Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
24 Sept	Wed	7	Network Analysis	3.1 – 3.3			
25 Sept	Thu						
26 Sept	Fri		Recitation		HW 3		
27 Sept	Sat						
28 Sept	Sun						
29 Sept	Mon	8	Network Analysis	3.4 – 3.5		LAB 3	
30 Sept	Tue						
1 Oct	Wed	9	Equivalent Circuits	3.6			

Learning is good only when humble

2 Nephi 9: 28-29

28 O that cunning plan of the evil one! O the vainness,
and the frailties, and the foolishness of men! When
they are learned they think they are wise, and they
hearken not unto the counsel of God, for they set it
aside, supposing they know of themselves, wherefore,
their wisdom is foolishness and it profiteth them not.
And they shall perish.

29 But to be learned is good if they hearken unto the
counsels of God.

Lecture 7 – Network Analysis

Node Voltage and Mesh Current Methods

Network Analysis

- Determining the unknown branch currents and node voltages
  - Important to clearly define all relevant variables
  - Construct concise set of equations
    - There are methods to follow in order to create these equations
    - This is the subject of the next few lectures

Network Analysis

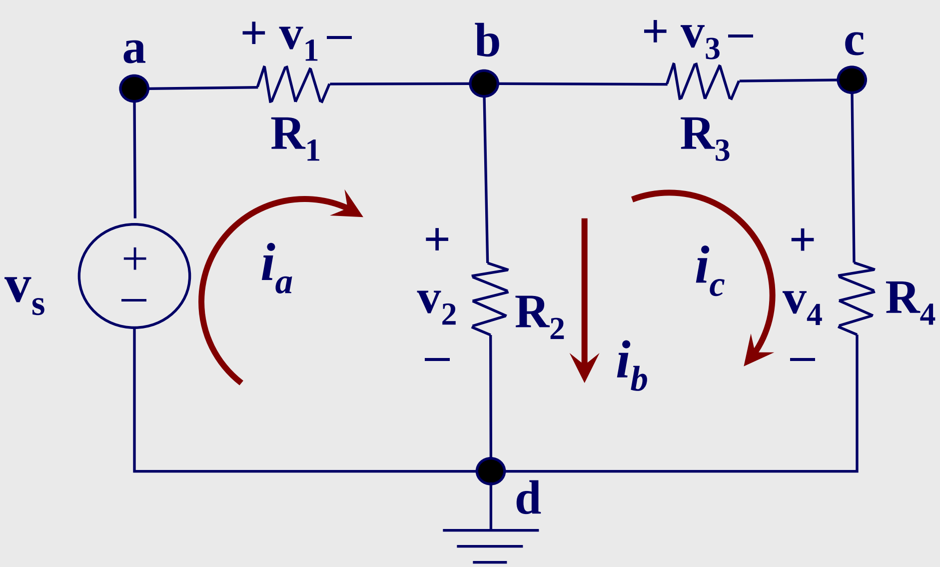
- Network Analysis Methods:
  - Node voltage method
  - Mesh current method
  - Superposition
  - Equivalent circuits
    - Source transformation
    - Thévenin equivalent
    - Norton equivalent

Node Voltage Method

Network Analysis

Node Voltage Method

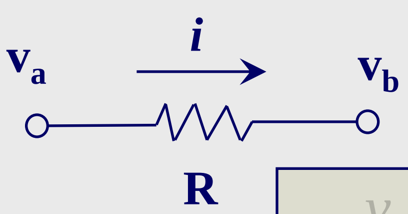
- Identify all node and branch voltages



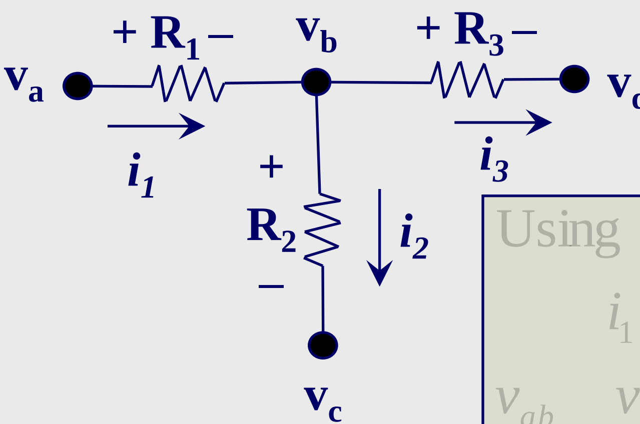
Node Voltages	Branch Voltages
$v_a = v_s$	$v_s = v_a - v_d$ $= v_a$
$v_b = v_2$	$v_1 = v_a - v_b$
$v_c = v_4$	$v_2 = v_b - v_d$ $= v_b$
$v_d = 0$	$v_3 = v_b - v_c$
	$v_4 = v_c - v_d$ $= v_c$

Node Voltage Method

- The most general method for electrical circuit analysis
  - Based on defining the voltage at each node
  - One node is selected as a **reference node** (often ground)
    - All other voltages given relative to reference node
    - **n – 1** equations of **n – 1** independent variables (node voltages)
  - Once node voltages are determined, Ohm's law can determine branch currents
    - Branch currents are expressed in terms of one or more node voltages



$$i = \frac{v_a - v_b}{R} = \frac{v_{ab}}{R}$$



Using KCL :

$$i_1 - i_2 - i_3 = 0$$

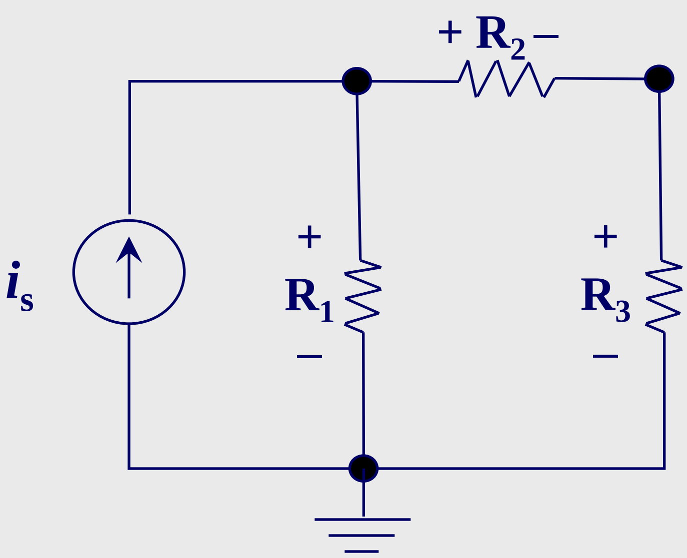
$$\frac{v_{ab}}{R_1} - \frac{v_{bc}}{R_2} - \frac{v_{bd}}{R_3} = 0$$

Node Voltage Method

1. Label all currents and voltages (choose arbitrary orientations unless orientations are already given)
2. Select a reference node (usually ground)
   - This node usually has most elements tied to it
3. Define the remaining **n – 1** node voltages as the independent or dependent variables
   - Each of the **m** voltage sources is associated with a dependent variable
   - If a node is not connected to a source then its voltage is treated as an independent variable
4. Apply KCL at each node labeled as an **independent** variable
   - Express currents in terms of node voltages
5. Solve the linear system of **n – 1 – m** unknowns

