Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
24 Nov	Mon	24	DAC	15.4			
25 Nov	Tue		Recitation		HW 10		
26 Nov	Wed		Thanksgiving				
27 Nov	Thu		Thanksgiving				
28 Nov	Fri		Thanksgiving				
29 Nov	Sat						
30 Nov	Sun						
1 Dec	Mon		Final Review			LAB 8	
2 Dec	Tue						

Conversion

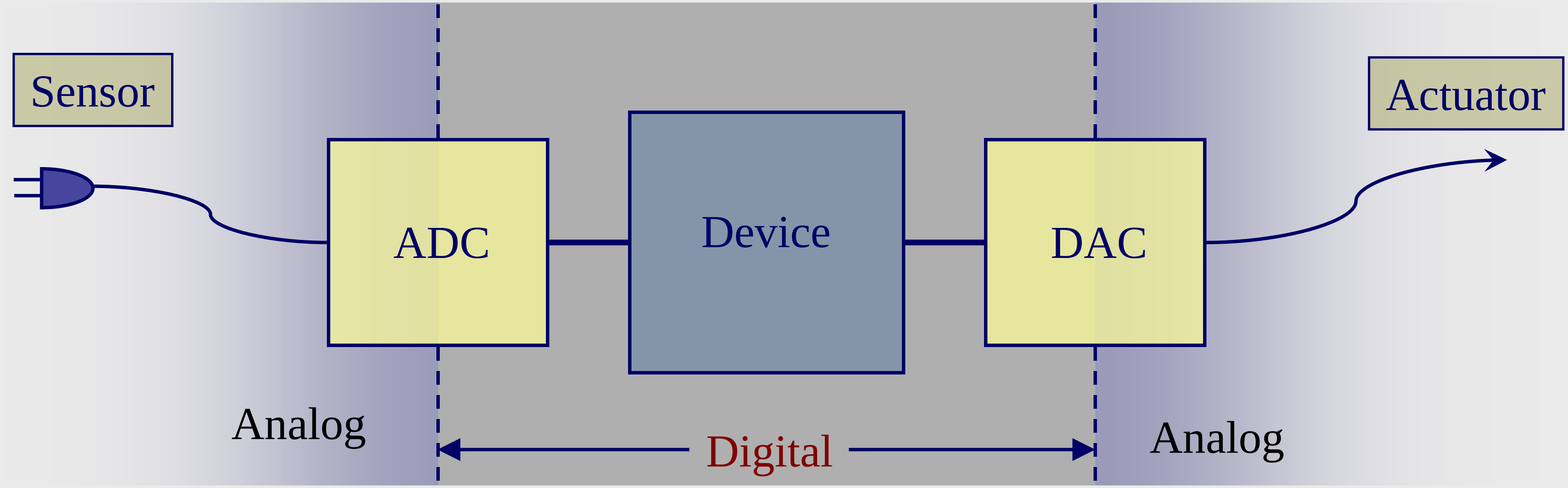
Luke 22:32

32 But I have prayed for thee, that thy faith fail not: and when thou art **converted**, strengthen thy brethren.

Lecture 24 – Digital to Analog Converters (DACs)

ADC/DAC

- Power coming from the wall is **analog**, but most devices (appliances, computers, etc.) are **digital** thus there must be a conversion
  - **Analog to digital (ADC) –** coming into a device
  - **Digital to analog (DAC) –** going out of a device



