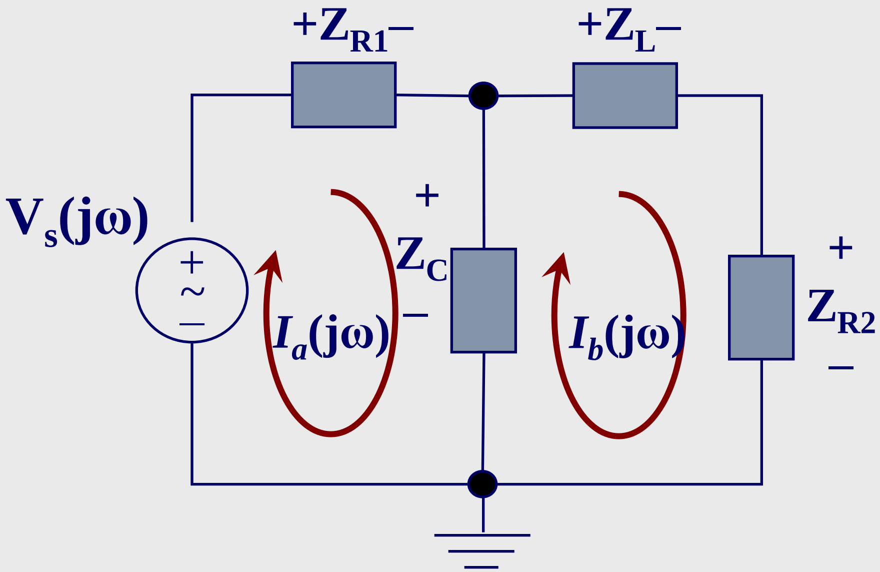
KVL at b :

$$-V_C(j\omega) + V_L(j\omega) + V_{R2}(j\omega) = 0$$
$$-(I_a - I_b) Z_C + I_b Z_L + I_b Z_{R2} = 0$$
$$-I_a Z_C + I_b (Z_C + Z_L + Z_{R2}) = 0$$

AC RLC Circuits

6. find $i_a(t)$ and $i_b(t)$

$v_s(t) = 15\cos(1500t)$V, $R_1 = 100\Omega$, $R_2 = 75\Omega$, $L = 0.5$H, $C = 1$uF



4. Solve using network analysis
   - Mesh current

$$I_a(100 - j667) + I_b(j667) = 15$$

$$I_a(j667) + I_b(75 + j83) = 0$$

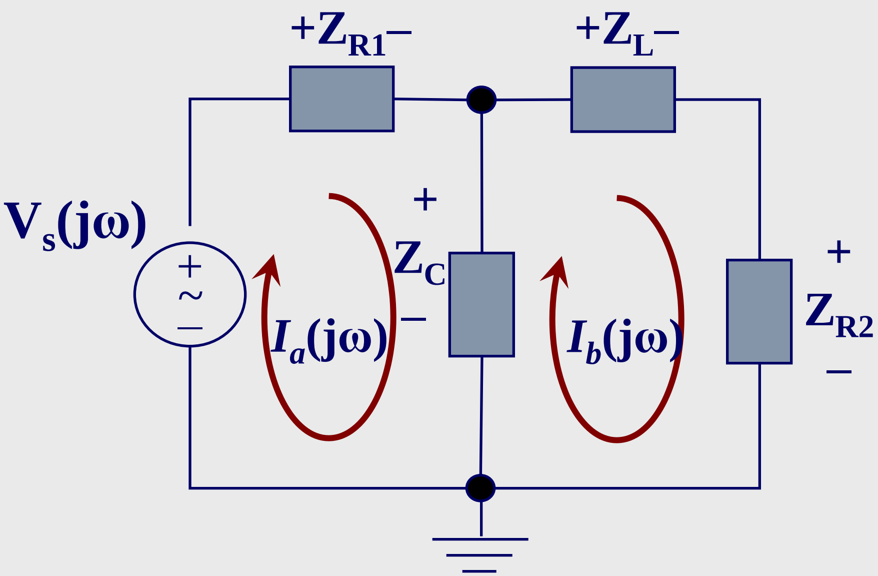
$$I_1 = 0.0032\angle 0.917\ A$$

$$I_2 = 0.019\angle -1.49\ A$$

AC RLC Circuits

6. find $i_a(t)$ and $i_b(t)$

$\mathbf{v_s(t)} = 15\cos(\mathbf{1500}t)\text{V}$, $\mathbf{R_1} = 100\Omega$, $\mathbf{R_2} = 75\Omega$, $\mathbf{L} = 0.5\text{H}$, $\mathbf{C} = 1\text{uF}$



5. Convert to Time domain

$$I_1 = 0.0032\angle 0.917\ A$$

$$i_1(t) = 3.2\cos(1500t + 0.917)\ mA$$

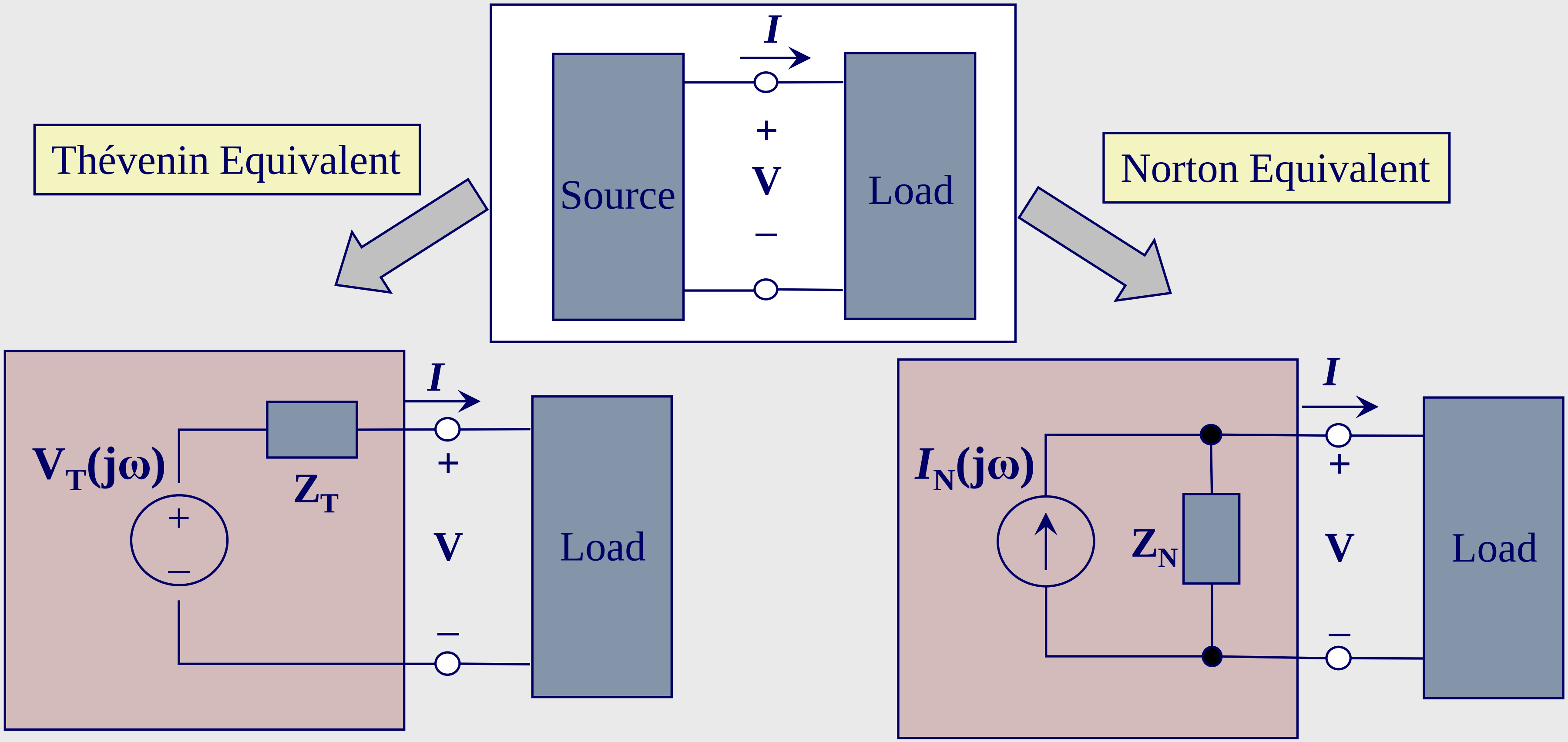
$$I_2 = 0.019\angle -1.49\ A$$

$$i_2(t) = 19\cos(1500t - 1.49)\ mA$$

AC Equivalent Circuits

Thévenin and **Norton** equivalent circuits apply in AC analysis

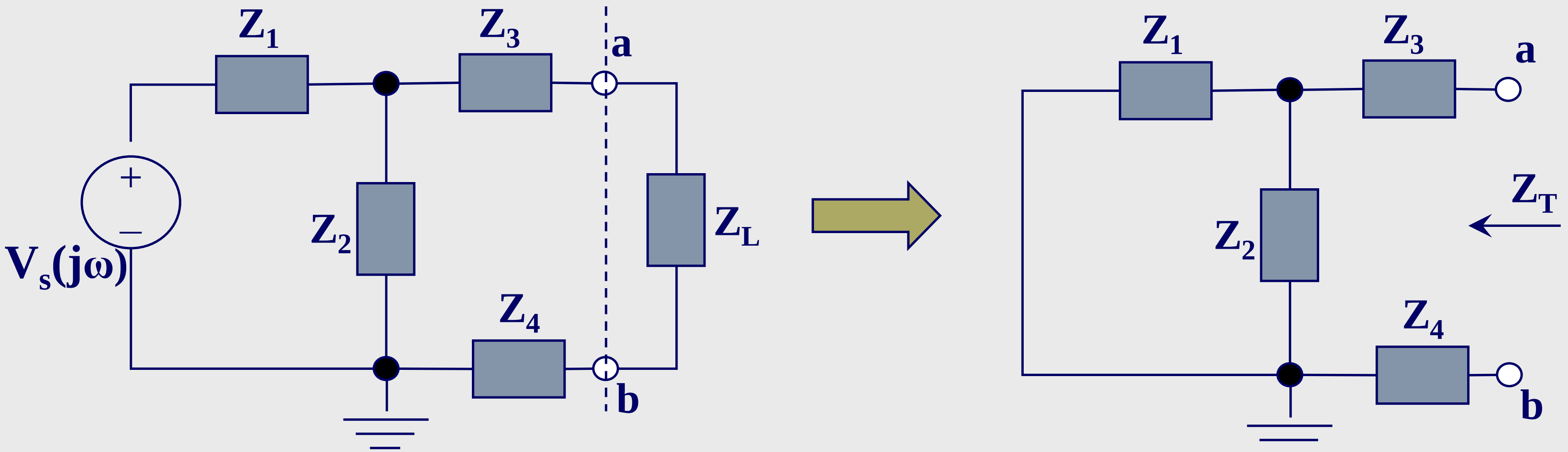
- Equivalent voltage/current will be **complex** and **frequency dependent**



AC Equivalent Circuits

<u>Computation of Thévenin and Norton Impedances</u>:

1. Remove the load (open circuit at load terminal)
2. Zero all independent sources
   - Voltage sources → short circuit (**v** = 0)
   - Current sources → open circuit (***i*** = 0)
3. Compute equivalent impedance **across load terminals** (with load removed)

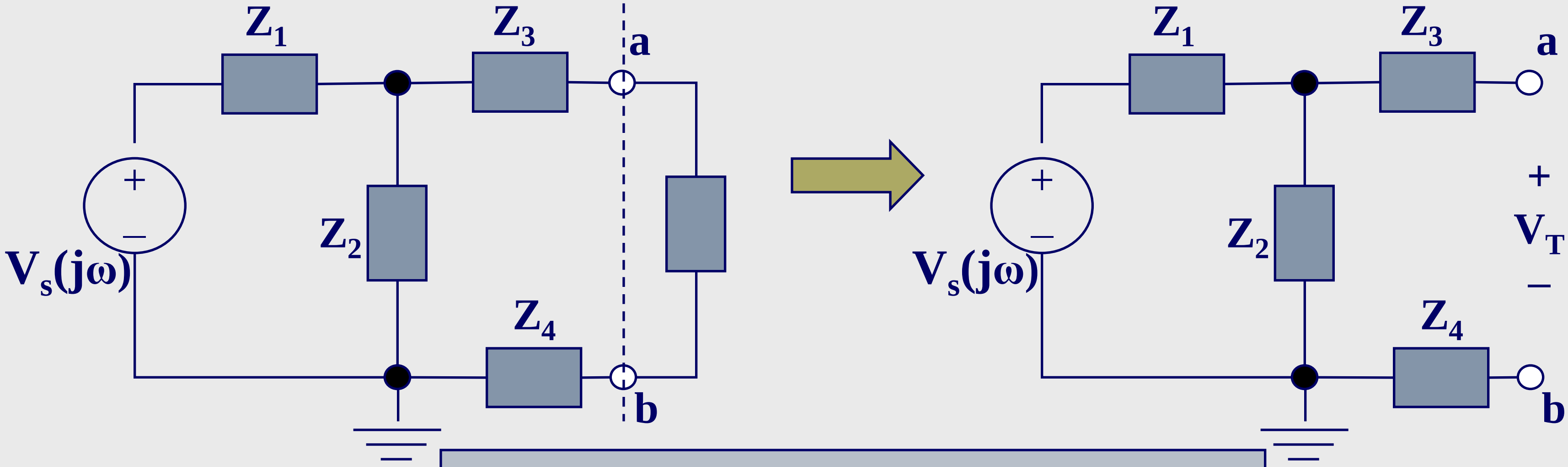


NB: same procedure as equivalent resistance

AC Equivalent Circuits

<u>Computing Thévenin voltage</u>:

1. Remove the load (open circuit at load terminals)
2. Define the open-circuit voltage ($\mathbf{V_{oc}}$) across the load terminals
3. Chose a network analysis method to find $\mathbf{V_{oc}}$
   - node, mesh, superposition, etc.
4. Thévenin voltage $\mathbf{V_T} = \mathbf{V_{oc}}$

