# Schedule…

| Date | Day | Class No. | Title | Chapters | HW Due date | Lab Due date | Exam |
|---|---|---|---|---|---|---|---|
| 22 Sept | Mon | 6 | Things Practical | 2.6 – 2.8 | | LAB 2 | |
| 23 Sept | Tue | | | | | | |
| 24 Sept | Wed | 7 | Network Analysis | 3.1 – 3.3 | | | |
| 25 Sept | Thu | | | | | | |
| 26 Sept | Fri | | Recitation | | HW 3 | | |
| 27 Sept | Sat | | | | | | |
| 28 Sept | Sun | | | | | | |
| 29 Sept | Mon | 8 | Equivalent Circuits | 3.4 – 3.5 | | LAB 3 | |
| 30 Sept | Tue | | | | | | |

# Spiritual – Temporal

**1 Nephi 15:32**

32 And it came to pass that I said unto them that it was a representation of things both **temporal** and **spiritual**; for the day should come that they must be judged of their works, yea, even the works which were done by the temporal body in their days of probation.

**Mosiah 2:41**

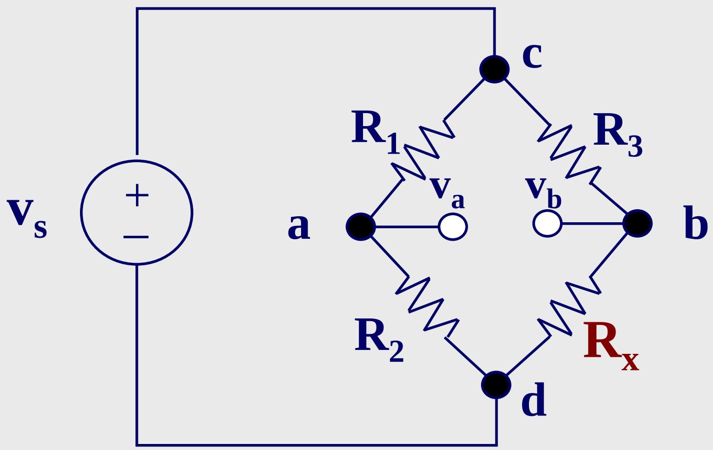
41 And moreover, I would desire that ye should consider on the blessed and happy state of those that keep the commandments of God. For behold, they are blessed in all things, both **temporal** and **spiritual**; and if they hold out faithful to the end they are received into heaven, that thereby they may dwell with God in a state of never-ending happiness. O remember, remember that these things are true; for the Lord God hath spoken it.

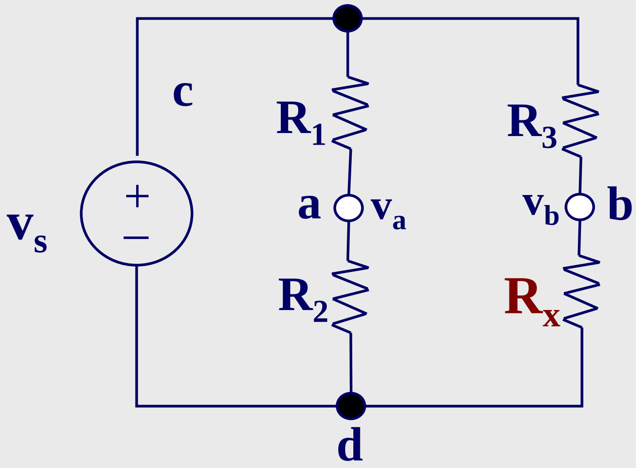
# Lecture 6 – The Wheatstone Bridge

An Applications of Things Electrical

# Wheatstone Bridge

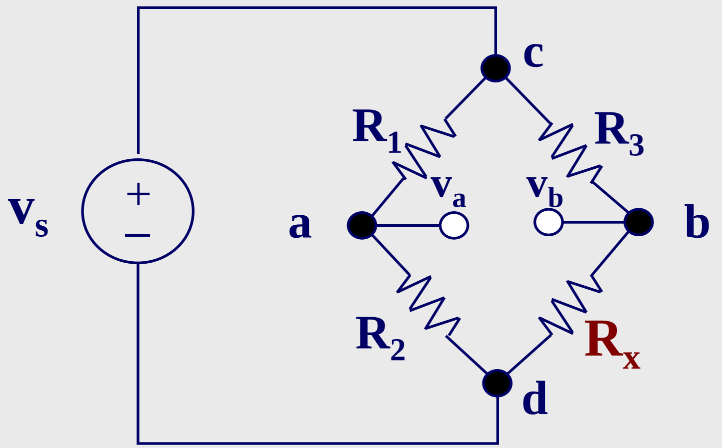
- $\mathbf{R_x}$ is an unknown resistance to be determined



# Wheatstone Bridge

- The circuit consists of the parallel combination of 3 subcircuits with the same voltage:
  a) The voltage source
  b) Series combination of $\mathbf{R_1}$ and $\mathbf{R_2}$
  c) Series combination of $\mathbf{R_3}$ and $\mathbf{R_x}$
- Voltage divider between:
  a) $\mathbf{R_1}$ and $\mathbf{R_2}$
     - $\mathbf{v_2 = v_{ad}}$
  b) $\mathbf{R_3}$ and $\mathbf{R_x}$
     - $\mathbf{v_x = v_{bd}}$



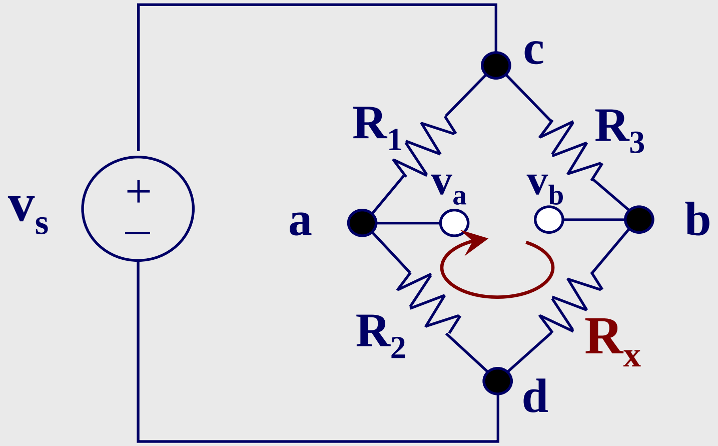

$$v_{ad} = v_s \frac{R_2}{R_1 + R_2}$$

$$v_{bd} = v_s \frac{R_x}{R_3 + R_x}$$

# Wheatstone Bridge

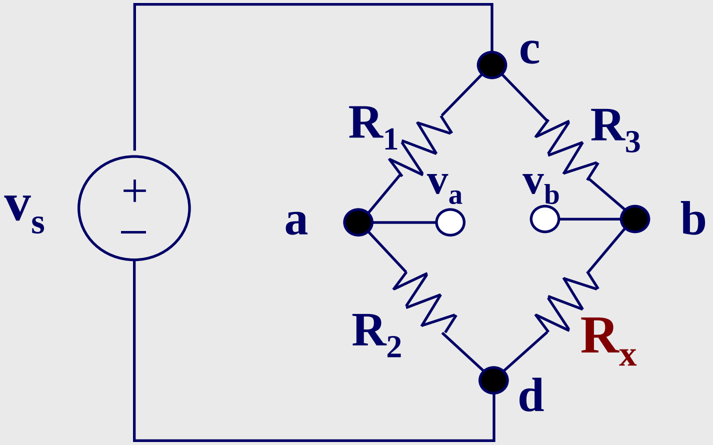
- KVL around the bottom loop:

$$v_{ab} = v_{ad} - v_{bd}$$
$$= v_s\left(\frac{R_2}{R_1 + R_2} - \frac{R_x}{R_3 + R_x}\right)$$