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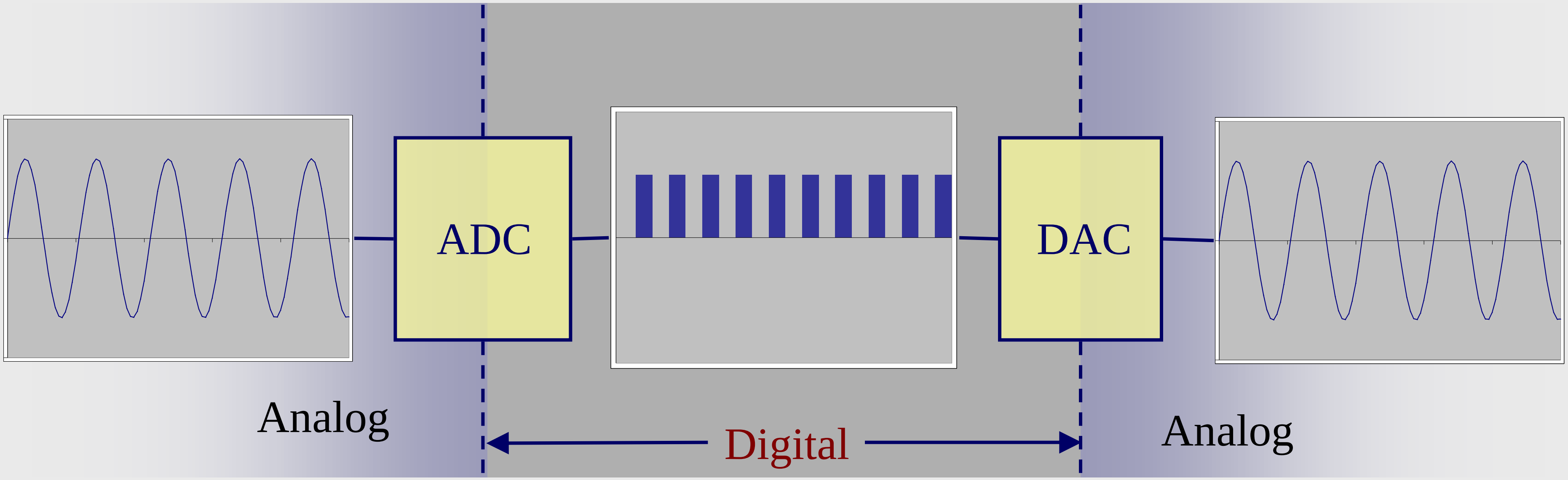


2.23094… 001110011… 2.23094…

Digital to Analog Converter (DAC)

DAC: converts a **binary word** to an analog output voltage or current

Binary word (B): a sequence of n **1**s and **0**s

$\mathbf{B = b_{n-1}b_{n-2}\ldots b_2b_1b_0}$

- EX:

  B = 10100101 (n = 8)

Word length (n): the number of bits in the sequence of **1**s and **0**s representing an output

- EX:

  0110 – 4-bit word length

  100101 – 6-bit word length

Digital to Analog Converter (DAC)

DAC: converts a **binary word** to an analog output voltage or current

Resolution δv: minimum step size by which the output voltage (or current) can increment

Output voltage v_a: the analog value represented by the binary word **B**

- EX: let n=4

  $v_a = (2^3 \cdot b_3 + 2^2 \cdot b_2 + 2^1 \cdot b_1 + 2^0 \cdot b_0)\delta v$

Max output voltage v_{aMax}: the maximum analog value

- EX: let n=4

  $v_{aMax} = (2^3 + 2^2 + 2^1 + 2^0)\delta v$

  $= (2^n - 1)\,\delta v$

Example: $\delta v = 1V$, **B** = 10110 (n = 5)

Find v_{aMax} and v_a

$v_{aMax} = (2^n - 1)\,\delta v$

$= (2^5 - 1) \cdot 1$

$= 31$

$v_a = (2^4 \cdot b_4 + 2^3 \cdot b_3 + 2^2 \cdot b_2 + 2^1 \cdot b_1 + 2^0 \cdot b_0)\delta v$