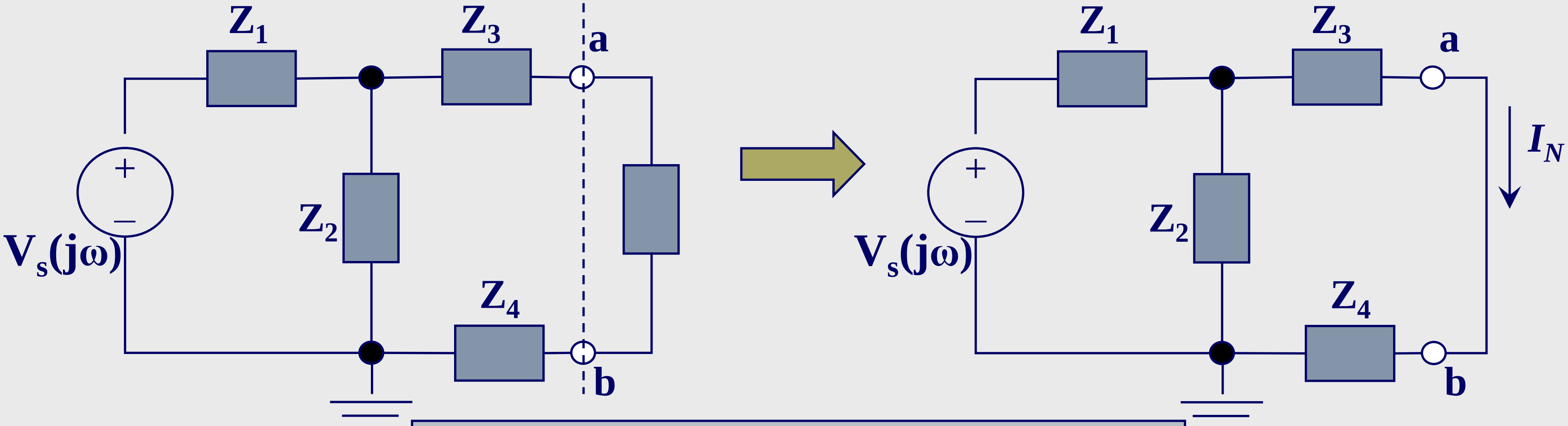


NB: same procedure as equivalent resistance

AC Equivalent Circuits

<u>Computing Norton current</u>:

1. Replace the load with a short circuit
2. Define the short-circuit current ($\boldsymbol{I_{sc}}$) across the load terminals
3. Chose a network analysis method to find $\boldsymbol{I_{sc}}$
   - node, mesh, superposition, etc.
4. Norton current $\boldsymbol{I_N} = \boldsymbol{I_{sc}}$

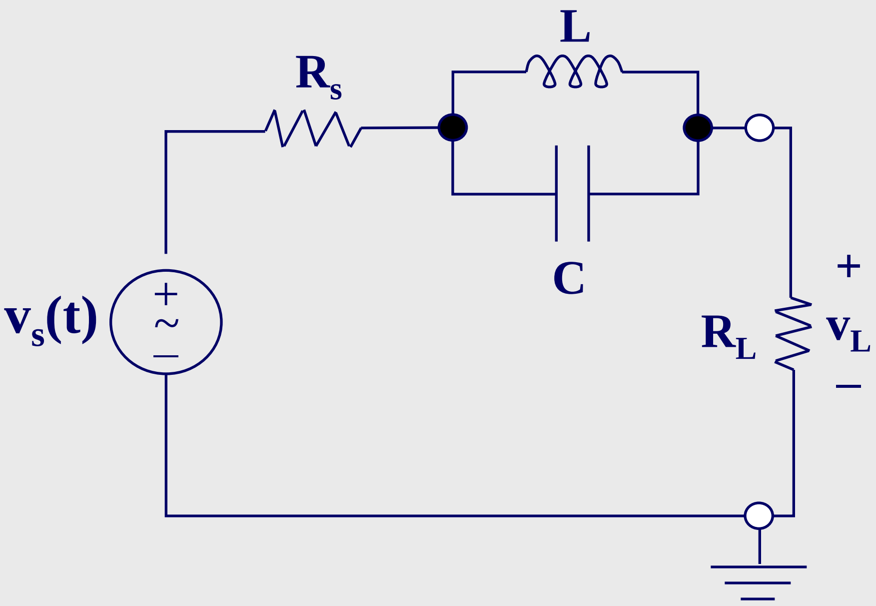


NB: same procedure as equivalent resistance

AC Equivalent Circuits

7. find the Thévenin equivalent

$\boldsymbol{\omega} = 10^3$ Hz, $\mathbf{R_s} = 50\Omega$, $\mathbf{R_L} = 50\Omega$, $\mathbf{L} = 10$mH, $\mathbf{C} = 0.1$uF

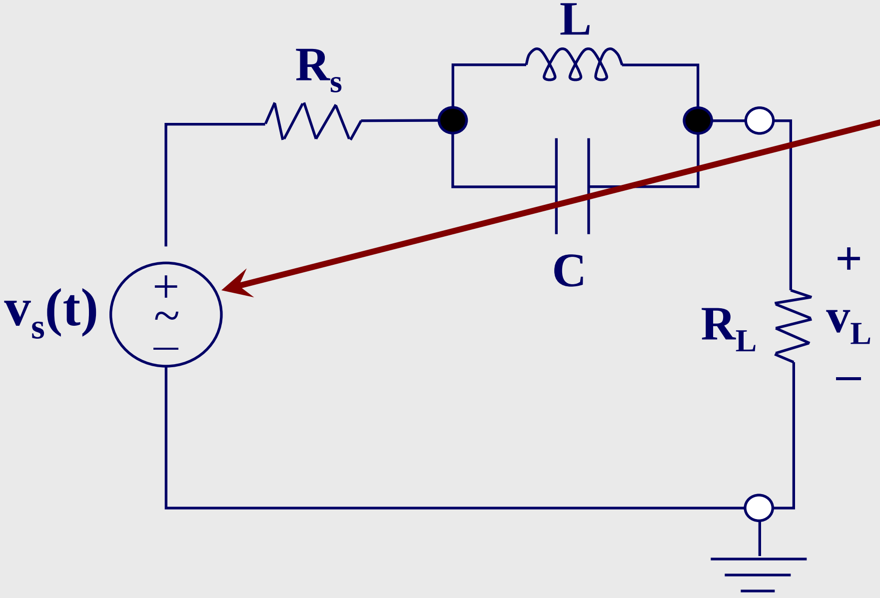


AC Equivalent Circuits

7. find the Thévenin equivalent

$\boldsymbol{\omega} = 10^3$ Hz, $\mathbf{R_s} = 50\Omega$, $\mathbf{R_L} = 50\Omega$, $\mathbf{L} = 10$mH, $\mathbf{C} = 0.1$uF

1. Note frequencies of AC sources

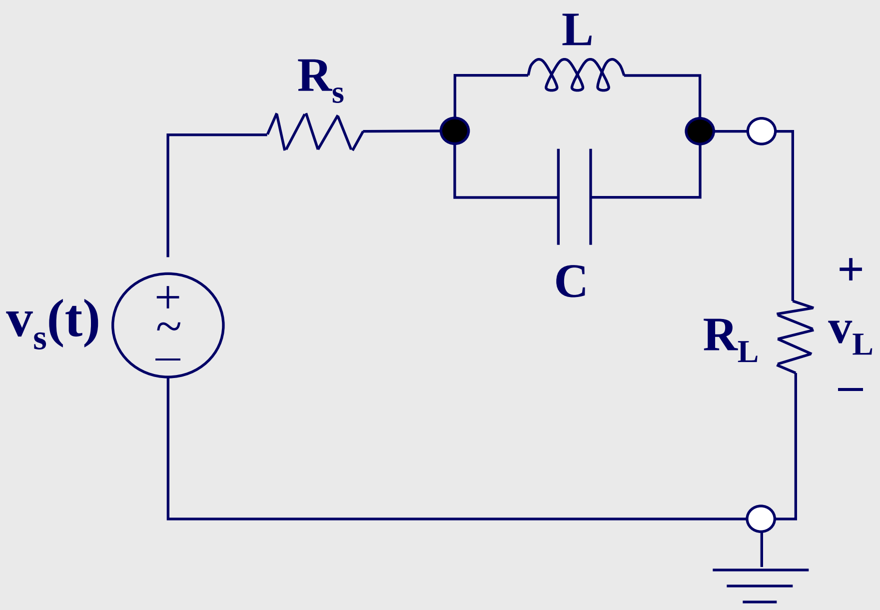


Only one AC source - $\boldsymbol{\omega} = 10^3$ rad/s

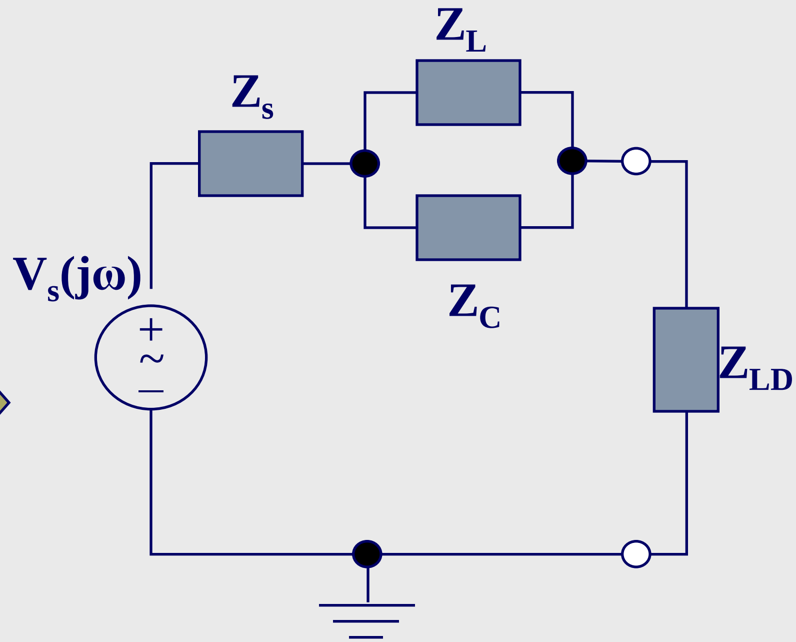
AC Equivalent Circuits

7. find the Thévenin equivalent

$\boldsymbol{\omega} = 10^3$ Hz, $\mathbf{R_s} = 50\Omega$, $\mathbf{R_L} = 50\Omega$, $\mathbf{L} = 10$mH, $\mathbf{C} = 0.1$uF



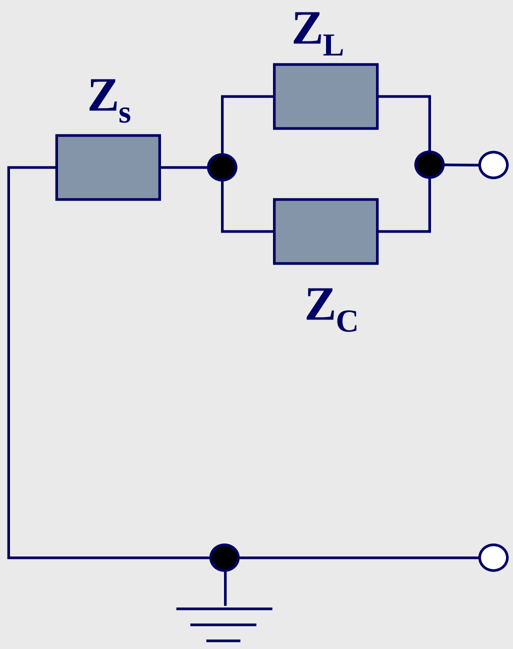
1. Note frequencies of AC sources
2. Convert to phasor domain



AC Equivalent Circuits

7. find the Thévenin equivalent

$\boldsymbol{\omega}$ = 10^3 Hz, $\mathbf{R_s}$ = 50Ω, $\mathbf{R_L}$ = 50Ω, **L** = 10mH, **C** = 0.1uF



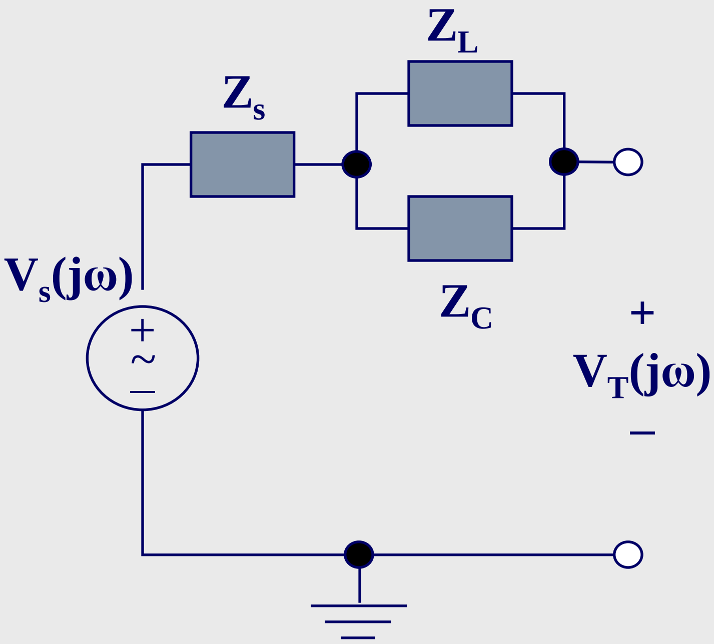
1. Note frequencies of AC sources
2. Convert to phasor domain
3. Find $\mathbf{Z_T}$
   - Remove load & zero sources

$$
\begin{aligned}
Z_T &= Z_S + Z_C \parallel Z_L \\
&= R_S + \frac{(j\omega L)(1/j\omega C)}{(j\omega L)+(1/j\omega C)} \\
&= R_S + j\frac{\omega L}{1-\omega^2 LC} \\
&= 50 + j65.414 \\
&= 82.33\angle 0.9182
\end{aligned}
$$

AC Equivalent Circuits

7. find the Thévenin equivalent

$\boldsymbol{\omega} = 10^3$ Hz, $\mathbf{R_s} = 50\Omega$, $\mathbf{R_L} = 50\Omega$, $\mathbf{L} = 10$mH, $\mathbf{C} = 0.1$uF



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Find $\mathbf{Z_T}$
   - Remove load & zero sources
4. Find $\mathbf{V_T(j\omega)}$
   - Remove load

NB: Since no current flows in the circuit once the load is removed:

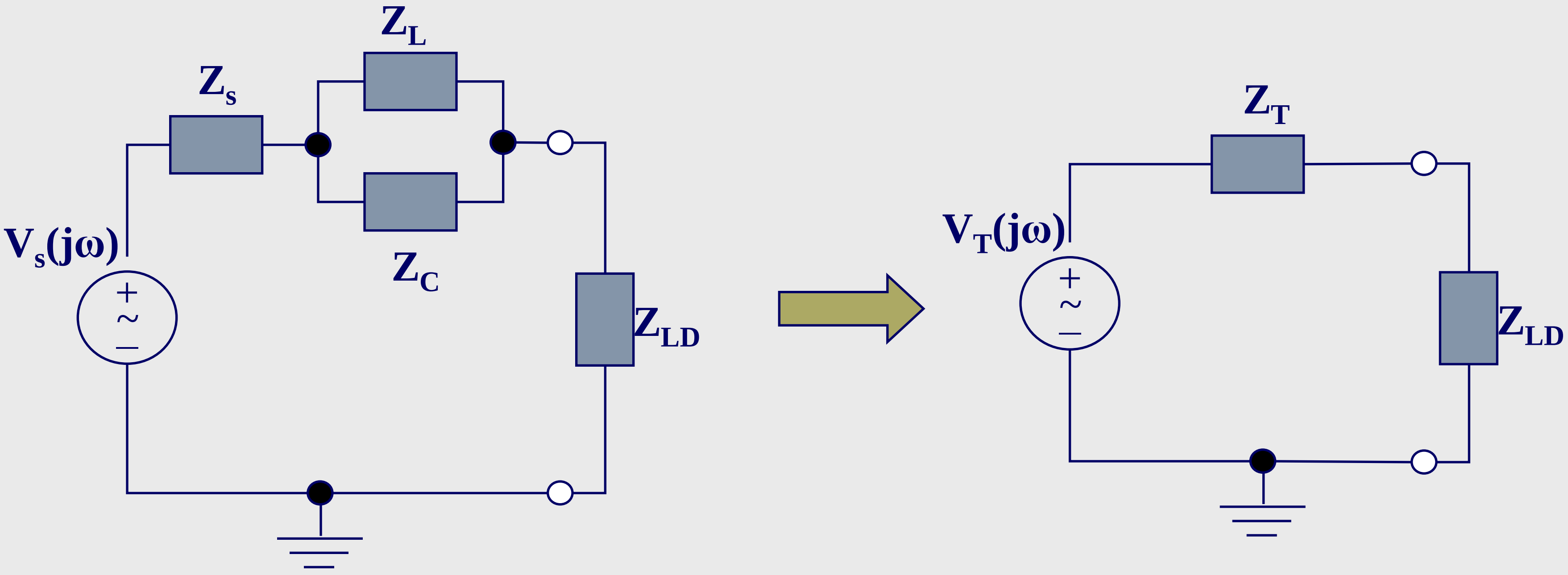
$$Z_T = 82.33\angle 0.9182$$

$$V_T = V_S$$

AC Equivalent Circuits

7. find the Thévenin equivalent

   $\boldsymbol{\omega} = 10^3$ Hz, $\mathbf{R_s} = 50\Omega$, $\mathbf{R_L} = 50\Omega$, $\mathbf{L} = 10$mH, $\mathbf{C} = 0.1$uF



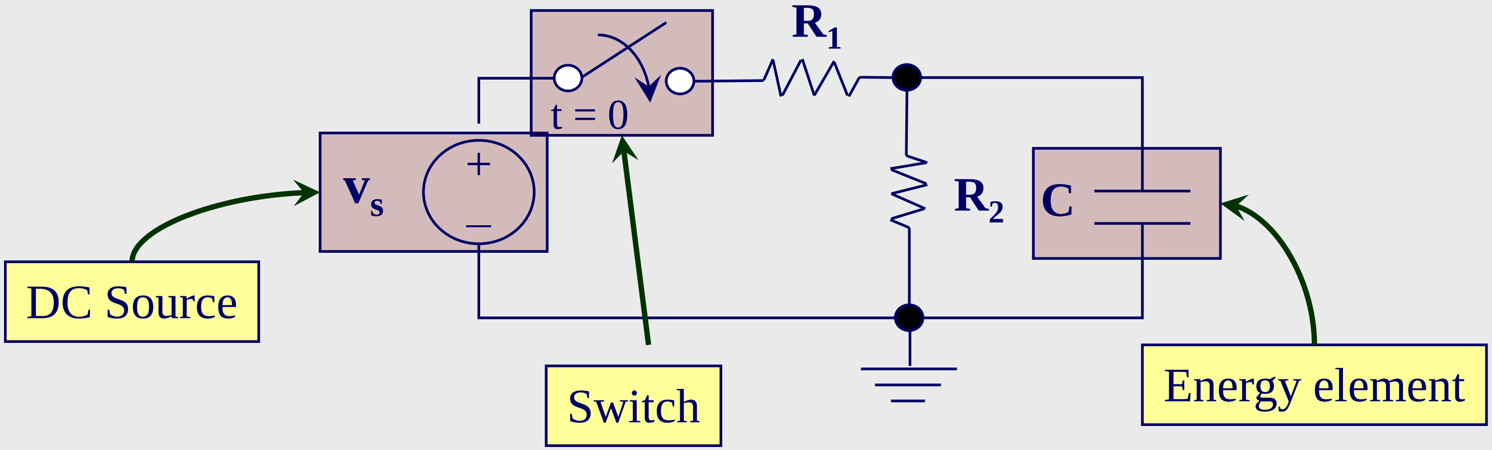
$$V_T = V_S$$

$$Z_T = 82.33\angle 0.9182$$

DC Transient Response

<u>Transient response of a circuit consists of 3 parts</u>:

1. Steady-state response **prior** to the **switching** on/off of a **DC source**
2. Transient response – the circuit **adjusts** to the **DC source**
3. Steady-state response **following** the transient response

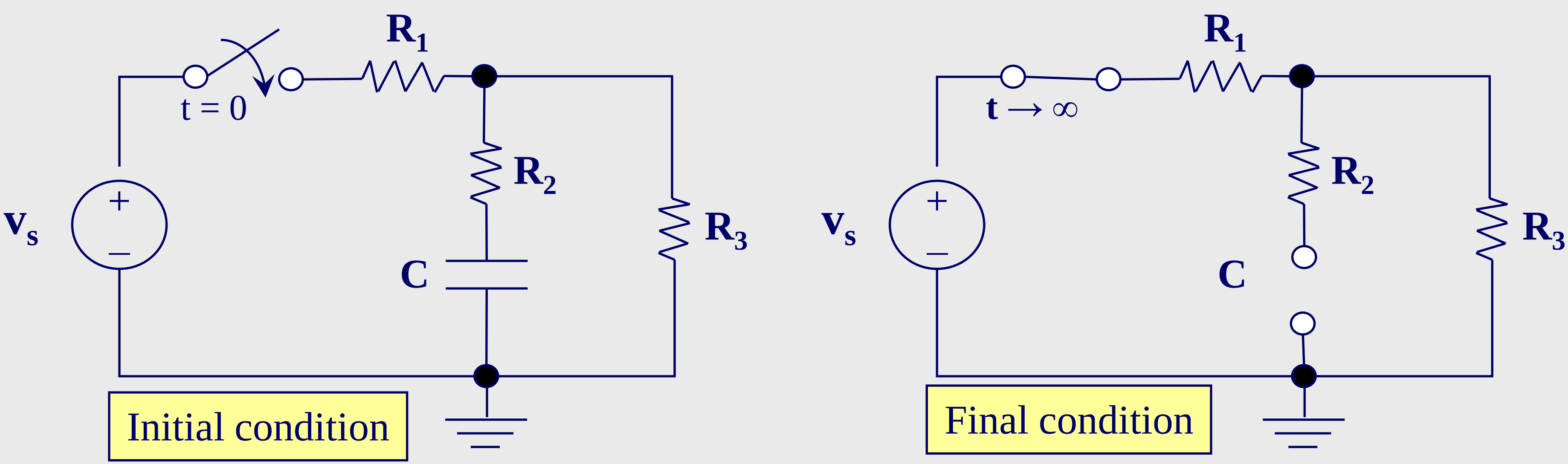


DC Transient Response – DC Steady-State

Initial condition x(0): DC steady state **before** a switch is first activated

- **x(0⁻)**: right before the switch is closed
- **x(0⁺)**: right after the switch is closed

Final condition x(∞): DC steady state a long time **after** a switch is activated



Initial condition

Final condition

DC Transient Response – DC Steady-State

Remember – capacitor voltages and inductor currents **cannot change instantaneously**

- Capacitor voltages and inductor currents don’t change right before closing and right after closing a switch

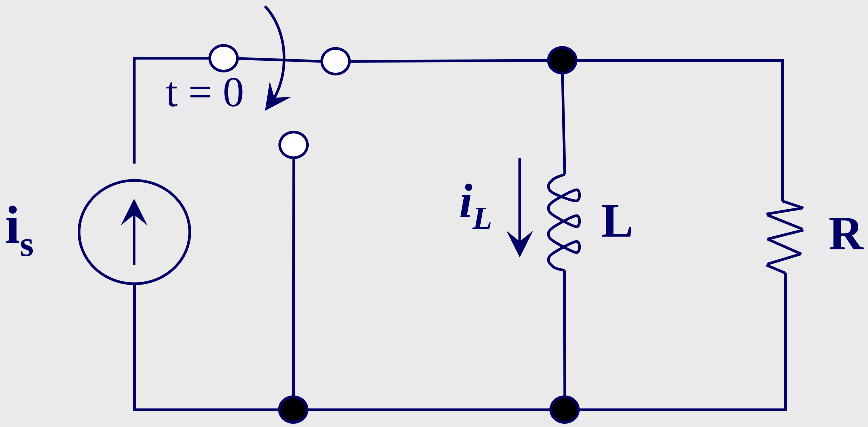
$$v_C(0^+) = v_C(0^-)$$

$$i_L(0^+) = i_L(0^-)$$

DC Transient Response – DC Steady-State

8. find the initial and final current conditions at the inductor

i_s = 10mA

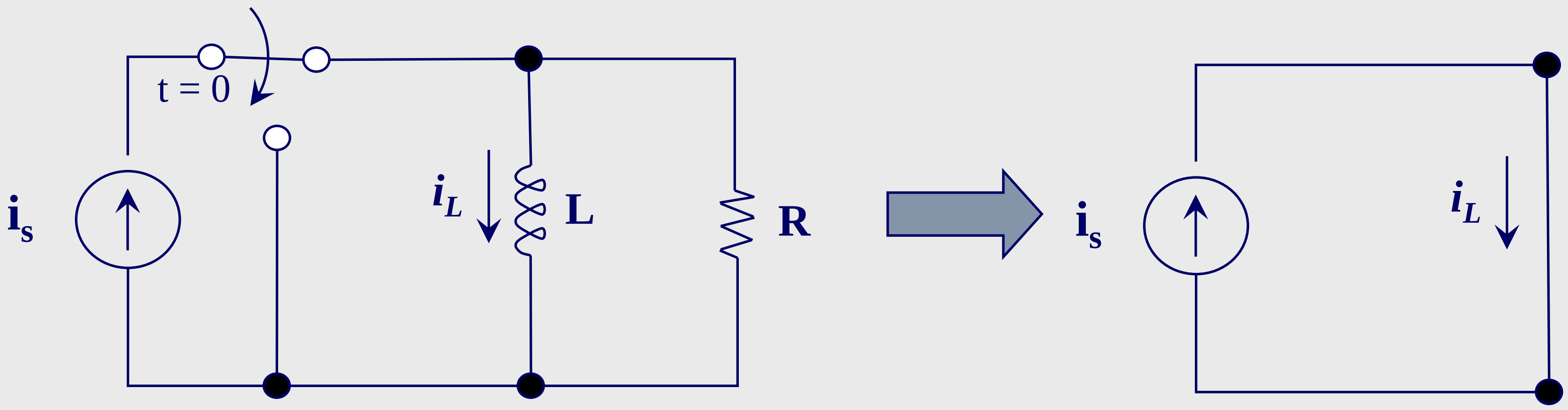


DC Transient Response – DC Steady-State

8. find the initial and final current conditions at the inductor

$\boldsymbol{i_s}$ = 10mA

1. Initial conditions – assume the current across the inductor is in steady-state.



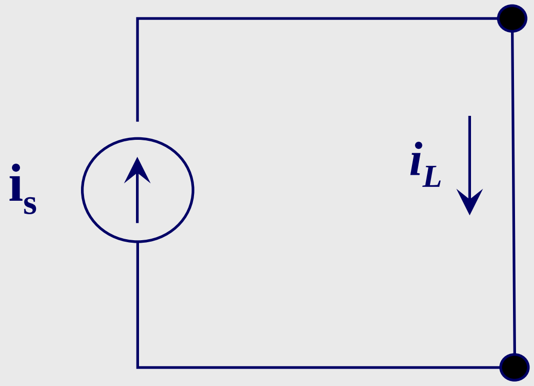
NB: in DC steady state inductors act like **short circuits**, thus no current flows through **R**

