Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
17 Nov	Mon	22	Combinational Logic	13.3 – 13.5		LAB 10	
18 Nov	Tue						
19 Nov	Wed	23	Sequential Logic	14.1			
20 Nov	Thu						
21 Nov	Fri		Recitation		HW 9		
22 Nov	Sat						
23 Nov	Sun						
24 Nov	Mon	24	ADC – DAC	15.4			
25 Nov	Tue		Recitation		HW 10		

Remember and be Thankful

2 Nephi 1:9, 20:

9 Wherefore, I, Lehi, have obtained a promise, that inasmuch as those whom the Lord God shall bring out of the land of Jerusalem shall keep his commandments, they shall prosper upon the face of this land; and they shall be kept from all other nations, that they may possess this land unto themselves. And if it so be that they shall keep his commandments they shall be blessed upon the face of this land, and there shall be none to molest them, nor to take away the land of their inheritance; and they shall dwell safely forever.

20 And he hath said that: **Inasmuch as ye shall keep my commandments ye shall prosper in the land; but inasmuch as ye will not keep my commandments ye shall be cut off from my presence.**

Lecture 22 – Boolean Algebra & Combinational Logic

Boolean Algebra

Boolean Algebra: the mathematics associated with binary numbers

- Developed by George Boole in 1854

Variables in boolean algebra can take only one of two possible values:

0 → FALSE

1 → TRUE

Rules of Boolean Algebra

1. $0 + X = X$
2. $1 + X = 1$
3. $X + X = X$
4. $X + \overline{X} = 1$
5. $0 \cdot X = 0$
6. $1 \cdot X = X$
7. $X \cdot X = X$
8. $X \cdot \overline{X} = 0$
9. $\overline{\overline{X}} = X$

10. $X + Y = Y + X$
11. $X \cdot Y = Y \cdot X$
12. $X + (Y + Z) = (X + Y) + Z$
13. $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$
14. $X \cdot (Y + Z) = X \cdot Y + X \cdot Z$

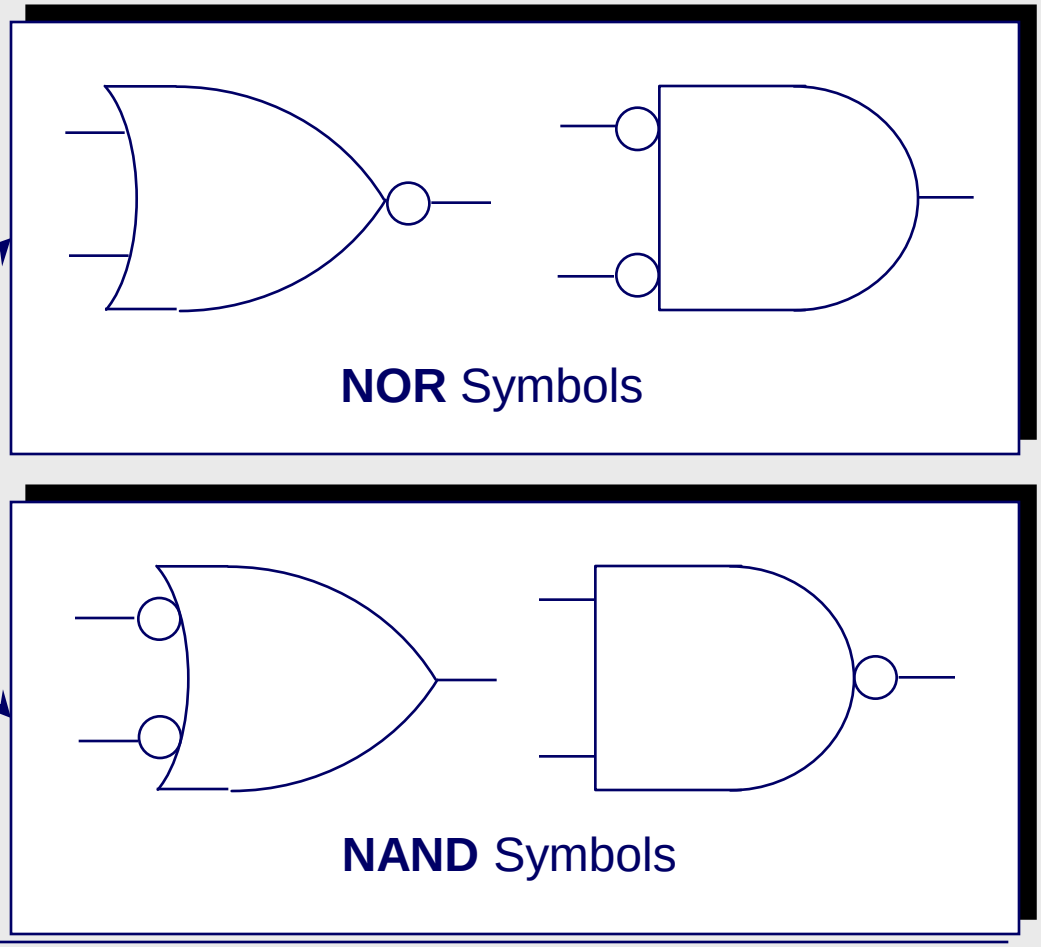
15. $X + X \cdot Z = X$
16. $X \cdot (X + Y) = X$
17. $(X + Y) \cdot (X + Z) = X + Y \cdot Z$
18. $X + \overline{X} \cdot Y = X + Y$
19. $X \cdot Y + Y \cdot Z + \overline{X} \cdot Z = X \cdot Y + \overline{X} \cdot Z$