$= (16 \cdot 1 + 8 \cdot 0 + 4 \cdot 1 + 2 \cdot 1 + 1 \cdot 0) \cdot 1$

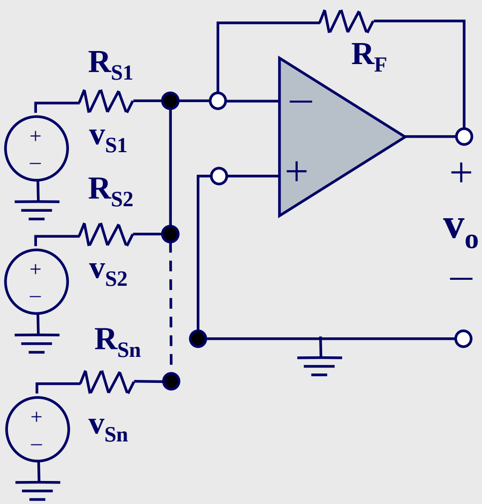
$= (16 + 4 + 2)$

$= 22$

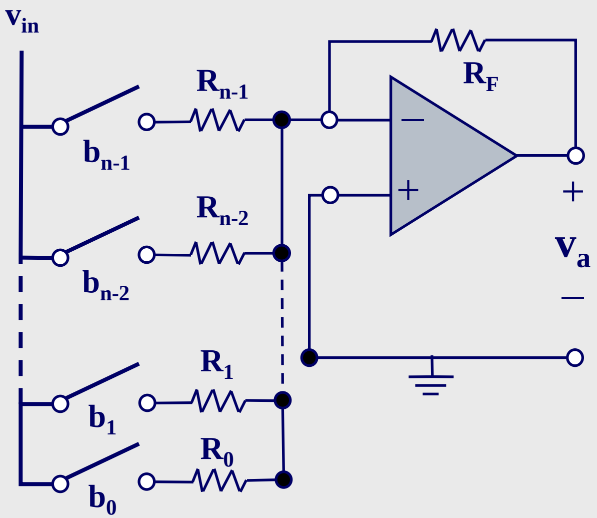
Digital to Analog Converter (DAC)

Building a DAC:

use a summing amplifier



Summing amplifier

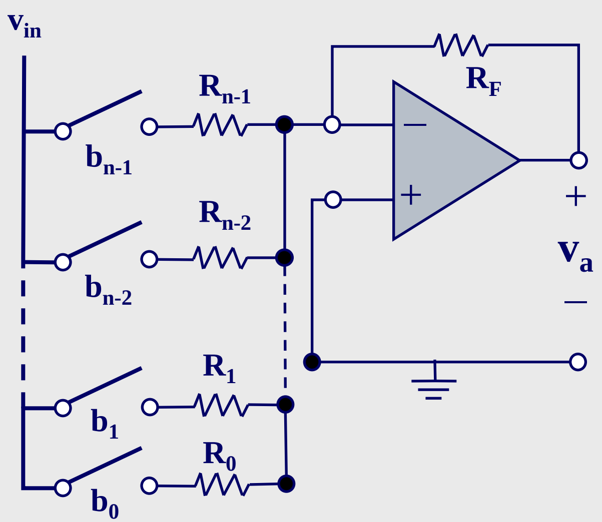


DAC

Digital to Analog Converter (DAC)

Building a DAC:

the analog output ($\mathbf{v_a}$) is proportional to the binary word **B**



$$v_a = -\sum_{i=0}^{n-1} \frac{R_F}{R_i} \cdot b_i \cdot v_{in}$$

$\mathbf{v_{si}}$ in summing amplifier

Choose

$$R_i = \frac{R_0}{2^i}$$

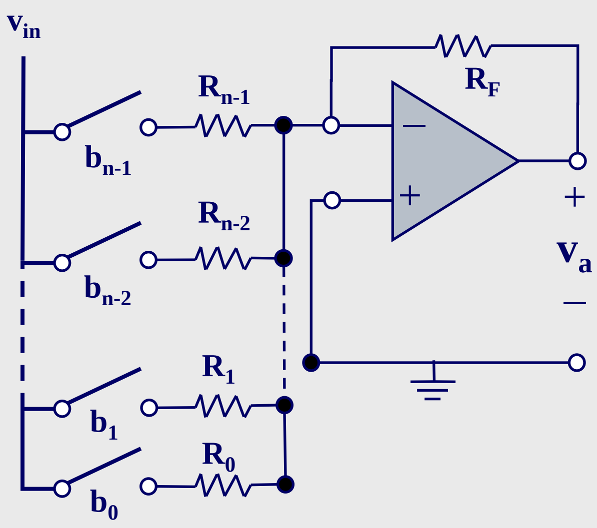
THUS

$$v_a = -\frac{R_F}{R_0} \cdot v_{in} \sum_{i=0}^{n-1} 2^i \cdot b_i$$

Digital to Analog Converter (DAC)