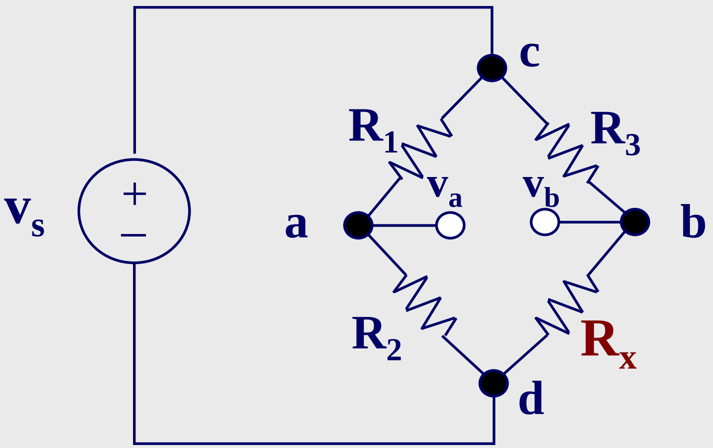
# Wheatstone Bridge

- **<u>Example1</u>**: when is $\mathbf{v_{ab}} = 0$?



# Wheatstone Bridge

- **Example1**: when is $\mathbf{v_{ab}} = 0$?




$$v_{ab} = v_s\left(\frac{R_2}{R_1+R_2} - \frac{R_x}{R_3+R_x}\right)$$

$$v_s\left(\frac{R_2}{R_1+R_2} - \frac{R_x}{R_3+R_x}\right) = 0$$

$$\frac{R_2}{R_1+R_2} = \frac{R_x}{R_3+R_x}$$

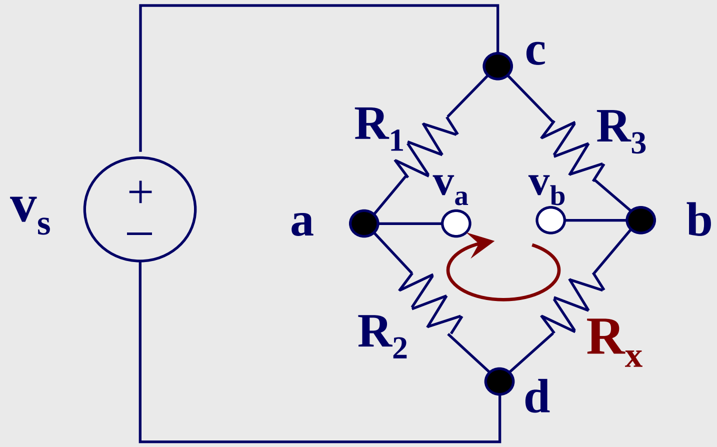
$$R_2 \cdot (R_3 + R_x) = R_x \cdot (R_1 + R_2)$$

$$R_2R_3 + R_2R_x = R_xR_1 + R_2R_x$$

$$R_2R_3 = R_xR_1$$

# Wheatstone Bridge

- Find $\mathbf{R_x}$:



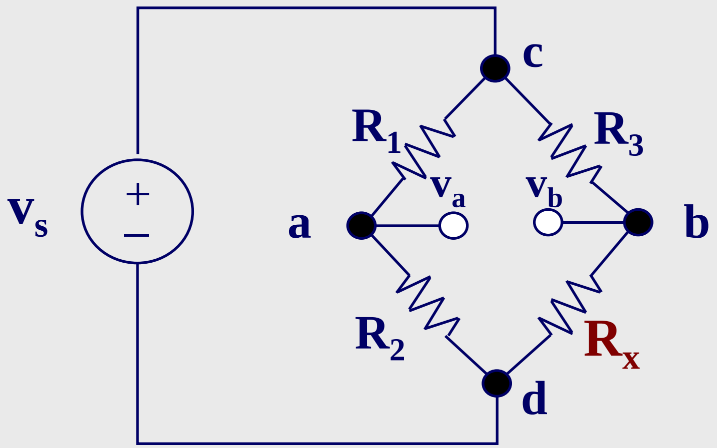

$$v_{ab} = v_s\left(\frac{R_2}{R_1+R_2} - \frac{R_x}{R_3+R_x}\right)$$

$$\vdots$$

$$R_x = \frac{R_3\left(\dfrac{R_2}{R_1+R_2} - \dfrac{v_{ab}}{v_s}\right)}{\left(1 - \dfrac{R_2}{R_1+R_2} + \dfrac{v_{ab}}{v_s}\right)}$$

# Wheatstone Bridge

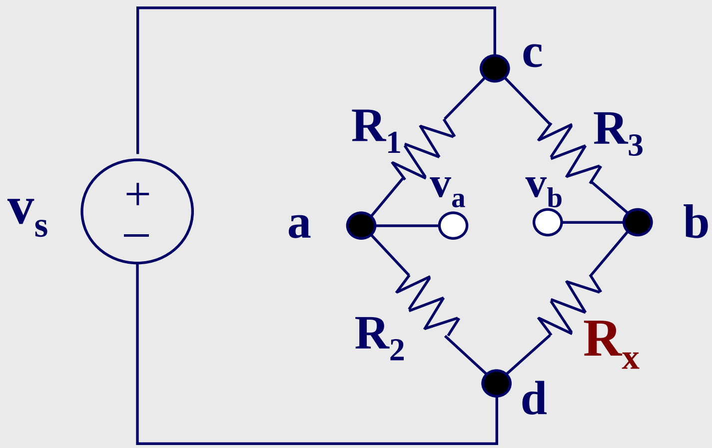
- **Example2**: find $\mathbf{R_x}$
  - $\mathbf{R_1} = \mathbf{R_2} = \mathbf{R_3} = 1\text{k}\Omega$, $\mathbf{v_s} = 12\text{V}$, $\mathbf{v_{ab}} = 12\text{mV}$




$$R_x = \frac{R_3\left(\frac{R_2}{R_1+R_2} - \frac{v_{ab}}{v_s}\right)}{\left(1 - \frac{R_2}{R_1+R_2} + \frac{v_{ab}}{v_s}\right)}$$

# Wheatstone Bridge

- **Example2**: find $\mathbf{R_x}$
  - $\mathbf{R_1} = \mathbf{R_2} = \mathbf{R_3} = 1\text{k}\Omega$, $\mathbf{v_s} = 12\text{V}$, $\mathbf{v_{ab}} = 12\text{mV}$




$$R_x = \frac{R_3\left(\dfrac{R_2}{R_1+R_2} - \dfrac{v_{ab}}{v_s}\right)}{\left(1 - \dfrac{R_2}{R_1+R_2} + \dfrac{v_{ab}}{v_s}\right)}$$

$$= \frac{10^3\left(\dfrac{10^3}{2\times 10^3} - \dfrac{12\times 10^{-3}}{12}\right)}{\left(1 - \dfrac{10^3}{2\times 10^3} - \dfrac{12\times 10^{-3}}{12}\right)}$$

$$= \frac{10^3\left(\dfrac{1}{2} - 10^{-3}\right)}{\left(1 - \dfrac{1}{2} - 10^{-3}\right)}$$

$$= \frac{499}{0.501}$$

$$= 996\Omega$$

# Resistance Strain Gauges

- **<u>Strain gauge</u>**: device bonded to the surface of an object and whose resistance varies as a function of surface strain
  - Used to perform measurements of:
    - Strain
    - Stress,
    - Force
    - Torque
    - Pressure
- **NB**: cylindrical resistance: $R = \dfrac{L}{\sigma A}$

| |
|---|
| ***L*** – length of cylindrical resistor<br>***σ*** – conductivity of the resistor<br>***A*** – resistor cross-sectional area |