DeMorgan's Law

To distribute the bar, change the operation.

$$\overline{A+B} = \overline{A} \cdot \overline{B}$$

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$



Boolean Algebra

Example1: simplify the following function

$$OUT = \overline{A}\,\overline{B}D + \overline{A}BD + BCD + ACD$$

Boolean Algebra

Example1: simplify the following function

$$OUT = \overline{A}\,\overline{B}D + \overline{A}BD + BCD + ACD \quad \text{Rule 14}$$

$$OUT = \overline{A}D\left(\overline{B} + B\right) + BCD + ACD \quad \text{Rule 4}$$

$$OUT = \overline{A}D + BCD + ACD \quad \text{Rule 14}$$

$$OUT = D\left(\overline{A} + AC\right) + BCD \quad \text{Rule 18}$$

$$OUT = D\left(\overline{A} + C\right) + BCD \quad \text{Rule 14}$$

$$OUT = \overline{A}D + CD + BCD \quad \text{Rule 14}$$

$$OUT = CD(1 + B) + \overline{A}D \quad \text{Rule 2}$$

$$OUT = CD + \overline{A}D \quad \text{Rule 14}$$

$$OUT = D\left(\overline{A} + C\right)$$

Boolean Algebra

Example2: Simplify the equation created by the following truth table

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Boolean Algebra

Example2: Simplify the equation created by the following truth table

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$Z = \overline{A}\,\overline{B}C + \overline{A}BC + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} + ABC$$

Boolean Algebra

Example2: Simplify the equation created by the following truth table

A	B	C	Z
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$Z = \overline{A}\,\overline{B}C + \overline{A}BC + A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} + ABC$$

$$Z = \overline{A}C\left(\overline{B} + B\right) + A\overline{B}\left(\overline{C} + C\right) + AB\left(\overline{C} + C\right)$$

$$Z = \overline{A}C + A\overline{B} + AB$$

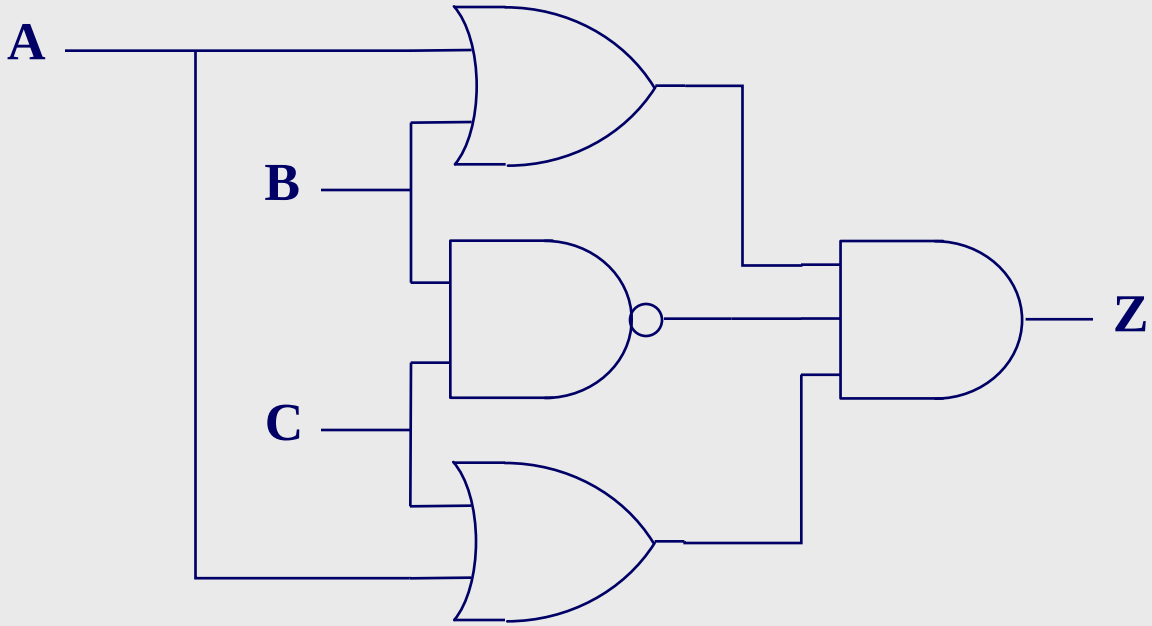
$$Z = \overline{A}C + A\left(\overline{B} + B\right)$$

