Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
17 Sept	Wed	5	Ohm's Law	2.6			
18 Sept	Thu					LAB 1	
19 Sept	Fri		Recitation		HW 2		
20 Sept	Sat						
21 Sept	Sun						
22 Sept	Mon	6	Things Practical	2.6 – 2.8		LAB 2	
23 Sept	Tue						
24 Sept	Wed	7	Network Analysis	3.1 – 3.2			

Divided We Fall

Matthew 12:25-26

25 And Jesus knew their thoughts, and said unto them, Every kingdom **divided** against itself is brought to desolation; and every city or house **divided** against itself shall not stand:

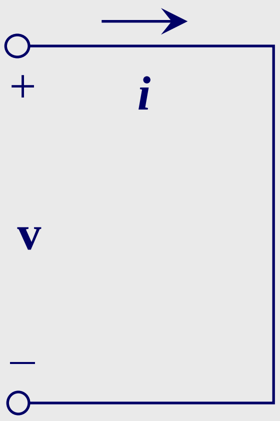
Nephi 7:2

2 And the people were **divided** one against another; and they did **separate** one from another into tribes, every man according to his family and his kindred and friends; and thus they did destroy the government of the land.

Lecture 5 – Resistance & Ohm's Law

Open and Short Circuits

- **Short circuit**: a circuit element across which the voltage is zero regardless of the current flowing through it
  - Resistance approaches zero
  - Flow of current is unimpeded
  - ex: an ideal wire
    - In reality there is a small resistance



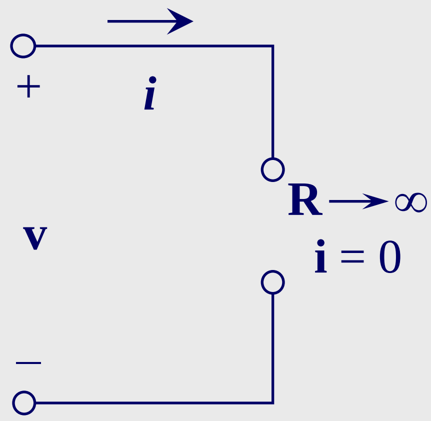
R = 0
v = 0

Open and Short Circuits

Undesirable short circuit – accidental connection between two nodes that are meant to be at different voltages. The resulting excessive current causes: overheating, fire, or explosion

Open and Short Circuits

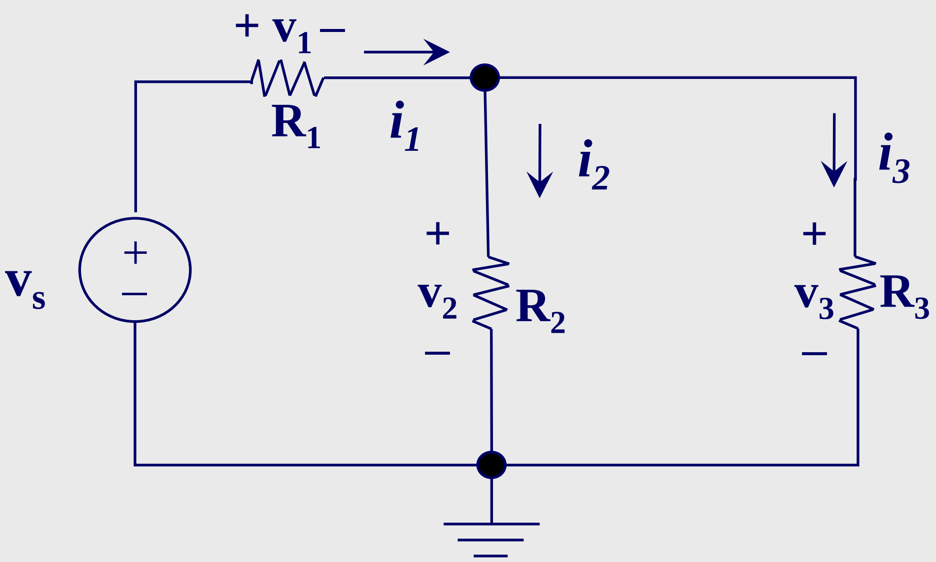
- **Open circuit**: a circuit element through which zero current flows regardless of the voltage applied to it
  - Resistance approaches infinity
  - No current flows
  - ex: a break in a circuit
    - At sufficiently high voltages arcing occurs



Open and Short Circuits

<u>Example</u>: What happens to $\mathbf{R_2}$ and $\mathbf{R_3}$ if $\mathbf{R_1}$ is shorted?

- $\mathbf{V_s}$ **= 2**V, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = $\mathbf{R_3}$ = 4Ω, $\boldsymbol{i_1}$ = 500mA
- $\mathbf{R_2}$ and $\mathbf{R_3}$ = ¼ W rating, $\mathbf{R_1}$ = ½ W rating



Open and Short Circuits

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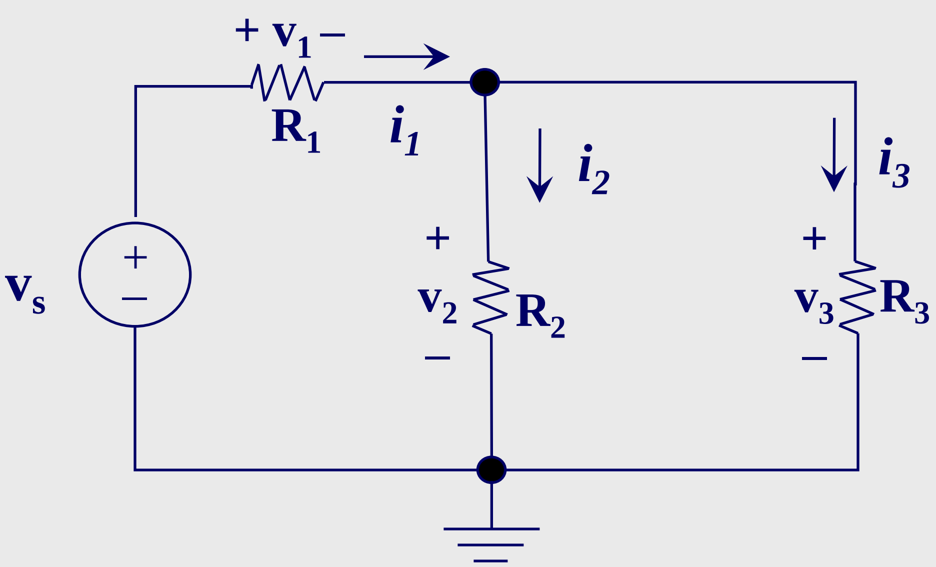


$$\begin{aligned} v_1 &= i_1 \cdot R_1 \\ &= 500\,mA\,(2\Omega) \\ &= 1V \end{aligned}$$