  - Compression/elongation will change resistance

Compression $\longrightarrow$ lower resistance

Stretch $\longrightarrow$ higher resistance

# Resistance Strain Gauges

- **Gauge factor (GF)**: the relationship between change in resistance and change in length
  - value of about 2 is common

$$GF = \frac{\Delta R / R_0}{\Delta L / L}$$

**$R_0$** – the **zero strain resistance**

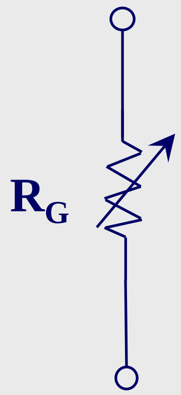
- **Strain (ε)**: the fractional change in length of an object
  - Max strain that can be measured is about 0.4 – 0.5 percent
    - i.e. ε = 0.004 to 0.005
    - For a **120Ω** resistor: +/– **1.2 Ω**

$$\varepsilon = \frac{\Delta L}{L}$$

# Resistance Strain Gauges

- **<u>Change in resistance due to strain</u>**:

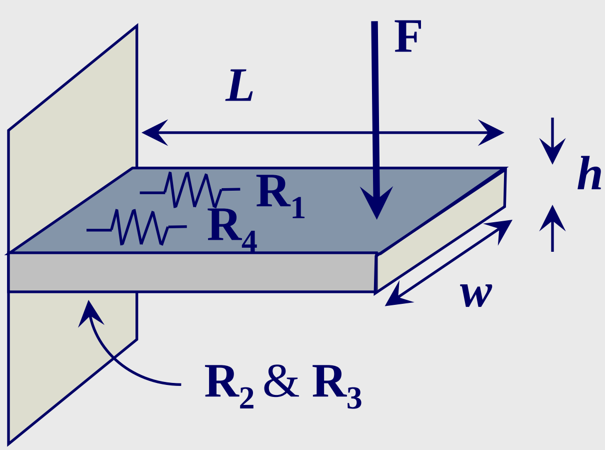
$$\Delta R = R_0 GF \varepsilon$$

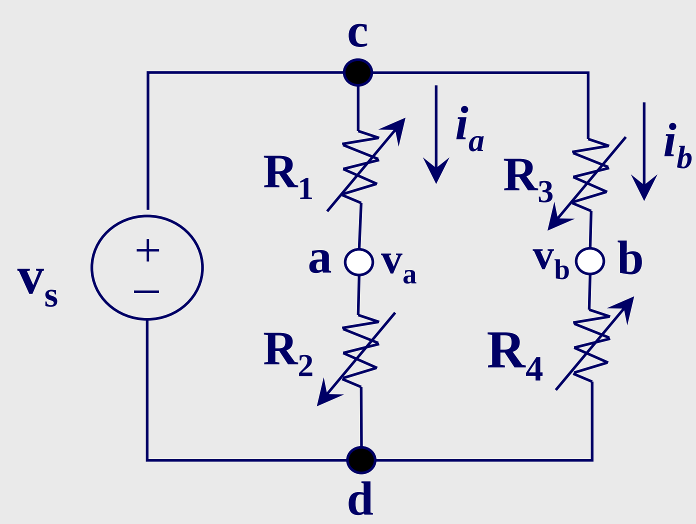



Circuit symbol for a strain gauge

# Wheatstone Bridge

- Wheatstone bridge commonly used to measure force using strain gauge resistors
  - **Example**: force applied to a cantilever beam
    - Two strain gauges (**$R_1$** and **$R_4$**) on top
    - Two strain gauges (**$R_2$** and **$R_3$**) on bottom



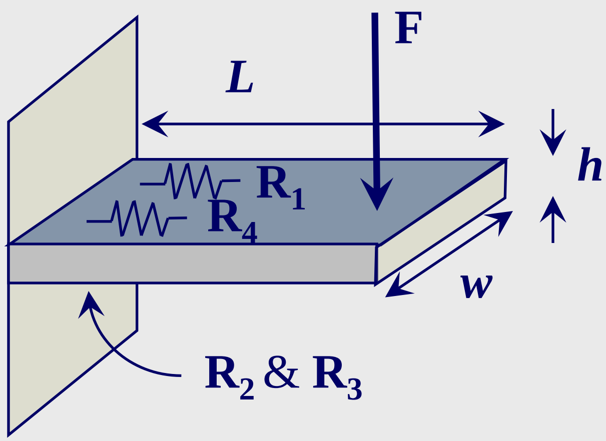



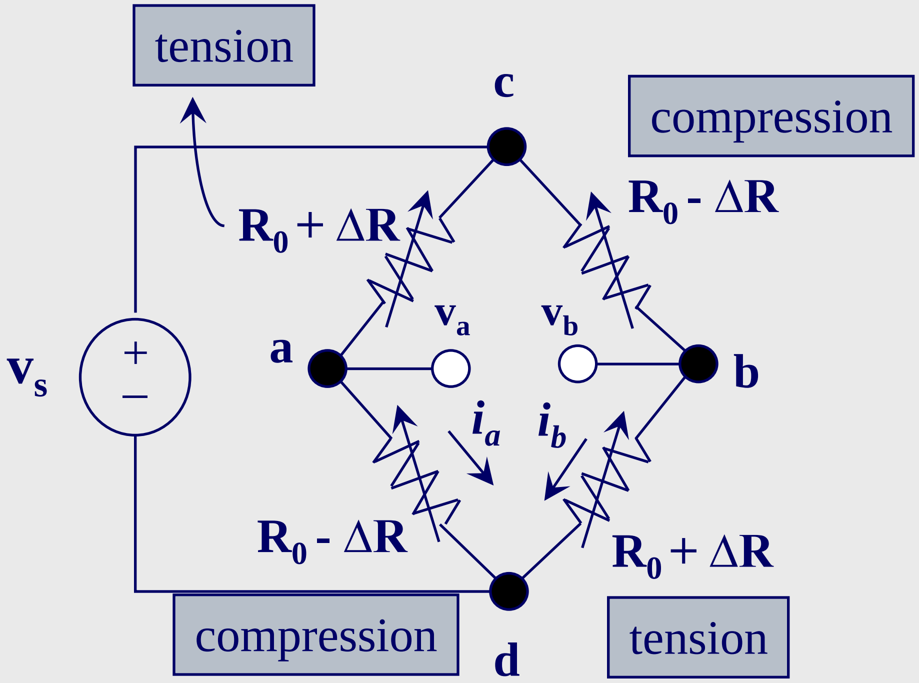
# Wheatstone Bridge

- Under the strain of force **F** we have:

$$R_1 = R_4 = R_0 + \Delta R$$

$$R_2 = R_3 = R_0 - \Delta R$$



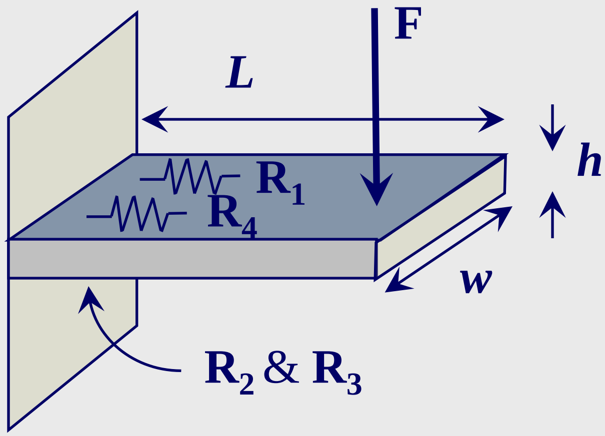



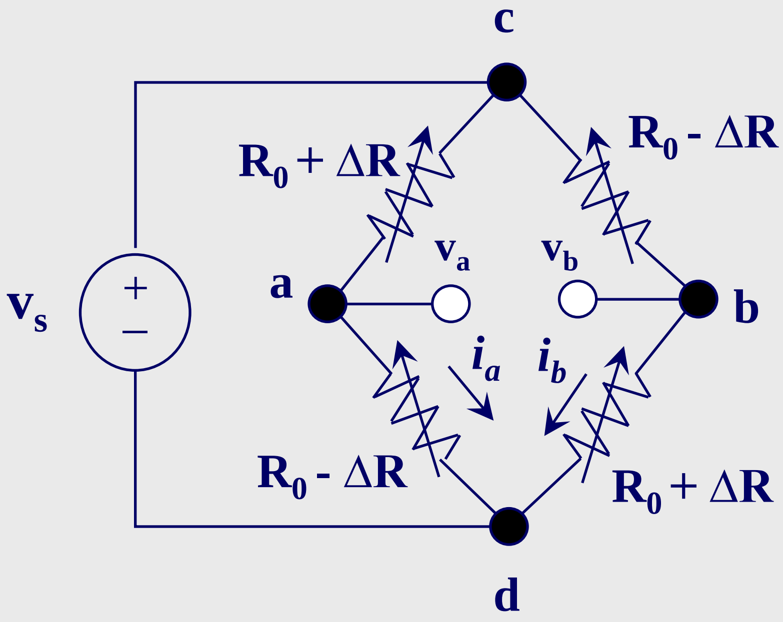
# Wheatstone Bridge

- From elementary statics it can be shown that:

$$\varepsilon = \frac{6LF}{wh^2Y}$$

***Y*** is the beam's modulus of elasticity



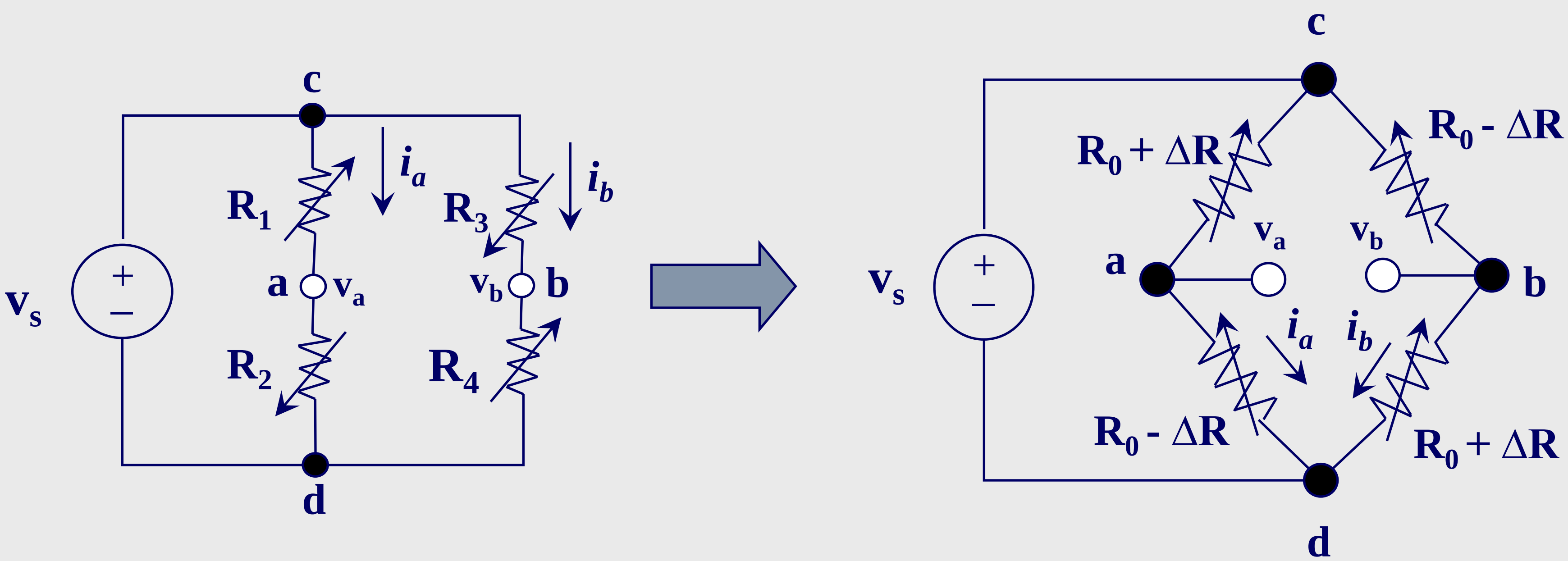



# Wheatstone Bridge

- Using Ohm's Law:

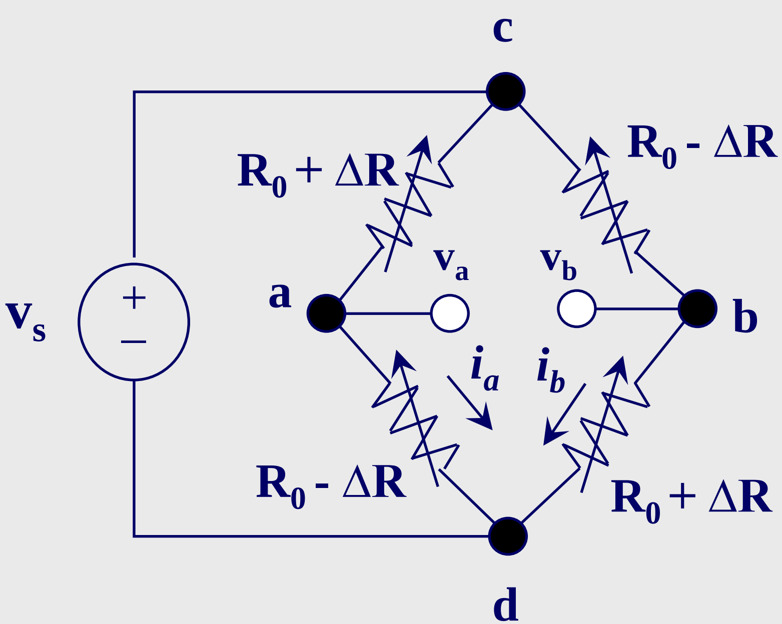
$$i_a = \frac{v_s}{R_1 + R_2}$$

$$i_b = \frac{v_s}{R_3 + R_4}$$



# Wheatstone Bridge

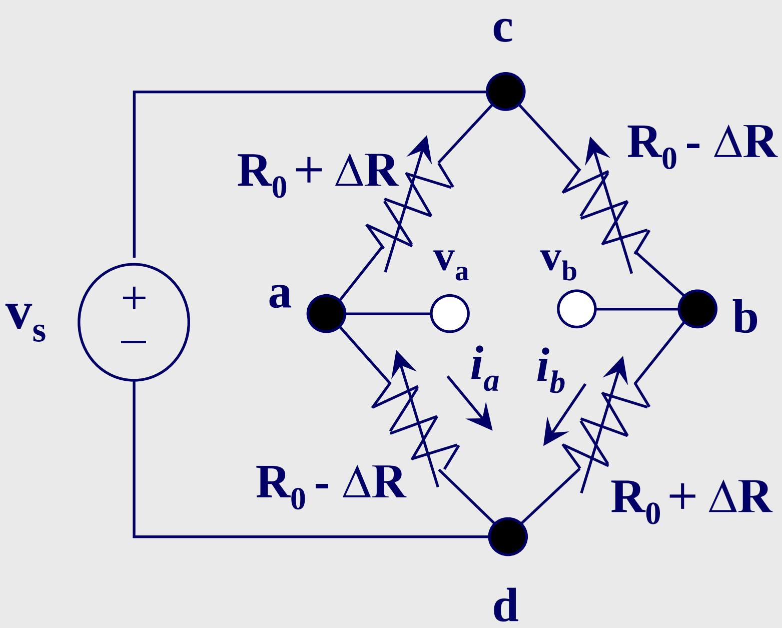
- Using Ohm's Law:




$$
\begin{aligned}
v_{out} &= v_{ba} = v_b - v_a \\
&= i_b R_4 - i_a R_2 \\
&= \frac{v_s R_4}{R_3 + R_4} - \frac{v_s R_2}{R_1 + R_2} \\
&= v_s \frac{R_0 + \Delta R}{R_0 + \Delta R + R_0 - \Delta R} - v_s \frac{R_0 - \Delta R}{R_0 + \Delta R + R_0 - \Delta R} \\
&= v_s \frac{\Delta R}{R_0} \\
&= v_s GF\varepsilon
\end{aligned}
$$

# Wheatstone Bridge

- Find $\mathbf{v_o}$ in terms of force $\mathbf{F}$:

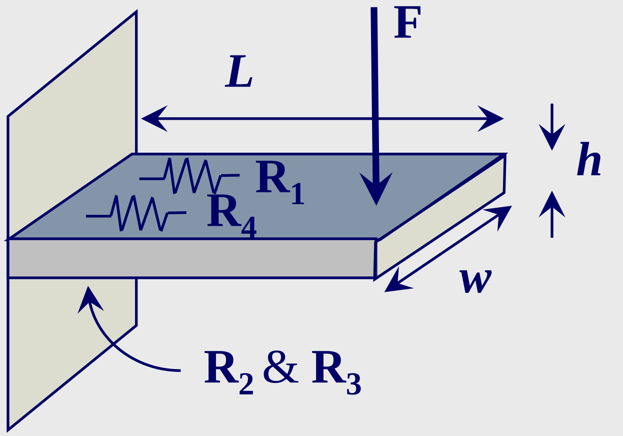



$$
\begin{aligned}
v_o &= v_s G F \varepsilon \\
&= v_s G F \frac{6LF}{wh^2 Y} \\
&= \left( \frac{6 v_s G F L}{wh^2 Y} \right) F \\
&= kF
\end{aligned}
$$

***k*** is a calibration constant

# Wheatstone Bridge

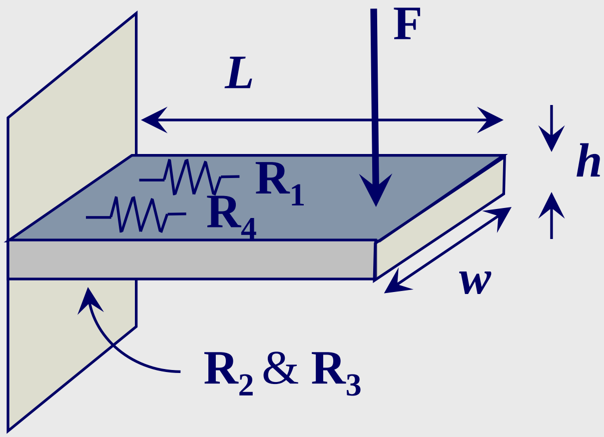
- **Example3**: using the Wheatstone bridge as a strain measurement tool find $\mathbf{v_o}$
  - the bridge measures forces ranging from 0 to 500 N
  - $L$ = 0.3m, $w$ = 0.05m, $h$ = 0.01m, **GF** = 2, $Y$ = 69x10$^9$N/m$^2$, $\mathbf{v_s}$ = 12V




$$
\begin{aligned}
v_o &= kF \\
&= \left(\frac{6 v_s GFL}{wh^2 Y}\right)F \\
&= \left(\frac{6(12)(2)(0.3)}{(0.05)(0.01)^2 69\times 10^9}\right)F \\
&= 1.25F\times 10^{-4} \quad V/N \\
&= 0.125 F \, mV/N
\end{aligned}
$$

# Wheatstone Bridge

- **<u>Example3</u>**: using the Wheatstone bridge as a strain measurement tool find $\mathbf{v_o}$
  - the bridge measures forces ranging from 0 to 500 N
  - $L$ = 0.3m, $w$ = 0.05m, $h$ = 0.01m, **GF** = 2, $Y$ = 69x10$^9$N/m$^2$, $\mathbf{v_s}$ = 12V




Maximum voltage :

$$v_o = 0.125 F\, mV/N$$
$$= 1.25 \times 10^{-4}(500)$$
$$= 62.5\, mV$$

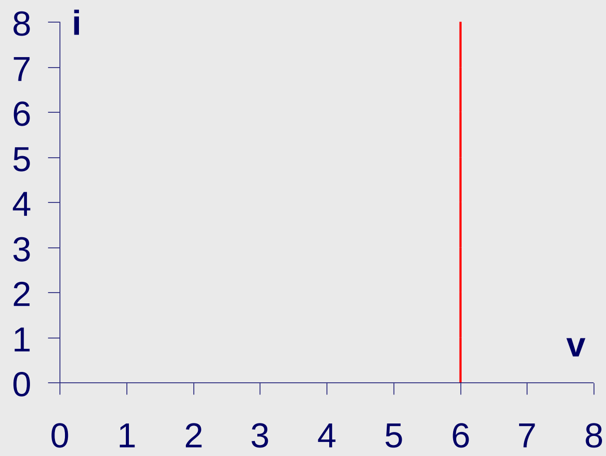
# Practical Sources

# Ideal Sources

- Ideal current source
  - 3A source


Provides a prescribed voltage across its terminals irrespective of the current flowing through it.

- Ideal voltage source
  - 6V source




Provides a prescribed current irrespective of the voltage across it.

# Practical Sources

- Actual voltage sources have limitations
  - There is a limit to the number of total electrons any battery can motivate through a circuit
    - How to measure limitations?
      - Total number of electrons? (huge number)
      - Use coulombs? (also too huge)
      - **amp-hour** – unit invented for this purpose
    - 1 amp-hour = 1 amp for 1 hour
      = 2 amps for ½ hour
      = 1/3 amp for 3 hours

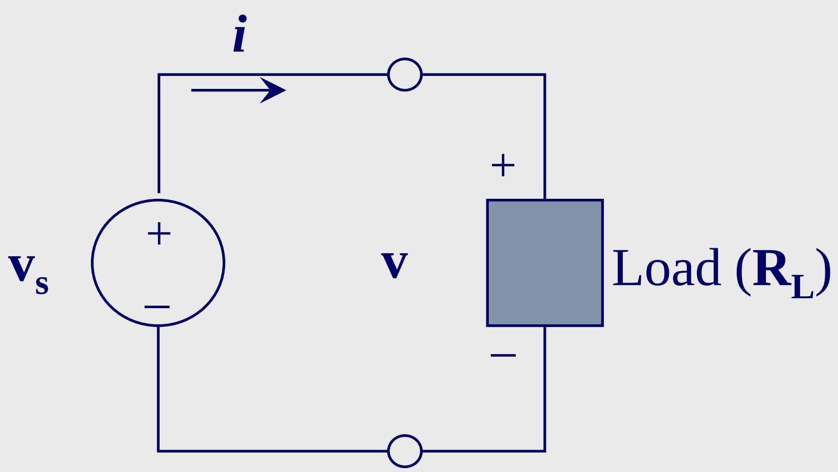
1 amp = 1 coulombs/second
1 amp-hour = 3600 coulombs