$$Z = \overline{A}C + A$$

$$Z = A + C$$

Boolean Algebra

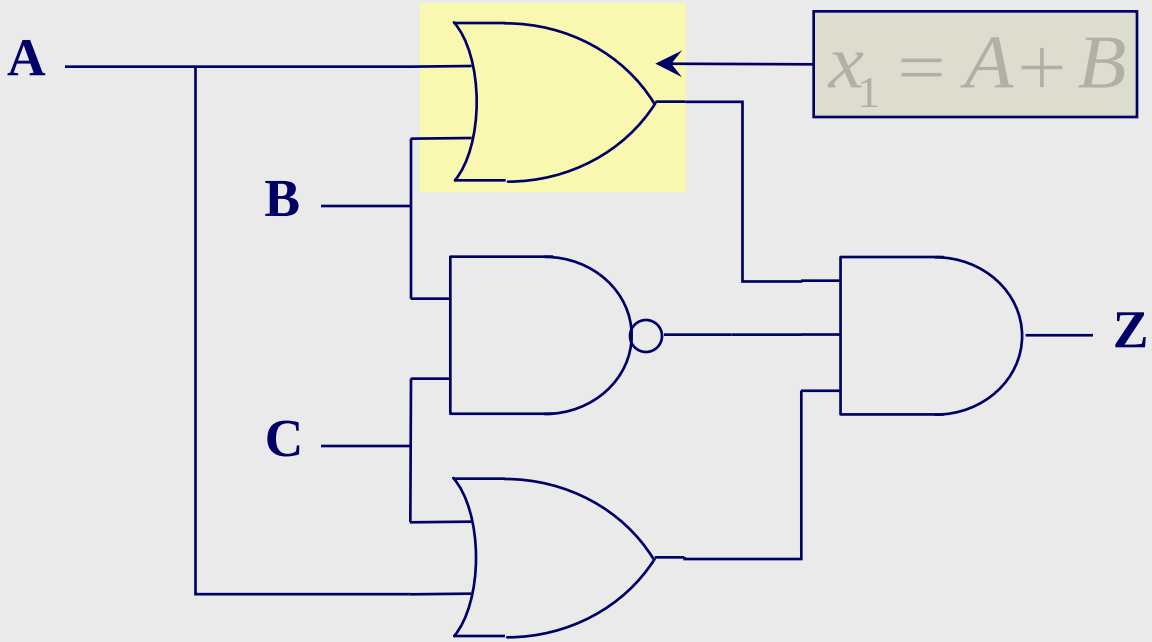
Example3: Determine the truth table



A	B	C	Z
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?