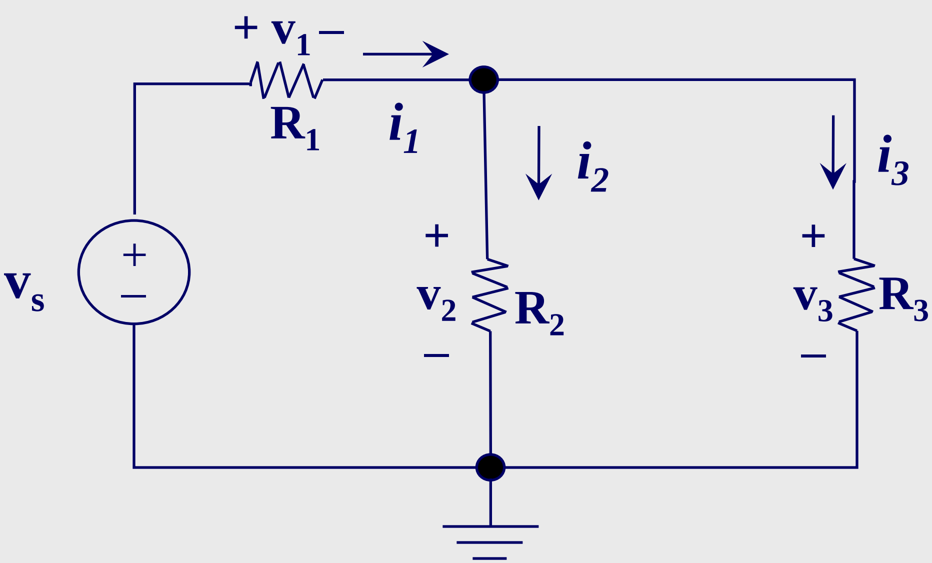
$$\begin{aligned} -v_s + v_1 + v_2 &= 0 \\ v_2 &= v_2 - v_1 \\ &= 1V \end{aligned}$$

$$\begin{aligned} -i_2 &= \frac{v_2}{R_2} \\ &= \frac{1}{4} \\ &= 250\,mA \end{aligned}$$

Open and Short Circuits

Example: What happens to $\mathbf{R_2}$ and $\mathbf{R_3}$ if $\mathbf{R_1}$ is shorted?

- $\mathbf{V_s} = \mathbf{2}$V, $\mathbf{R_1} = 2\Omega$, $\mathbf{R_2} = \mathbf{R_3} = 4\Omega$, $\boldsymbol{i_1} = 500$mA
- $\mathbf{R_2}$ and $\mathbf{R_3}$ = ¼ W rating, $\mathbf{R_1}$ = ½ W rating



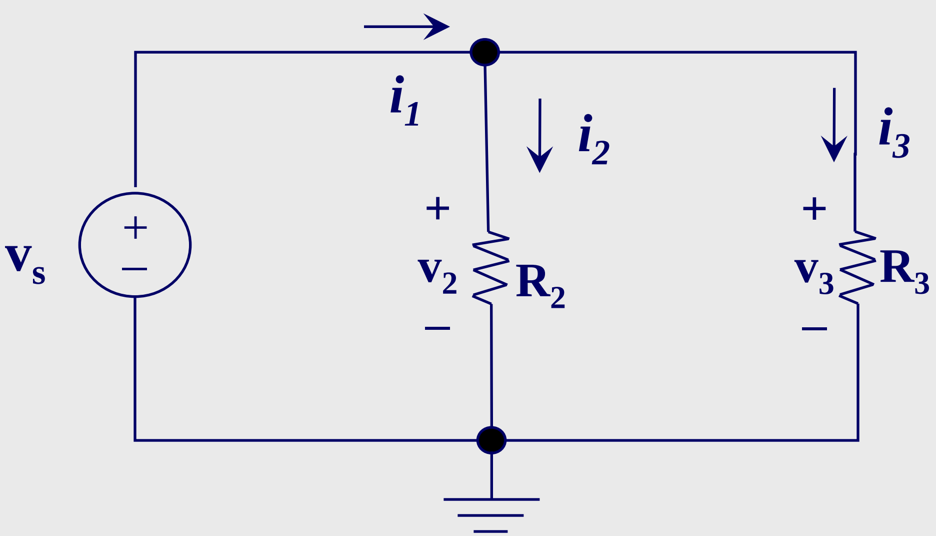
$$P_1 = i_1 \cdot v_1 = 500\,mA\,(1V) = 500\,mW = \frac{1}{2}W$$

$$P_2 = i_2 \cdot v_2 = 250\,mA\,(1V) = 250\,mW = \frac{1}{4}W$$

Open and Short Circuits

Example: What happens to $\mathbf{R_2}$ and $\mathbf{R_3}$ if $\mathbf{R_1}$ is shorted?