<u>Example1</u>: $\mathbf{v_{in}}$ = 5V, **B** = 1011, $\mathbf{R_0}$ = 32kΩ, $\mathbf{R_F}$ = 64kΩ

Find $\mathbf{R_n}$ and $\mathbf{v_a}$



Digital to Analog Converter (DAC)

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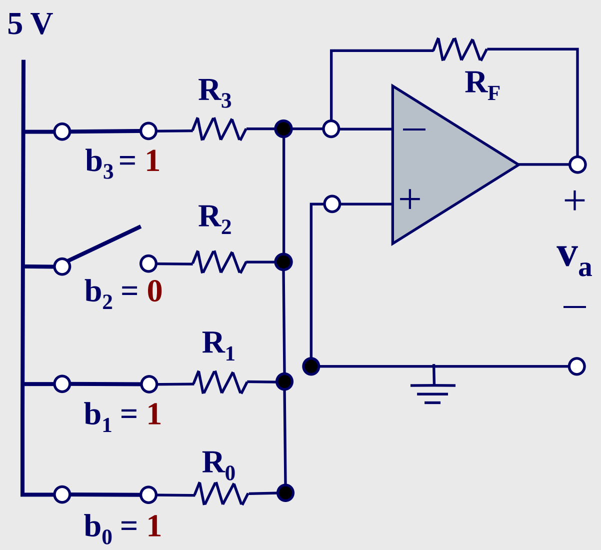


$$R_1 = \frac{R_0}{2^1} = \frac{32k\Omega}{2} = 16k\Omega$$

$$R_2 = \frac{R_0}{2^2} = \frac{32k\Omega}{4} = 8k\Omega$$

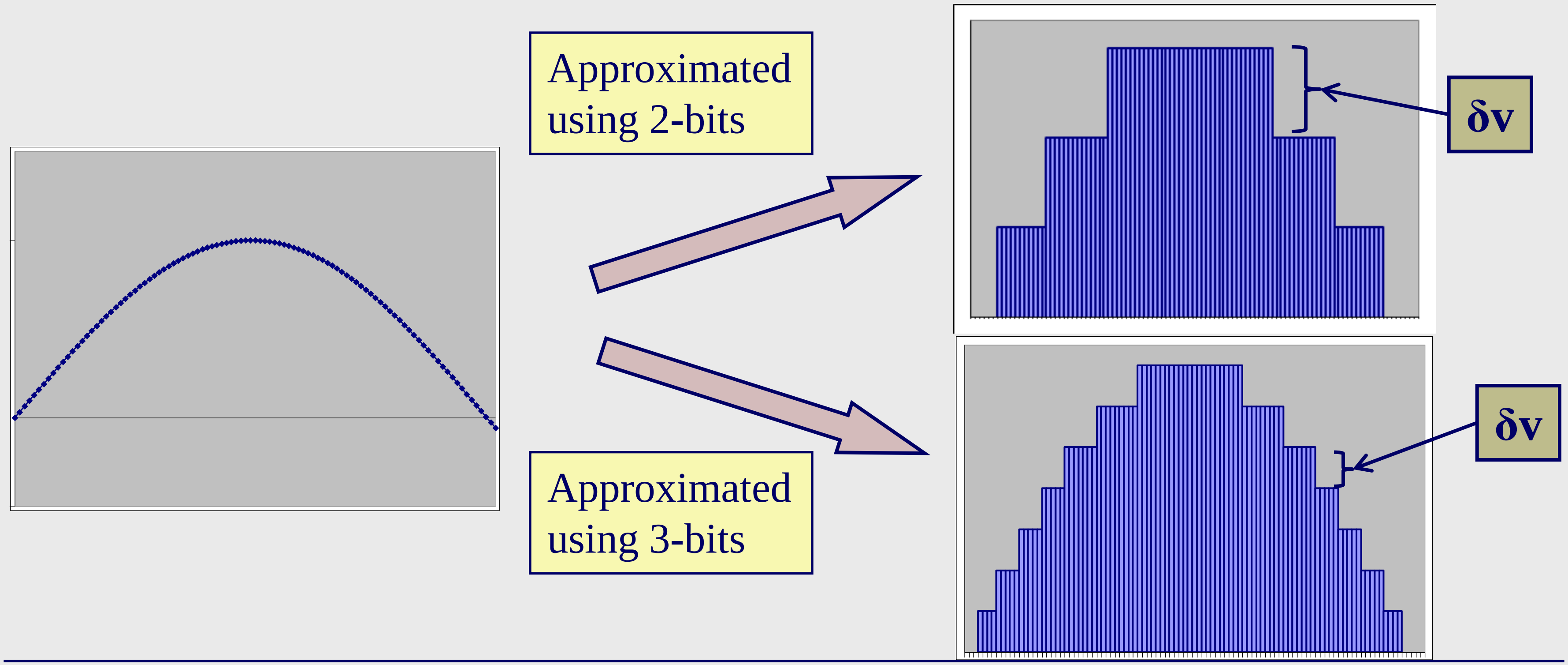
$$R_3 = \frac{R_0}{2^3} = \frac{32k\Omega}{8} = 4k\Omega$$