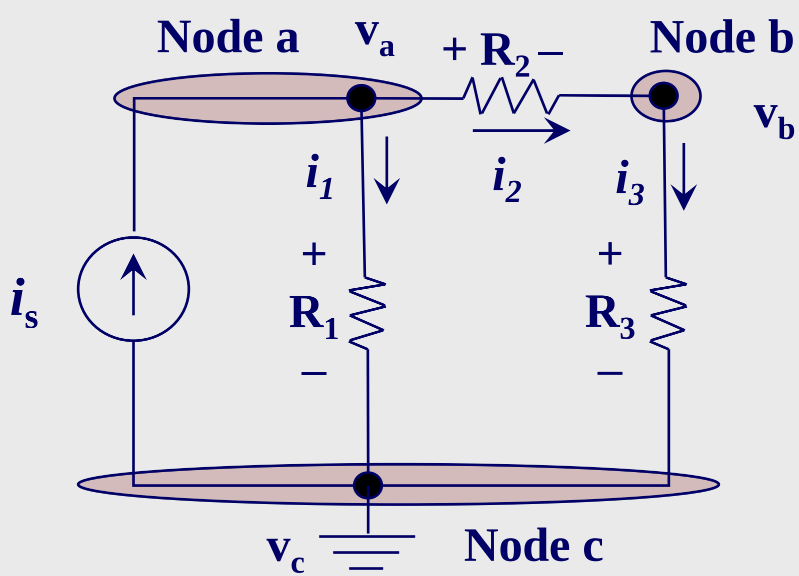
Node Voltage Method

- **Example1**: find expressions for each of the node voltages and the currents



Node Voltage Method

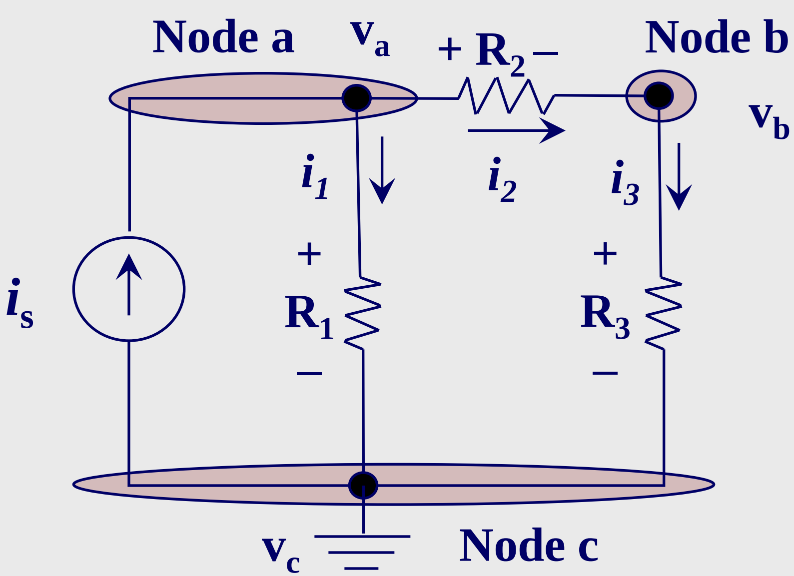
- **<u>Example1</u>**: find expressions for each of the node voltages and the currents



1. Label currents and voltages (polarities "arbitrarily" chosen)
2. Choose Node c (**v_c**) as the reference node (**v_c** = 0)
3. Define remaining n – 1 (2) voltages
   - **v_a** is **independent** since it is not associated with a **voltage** source
   - **v_b** is **independent**
4. Apply KCL at nodes **a** and **b** (node **c** is not independent) to find expressions for i_1, i_2, i_3

Node Voltage Method

- **Example1**: find expressions for each of the node voltages and the currents



4. Apply KCL to find expressions for i_1, i_2, i_3

KCL at Node a :

$$i_s - i_1 - i_2 = 0$$

KCL at Node b :

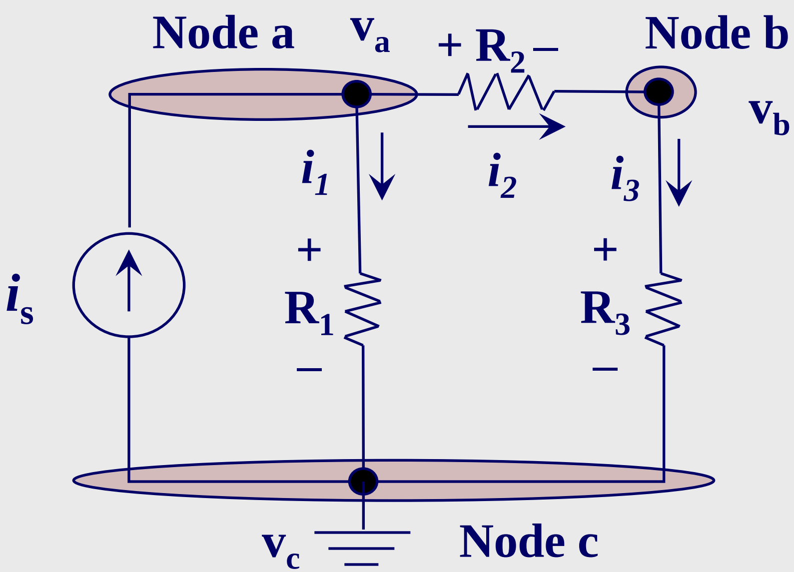
$$i_2 - i_3 = 0$$

$$i_2 = i_3$$

NB: whenever a node connects **only 2 branches** the same current flows in the 2 branches (EX: $i_2 = i_3$)

Node Voltage Method

- **Example1**: find expressions for each of the node voltages and the currents



4. Express currents in terms of node voltages

$$i_1 = \frac{v_{ac}}{R_1} \qquad i_2 = \frac{v_{ab}}{R_2} \qquad i_3 = \frac{v_{bc}}{R_3}$$

KCL at Node a:

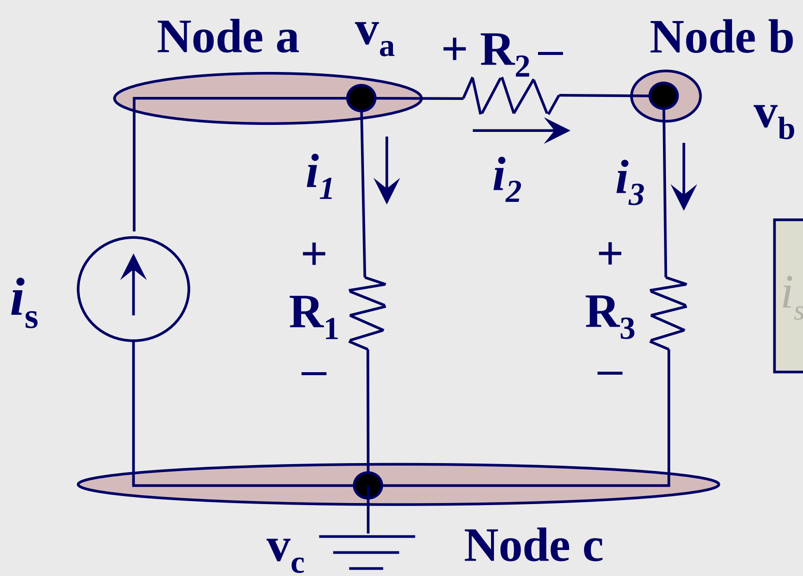
$$i_s - \frac{v_a}{R_1} - \frac{v_{ab}}{R_2} = 0$$

KCL at Node b:

$$\frac{v_{ab}}{R_2} = \frac{v_b}{R_3}$$

Node Voltage Method

- **Example1**: find expressions for each of the node voltages and the currents



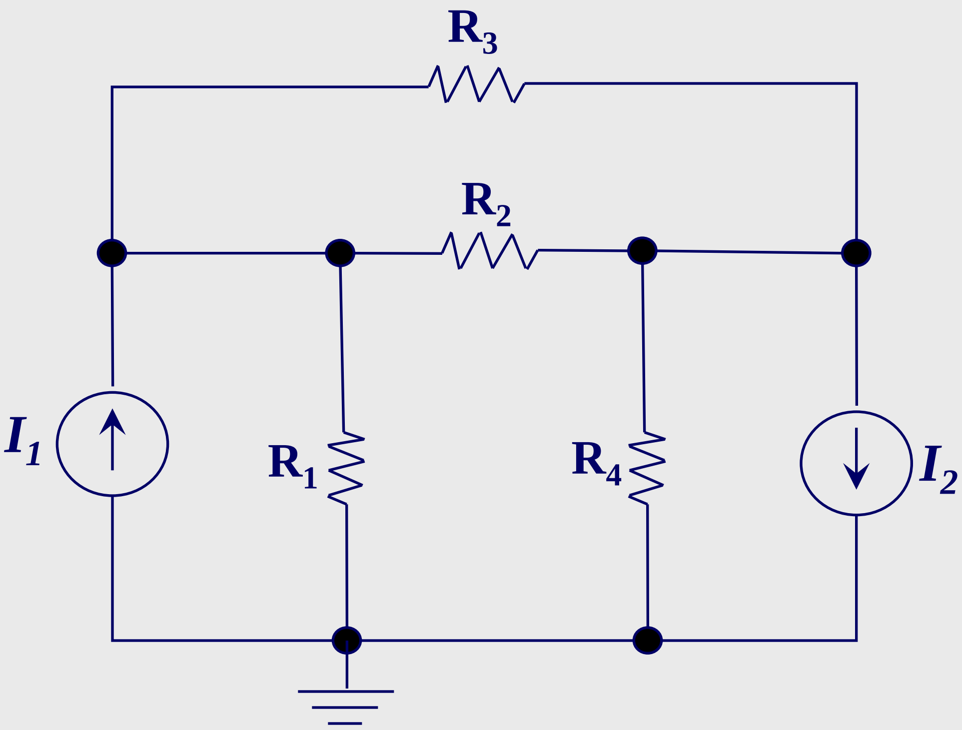
5. Solve the **n – 1 – m** equations

$$i_s = v_a\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - v_b\left(\frac{1}{R_2}\right)$$