- $\mathbf{V_s}$ **= 2**V, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = $\mathbf{R_3}$ = 4Ω, i_1 = 500mA
- $\mathbf{R_2}$ and $\mathbf{R_3}$ = ¼ W rating, $\mathbf{R_1}$ = ½ W rating



$$-v_s + v_2 = 0$$
$$v_2 = v_s$$
$$= 2V$$

$$-i_2 = \frac{v_2}{R_2}$$
$$= \frac{2}{4}$$
$$= 500\,mA$$

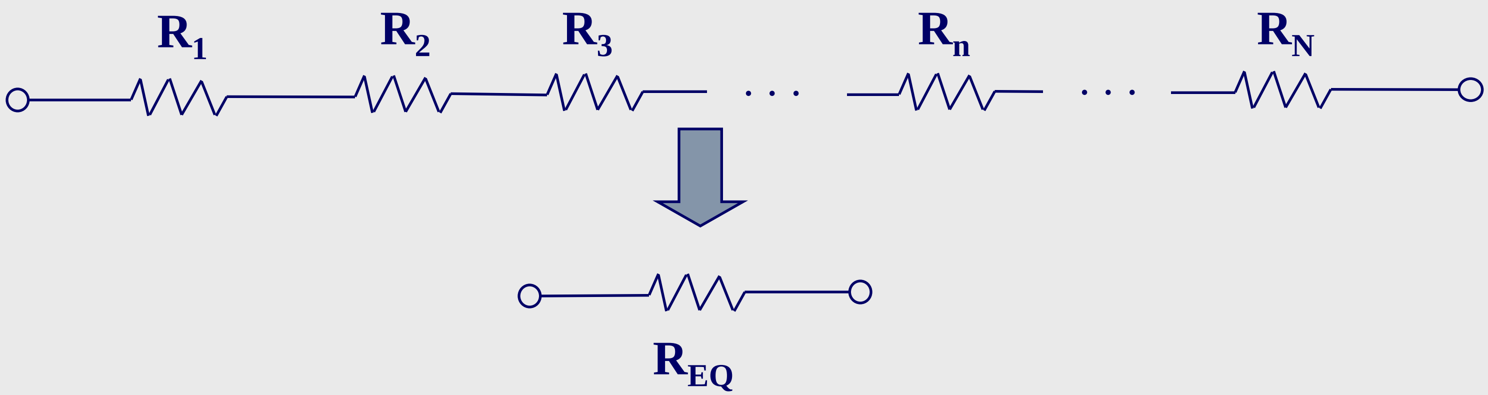
$$P_2 = i_2 \cdot v_2$$
$$= 500\,mA\,(2V)$$
$$= 1W$$

BAD! $\mathbf{R_2}$ and $\mathbf{R_3}$ are damaged

Series Resistors

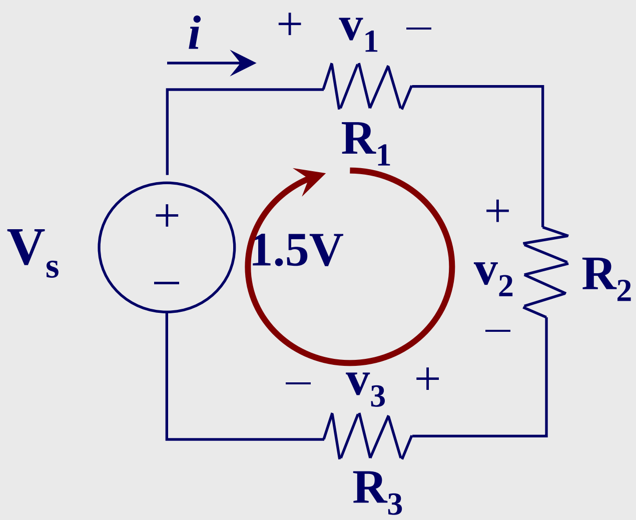
- **<u>Series Rule</u>**: two or more circuit elements are said to be **in series** if the current from one element *exclusively* flows into the next element.
  - <u>From KCL</u>: all series elements have the same current

$$R_{EQ} = \sum_{n=1}^{N} R_n$$



Series Resistors

- **Demonstration of series rule**: apply KVL and Ohm's law on the circuit



$$-V_s + v_1 + v_2 + v_3 = 0$$

$$\begin{aligned} V_s &= v_1 + v_2 + v_3 \\ &= (i \cdot R_1) + (i \cdot R_2) + (i \cdot R_3) \\ &= i \cdot (R_1 + R_2 + R_3) \\ &= i \cdot R_{EQ} \end{aligned}$$

$$R_{EQ} = \frac{V_s}{i}$$

Series Resistors

- **Voltage divider**: the voltage across each resistor in a series circuit is directly proportional to the ratio of its resistance to the total series resistance of the circuit

$$v_n = \frac{R_n}{R_1 + R_2 + R_3 + \cdots + R_n + \cdots + R_N} V_s$$

$$= \frac{R_n}{R_{EQ}} V_s$$

NB: the ratio $\frac{R_n}{R_{EQ}}$ will always be <= 1

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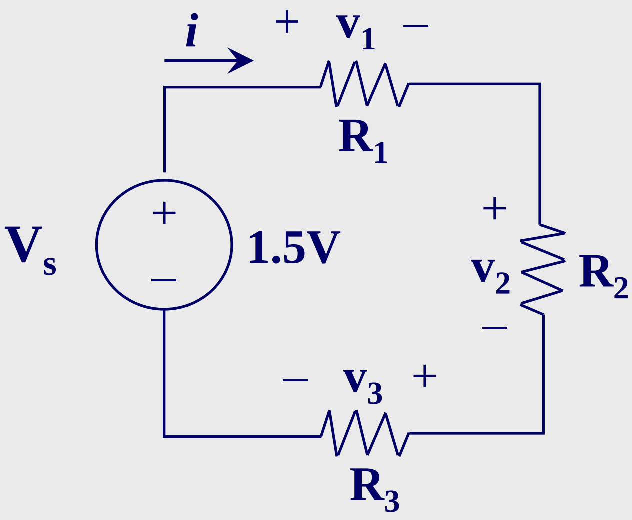
Series Resistors

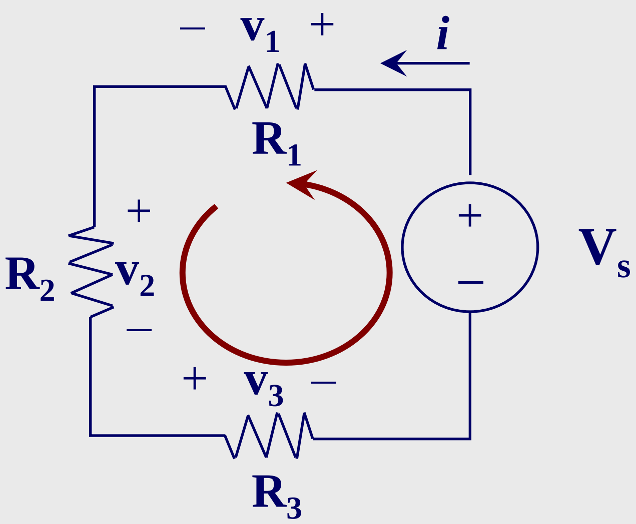
- **<u>Voltage divider</u>**: the voltage across each resistor in a series circuit is directly proportional to the ratio of its resistance to the total series resistance of the circuit



$$v_1 = \frac{R_1}{R_{EQ}} V_s \qquad v_2 = \frac{R_2}{R_{EQ}} V_s \qquad v_3 = \frac{R_3}{R_{EQ}} V_s$$