Boolean Algebra

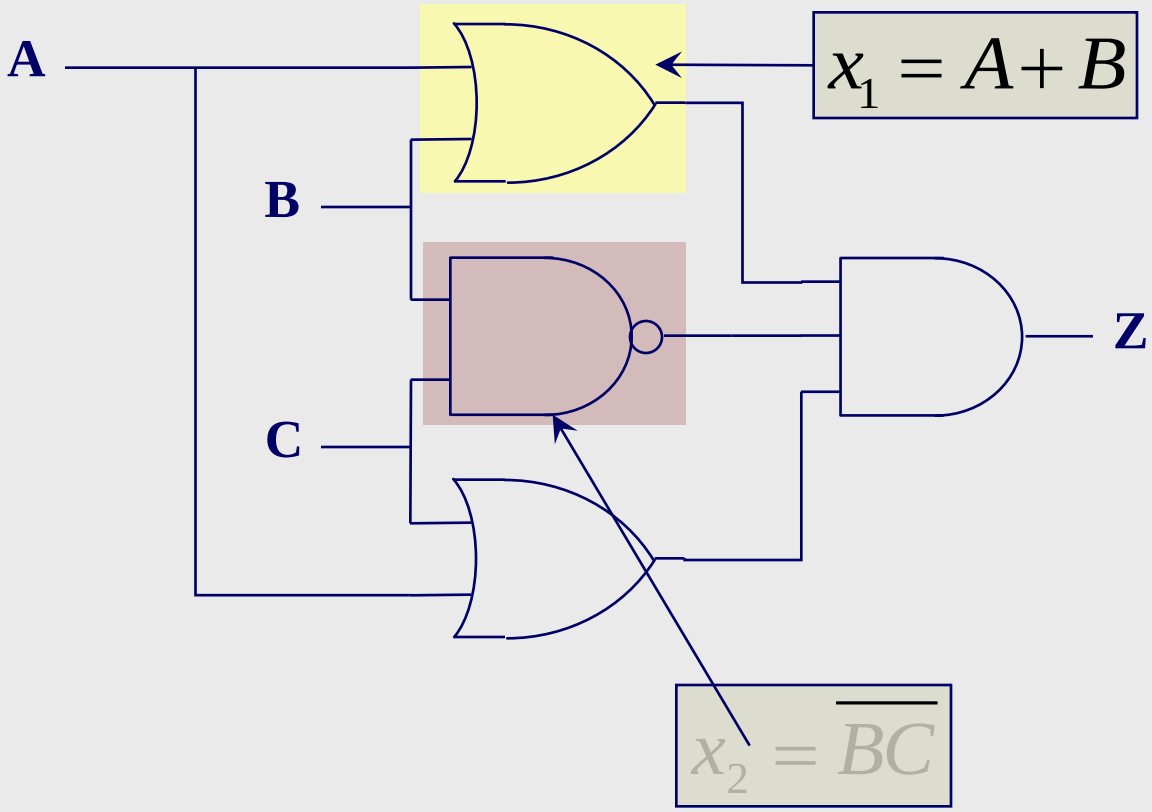
Example3: Determine the truth table



A	B	C	x_1	Z
0	0	0	0	?
0	0	1	0	?
0	1	0	1	?
0	1	1	1	?
1	0	0	1	?
1	0	1	1	?
1	1	0	1	?
1	1	1	1	?

Boolean Algebra

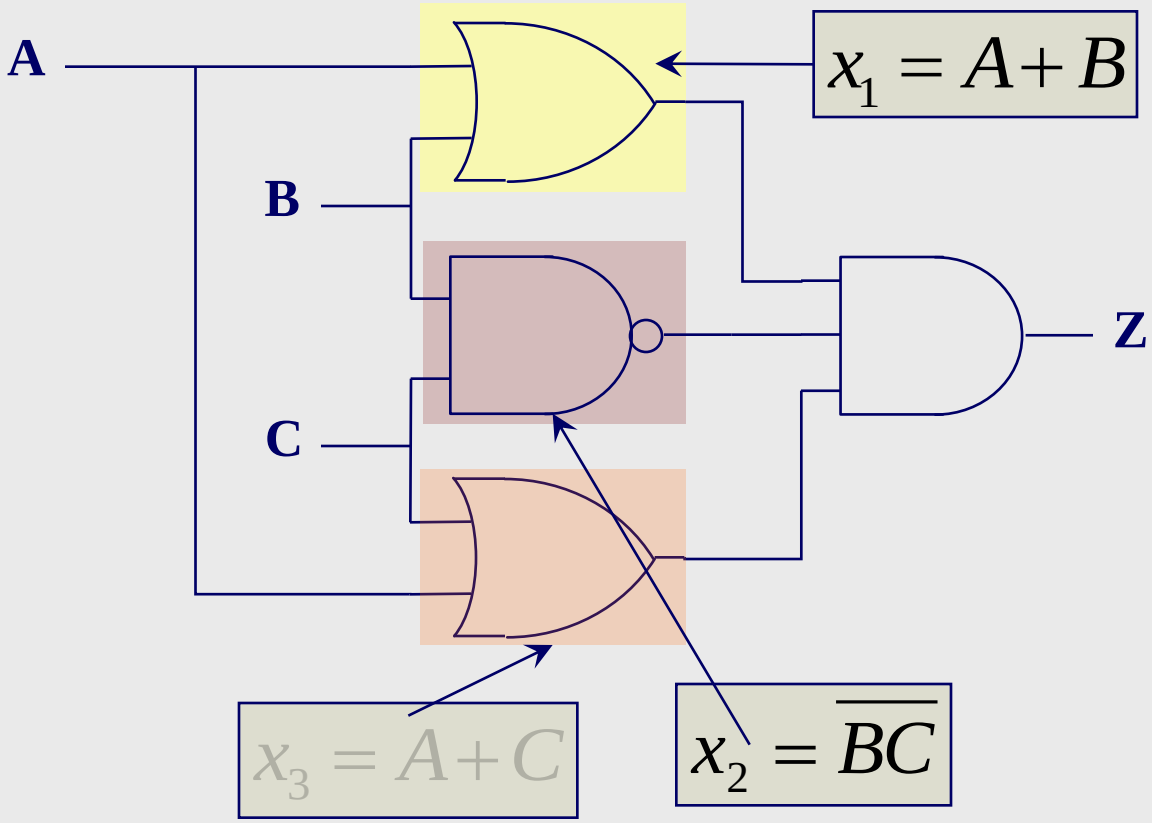
Example3: Determine the truth table



A	B	C	x_1	x_2	Z
0	0	0	0	1	?
0	0	1	0	1	?
0	1	0	1	1	?
0	1	1	1	0	?
1	0	0	1	1	?
1	0	1	1	1	?
1	1	0	1	1	?
1	1	1	1	0	?