$$v_a\left(\frac{1}{R_2}\right) = v_b\left(\frac{1}{R_2}+\frac{1}{R_3}\right)$$

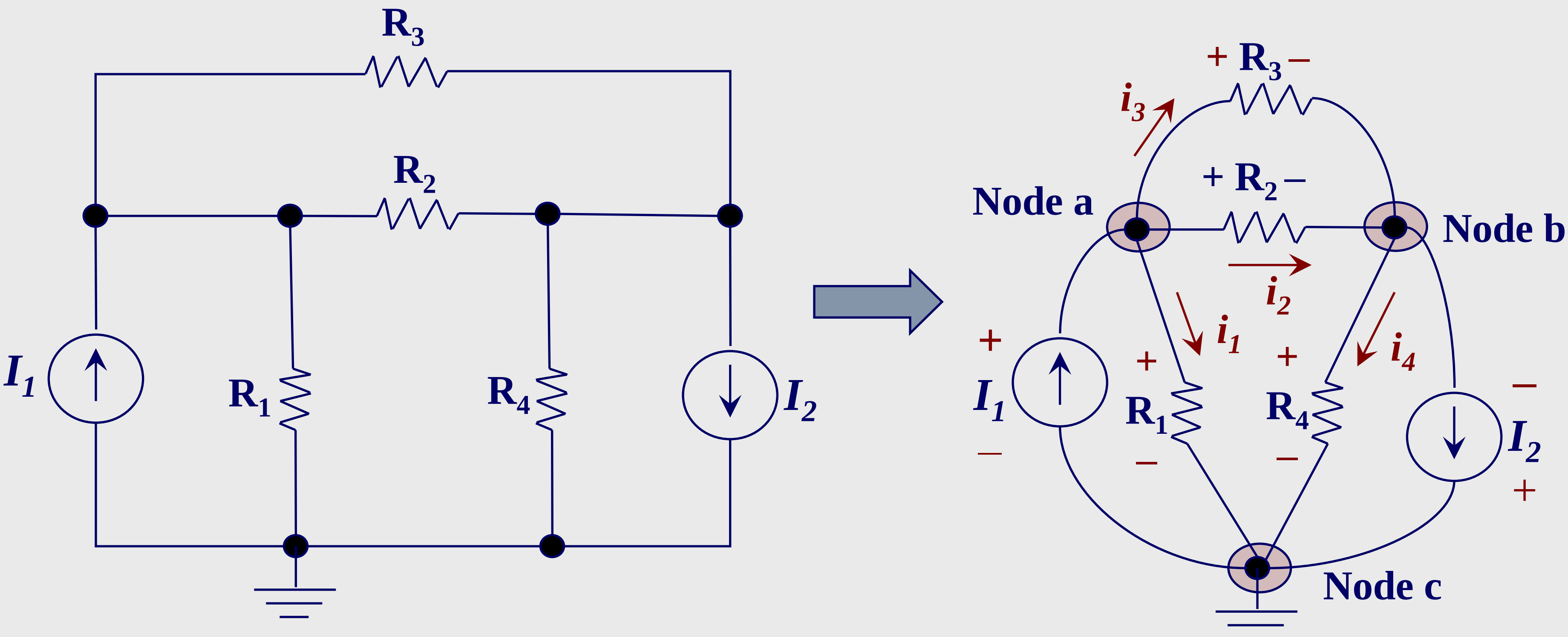
Node Voltage Method

- **<u>Example2</u>**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



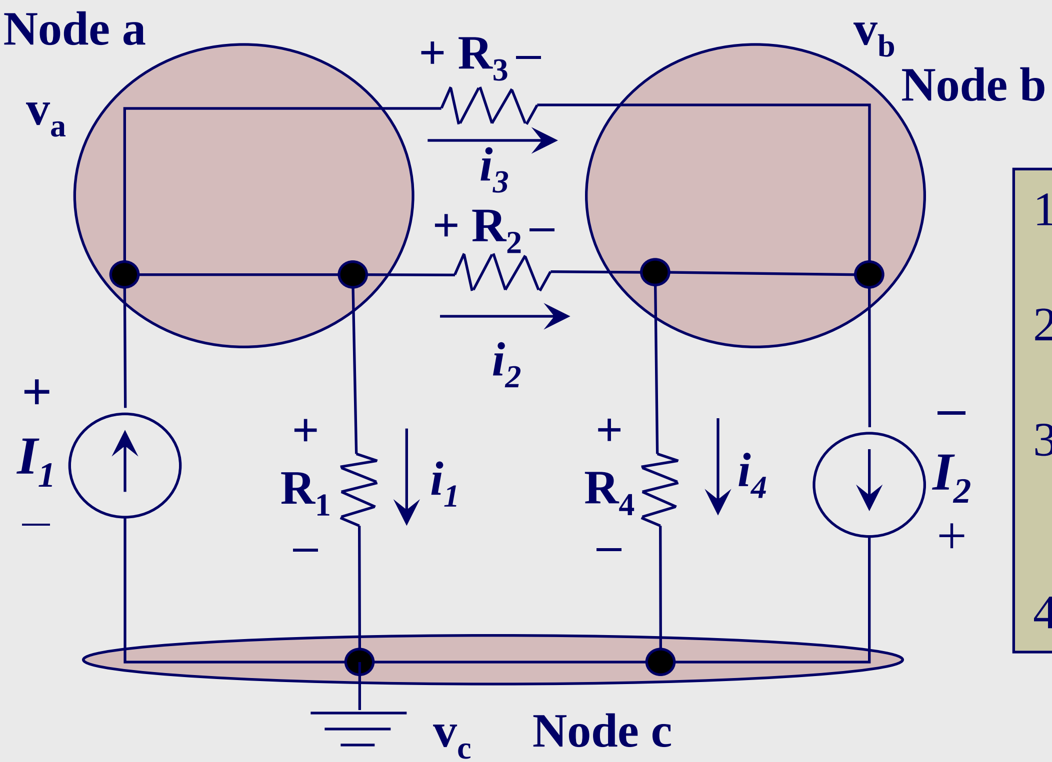
Node Voltage Method

- **<u>Example2</u>**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



Node Voltage Method

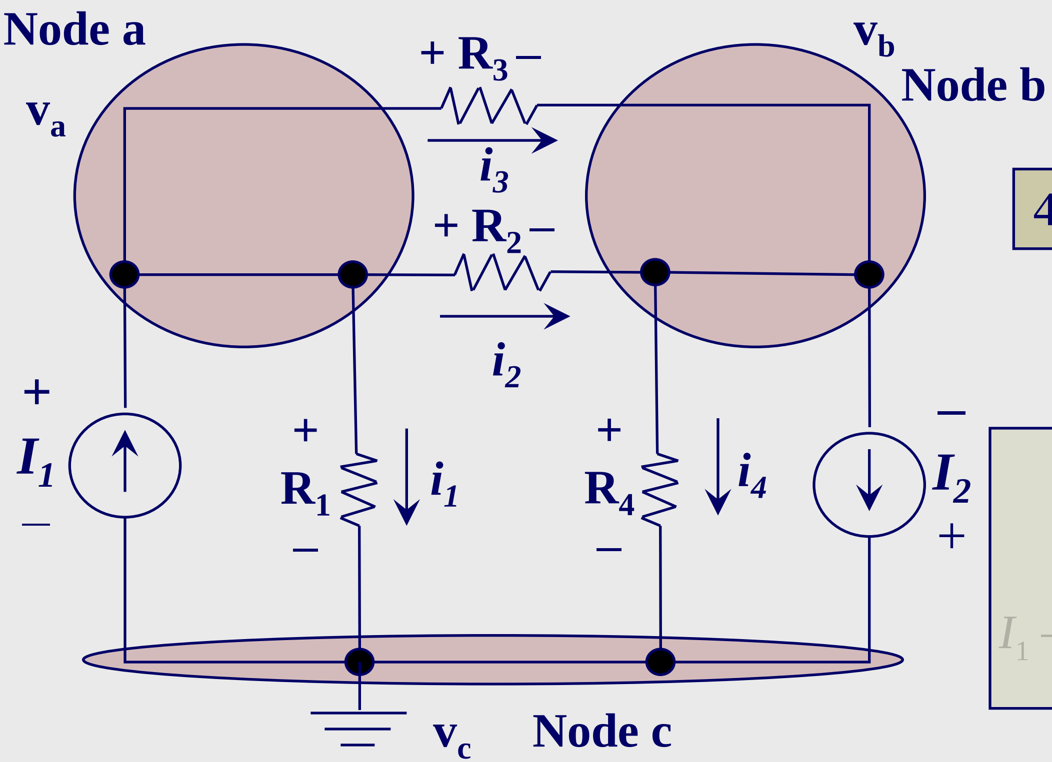
- **<u>Example2</u>**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