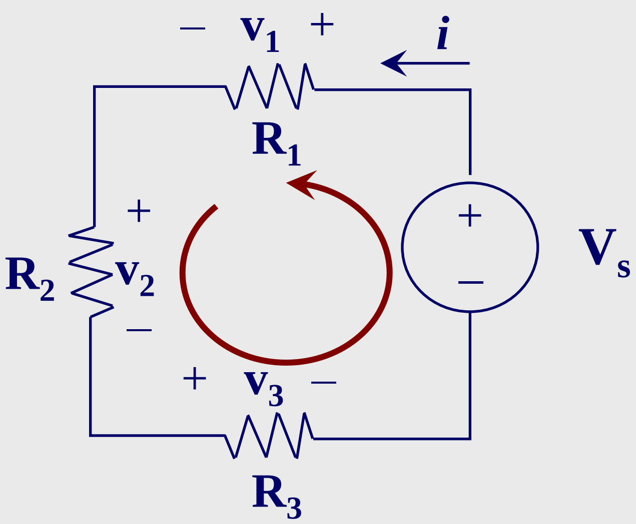
Series Resistors

- **Example1**: Determine $\mathbf{v_3}$
  - $\mathbf{V_s}$ **=** 3V, **R1** = 10Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 8Ω



Series Resistors

- **Example1**: Determine $\mathbf{v_3}$
  - $\mathbf{V_s}$ **=** 3V, **R1** = 10Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 8Ω



Using Series Rule :

$$R_{EQ} = R_1 + R_2 + R_3$$
$$= 10 + 6 + 8$$
$$= 24\Omega$$

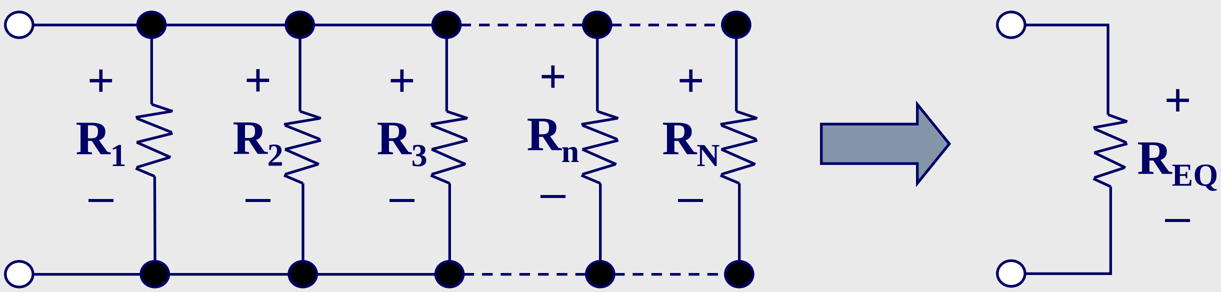
Using Voltage Divider :

$$v_3 = \frac{R_3}{R_{EQ}} \cdot V_s$$
$$= \frac{8}{24} \cdot 3$$
$$= 1V$$

Parallel Resistors

- **<u>Parallel Rule</u>**: two or more circuit elements are said to be **in parallel** if the elements share the *same* terminal**<u>s</u>**
  - <u>From KVL</u>: the elements will have the same voltage

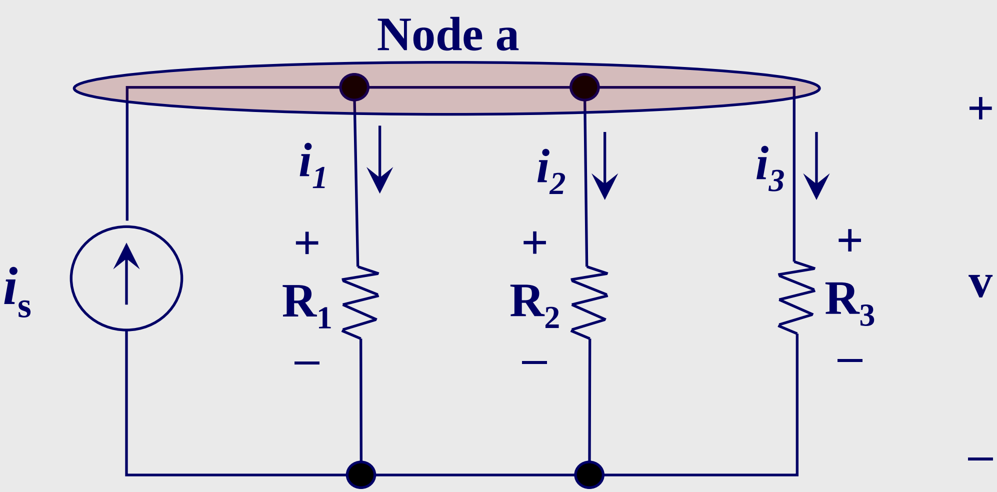
$$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \frac{1}{R_N} \qquad R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$



NB: the parallel combination of two resistors is often written: $\mathbf{R_1 \parallel R_2}$