Boolean Algebra

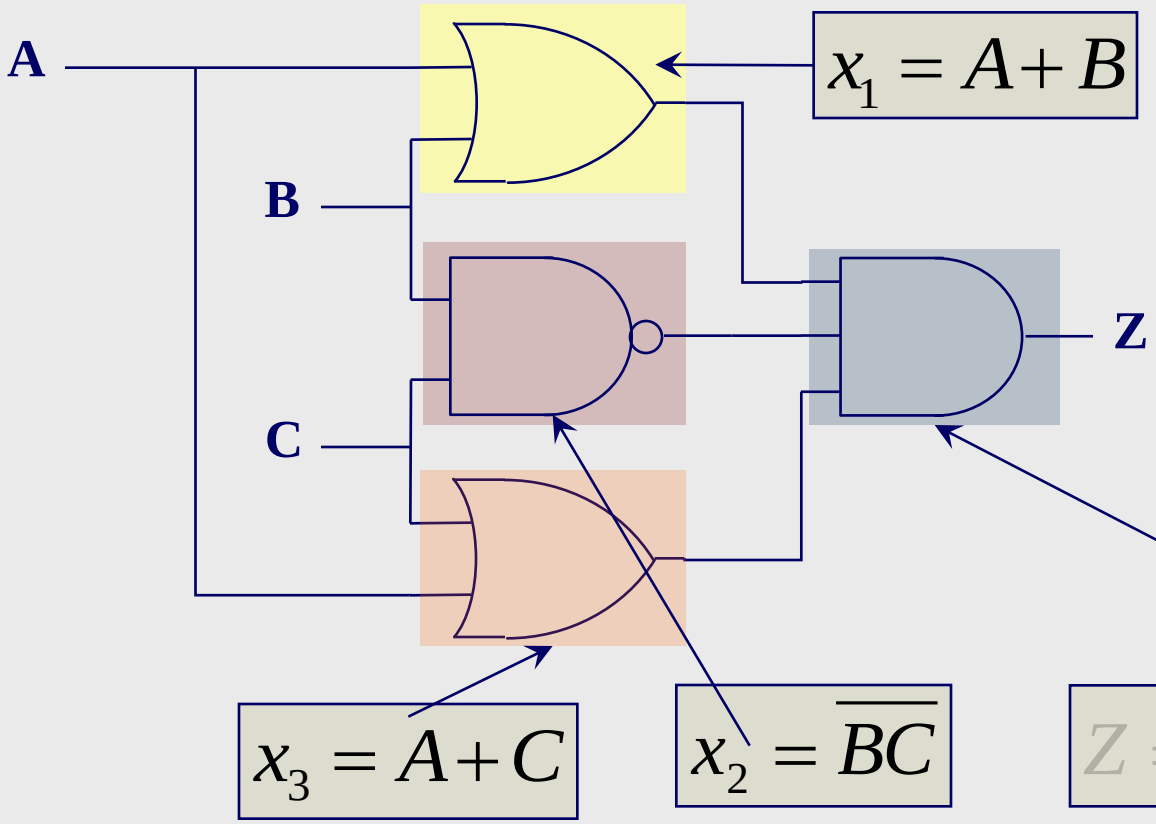
Example3: Determine the truth table



A	B	C	x_1	x_2	x_3	Z
0	0	0	0	1	0	?
0	0	1	0	1	1	?
0	1	0	1	1	0	?
0	1	1	1	0	1	?
1	0	0	1	1	1	?
1	0	1	1	1	1	?
1	1	0	1	1	1	?
1	1	1	1	0	1	?