1. Label currents and voltages (polarities "arbitrarily" chosen)
2. Choose **Node c** (**v_c**) as the reference node (**v_c** = 0)
3. Define remaining n – 1 (2) voltages
   - **v_a** is **independent**
   - **v_b** is **independent**
4. Apply KCL at nodes **a** and **b**

Node Voltage Method

- **Example2**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



4. Apply KCL at nodes **a** and **b**

KCL at Node a :

$$I_1 - i_1 - i_2 - i_3 = 0$$

$$I_1 - \frac{v_a}{R_1} - \frac{v_{ab}}{R_2} - \frac{v_{ab}}{R_3} = 0$$

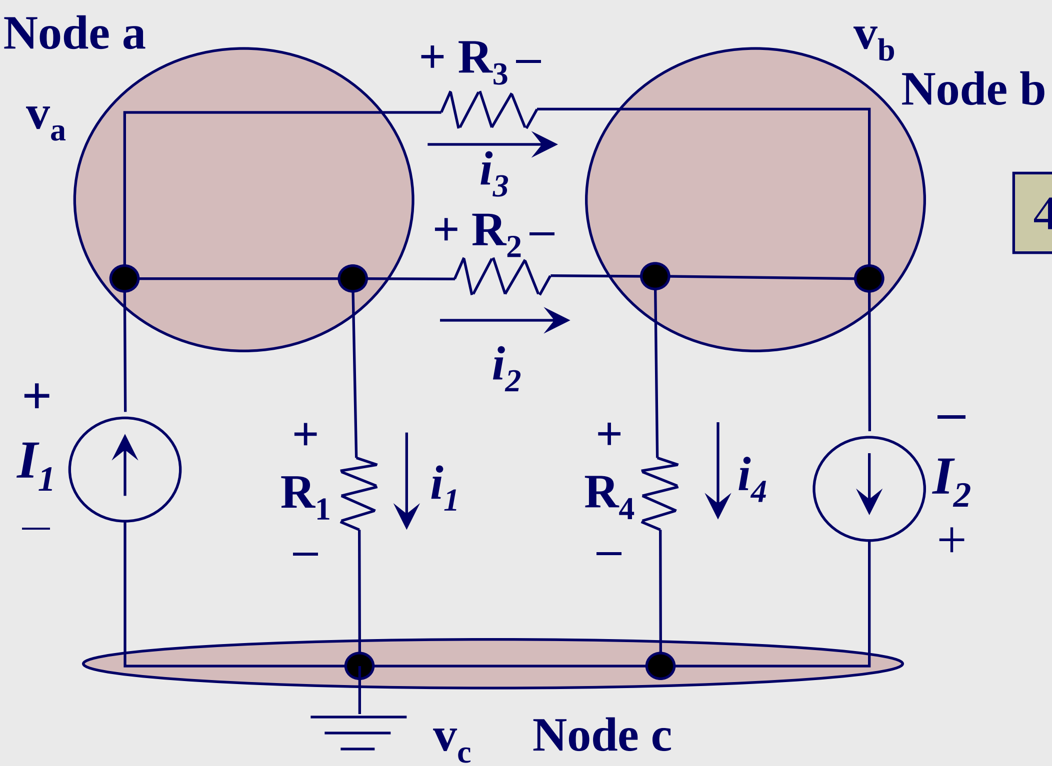
KCL at Node b :

$$i_2 + i_3 - i_4 - I_2 = 0$$

$$\frac{v_{ab}}{R_2} + \frac{v_{ab}}{R_3} - \frac{v_b}{R_4} - I_2 = 0$$

Node Voltage Method

- **Example2**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, R_1 = 1kΩ, R_2 = 2kΩ, R_3 = 10kΩ, R_4 = 2kΩ



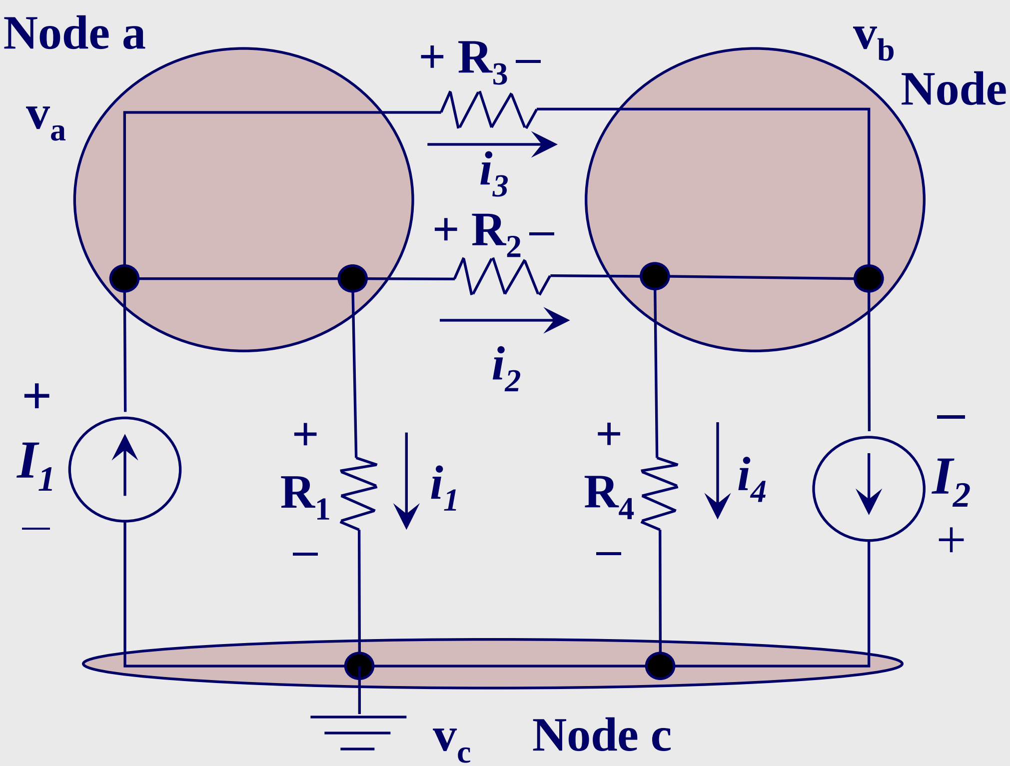
4. Express currents in terms of voltages

$$I_1 = v_a\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-v_b\left(\frac{1}{R_2}+\frac{1}{R_3}\right)$$

$$I_2 = v_a\left(\frac{1}{R_2}+\frac{1}{R_3}\right)-v_b\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)$$

Node Voltage Method

- **<u>Example2</u>**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, R_1 = 1kΩ, R_2 = 2kΩ, R_3 = 10kΩ, R_4 = 2kΩ



5. Solve the n – 1 – m equations

$$I_1 = v_a\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - v_b\left(\frac{1}{R_2}+\frac{1}{R_3}\right)$$

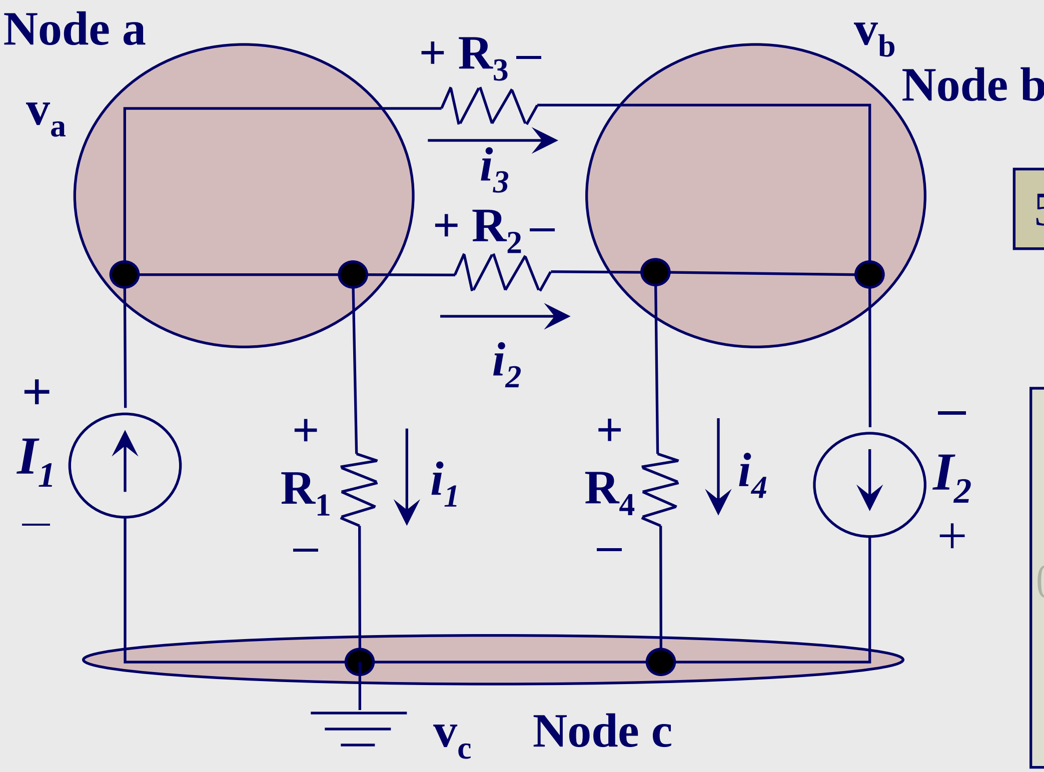
$$0.01 = v_a(10^{-3}+5\times10^{-4}+10^{-4}) - v_b(5\times10^{-4}+10^{-4})$$