Parallel Resistors

- **Demonstration of parallel rule**: apply KCL and Ohm's law on the circuit



KCL at node a :

$$i_s - i_1 - i_2 - i_3 = 0$$

$$i_s = i_1 + i_2 + i_3$$

$$= \left(\frac{v}{R_1}\right) + \left(\frac{v}{R_2}\right) + \left(\frac{v}{R_3}\right)$$

$$= v \cdot \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$= v \cdot \left(\frac{1}{R_{EQ}}\right)$$

$$R_{EQ} = \frac{v}{i_s}$$

Parallel Resistors

- **Current divider**: the current in a parallel circuit divides in proportion to the resistances of the individual parallel elements

$$i_n = \frac{\frac{1}{R_n}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\cdots+\frac{1}{R_n}+\cdots+\frac{1}{R_N}} i_s$$

$$= \frac{\frac{1}{R_n}}{\frac{1}{R_{EQ}}} i_s$$

$$= \frac{R_{EQ}}{R_n} i_s$$

NB: the ratio $\frac{R_{EQ}}{R_n}$ will always be <= 1

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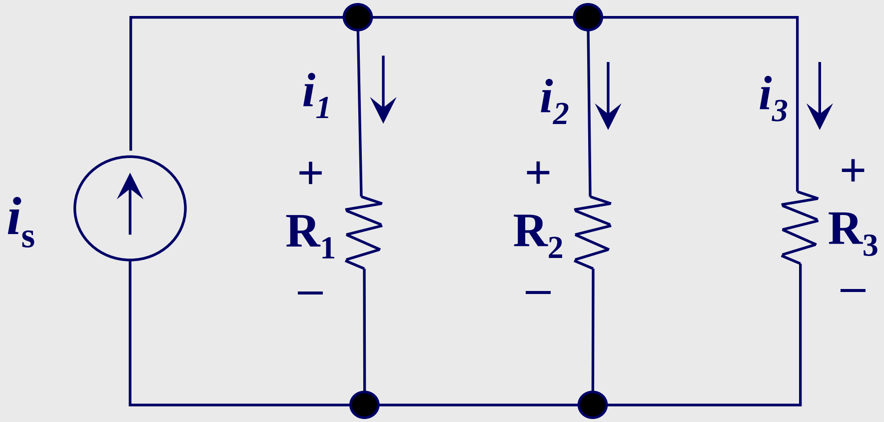
Parallel Resistors

- **Current divider**: the current in a parallel circuit divides in proportion to the resistances of the individual parallel elements



$$i_1 = \frac{1/R_1}{1/R_{EQ}} i_s = \frac{R_{EQ}}{R_1} i_s$$

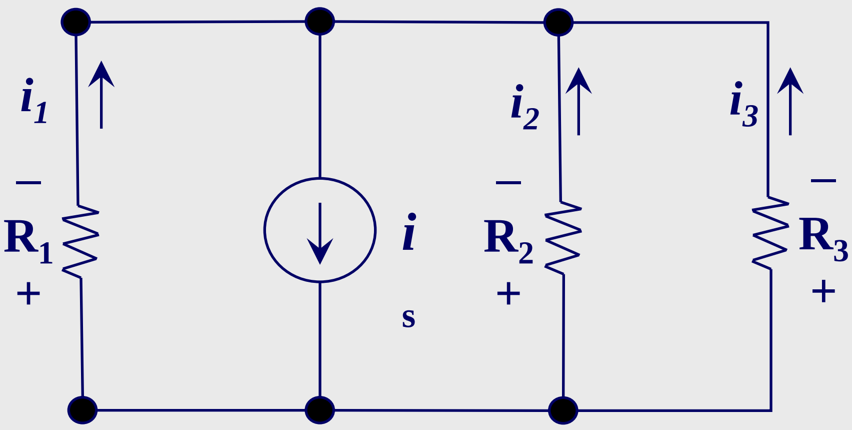
$$i_2 = \frac{R_{EQ}}{R_2} i_s$$

$$i_3 = \frac{R_{EQ}}{R_3} i_s$$

NB: it makes sense that the smaller the resistor, the larger the amount of current that will flow.

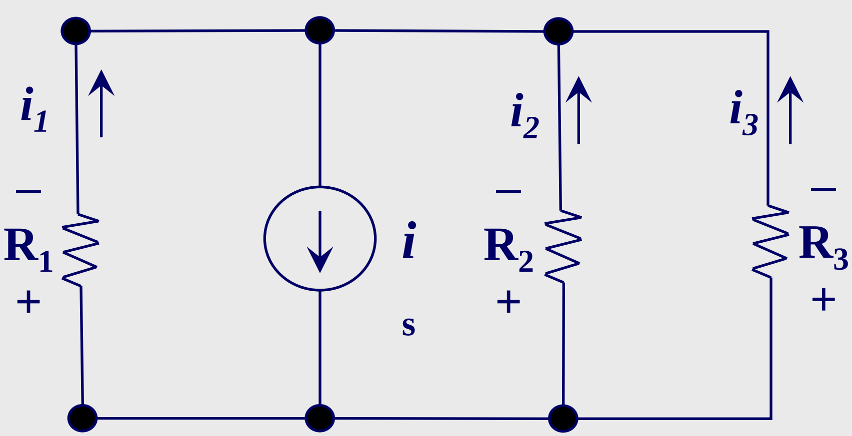
Parallel Resistors

- **Example2**: find $\boldsymbol{i_1}$
  - $\mathbf{R_1} = 10\Omega$, $\mathbf{R_2} = 2\ \Omega$, $\mathbf{R_3} = 20\ \Omega$, $\boldsymbol{I_s} = 4A$



Parallel Resistors

- **Example2**: find i_1
  - $\mathbf{R_1} = 10\Omega$, $\mathbf{R_2} = 2\ \Omega$, $\mathbf{R_3} = 20\ \Omega$, $I_s = 4\text{A}$



NB : all elements share terminals

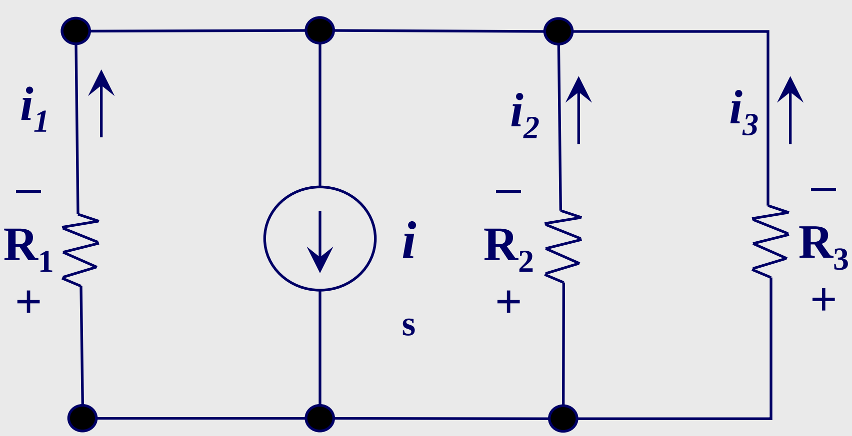
$$R_{EQ} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{10}+\frac{1}{2}+\frac{1}{20}} = \frac{1}{\frac{2+10+1}{20}} = \frac{20}{13}\Omega$$