$$\begin{aligned} v_a &= -\frac{R_F}{R_0} \cdot v_{in} \sum_{i=0}^{n-1} 2^i \cdot b_i \\ &= -\frac{64k\Omega}{32k\Omega} \cdot 5 \cdot \left(2^3 \cdot b_3 + 2^2 \cdot b_2 + 2^1 \cdot b_1 + 2^0 \cdot b_0\right) \\ &= -10 \cdot \left(8 \cdot 1 + 4 \cdot 0 + 2 \cdot 1 + 1 \cdot 1\right) \\ &= -10 \cdot \left(11\right) \\ &= -110\,V \end{aligned}$$

Digital to Analog Converter (DAC)

<u>Building a DAC</u>:

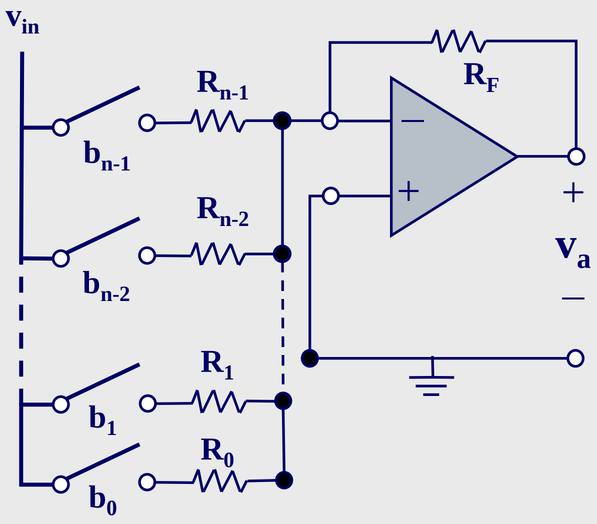
- the analog output (**v_a**) has a step-like appearance because of the discrete nature of a binary signal
- The **resolution** (coarseness of the "staircase") can be adjusted by changing the **word length** (the number of bits)



Digital to Analog Converter (DAC)

<u>Example2</u>: find the smallest resolution **δv** of an 8-bit DAC

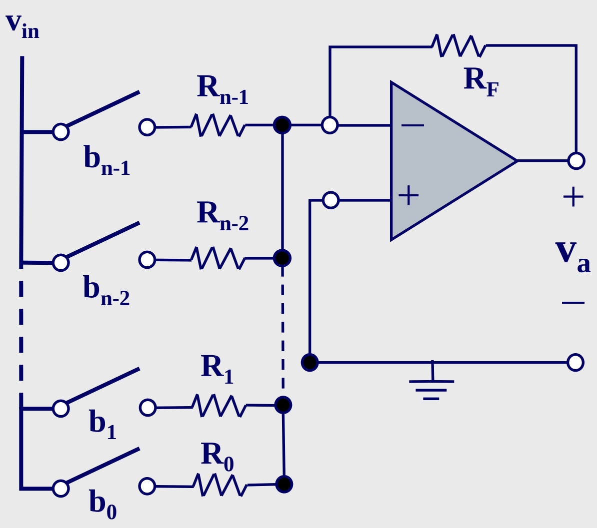
$v_{aMax} = 12V$



Digital to Analog Converter (DAC)