$$10 = v_a(1+0.5+0.1) - v_b(0.5+0.1)$$

$$10 = 1.6v_a - 0.6v_b$$

Node Voltage Method

- **<u>Example2</u>**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



5. Solve the n – 1 – m equations

$$I_2 = v_a\left(\frac{1}{R_2}+\frac{1}{R_3}\right) - v_b\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)$$

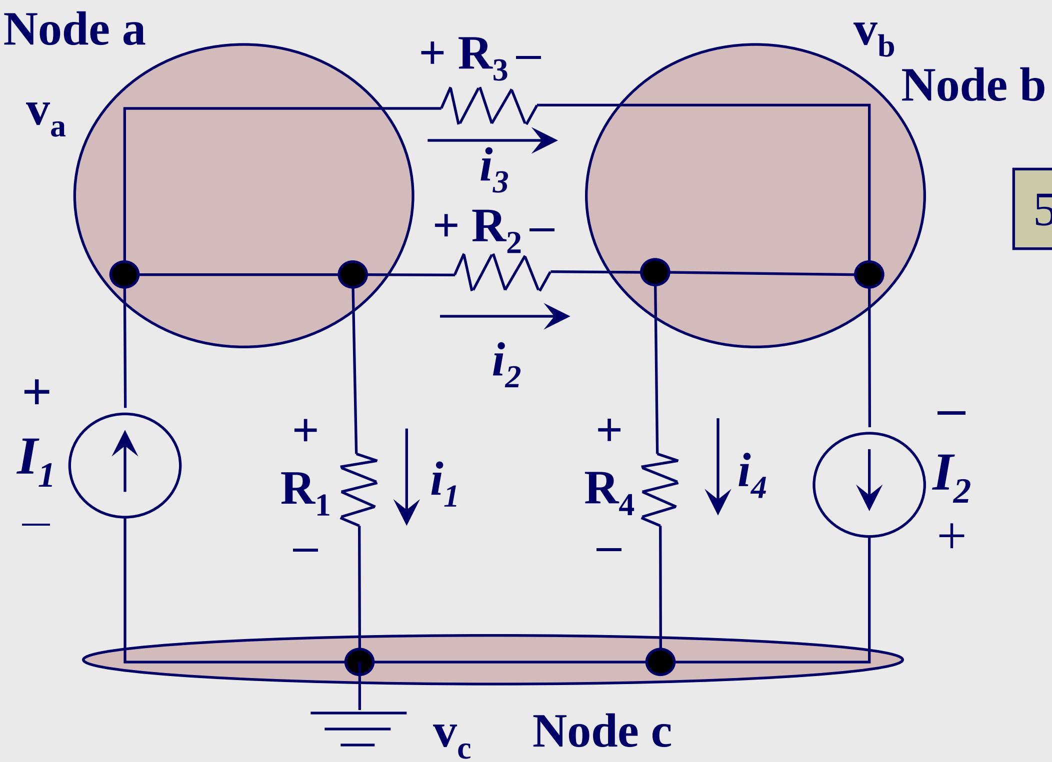
$$0.05 = v_a\left(5\times10^{-4}+10^{-4}\right) - v_b\left(5\times10^{-4}+10^{-4}+5\times10^{-4}\right)$$

$$50 = v_a(0.5+0.1) - v_b(0.5+0.1+0.5)$$

$$50 = 0.6v_a - 1.1v_b$$

Node Voltage Method

- **Example2**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



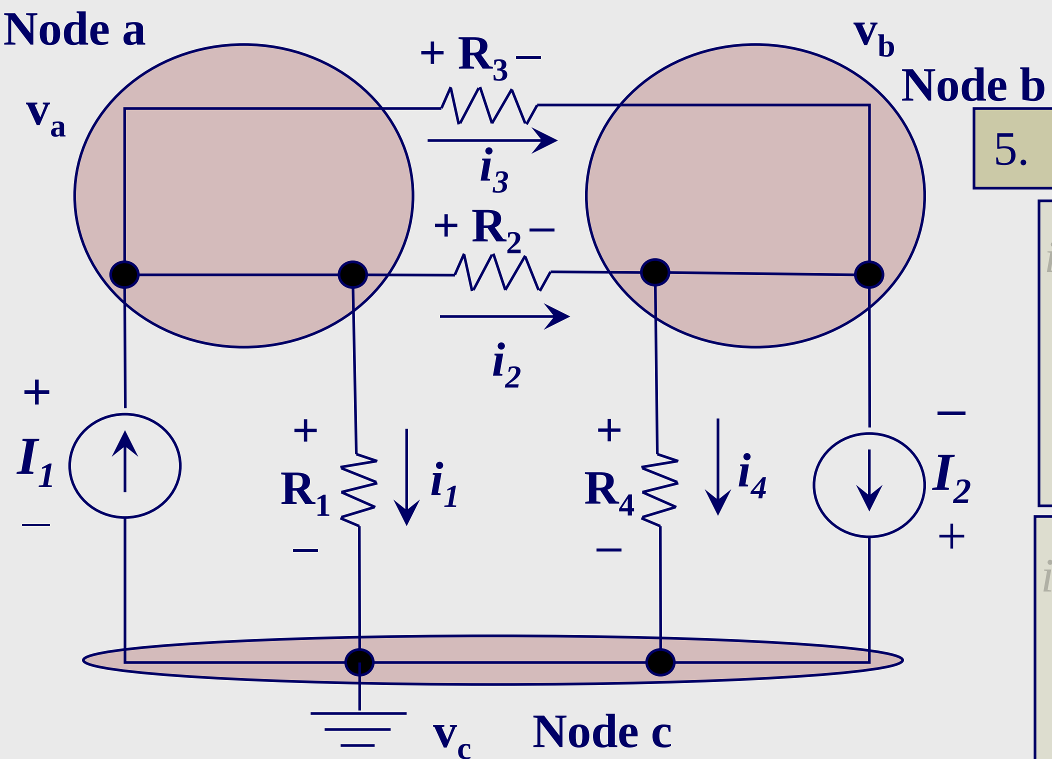
5. Solve the n – 1 – m equations

$$10 = 1.6v_a - 0.6v_b$$
$$50 = 0.6v_a - 1.1v_b$$

$$v_a = -13.57V$$
$$v_b = -52.86V$$

Node Voltage Method

- **Example2**: solve for all unknown currents and voltages
  - I_1 = 10mA, I_2 = 50mA, $\mathbf{R_1}$ = 1kΩ, $\mathbf{R_2}$ = 2kΩ, $\mathbf{R_3}$ = 10kΩ, $\mathbf{R_4}$ = 2kΩ