  - Batteries have ratings indicating their current limitations
    - Car battery – 12V, 70 amp-hours (A-h) @ 3.5 A (for 20 hours)
    - D – cell (1.5V) carbon-zinc battery – 4.6 amp-hours @ 100mA (for 46 hours)
    - 9 – volt carbon-zinc battery – 400 mA-hours @ 8mA (for 50 hours)

# Practical Sources

- Actual voltage sources have limitations
  - As the load resistance (**$R_L$**) decreases, the voltage source (**$v_s$**) is required to provide increasing amounts of current (***i***) in order to maintain **$v_s$**

$$i = \frac{v_s}{R_L}$$

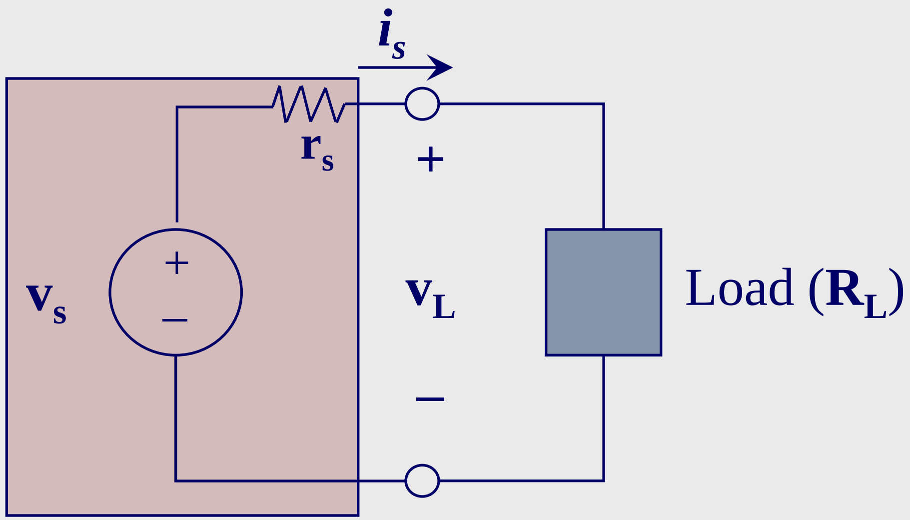



As **$R_L$** → 0, **$v_s$** has to provide an **infinite** amount of current!

# Practical Sources

- Actual voltage sources have limitations

A series resistance $\mathbf{r_s}$ poses a limit to the maximum current the voltage source can provide



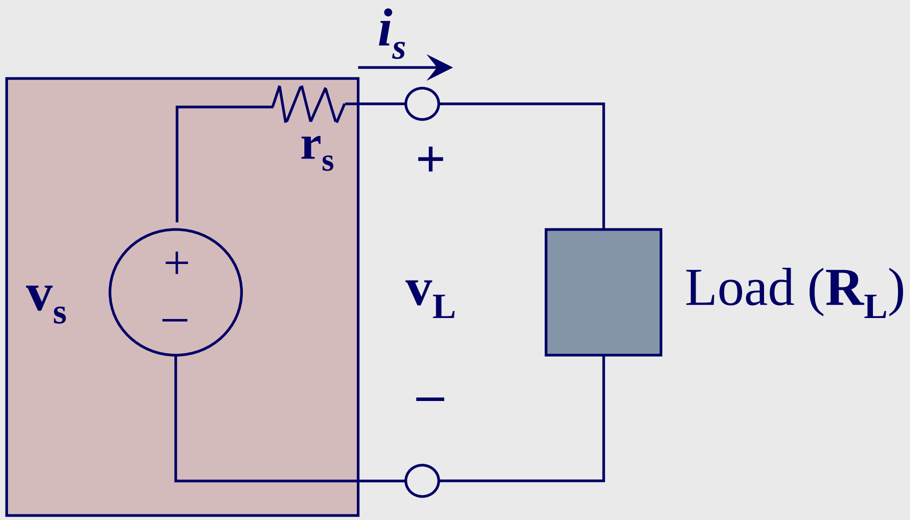
Practical voltage source

$$i_s = \frac{v_s}{r_s + R_L}$$

# Practical Sources

- Actual voltage sources have limitations

A series resistance $\mathbf{r_s}$ poses a limit to the maximum current the voltage source can provide




Practical voltage source

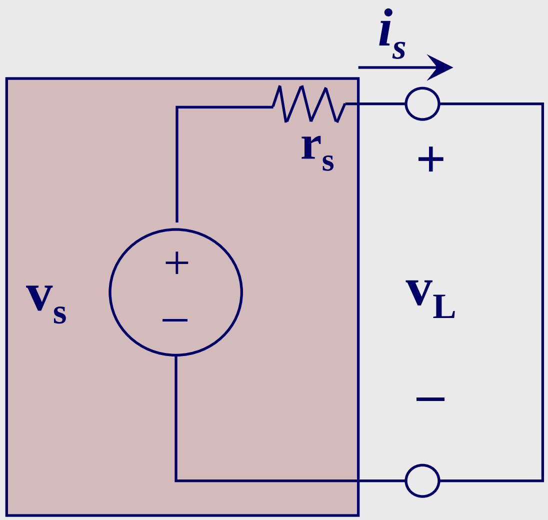
$$v_L = i_s R_L = v_s \frac{R_L}{r_s + R_L}$$

**NB**: $\mathbf{v_s \neq v_L}$

# Practical Sources

- Actual voltage sources have limitations

A series resistance $\mathbf{r_s}$ poses a limit to the maximum current the voltage source can provide




Practical voltage source

$$i_s = \frac{v_s}{r_s + R_L}$$

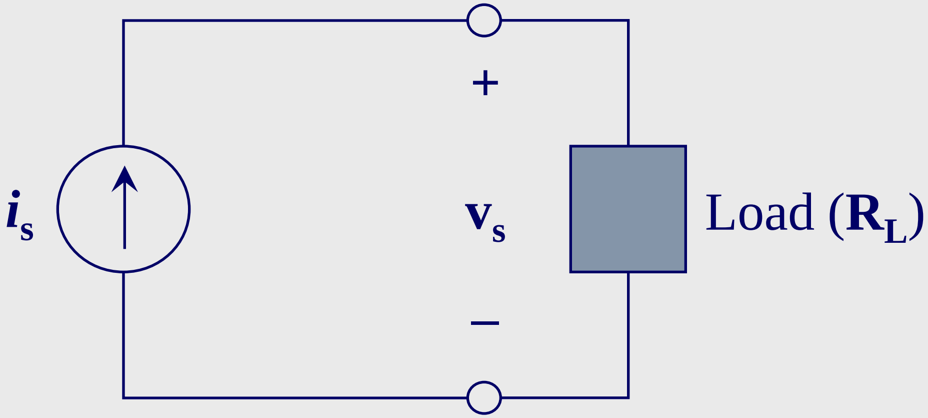
$$\lim_{R_L \to 0} i_s = \frac{v_s}{r_s}$$

$$i_{S\max} = \frac{v_s}{r_s}$$

# Practical Sources

- Actual current sources have limitations
  - As the load resistance (**$R_L$**) increases, the current source (***$i_s$***) is required to provide increasing amounts of current (**v**) in order to maintain ***$i_s$***