<u>Example2</u>: find the smallest resolution δv of an 8-bit DAC

$v_{aMax} = 12V$



$$v_{a\max} = \left(2^n - 1\right)\delta v$$

$$\delta v = \frac{v_{a\max} - v_{a\min}}{\left(2^n - 1\right)}$$

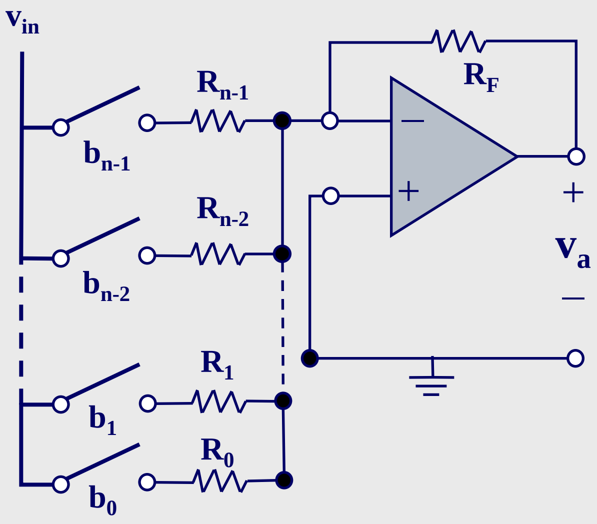
$$= \frac{12 - 0}{\left(2^8 - 1\right)}$$

$$= 47.1\,mV$$

Digital to Analog Converter (DAC)

<u>Example3</u>: find an expression for the number of bits required in a DAC for a given **range** and **resolution**

$$\textbf{range} = v_{aMax} - v_{aMin}$$
$$\textbf{resolution} = \delta v$$

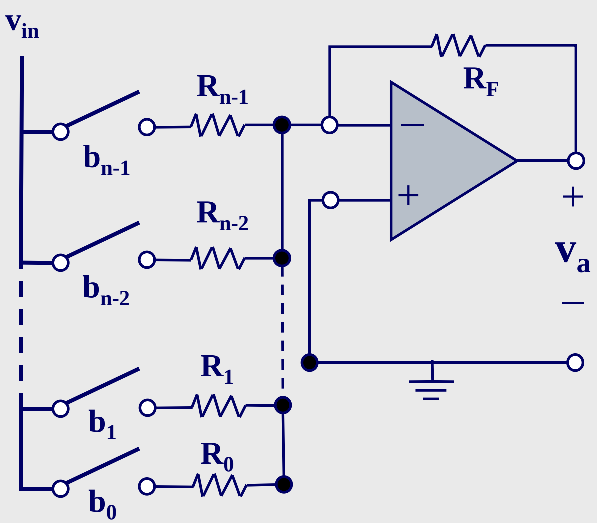


Digital to Analog Converter (DAC)

<u>Example3</u>: find an expression for the number of bits required in a DAC for a given **range** and **resolution**

range = $v_{aMax} – v_{aMin}$

resolution = δv



$$\delta v = \frac{v_{a\max} - v_{a\min}}{\left(2^n - 1\right)}$$

$$2^n - 1 = \frac{v_{a\max} - v_{a\min}}{\delta v}$$

$$2^n = \frac{v_{a\max} - v_{a\min}}{\delta v} + 1$$

$$n = \log_2\left(\frac{v_{a\max} - v_{a\min}}{\delta v} + 1\right)$$

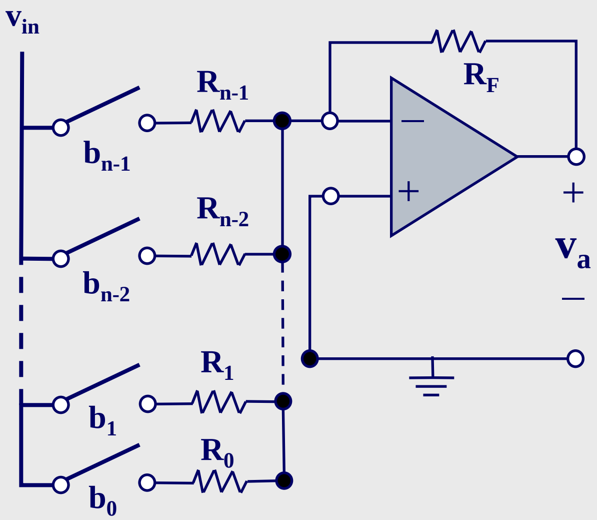
$$n = \left\lceil \frac{\log_{10}\left[\left(v_{a\max} - v_{a\min}\right)/\delta v + 1\right]}{\log_{10} 2} \right\rceil$$