DC Transient Response – DC Steady-State

8. find the initial and final current conditions at the inductor

$\boldsymbol{i_s}$ = 10mA

1. Initial conditions – assume the current across the inductor is in steady-state.



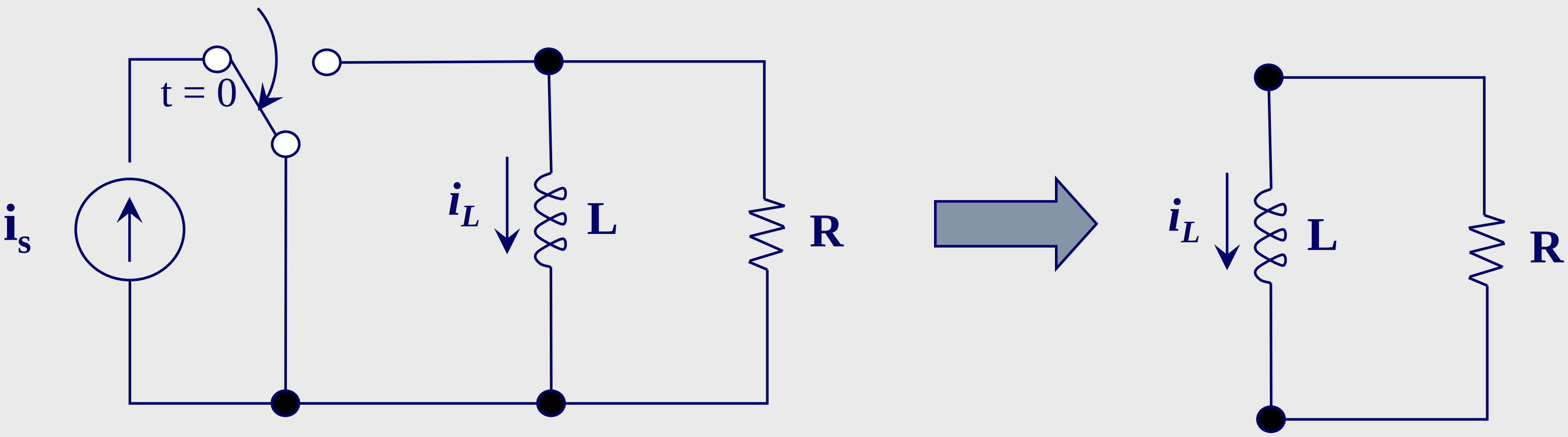
$$i_L(0^-) = i_s$$
$$= 10mA$$

DC Transient Response – DC Steady-State

8. find the initial and final current conditions at the inductor

$i_s = 10\text{mA}$

1. Initial conditions – assume the current across the inductor is in steady-state.
2. Throw the switch



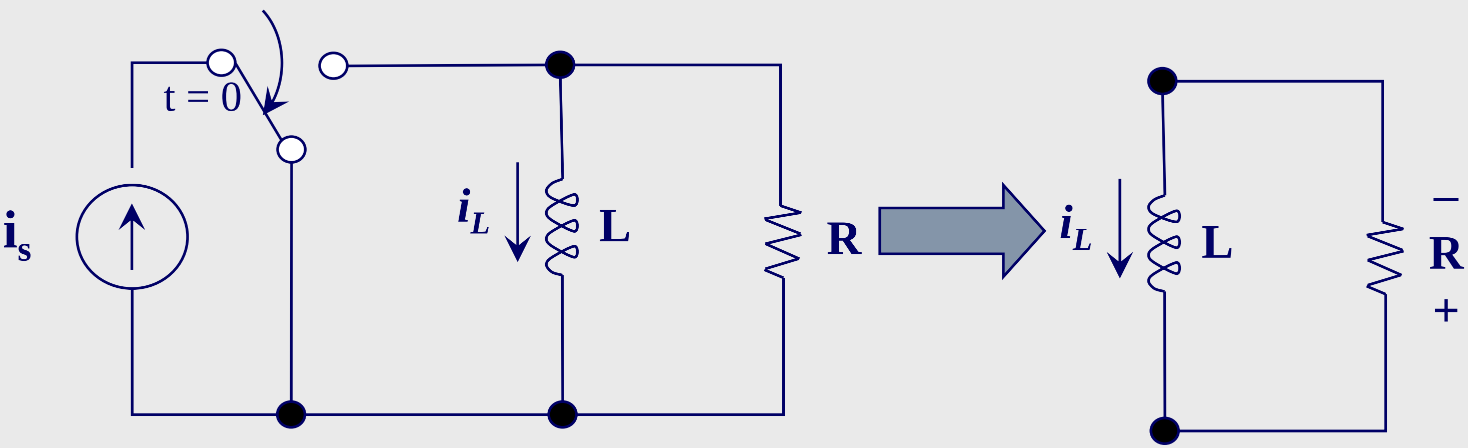
NB: inductor current cannot change instantaneously

DC Transient Response – DC Steady-State

8. find the initial and final current conditions at the inductor

i_s = 10mA

1. Initial conditions – assume the current across the inductor is in steady-state.
2. Throw the switch



$$i_L(0^+) = i_L(0^-) = 10mA$$

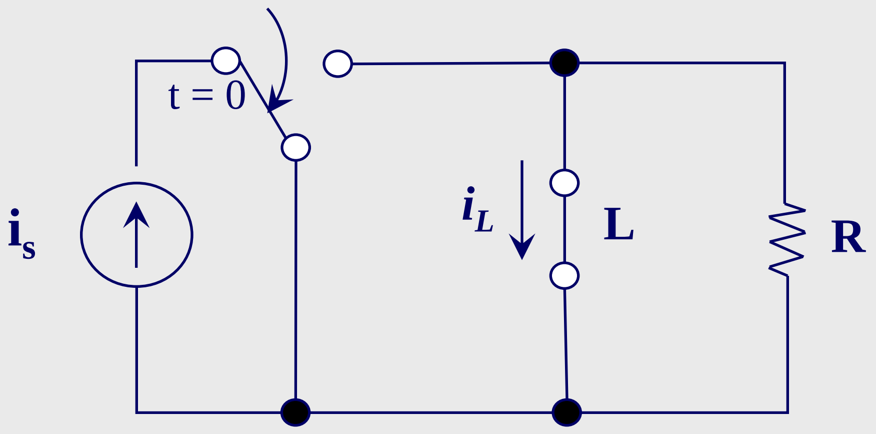
NB: polarity of **R**

NB: inductor current cannot change instantaneously

DC Transient Response – DC Steady-State

8. find the initial and final current conditions at the inductor

$i_s = 10\text{mA}$



1. Initial conditions – assume the current across the inductor is in steady-state.
2. Throw the switch
3. Final conditions

$$i_L(\infty) = 0A$$

NB: in DC steady state inductors act like **short circuits**

DC Transient Response

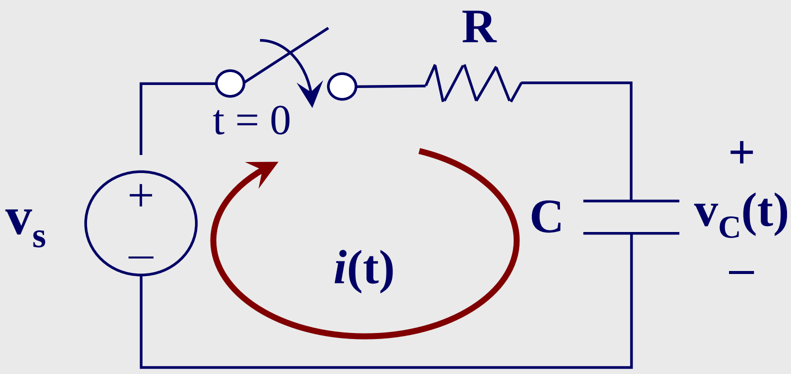
Solving 1st order transient response:

1. Solve the **DC steady-state** circuit:
   - Initial condition $\mathbf{x(0^-)}$: before switching (on/off)
   - Final condition $\mathbf{x(\infty)}$: After any transients have died out ($t \rightarrow \infty$)
2. Identify $\mathbf{x(0^+)}$**:** the circuit **initial conditions**
   - Capacitors: $\mathbf{v_C(0^+) = v_C(0^-)}$
   - Inductors: $\boldsymbol{i_L(0^+) = i_L(0^-)}$
3. Write **a differential equation** for the circuit at time $t = 0^+$
   - Reduce the circuit to its **Thévenin or Norton equivalent**
     - The energy storage element (capacitor or inductor) is the load
   - The differential equation will be either in terms of $\mathbf{v_C(t)}$ or $\boldsymbol{i_L(t)}$
   - Reduce this equation to standard form
4. Solve for the **time constant**
   - Capacitive circuits: $\boldsymbol{\tau = R_T C}$
   - Inductive circuits: $\boldsymbol{\tau = L/R_T}$
5. Write the **complete response** in the form:
   - $\mathbf{x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}}$

DC Transient Response

9. find $\mathbf{v_c(t)}$ for all t

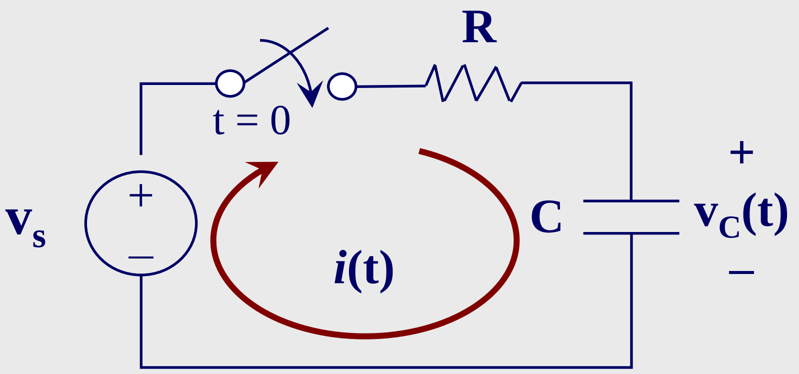
   $\mathbf{v_s}$ = 12V, $\mathbf{v_C(0^-)}$ = 5V, **R** = 1000Ω, **C** = 470uF



DC Transient Response

9. find $\mathbf{v_c(t)}$ for all t

$\mathbf{v_s}$ = 12V, $\mathbf{v_C(0^-)}$ = 5V, **R** = 1000Ω, **C** = 470uF



1. DC steady-state
   a) Initial condition: $\mathbf{v_C(0)}$
   b) Final condition: $\mathbf{v_C(\infty)}$

$$v_C(t<0) = v_C(0^-) = 5V$$

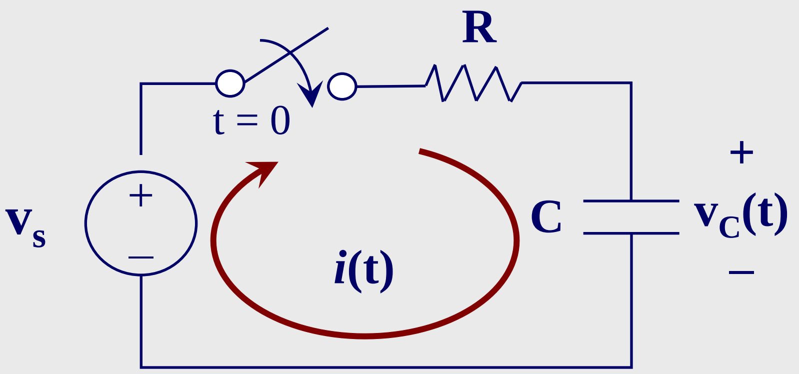
$$v_C(\infty) = v_S$$
$$= 12V$$

NB: as t → ∞ the capacitor acts like an open circuit thus $\mathbf{v_C(\infty)} = \mathbf{v_S}$

DC Transient Response

9. find $\mathbf{v_c(t)}$ for all t

$\mathbf{v_s}$ = 12V, $\mathbf{v_C(0^-)}$ = 5V, **R** = 1000Ω, **C** = 470uF



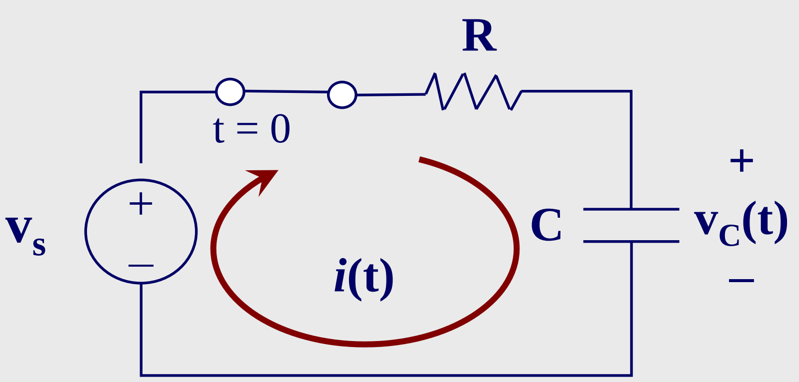
2. Circuit initial conditions: $\mathbf{v_C(0^+)}$

$$v_C(0^+) = v_C(0^-)$$
$$= 5V$$

DC Transient Response

9. find $\mathbf{v_c(t)}$ for all t

$\mathbf{v_s}$ = 12V, $\mathbf{v_C(0^-)}$ = 5V, **R** = 1000Ω, **C** = 470uF



3. Write differential equation (already in Thévenin equivalent) at t = 0

KVL :

$$-v_S + v_R(t) + v_C(t) = 0$$

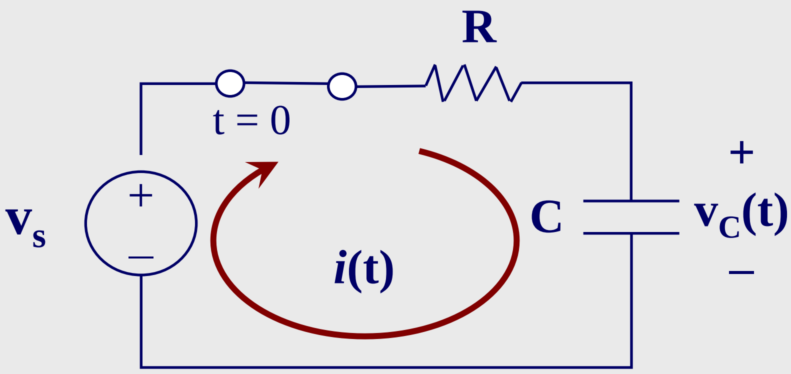
$$i_C(t)R + v_C(t) = v_S$$

$$RC\frac{dv_C(t)}{dt} + v_C(t) = v_S$$

DC Transient Response

9. find $\mathbf{v_c(t)}$ for all t

$\mathbf{v_s}$ = 12V, $\mathbf{v_C(0^-)}$ = 5V, **R** = 1000Ω, **C** = 470uF



4. Find the time constant **τ**

$$\tau = RC$$
$$= (1000)(470 \times 10^{-6})$$
$$= 0.47$$

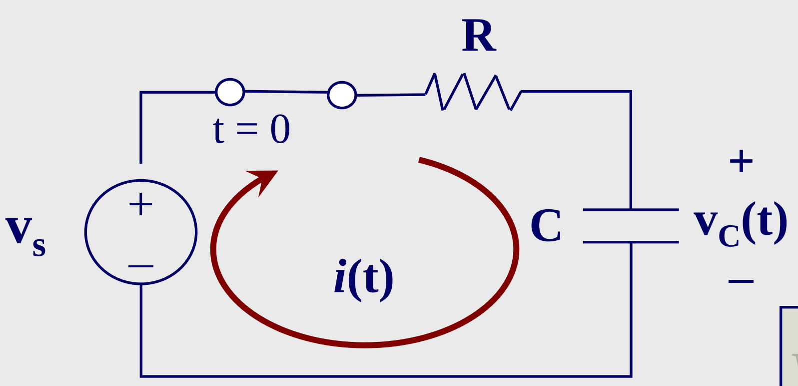
$$K_S = 1$$

$$F = v_S$$
$$= 12$$

DC Transient Response

9. find $\mathbf{v_c(t)}$ for all t

$\mathbf{v_s}$ = 12V, $\mathbf{v_C(0^-)}$ = 5V, **R** = 1000Ω, **C** = 470uF



5. Write the complete response
x(t) = x(∞) + [x(0) - x(∞)]e^{-t/τ}

$$v_C(t) = v_C(\infty) + \left[v_C(0) - v_C(\infty)\right]e^{-t/\tau}$$
$$= 12 + (5 - 12)e^{-t/0.47}$$
$$= 12 - 7e^{-t/0.47}$$