$$v_s = i \cdot R_L$$

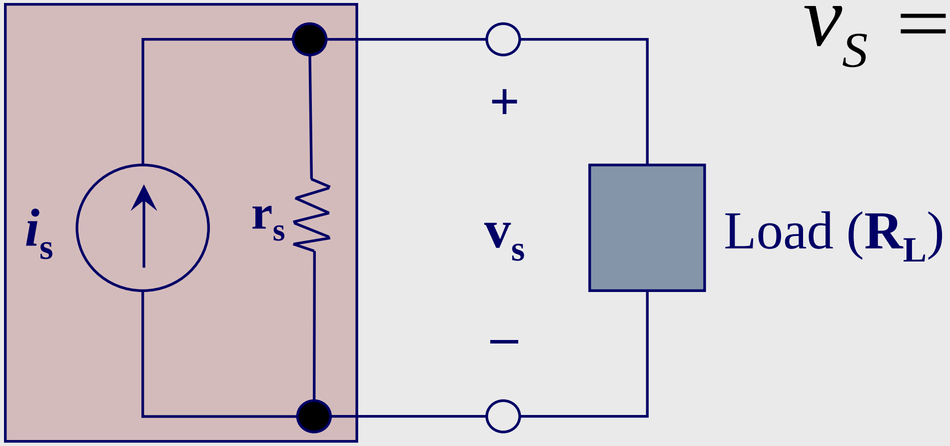



As **$R_L$** → ∞, ***$i_s$*** has to provide an **infinite** amount of voltage!

# Practical Sources

- Actual current sources have limitations

A series resistance $\mathbf{r_s}$ poses a limit to the maximum voltage the current source can provide



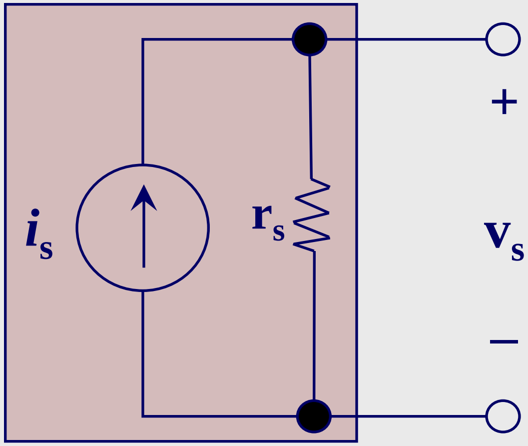

$$v_S = i_s(r_s \parallel R_L)$$

Practical current source

# Practical Sources

- Actual current sources have limitations

A series resistance $\mathbf{r_s}$ poses a limit to the maximum voltage the current source can provide




Practical current source

$$v_S = i_s (r_s \parallel R_L)$$

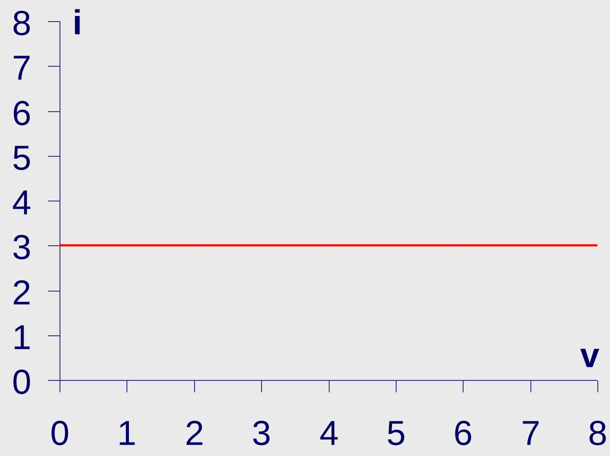
$$\lim_{R_L \to \infty} v_s = i_s r_s$$

$$v_{S\max} = i_s r_s$$

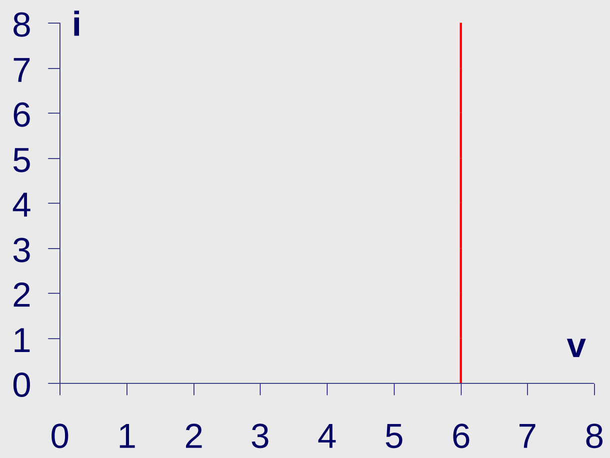
# Practical Sources

- Actual current and voltage sources have limitations

Ideal 3A current source



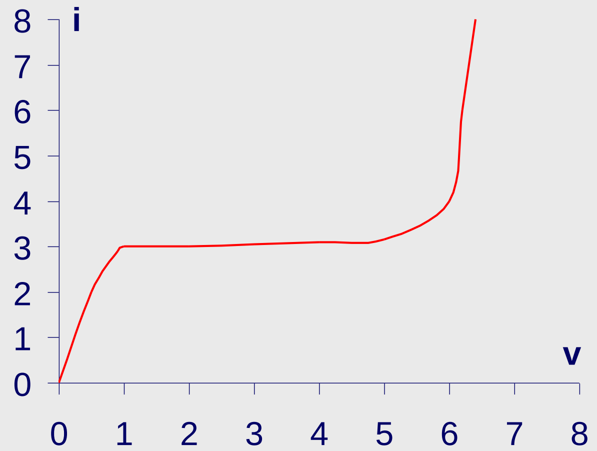

Ideal 6V votage source



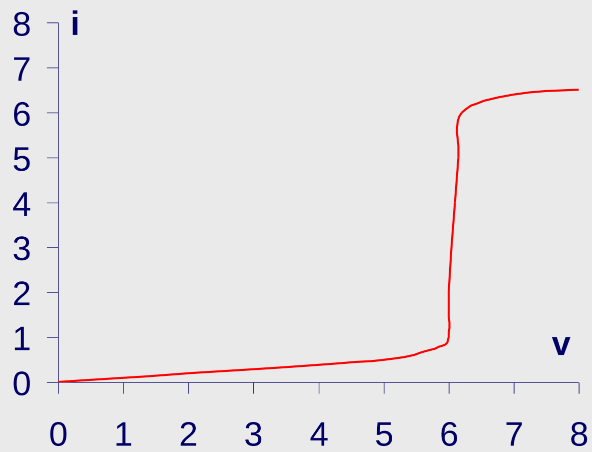
# Practical Sources

- Actual current and voltage sources have limitations

Practical 3A current source




Practical 6V votage source

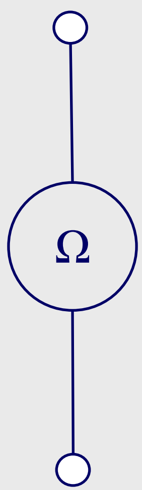


# Measuring Devices

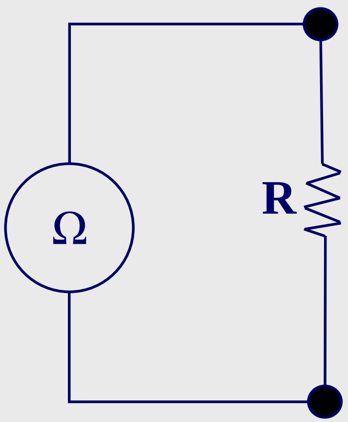
# Ohmmeter

- **<u>Ohmmeter</u>**: measures the resistance of a circuit element

**NB**: the resistance of an element can only be measured when the element is **disconnected** from **all** other circuit elements