Digital to Analog Converter (DAC)

<u>Example4</u>: find the number of bits required in a DAC given that:

range = 10V

resolution = 10 mV



Digital to Analog Converter (DAC)

<u>Example4</u>: find the number of bits required in a DAC given that:

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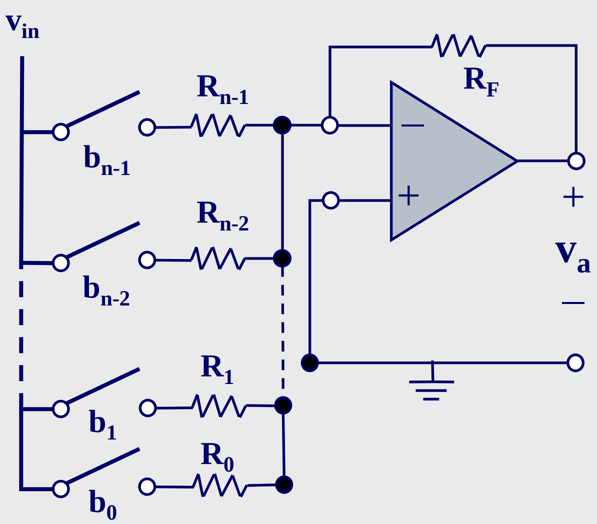


$$n = \left\lceil \frac{\log\left((v_{a\max} - v_{a\min})/\delta v + 1\right)}{\log 2} \right\rceil$$

$$= \left\lceil \frac{\log\left(10/10^{-2} + 1\right)}{\log 2} \right\rceil$$

$$= \left\lceil \frac{\log(1001)}{\log 2} \right\rceil$$

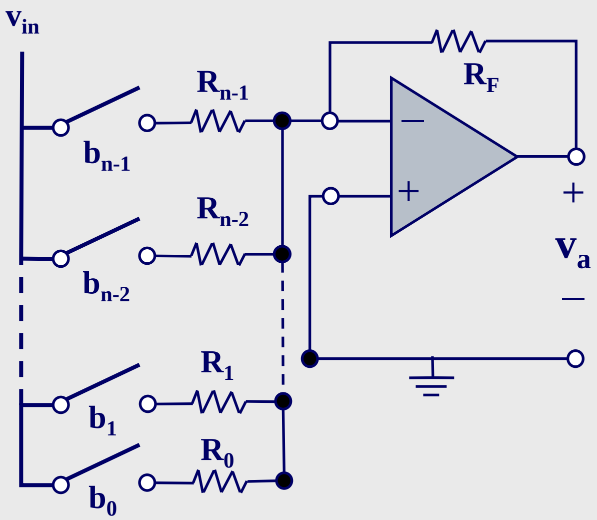
$$= \lceil 9.97 \rceil$$

$$= 10$$

Digital to Analog Converter (DAC)

<u>Example5</u>: find the resistor values for a DAC with:

range = 15V
δv = 1V
v_{in} = 5V
R_F = 2kΩ



Digital to Analog Converter (DAC)