Using current divider :

$$i_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_{EQ}}} i_s = \frac{\frac{1}{10}}{\frac{13}{20}} \cdot 4 = \frac{40}{65} = 0.615A$$

Parallel Resistors

- **Example2**: find i_1
  - $\mathbf{R_1} = 10\Omega$, $\mathbf{R_2} = 2\ \Omega$, $\mathbf{R_3} = 20\ \Omega$, $I_s = 4\text{A}$



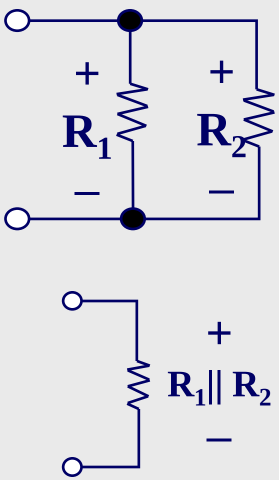
Verify that KCL holds with current dividing:

$$i_s - i_1 - i_2 - i_3 = 0$$

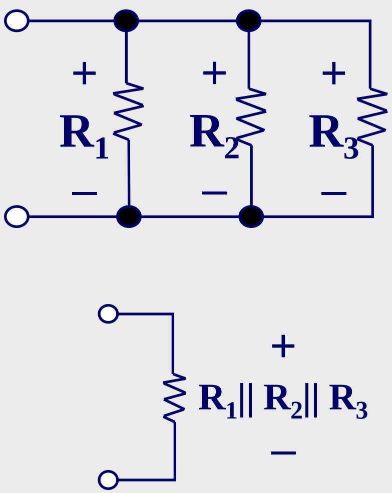
$$
\begin{aligned}
i_s &= i_1 + i_2 + i_3 \\
&= \frac{1/R_1}{1/R_{EQ}} \cdot i_s + \frac{1/R_2}{1/R_{EQ}} \cdot i_s + \frac{1/R_3}{1/R_{EQ}} \cdot i_s \\
&= i_s \cdot \left(\frac{1/R_1 + 1/R_2 + 1/R_3}{1/R_{EQ}} \right) \\
&= i_s \cdot \frac{1/R_{EQ}}{1/R_{EQ}} \\
&= i_s
\end{aligned}
$$

Parallel Resistors

- The parallel combination of resistors is often written:
  - For two resistors: $\mathbf{R_1 \,||\, R_2}$
  - For three resistors: $\mathbf{R_1 \,||\, R_2 \,||\, R_3}$
  - etc.
- In each case $\mathbf{R_{EQ}}$ for the parallel combination must be found



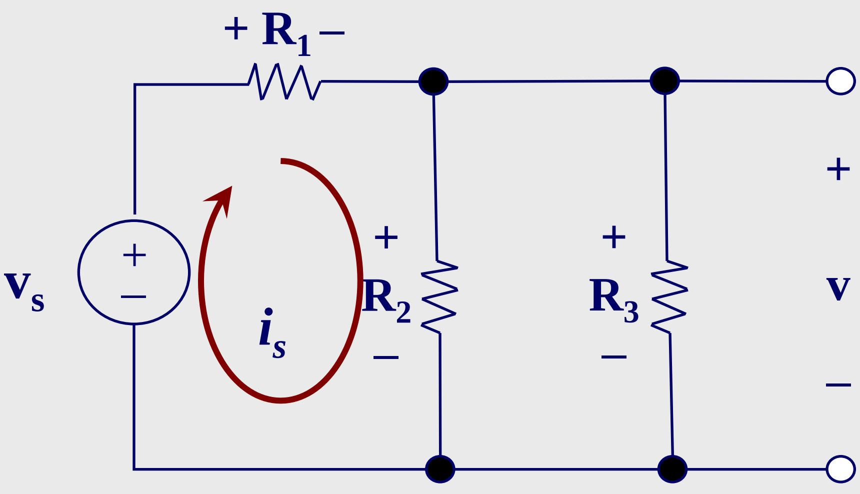
$$R_1 \,||\, R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{R_2 + R_1}{R_1 \cdot R_2}} = \frac{R_1 \cdot R_2}{R_2 + R_1}$$



$$R_1 \,||\, R_2 \,||\, R_3 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 \cdot R_2 \cdot R_3}} = \frac{R_1 \cdot R_2 \cdot R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

Parallel Resistors

- **<u>Example3</u>**: find **v**
  - $\mathbf{v_s}$ = 5V, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 1kΩ