Frequency Response

<u>Frequency Response H(jω)</u>: a measure of how the voltage/current/impedance of a load responds to the voltage/current of a source

$$H_V(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)}$$

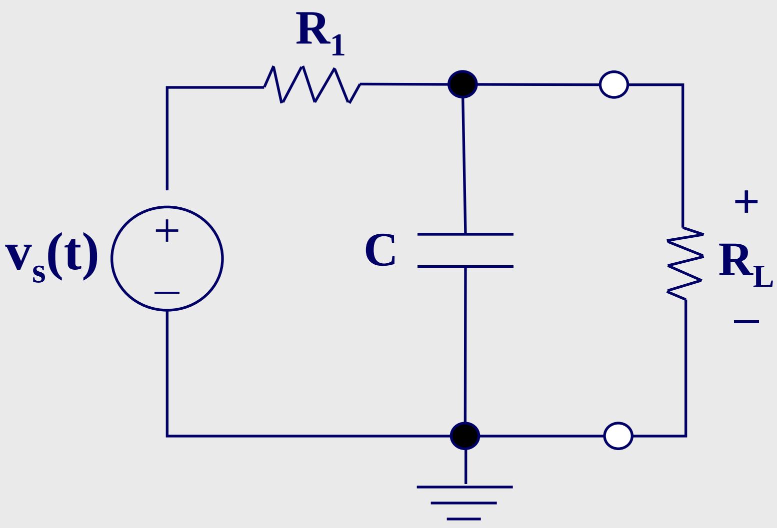
$$H_I(j\omega) = \frac{I_L(j\omega)}{I_S(j\omega)}$$

$$H_Z(j\omega) = \frac{V_L(j\omega)}{I_S(j\omega)}$$

Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

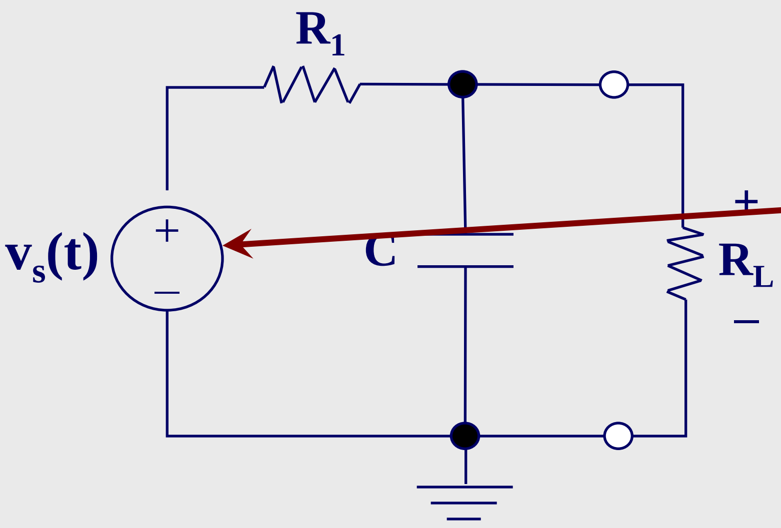
    $\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



1. Note frequencies of AC sources

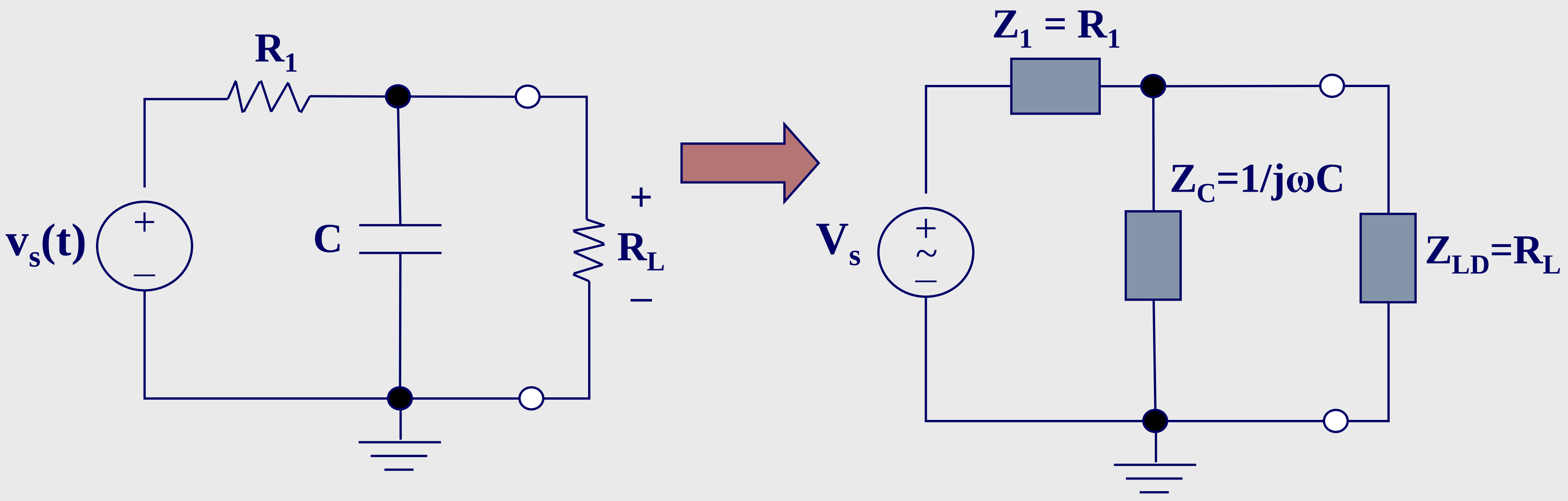
Only one AC source so frequency response $\mathbf{H_V(j\omega)}$ will be the function of a single frequency

Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1k\Omega$, $\mathbf{R_L} = 10k\Omega$, $\mathbf{C} = 10uF$

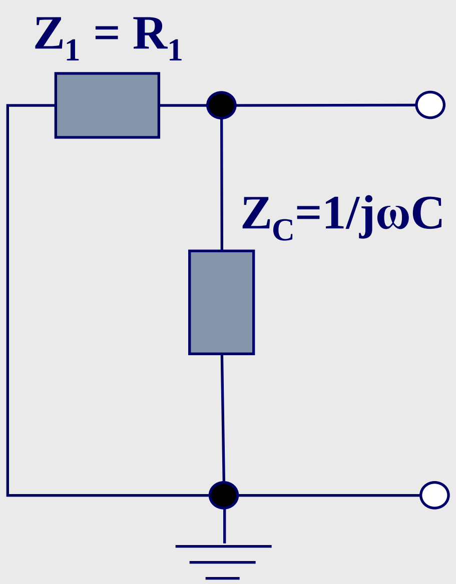
1. Note frequencies of AC sources
2. Convert to phasor domain



Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



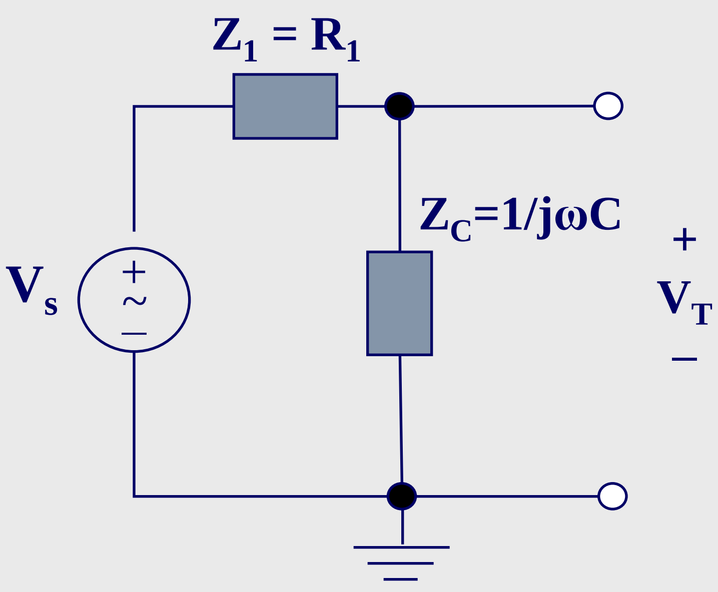
1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Thévenin equivalent

$$Z_T = Z_1 \parallel Z_C$$

Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Thévenin equivalent

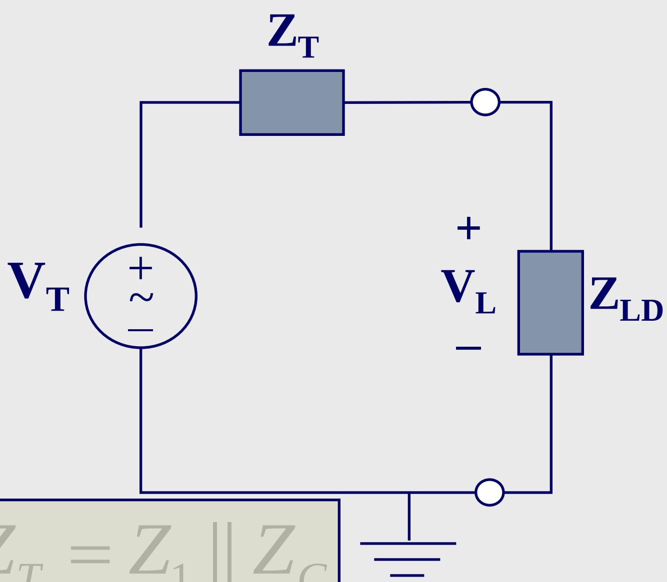
$$V_T(j\omega) = V_S(j\omega)\frac{Z_C}{Z_1 + Z_C}$$

$$Z_T = Z_1 \,||\, Z_C$$

Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



1. Note frequencies of AC sources
2. Convert to phasor domain
3. Solve using network analysis
   - Thévenin equivalent
4. Find an expression for the load voltage

$$Z_T = Z_1 \| Z_C$$

$$V_T(j\omega) = V_S(j\omega)\frac{Z_C}{Z_1 + Z_C}$$

$$V_L(j\omega) = V_T(j\omega)\frac{Z_{LD}}{Z_T + Z_{LD}}$$

$$= \left(V_S(j\omega)\frac{Z_C}{Z_1 + Z_C}\right)\frac{Z_{LD}}{(Z_1 \| Z_C) + Z_{LD}}$$

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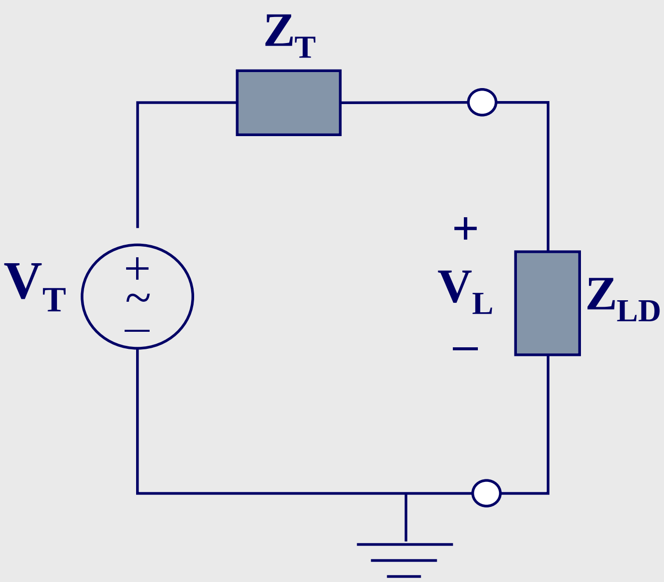
Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



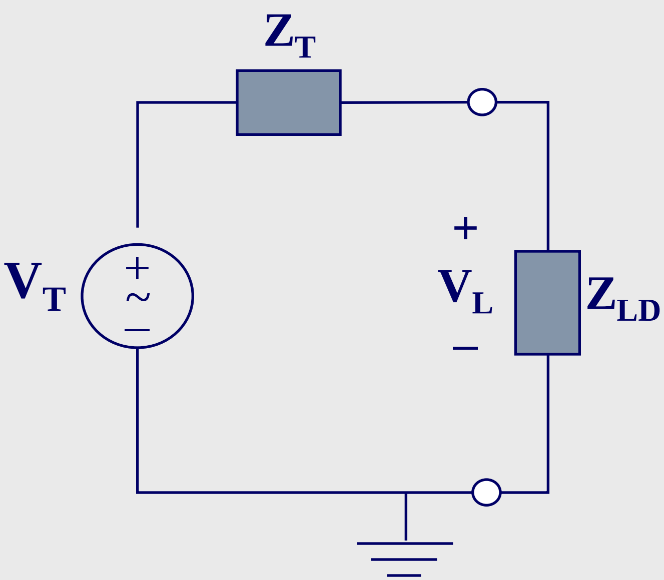
5. Find an expression for the frequency response

$$H_V(j\omega) = \frac{V_L(j\omega)}{V_S(j\omega)} = \frac{Z_C}{Z_1 + Z_C} \cdot \frac{Z_{LD}}{(Z_1 \parallel Z_C) + Z_{LD}}$$

Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



5. Find an expression for the frequency response

$$H_V(j\omega) = \frac{Z_C}{Z_1 + Z_C} \cdot \frac{Z_{LD}}{(Z_1 \| Z_C) + Z_{LD}}$$

$$= \frac{Z_{LD} Z_C}{Z_{LD}(Z_1 + Z_C) + Z_1 Z_C}$$

$$= \frac{10^4/(j\omega \times 10^{-5})}{10^4\left[10^3 + 1/(j\omega \times 10^{-5})\right] + 10^3/(j\omega \times 10^{-5})}$$

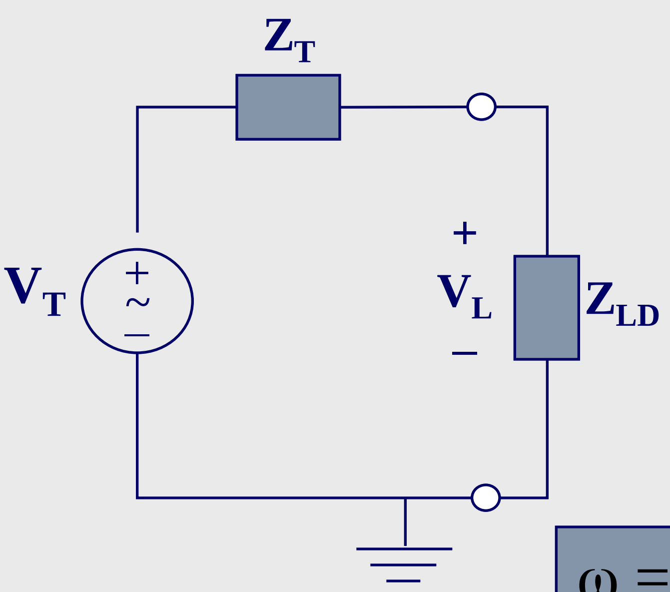
$$= \frac{100}{110 + j\omega}$$

$$= \frac{100}{\sqrt{110^2 + \omega^2}} \angle -\arctan\left(\frac{\omega}{110}\right)$$

Frequency Response

10. compute the frequency response $\mathbf{H_V(j\omega)}$

$\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_L} = 10\text{k}\Omega$, $\mathbf{C} = 10\text{uF}$



5. Find an expression for the frequency response
   - Look at response for low frequencies (ω = 10) and high frequencies (ω = 10000)

$$H_V(j\omega) = \frac{100}{\sqrt{110^2 + \omega^2}} \angle -\arctan\left(\frac{\omega}{110}\right)$$

ω = 10

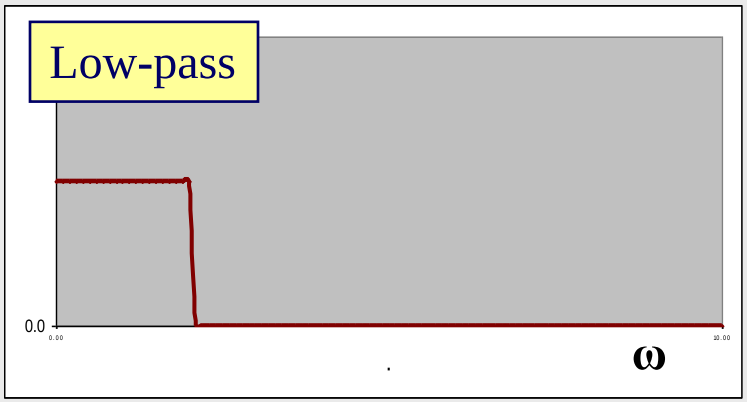
$$H_V(j\omega) = 0.905 \angle -0.0907$$

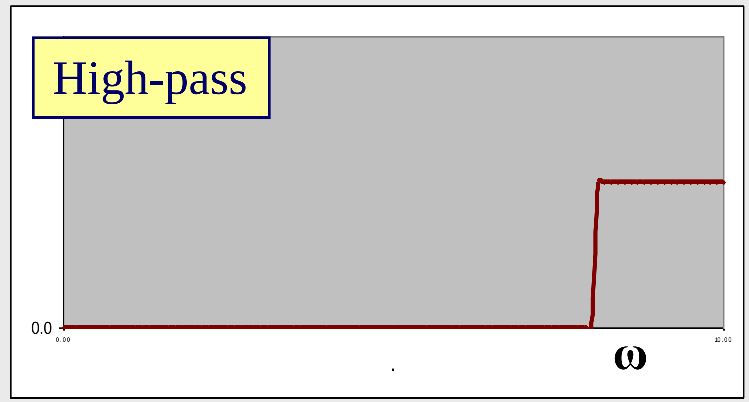
ω = 10000

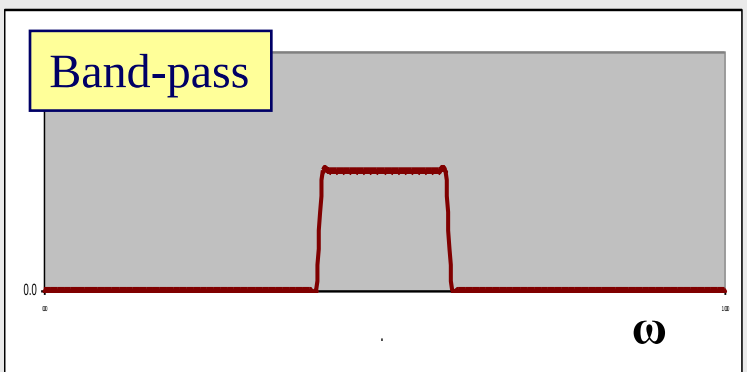
$$H_V(j\omega) = 0.010 \angle -\frac{\pi}{2}$$

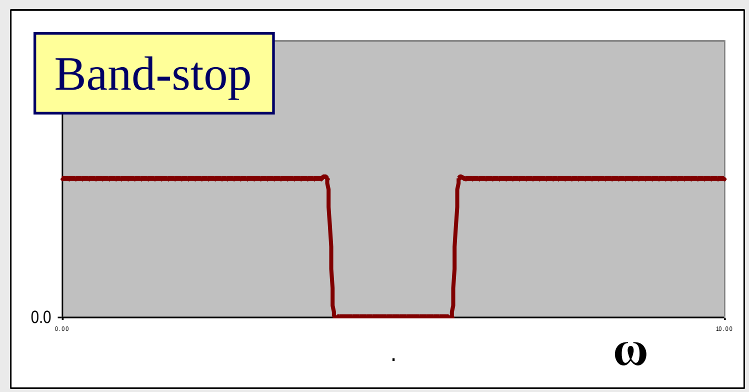
Basic Filters

<u>Electric circuit filter</u>: attenuates (reduces) or eliminates signals at unwanted frequencies







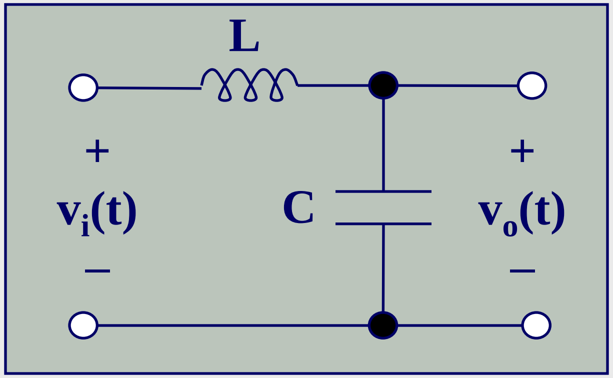


Basic Filters – Resonant Frequency

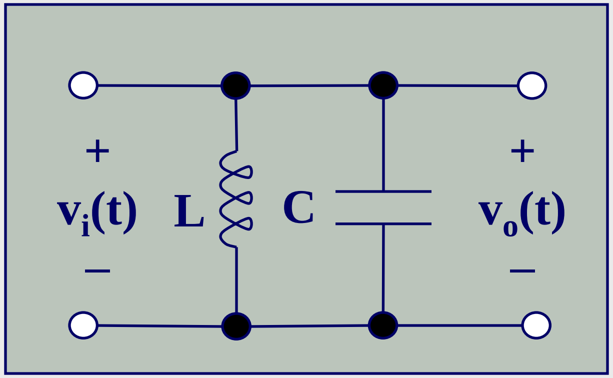
Resonant Frequency (ω_n): the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

$$\omega_n = \frac{1}{\sqrt{LC}}$$

Impedances in **series**



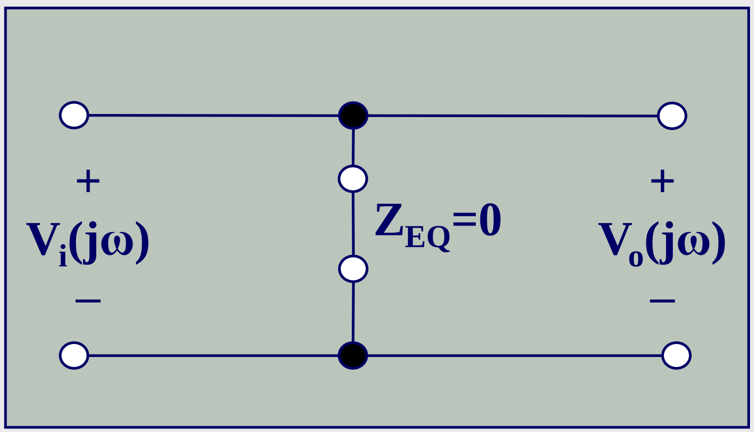
Impedances in **parallel**



Basic Filters – Resonant Frequency

<u>Resonant Frequency (ω_n)</u>: the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

Impedances in **series**



$$V_o(j\omega)=0$$

$$Z_L=\frac{j\sqrt{LC}}{C}$$

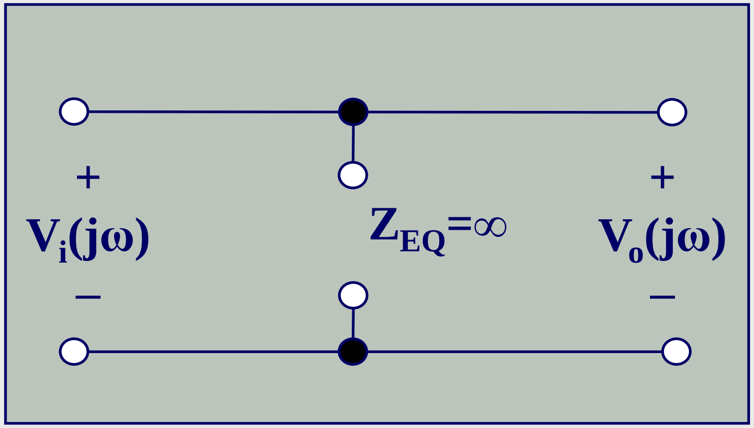
$$Z_C=-\frac{j\sqrt{LC}}{C}$$

$$\begin{aligned}Z_{EQ}&=Z_L+Z_C\\&=0\end{aligned}$$

Basic Filters – Resonant Frequency

Resonant Frequency (ω_n): the frequency at which capacitive impedance and inductive impedance are equal and opposite (in 2nd order filters)

Impedances in **parallel**



$$V_o(j\omega) = V_i(j\omega)$$

$$Z_L = \frac{j\sqrt{LC}}{C}$$

$$Z_C = -\frac{j\sqrt{LC}}{C}$$

$$\begin{aligned} Z_{EQ} &= Z_L \parallel Z_C \\ &= \frac{Z_L Z_C}{Z_L + Z_C} \\ &= \frac{L/C}{0} \\ &= \infty \end{aligned}$$

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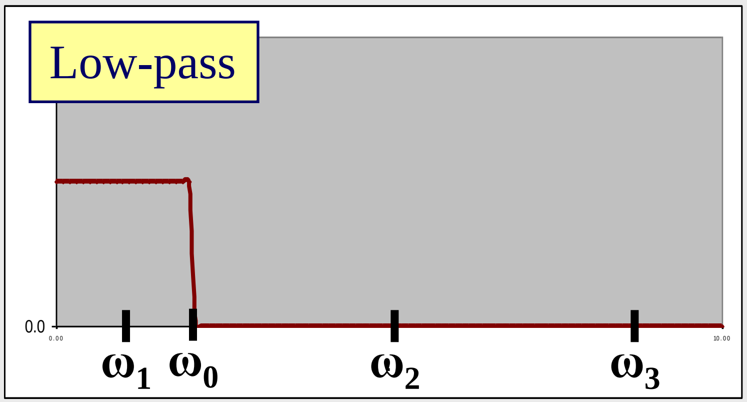
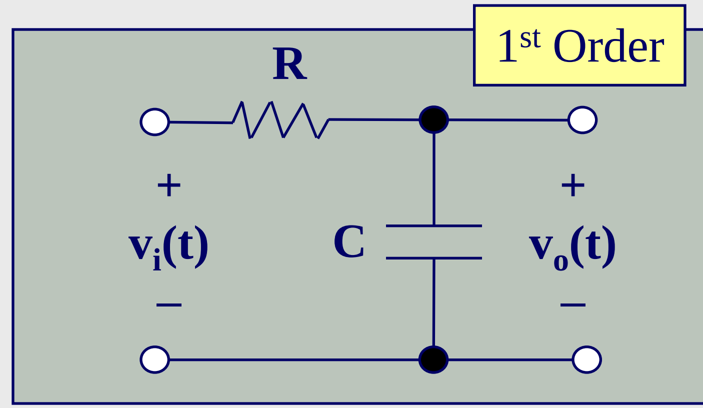
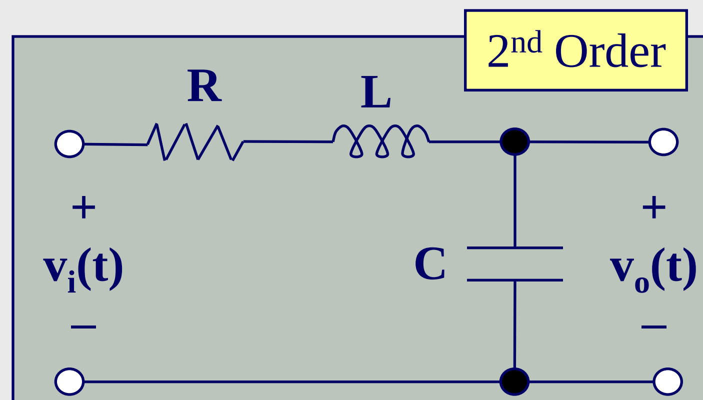
Basic Filters – Low-Pass Filters

<u>Low-pass Filters</u>: only allow signals under the **cutoff frequency (ω_0)** to pass