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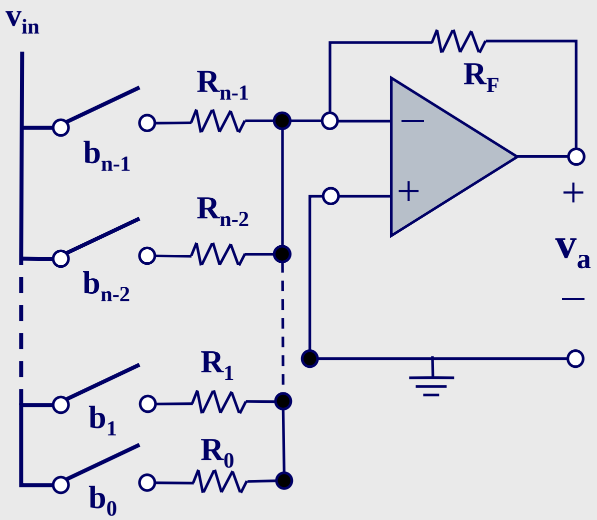

$$n = \left\lceil \frac{\log\left(\left(v_{a\max} - v_{a\min}\right)/\delta v + 1\right)}{\log 2} \right\rceil$$

$$= \left\lceil \frac{\log\left(15/1+1\right)}{\log 2} \right\rceil$$

$$= \lceil 4 \rceil$$

$$= 4$$

Digital to Analog Converter (DAC)

<u>Example5</u>: find the resistor values for a DAC with:

range = 15V
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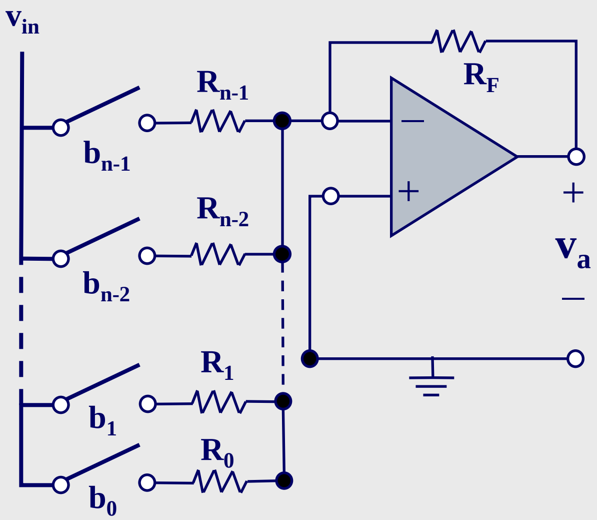
v_{in} = 5V
R_F = 2kΩ



$$v_a = -\frac{R_F}{R_0} \cdot v_{in} \sum_{i=0}^{n-1} 2^i \cdot b_i$$

$$v_{a\max} = \frac{R_F}{R_0} \cdot v_{in} \left(2^n - 1\right)$$

$$R_0 = \frac{R_F}{v_{a\max}} \cdot v_{in} \left(2^n - 1\right)$$

$$= \frac{2}{15} \left(5\right)\left(2^4 - 1\right)$$

$$= 10\ k\Omega$$

Digital to Analog Converter (DAC)

Example5: find the resistor values for a DAC with:

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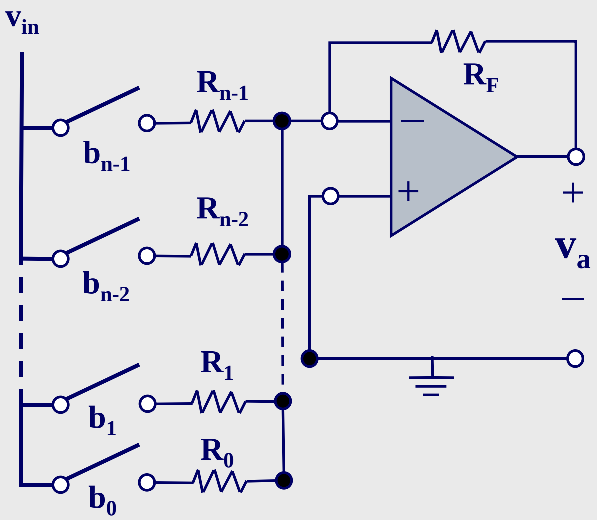

$$R_i = \frac{R_0}{2^i}$$

$$R_1 = \frac{R_0}{2^1} = \frac{10^3}{2} = 5k\Omega$$

$$R_2 = \frac{R_0}{2^2} = \frac{10^3}{4} = 2.5k\Omega$$

$$R_3 = \frac{R_0}{2^3} = \frac{10^3}{8} = 1.25k\Omega$$