Ohmmeter symbol

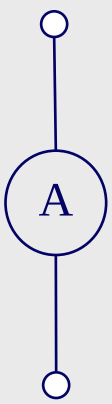



Ohmmeter connection setup for circuit element resistance measurement

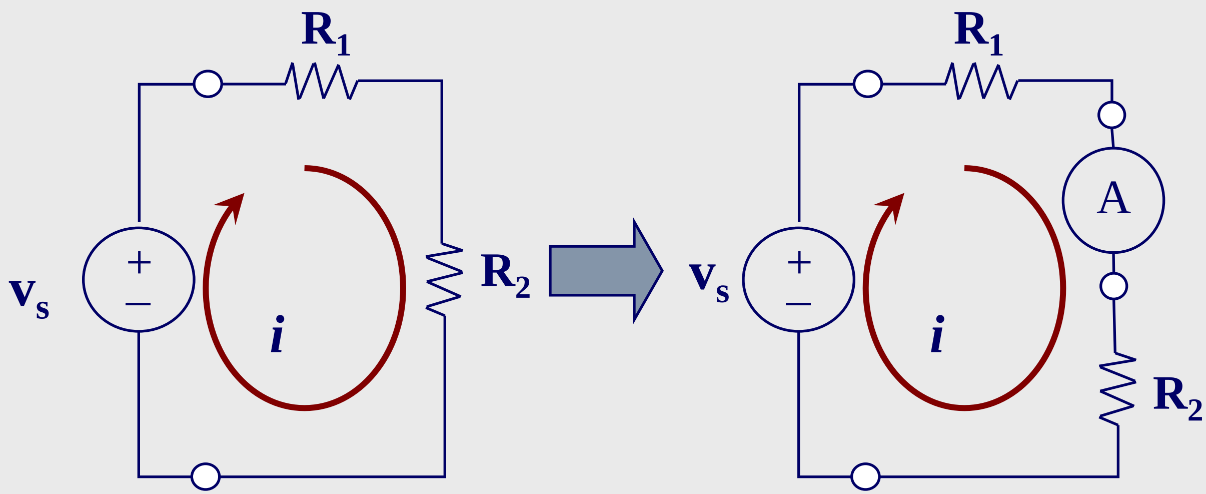
# Ammeter

- **Ammeter**: a device that can measure the current flowing though a circuit element when connected in **series** with that circuit element

**NB**: 1. the ammeter must be connected in **series** with the circuit element
2. the ammeter should not restrict the flow of current (i.e. cause a voltage drop) – an ideal ammeter has **zero** resistance



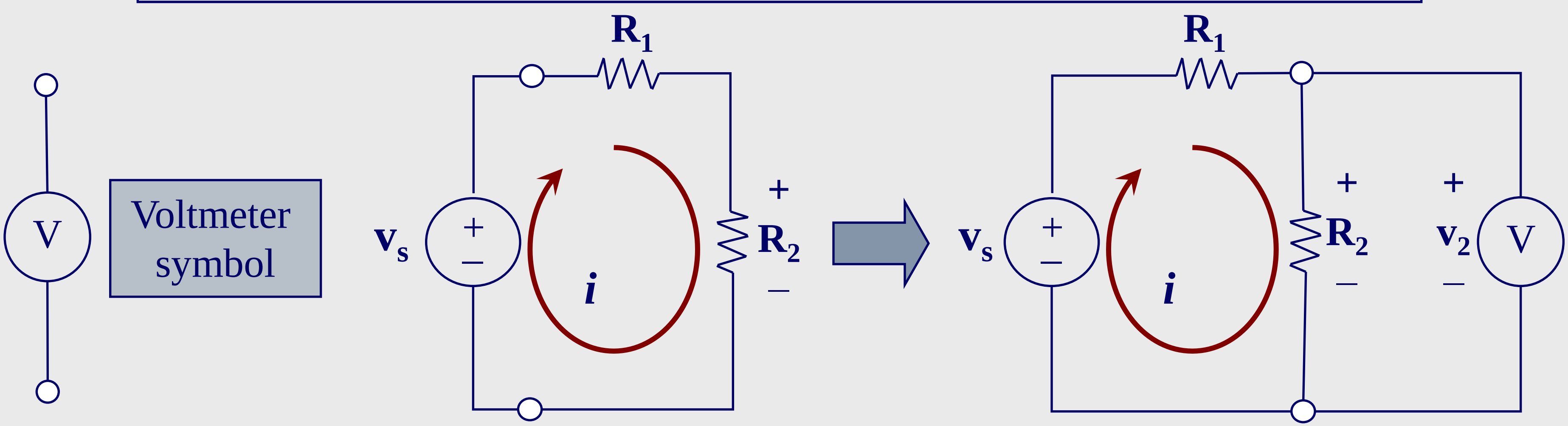

Ammeter symbol



# Voltmeter

- **<u>Voltmeter</u>**: a device that can measure the voltage across a circuit element when connected in **parallel** with that circuit element

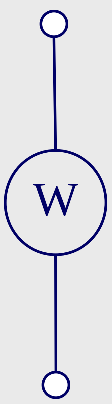
**NB**: 1. the voltmeter must be connected in **parallel** with the circuit element
2. the voltmeter should not draw any current away from the element
– an ideal voltmeter has **infinite** resistance



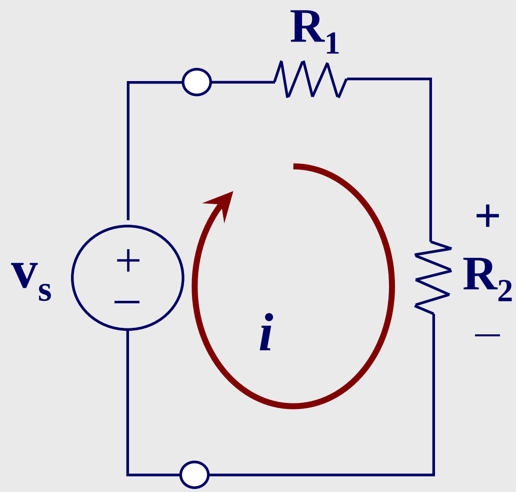
# Wattmeter

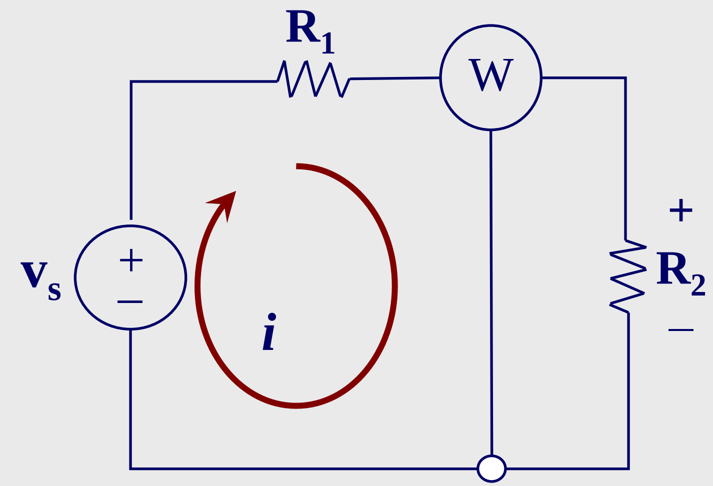
- **<u>Wattmeter</u>**: a device that can measure the power dissipated by a circuit element

**NB**: 1. the wattmeter must be connected in **parallel** with the circuit element, but also in **series** with the circuit.
– a wattmeter is simply the combination of a voltmeter and an ammeter




Wattmeter symbol



# Practical Voltmeters and Ammeters

- In reality, voltmeters and ammeters have internal resistances
  - Practical ammeters will always add some resistance
  - Practical voltmeters will always draw some current