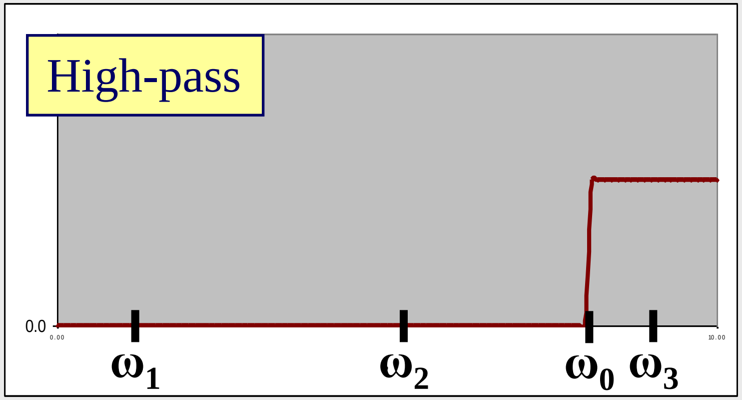


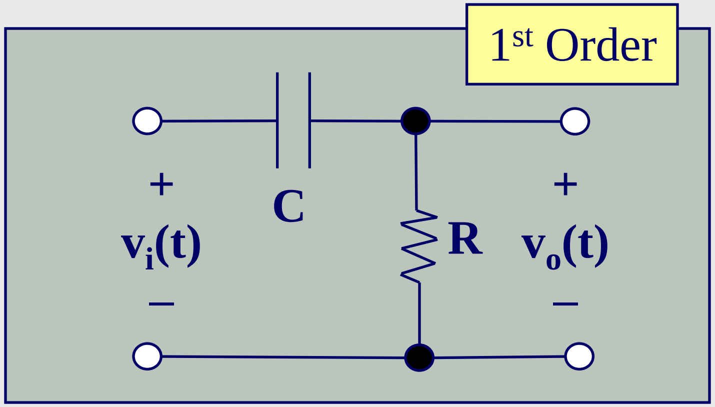
$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) \rightarrow \boxed{H_L(j\omega)} \rightarrow v_o(t) = \cos(\omega_1 t)$$



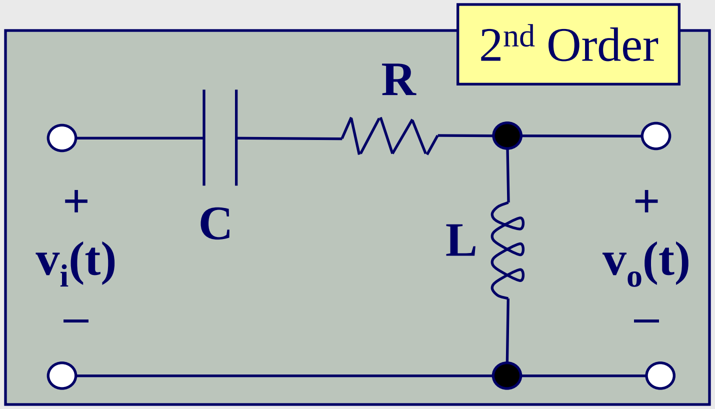
Basic Filters – High-Pass Filters

High-pass Filters: only allow signals above the **cutoff frequency (ω_0)** to pass



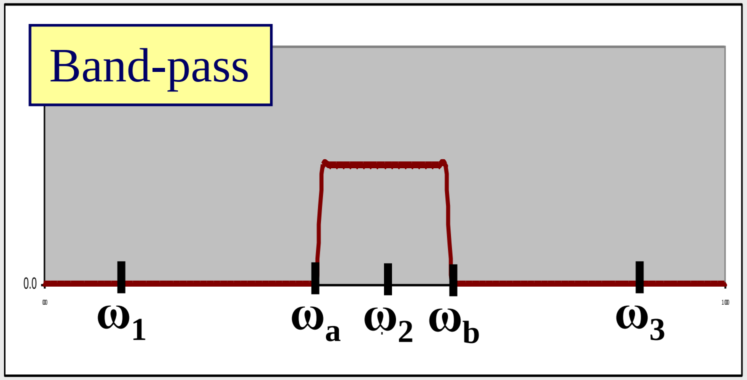


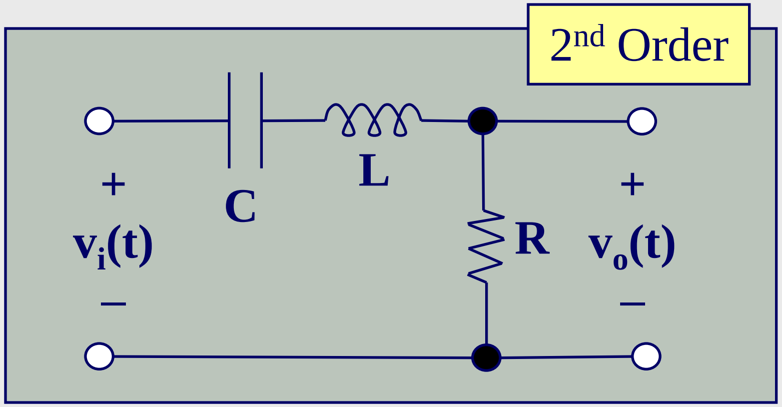
$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) \rightarrow H_H(j\omega) \rightarrow v_o(t) = \cos(\omega_3 t)$$



Basic Filters – Band-Pass Filters

Band-pass Filters: only allow signals between the **passband** (ω_a to ω_b) to pass

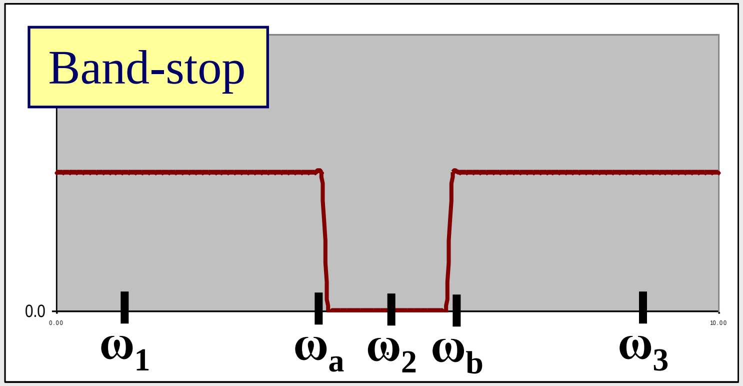


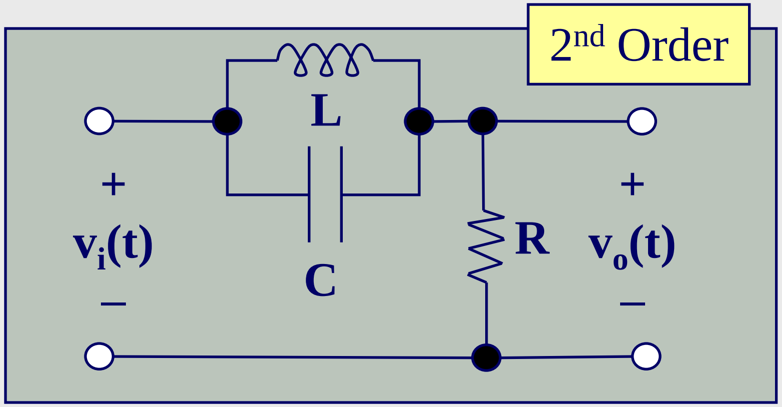


$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) \rightarrow \boxed{H_B(j\omega)} \rightarrow v_o(t) = \cos(\omega_2 t)$$

Basic Filters – Band-Stop Filters

<u>Band-stop Filters</u>: allow signals except those between the **stopband** ($\boldsymbol{\omega_a}$ to $\boldsymbol{\omega_b}$) to pass

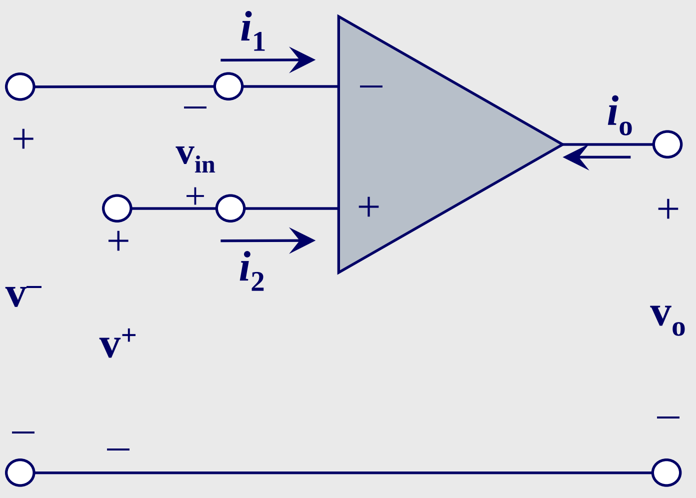


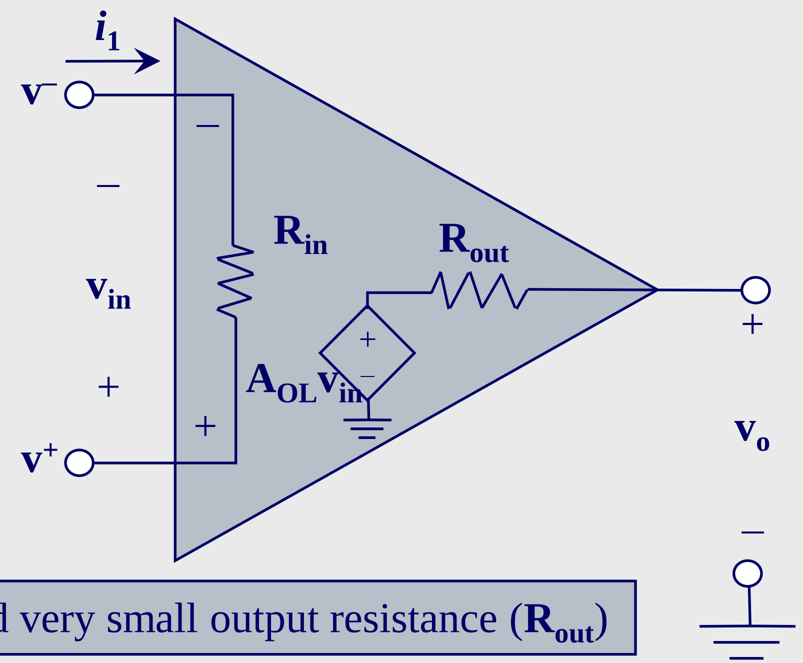


$$v_i(t) = \cos(\omega_1 t) + \cos(\omega_2 t) + \cos(\omega_3 t) \rightarrow \boxed{H_N(j\omega)} \rightarrow v_o(t) = \cos(\omega_1 t) + \cos(\omega_3 t)$$

Op-Amps – Open-Loop Mode

<u>Open-Loop Model</u>: an ideal op-amp acts like a **difference amplifier** (a device that amplifies the difference between two input voltages)



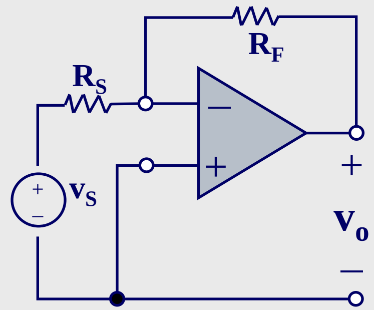
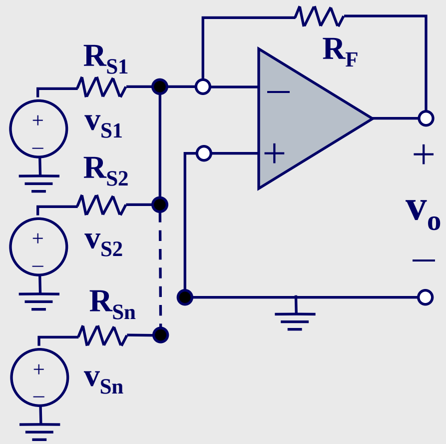


NB: op-amps have near-infinite input resistance (**R$_{in}$**) and very small output resistance (**R$_{out}$**)

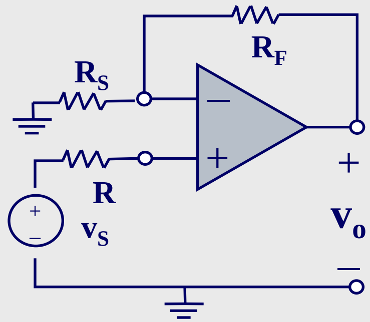
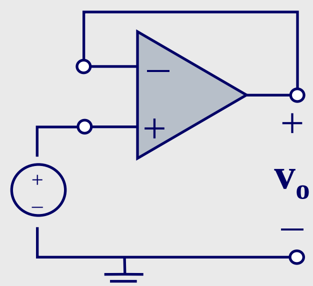
$$v_o \approx A_{OL} v_{in} = A_{OL}(v^+ - v^-)$$

A$_{OL}$ – open-loop voltage gain

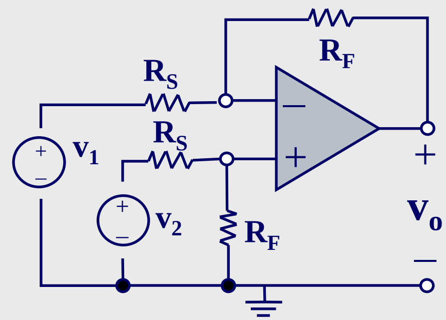
Op-Amps – Closed-Loop Mode

<table>
<tr><th></th><th>Circuit Diagram</th><th>A_{CL}</th></tr>
<tr><td>Inverting Amplifier</td><td>
</td><td>$$v_o = A_{CL} v_S = -\frac{R_F}{R_S} v_S$$</td></tr>
<tr><td>Summing Amplifier</td><td>
</td><td>$$v_o = \sum_{n=1}^{N} A_{OLn} v_{Sn} = -\sum_{n=1}^{N} \frac{R_F}{R_n} v_{Sn}$$</td></tr>
</table>

Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Noninverting Amplifier		$v_o = A_{CL} v_S = \left(1 + \frac{R_F}{R_S}\right) v_S$
Voltage Follower		$v_o = A_{CL} v_s = v_s$

Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Differential Amplifier		$v_o = \frac{R_F}{R_S}\left(v_{S2} - v_{S1}\right)$

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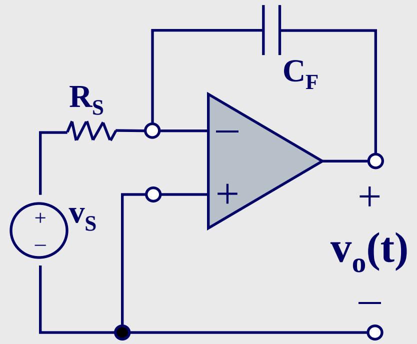
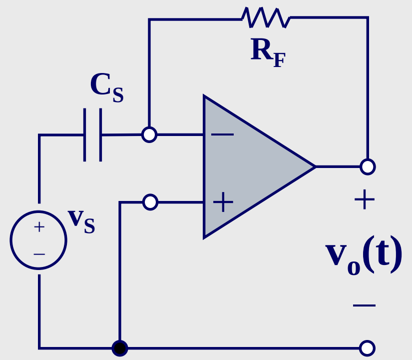
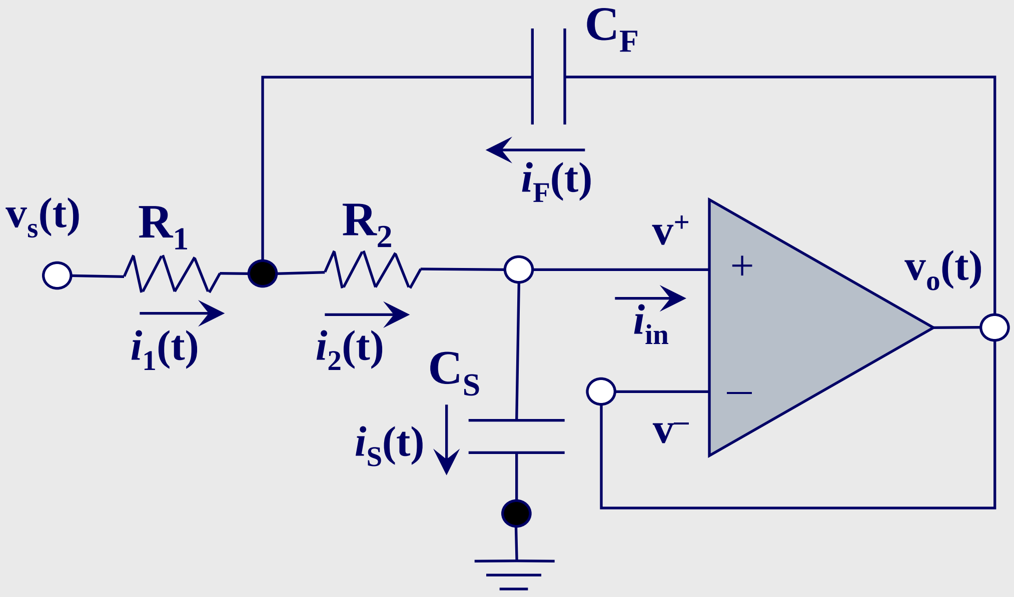
Op-Amps – Closed-Loop Mode

	Circuit Diagram	A_{CL}
Ideal Integrator		$v_o = -\frac{1}{R_S C_F}\int_{-\infty}^{t} v_S(\tau)d\tau$
Ideal Differentiator		$v_o = -R_F C_S \frac{dv_S(t)}{dt}$

Op-Amps

11. find an expression for the gain

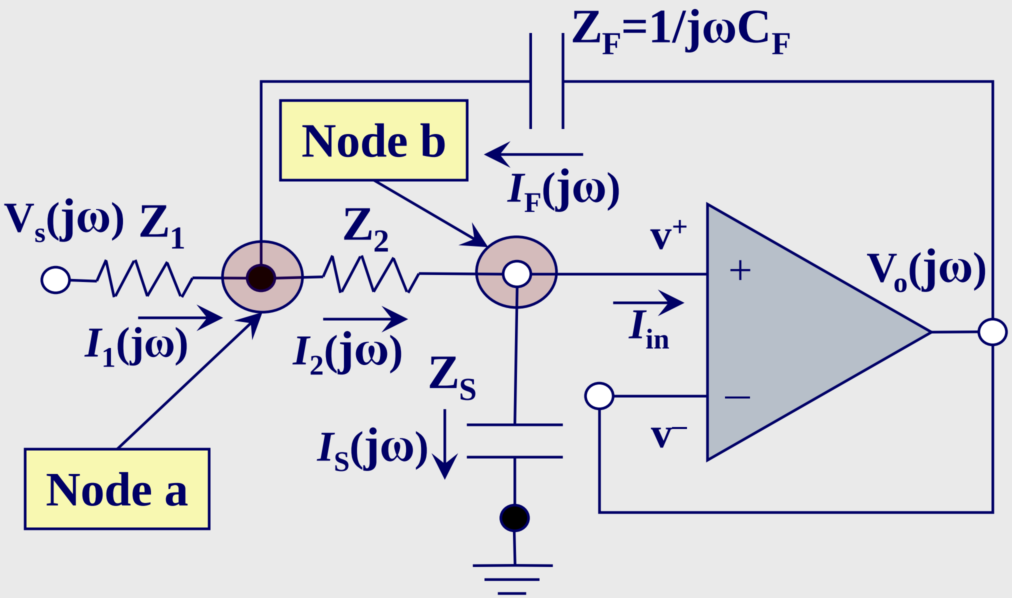
C$_F$ = 1/6 F, **R**$_1$ = 3Ω, **R**$_2$ = 2Ω, **C**$_S$ = 1/6 F



Op-Amps

11. find an expression for the gain

C$_F$ = 1/6 F, **R**$_1$ = 3Ω, **R**$_2$ = 2Ω, **C**$_S$ = 1/6 F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**

KCL at a:

$$I_1 + I_F - I_2 = 0$$

$$\frac{V_S - V_a}{Z_1} + \frac{V_o - V_a}{Z_F} - \frac{V_a - V_o}{Z_2} = 0$$

$$V_a\left(\frac{1}{Z_1} + \frac{1}{Z_F} + \frac{1}{Z_2}\right) - V_o\left(\frac{1}{Z_2} + \frac{1}{Z_F}\right) = V_S\left(\frac{1}{Z_1}\right)$$

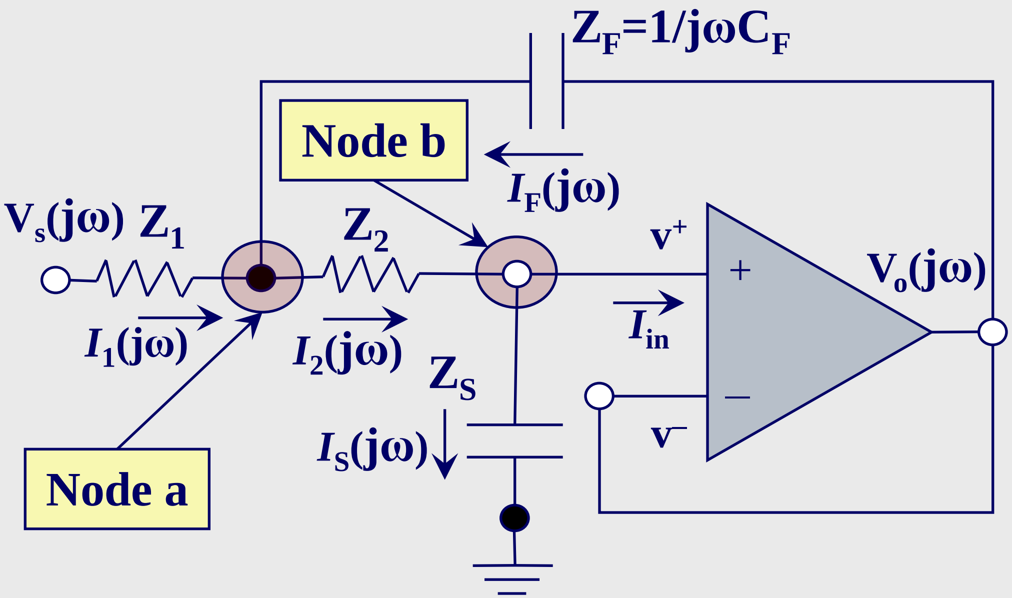
$$V_a\left(\frac{1}{3} + \frac{j\omega}{6} + \frac{1}{2}\right) - V_o\left(\frac{1}{2} + \frac{j\omega}{6}\right) = V_S\left(\frac{1}{3}\right)$$

NB: $\mathbf{v}_+ = \mathbf{v}^-$ and $I_{in} = 0$

Op-Amps

11. find an expression for the gain

$\mathbf{C_F} = 1/6$ F, $\mathbf{R_1} = 3\Omega$, $\mathbf{R_2} = 2\Omega$, $\mathbf{C_S} = 1/6$ F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**

KCL at b :

$$I_2 - I_S - I_{in} = 0$$

$$\frac{V_a - V_o}{Z_2} - \frac{V_o - 0}{Z_S} = 0$$

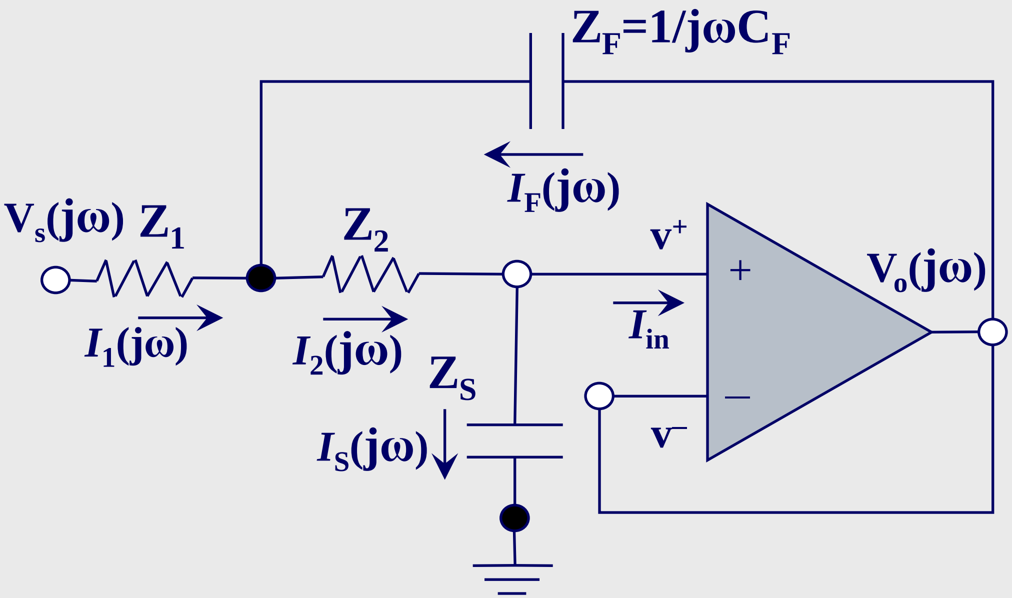
$$V_a\left(\frac{1}{Z_2}\right) - V_o\left(\frac{1}{Z_S} + \frac{1}{Z_2}\right) = 0$$

$$V_a\left(\frac{1}{2}\right) - V_o\left(\frac{j\omega}{6} + \frac{1}{2}\right) = 0$$

Op-Amps

11. find an expression for the gain

C_F = 1/6 F, **R_1** = 3Ω, **R_2** = 2Ω, **C_S** = 1/6 F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**
3. Express **V_o** in terms of **V_s**

$$V_a(5+j\omega) - V_o(3+j\omega) = 2V_S$$

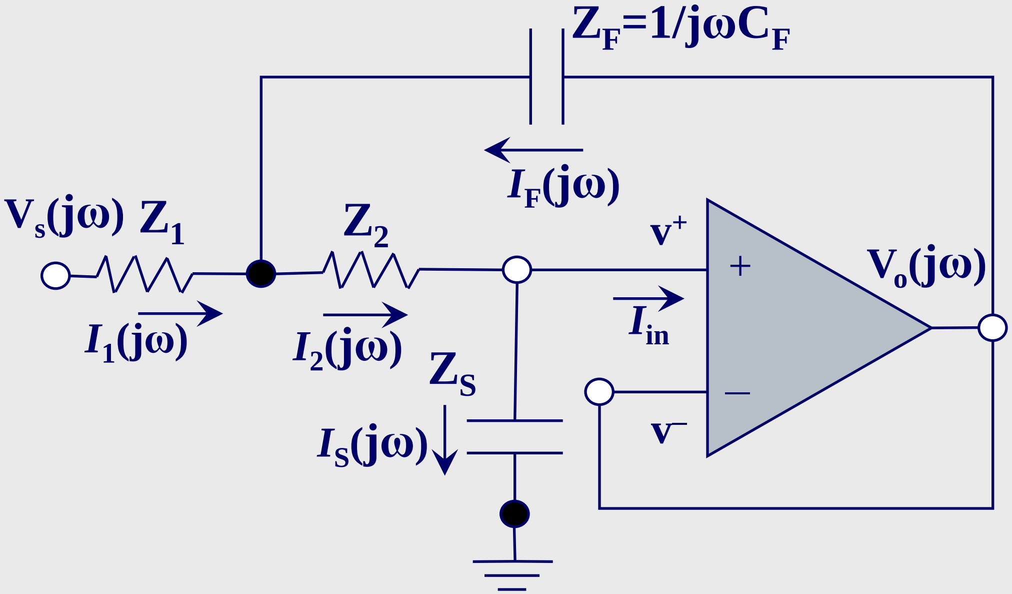
$$3V_a - V_o(3+j\omega) = 0$$

$$V_o = -\frac{6V_S}{\omega^2 - j5\omega - 6}$$

Op-Amps

11. find an expression for the gain

$\mathbf{C_F} = 1/6$ F, $\mathbf{R_1} = 3\Omega$, $\mathbf{R_2} = 2\Omega$, $\mathbf{C_S} = 1/6$ F



1. Transfer to frequency domain
2. Apply KCL at nodes **a** and **b**
3. Express $\mathbf{V_o}$ in terms of $\mathbf{V_S}$
4. Find the gain ($\mathbf{V_o}/\mathbf{V_S}$)

$$\frac{V_o}{V_S} = -\frac{6}{\omega^2 - j5\omega - 6}$$