5. Solve the n – 1 – m equations

$$i_1 = \frac{v_a}{R_1} = \frac{-13.57}{10^3} = -13.57mA$$

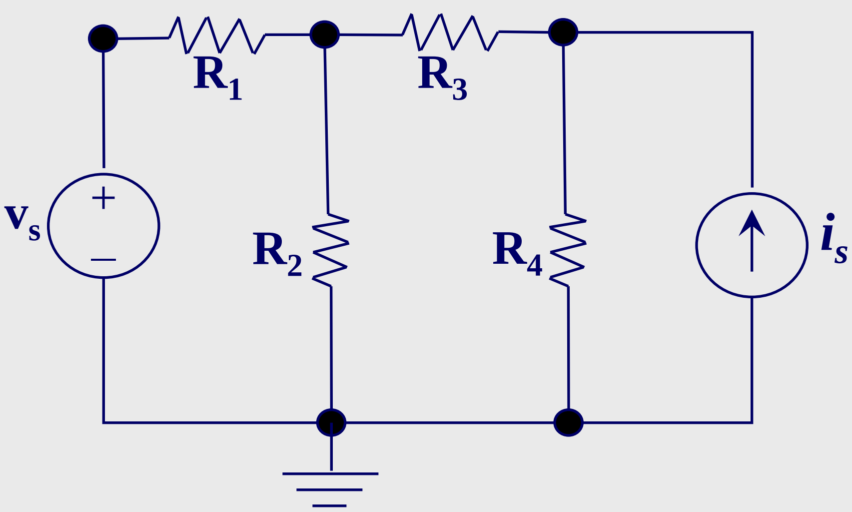
$$i_2 = \frac{v_{ab}}{R_2} = \frac{-13.57+52.86}{2\times10^3} = 19.65mA$$

$$i_3 = \frac{v_{ab}}{R_3} = \frac{-13.57+52.86}{10^4} = 3.93mA$$

$$i_4 = \frac{v_b}{R_4} = \frac{-52.86}{2\times10^3} = -26.43mA$$

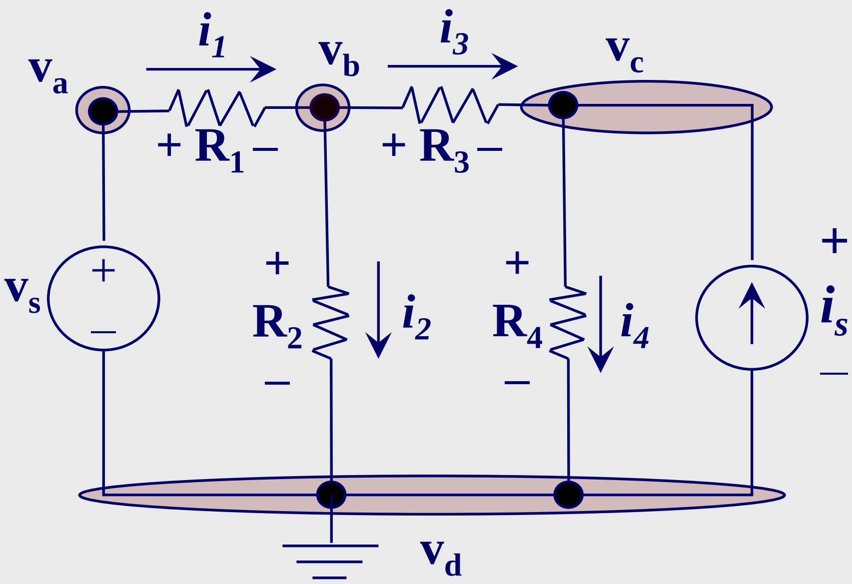
Node Voltage Method

- **Example5**: find all node voltages
  - $\mathbf{v_s} = 2V$, $\boldsymbol{i_s} = 3A$, $\mathbf{R_1} = 1\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



Node Voltage Method

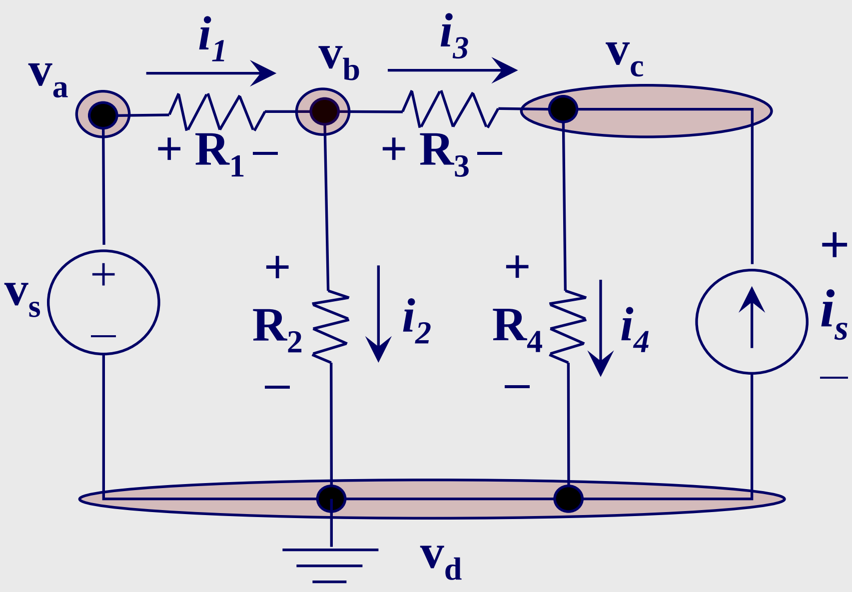
- **Example5**: find all node voltages
  - $\mathbf{v_s} = 2V$, $\boldsymbol{i_s} = 3A$, $\mathbf{R_1} = 1\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



1. Label currents and voltages (polarities "arbitrarily" chosen)
2. Choose **Node d** ($\mathbf{v_d}$) as the reference node ($\mathbf{v_d} = 0$)
3. Define remaining n – 1 (3) voltages
   - $\mathbf{v_a}$ is **dependent** ($\mathbf{v_a} = \mathbf{v_s}$)
   - $\mathbf{v_b}$ is **independent**
   - $\mathbf{v_c}$ is **independent**
4. Apply KCL at nodes **b**, and **c**

Node Voltage Method

- **Example5**: find all node voltages
  - $\mathbf{v_s}$ = 2V, i_s = 3A, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 4Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_4}$ = 3Ω



4. Apply KCL at nodes **b**, and **c**

KCL at Node b :

$$i_1 - i_2 - i_3 = 0$$

$$\frac{v_{sb}}{R_1} - \frac{v_b}{R_2} - \frac{v_{bc}}{R_3} = 0$$

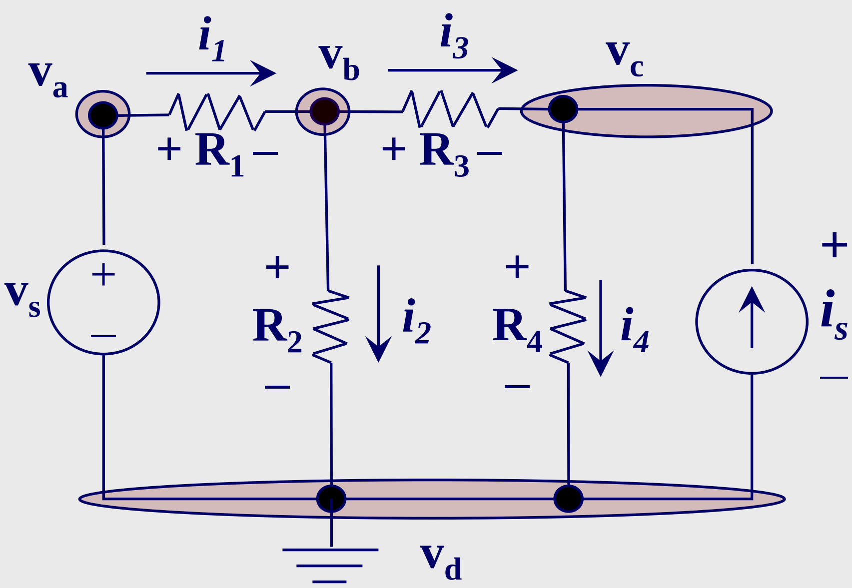
KCL at Node c :

$$i_s + i_3 - i_4 = 0$$

$$i_s + \frac{v_{bc}}{R_3} - \frac{v_c}{R_4} = 0$$

Node Voltage Method

- **<u>Example5</u>**: find all node voltages
  - $\mathbf{v_s}$ = 2V, $\boldsymbol{i_s}$ = 3A, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 4Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_4}$ = 3Ω