Boolean Algebra

Example3: Determine the truth table



A	B	C	x_1	x_2	x_3	Z
0	0	0	0	1	0	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	1	0	1	0
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	0	1	0

Boolean Algebra

Example3: Determine the truth table

$$
\begin{aligned}
Z &= (A+B)\overline{(BC)}(A+C) \\
&= A\overline{B}\,\overline{C} + A\overline{B}C + AB\overline{C} \\
&= A\overline{C}(\overline{B}+B) + A\overline{B}C \\
&= A\overline{C} + A\overline{B}C \\
&= A(\overline{C}+\overline{B}C) \\
&= A(\overline{C}+\overline{B})
\end{aligned}
$$

A	B	C	x_1	x_2	x_3	Z
0	0	0	0	1	0	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	1	0	1	0
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	0	1	0

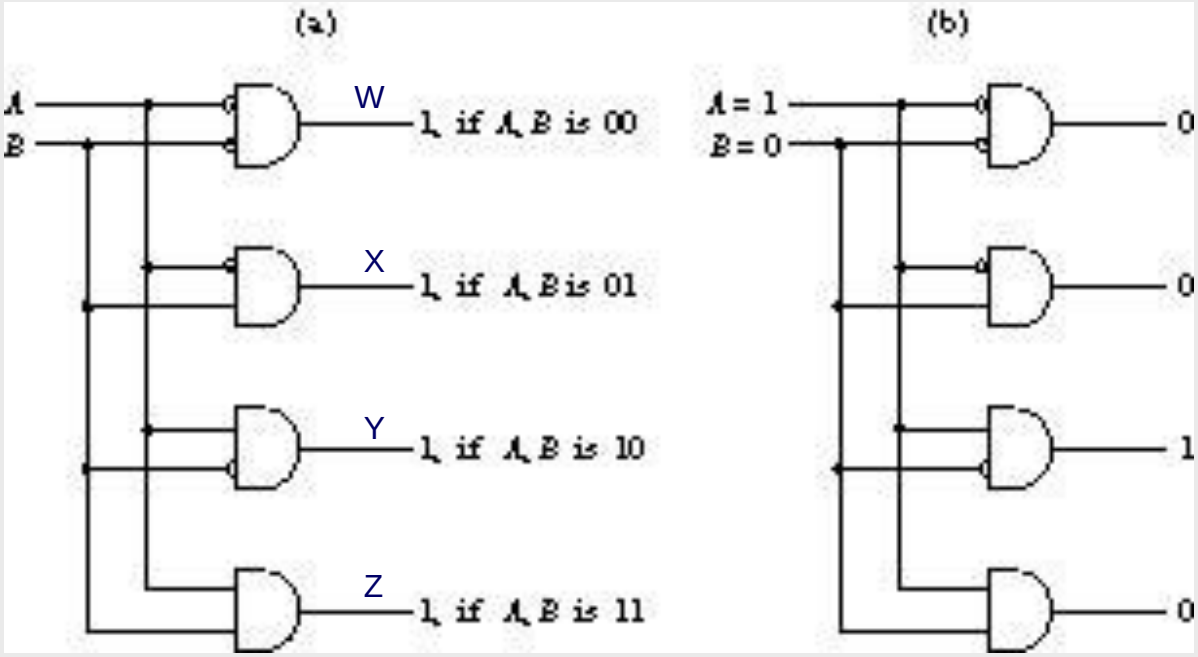
Combinational Logic

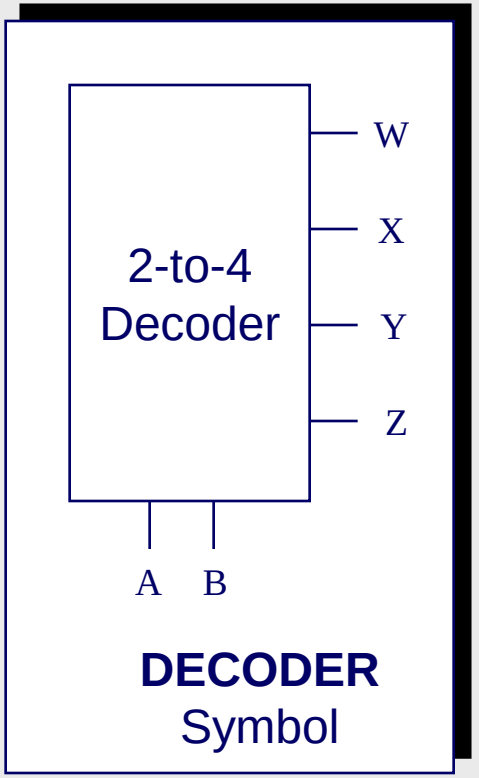
Decoders

Multiplexers

Decoders

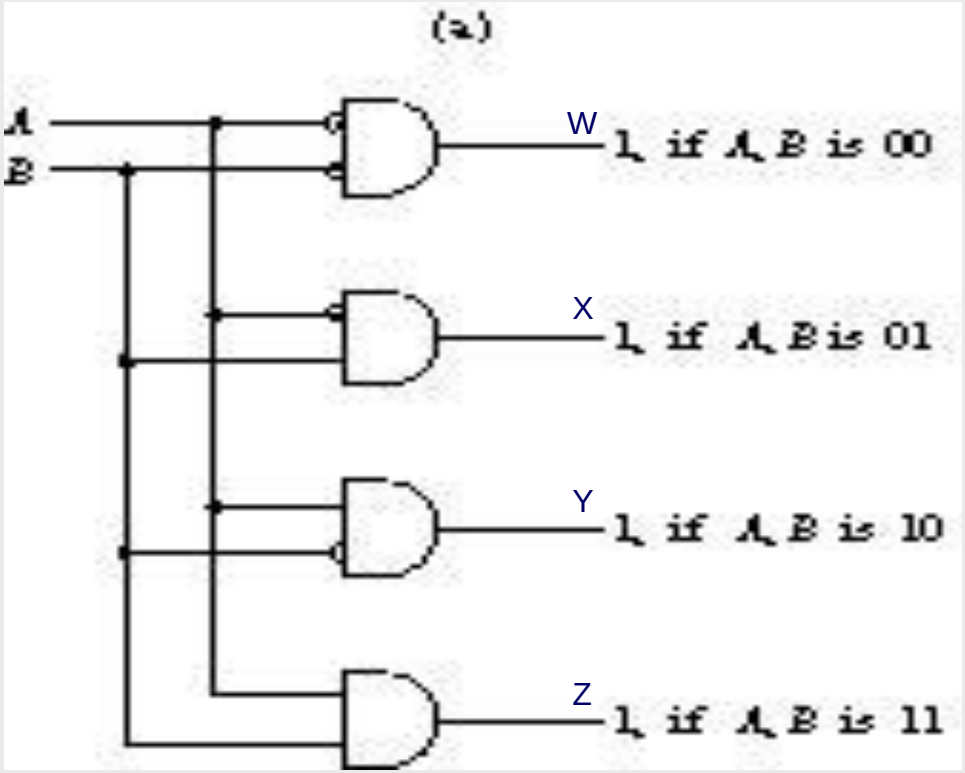
- Decode the input and signify its value by raising **just one** of its outputs.
- A decoder with **n** inputs has 2^n outputs