Parallel Resistors

- **<u>Example3</u>**: find **v**
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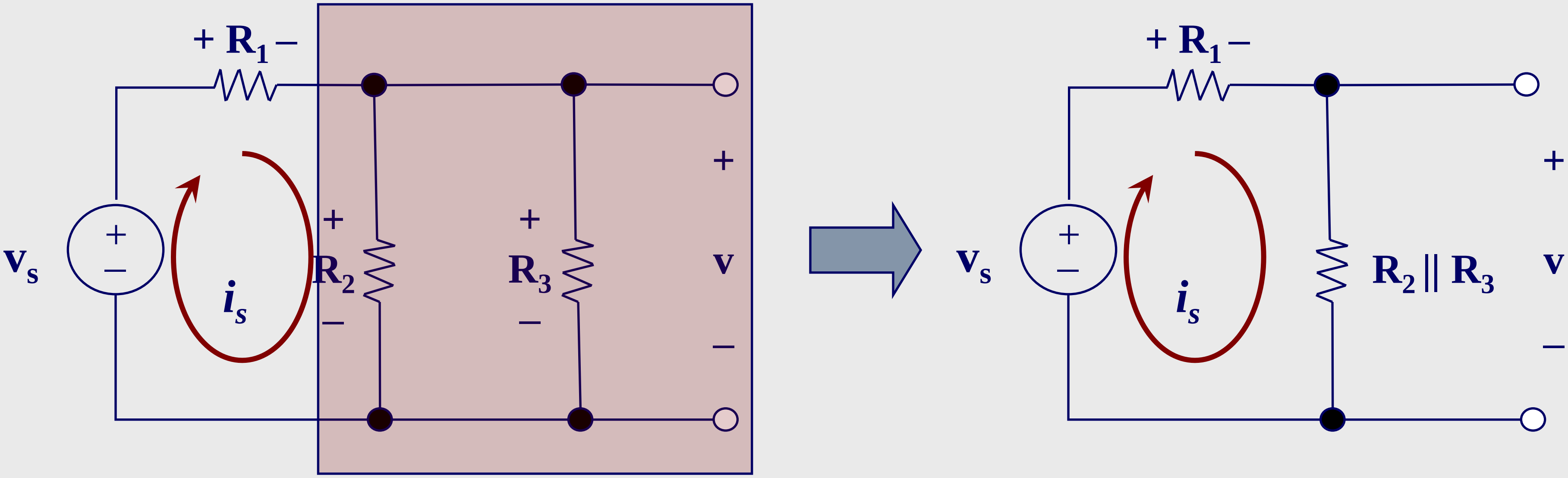
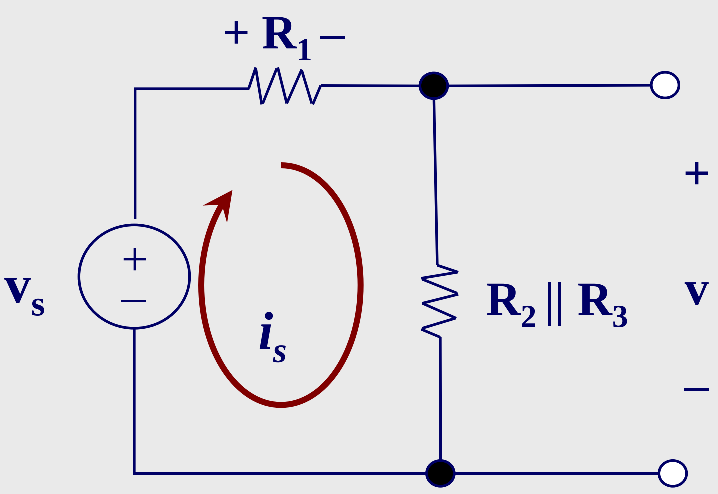


Parallel Resistors

- **Example3**: find **v**
  - $\mathbf{v_s} = 5\text{V}$, $\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_2} = 1\text{k}\Omega$



Using voltage divider :

$$v = \frac{R_2 \parallel R_3}{R_{EQ}} v_s$$

$$= \frac{R_2 \parallel R_3}{(R_1 + R_2 \parallel R_3)} v_s$$

$$= \frac{R_2 \cdot R_3}{R_2 + R_3} \cdot \frac{1}{\left[R_1 + \left(\frac{R_2 \cdot R_3}{R_2 + R_3}\right)\right]} v_s$$

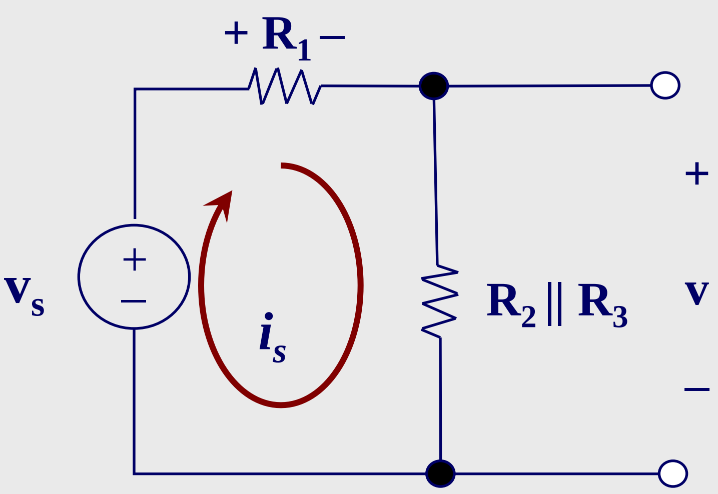
$$= \frac{R_2 \cdot R_3}{R_2 + R_3} \cdot \frac{1}{\left(\frac{R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3}{R_2 + R_3}\right)} v_s$$

$$= \frac{R_2 \cdot R_3}{(R_2 + R_3)} \cdot \frac{R_2 + R_3}{(R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} v_s$$

$$= \frac{R_2 \cdot R_3}{(R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} v_s$$

Parallel Resistors

- **Example3**: find **v**
  - $\mathbf{v_s}$ = 5V, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 1kΩ



Using voltage divider :

$$v = \frac{R_2 \cdot R_3}{(R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3)} v_s$$

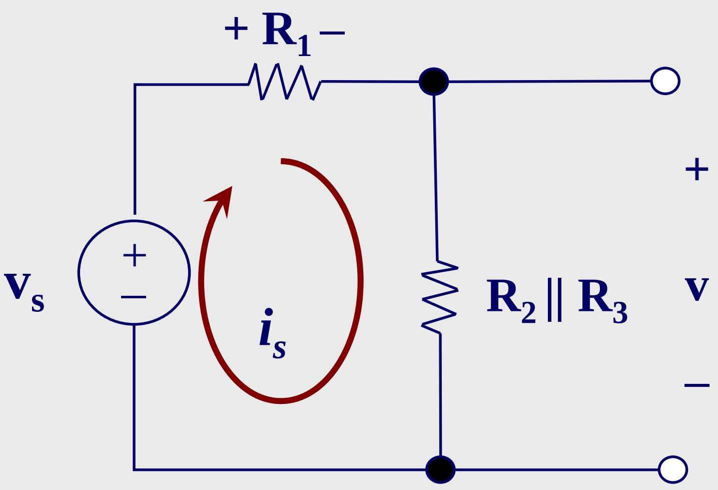
$$= \frac{R_3 \times 10^3 \Omega}{(10^6 \Omega^2 + R_3 \times 10^3 \Omega + R_3 \times 10^3 \Omega)} (5V)$$

$$= \frac{5R_3}{10^3 \Omega + 2R_3} (V)$$

NB: notice how $\mathbf{R_3}$ controls **v**

Parallel Resistors

- **Example3**: find **v**
  - $\mathbf{v_s}$ = 5V, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 1kΩ