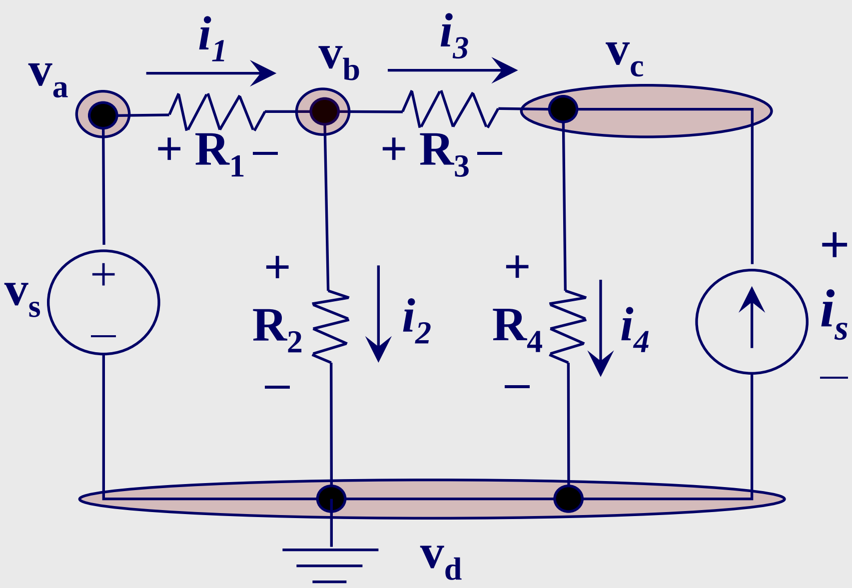
4. Express currents in terms of voltages

$$\frac{v_s}{R_1} = v_b\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - v_c\left(\frac{1}{R_3}\right)$$

$$i_s = -v_b\left(\frac{1}{R_3}\right) + v_c\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$$

Node Voltage Method

- **Example5**: find all node voltages
  - $\mathbf{v_s} = 2V$, $\boldsymbol{i_s} = 3A$, $\mathbf{R_1} = 1\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



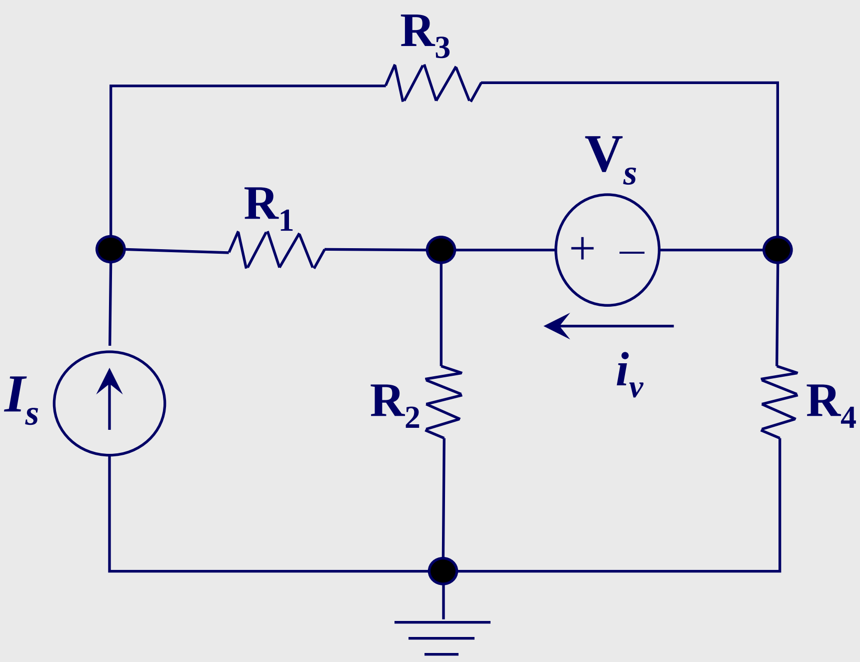
5. Solve the n – 1 – m equations

$$7v_b - 2v_c = 8$$
$$-3v_b + 5v_c = 18$$

$$v_b = 2.62V$$
$$v_c = 5.17V$$

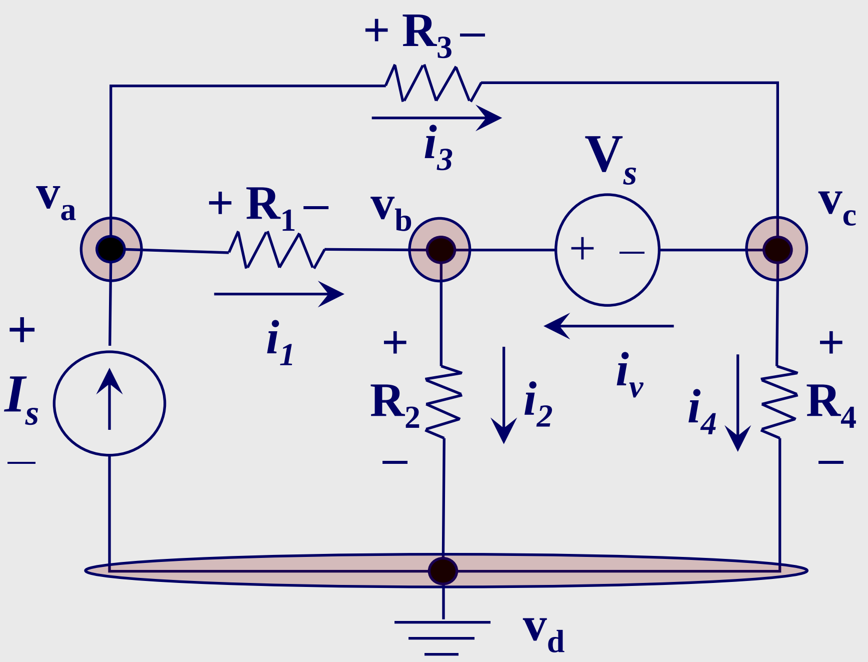
Node Voltage Method

- **<u>Example6</u>**: find the current $\boldsymbol{i_v}$
  - $\mathbf{V_s} = 3\text{V}$, $I_s = 2\text{A}$, $\mathbf{R_1} = 2\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



Node Voltage Method

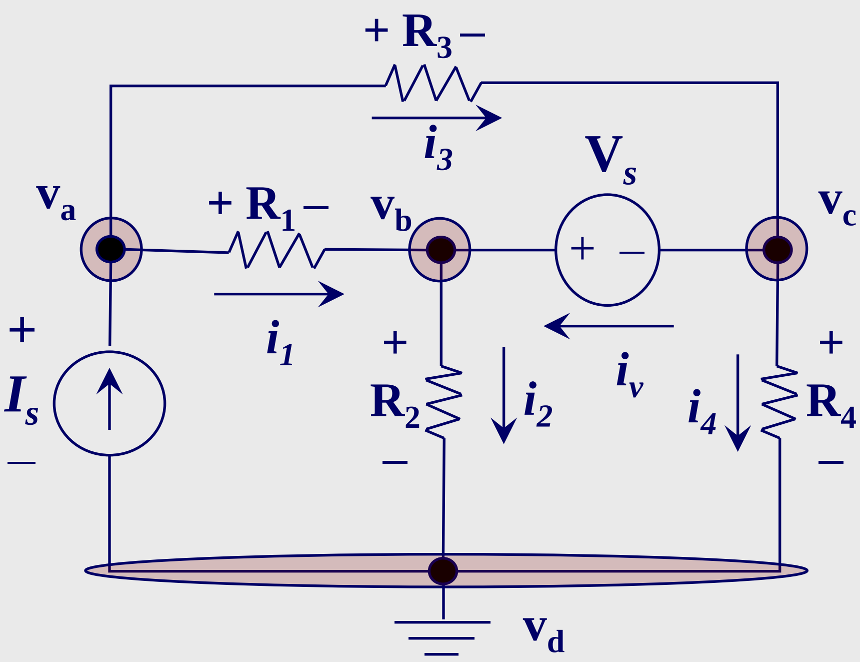
- **<u>Example6</u>**: find the current i_v
  - $\mathbf{V_s} = 3V$, $I_s = 2A$, $\mathbf{R_1} = 2\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



1. Label currents and voltages (polarities "arbitrarily" chosen)
2. Choose **Node d** ($\mathbf{v_d}$) as the reference node ($\mathbf{v_d} = 0$)
3. Define remaining n – 1 (3) voltages
   - $\mathbf{v_a}$ is **independent**
   - $\mathbf{v_b}$ is **dependent**
     (actually both $\mathbf{v_b}$ and $\mathbf{v_c}$ are dependent on each other so choose one to be dependent and one to be independent)
     ($\mathbf{v_b} = \mathbf{v_c} + \mathbf{V_s}$)
   - $\mathbf{v_c}$ is **independent**
4. Apply KCL at nodes **a**, and **c**

Node Voltage Method

- **Example6**: find the current $\boldsymbol{i_v}$
  - $\mathbf{V_s}$ = 3V, $\boldsymbol{I_s}$ = 2A, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 4Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_4}$ = 3Ω