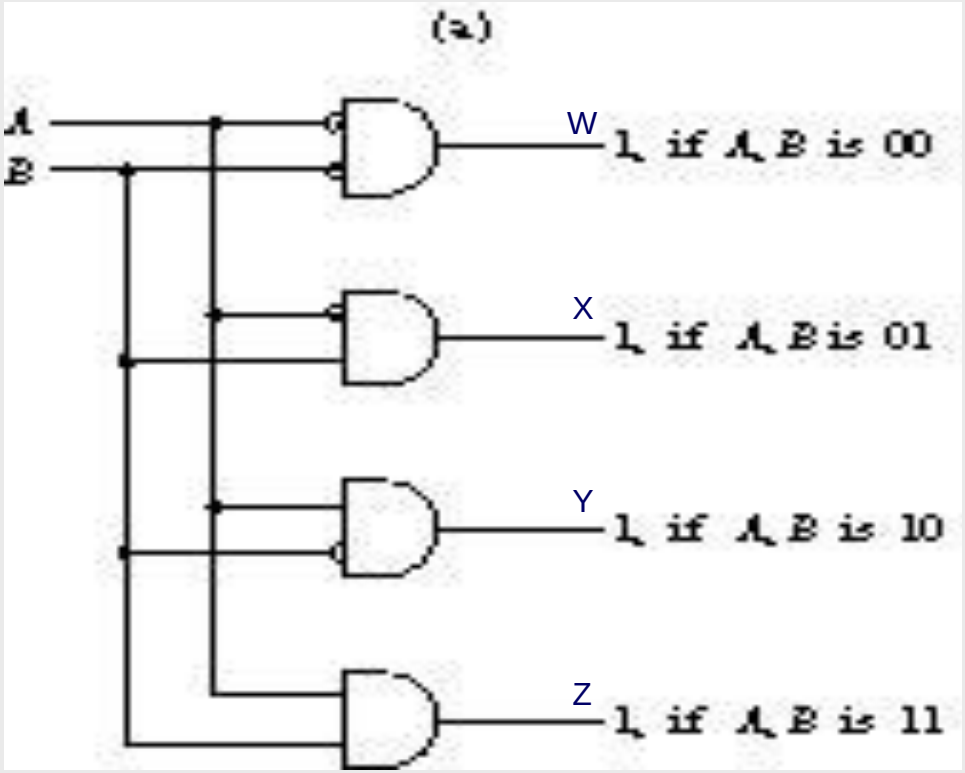
Decoders

- Write the truth table



Decoders

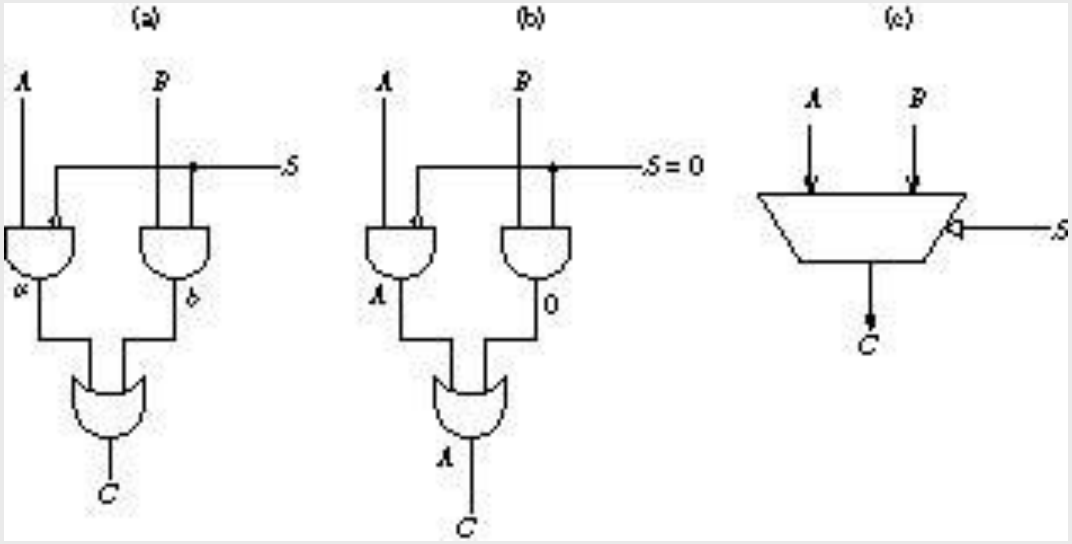
- Write the truth table

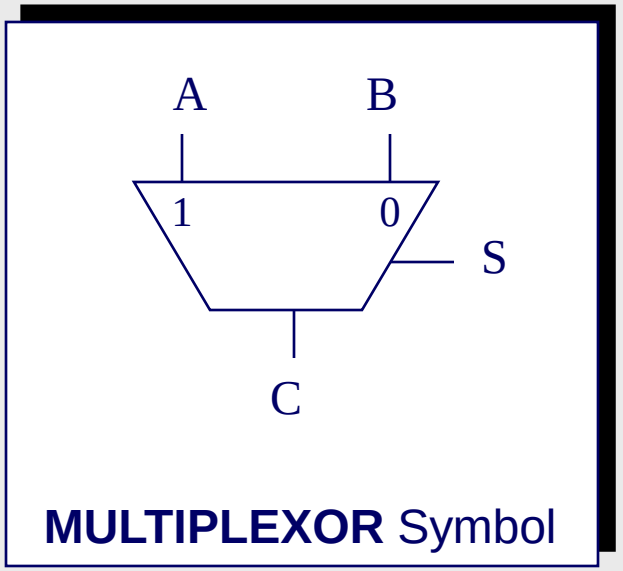


A	B	W	X	Y	Z
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

Multiplexors

- Connect one of its inputs to its output according to select signals