If $R_3 = 1k\Omega$:

$$v = \frac{5R_3}{10^3\Omega + 2R_3}(V) = \frac{5\times10^3\Omega}{10^3\Omega + 2\times10^3\Omega}(V) = \frac{5}{3}V$$

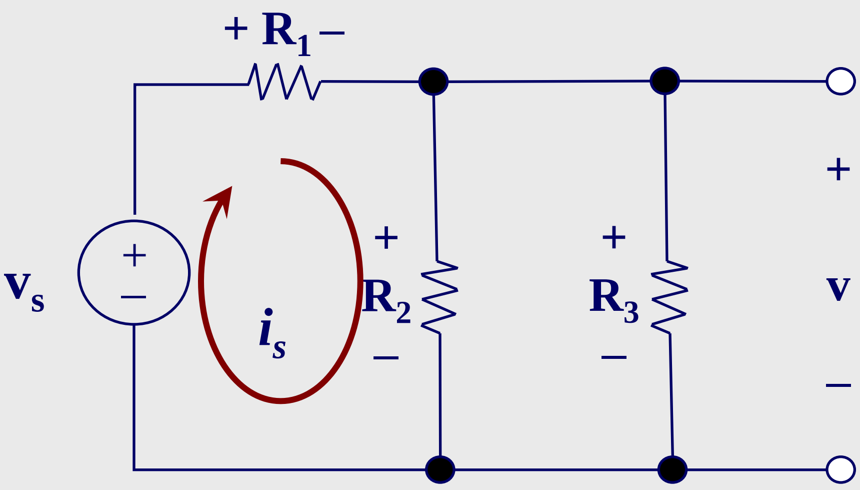
If $R_3 = 0.1k\Omega$:

$$v = \frac{5R_3}{10^3\Omega + 2R_3}(V) = \frac{0.5\times10^3\Omega}{10^3\Omega + 0.2\times10^3\Omega}(V) = \frac{5}{12}(V)$$

NB: notice how $\mathbf{R_3}$ controls **v**

Parallel Resistors

- **Example3**: find **v**
  - $\mathbf{v_s} = 5\text{V}$, $\mathbf{R_1} = 1\text{k}\Omega$, $\mathbf{R_2} = 1\text{k}\Omega$



$$\text{As } R_3 \to 0: \quad \text{v} \to 0$$

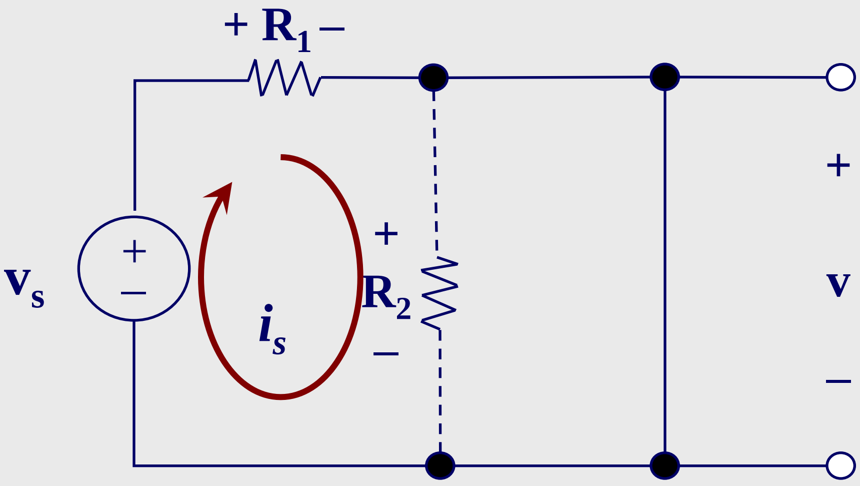
$$\text{As } R_3 \to \infty: \quad \text{v} \to \frac{5}{2}$$

$$\text{In other words}: \quad \text{v} \to v_2$$

NB: notice how $\mathbf{R_3}$ controls **v**

Parallel Resistors

- **Example3**: find **v**
  - $\mathbf{v_s} = 5V$, $\mathbf{R_1} = 1k\Omega$, $\mathbf{R_2} = 1k\Omega$

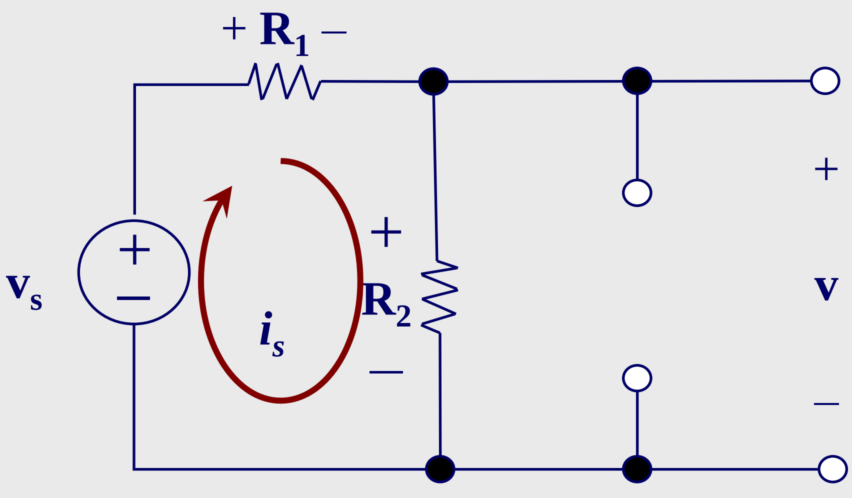


$$\text{As } R_3 \to 0:$$
$$v \to 0$$

In other words, no current goes through R_2

Parallel Resistors

- **<u>Example3</u>**: find **v**
  - $\mathbf{v_s}$ = 5V, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 1kΩ



As $R_3 \to \infty$:

$$v \to \frac{5}{2}$$

In other words:

$$v \to v_2$$

(no current goes through R_3)