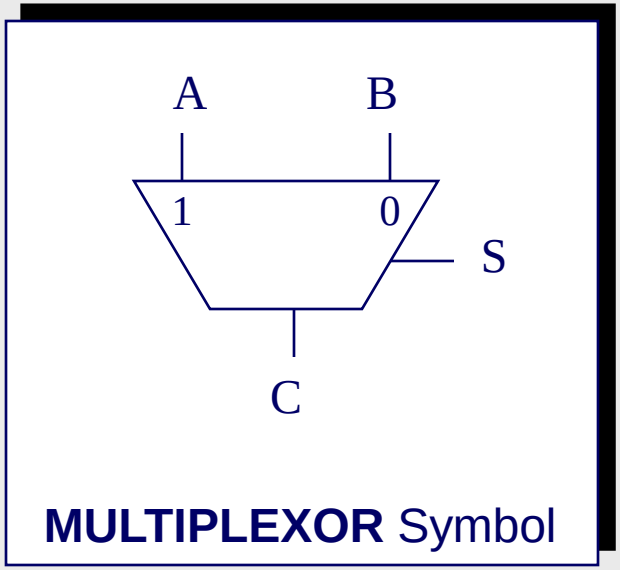


- Useful for *selecting* one from a collection of data inputs.
- Usually has 2^n inputs and **n** select lines.

Multiplexors

- Write the truth table

A	B	S	C
0	0	0	?
0	0	1	?
0	1	0	?
0	1	1	?
1	0	0	?
1	0	1	?
1	1	0	?
1	1	1	?



Multiplexors

- Write the truth table

A	B	S	C
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1