4. Apply KCL at nodes **a**, and **c**

KCL at Node a :

$$I_s - i_1 - i_3 = 0$$

$$I_s - \frac{v_{ab}}{R_1} - \frac{v_{ac}}{R_3} = 0$$

KCL at Node c :

$$i_3 - i_v - i_4 = 0$$

$$\frac{v_{ac}}{R_3} - i_v - \frac{v_c}{R_4} = 0$$

BUT at Node b :

$$i_1 + i_v - i_2 = 0$$

$$i_v = \frac{v_b}{R_2} - \frac{v_{ab}}{R_1}$$

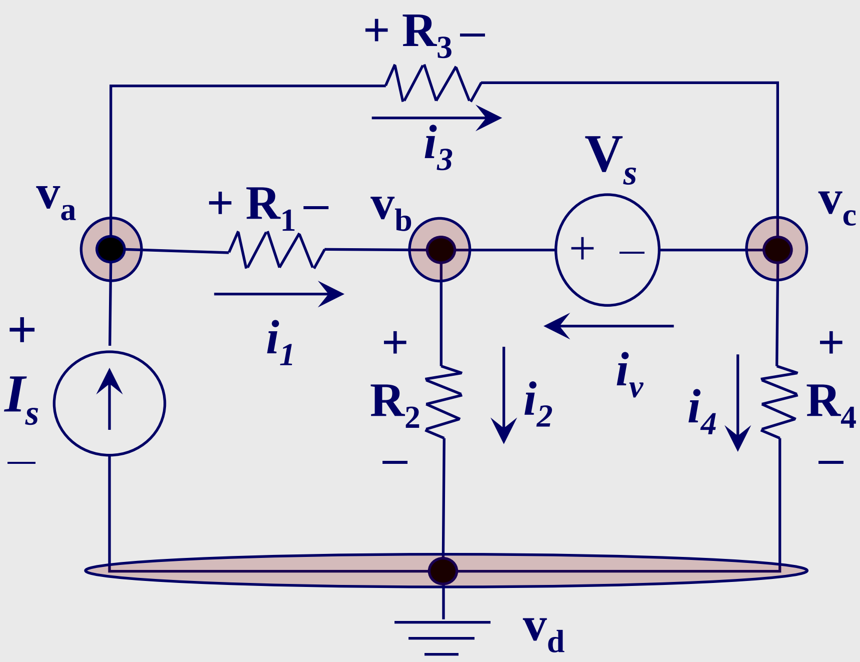
∴ KCL at Node c :

$$\frac{v_{ac}}{R_3} - i_v - \frac{v_c}{R_4} = 0$$

$$\frac{v_{ac}}{R_3} - \frac{v_b}{R_2} + \frac{v_{ab}}{R_1} - \frac{v_c}{R_4} = 0$$

Node Voltage Method

- **<u>Example6</u>**: find the current $\boldsymbol{i_v}$
  - $\mathbf{V_s} = 3\text{V}$, $\boldsymbol{I_s} = 2\text{A}$, $\mathbf{R_1} = 2\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



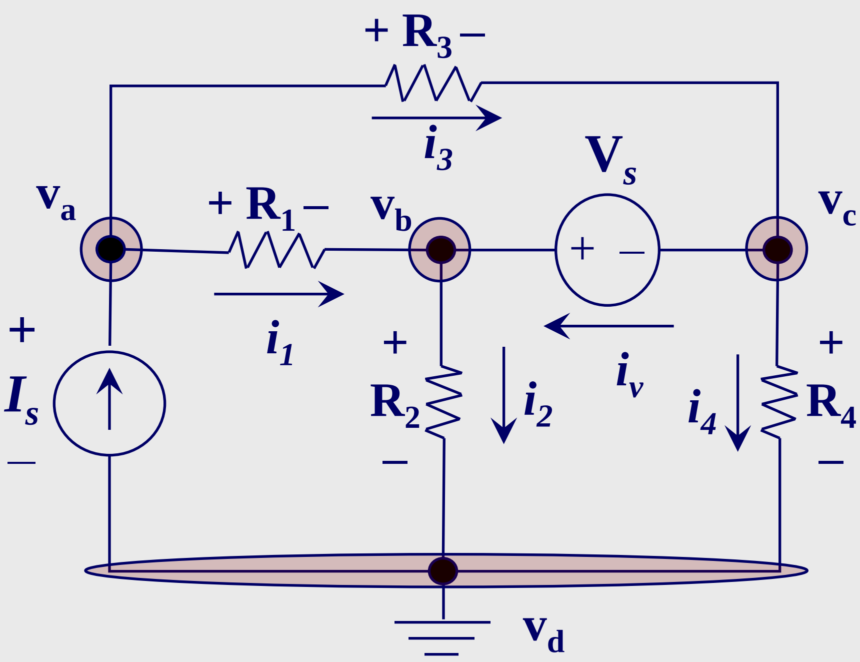
4. Express currents in terms of voltages

$$I_s = v_a\left(\frac{1}{R_1}+\frac{1}{R_3}\right) - v_b\left(\frac{1}{R_1}\right) - v_c\left(\frac{1}{R_3}\right)$$

$$0 = v_a\left(\frac{1}{R_1}+\frac{1}{R_3}\right) - v_b\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - v_c\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$$

Node Voltage Method

- **Example6**: find the current i_v
  - $\mathbf{V_s} = 3V$, $I_s = 2A$, $\mathbf{R_1} = 2\Omega$, $\mathbf{R_2} = 4\Omega$, $\mathbf{R_3} = 2\Omega$, $\mathbf{R_4} = 3\Omega$



5. Solve the n – 1 – m equations

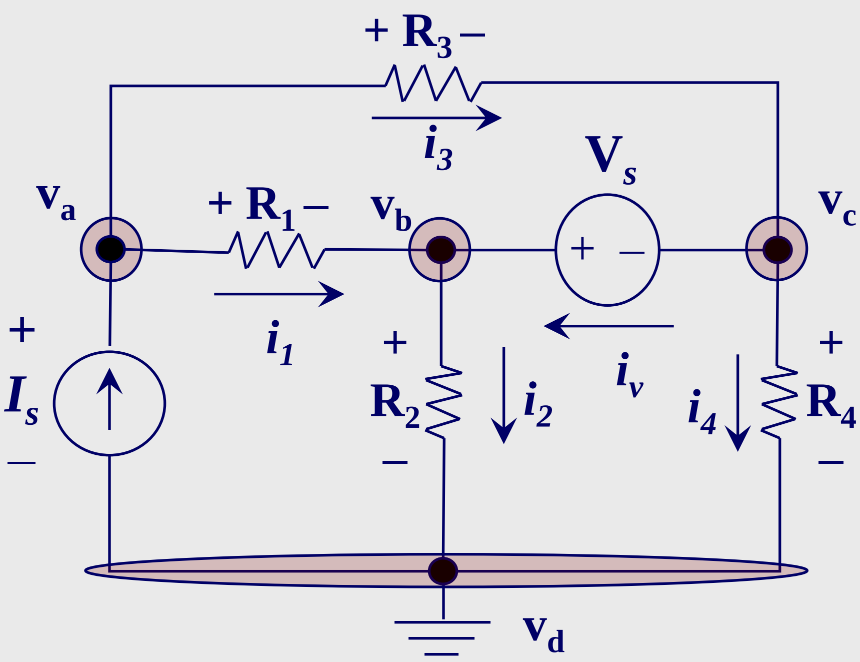
$$2v_a - v_b - v_c = 4$$
$$12v_a - 9v_b - 10v_c = 0$$

$$v_b - v_c = 3$$

$$v_a = 5.64V$$
$$v_b = 5.14V$$
$$v_c = 2.14V$$

Node Voltage Method

- **Example6**: find the current i_v
  - $\mathbf{V_s}$ = 3V, I_s = 2A, $\mathbf{R_1}$ = 2Ω, $\mathbf{R_2}$ = 4Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_4}$ = 3Ω



5. Solve the n – 1 – m equations

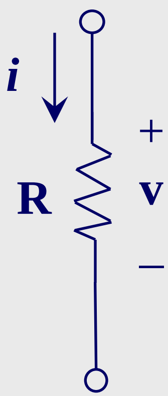
$$i_v = \frac{v_b}{R_2} - \frac{v_{ab}}{R_1}$$
$$= \frac{5.14}{4} - \frac{(5.64 - 5.14)}{2}$$
$$= 2.32\,A$$

Mesh Current Method

Network Analysis

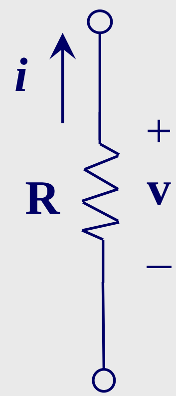
Mesh Current Method

- Write **n** equations of **n** unknowns in terms of mesh currents (where **n** is the number of meshes)
  - Use of KVL to solve unknown currents
  - Important to be **consistent** with current direction



Positive voltage

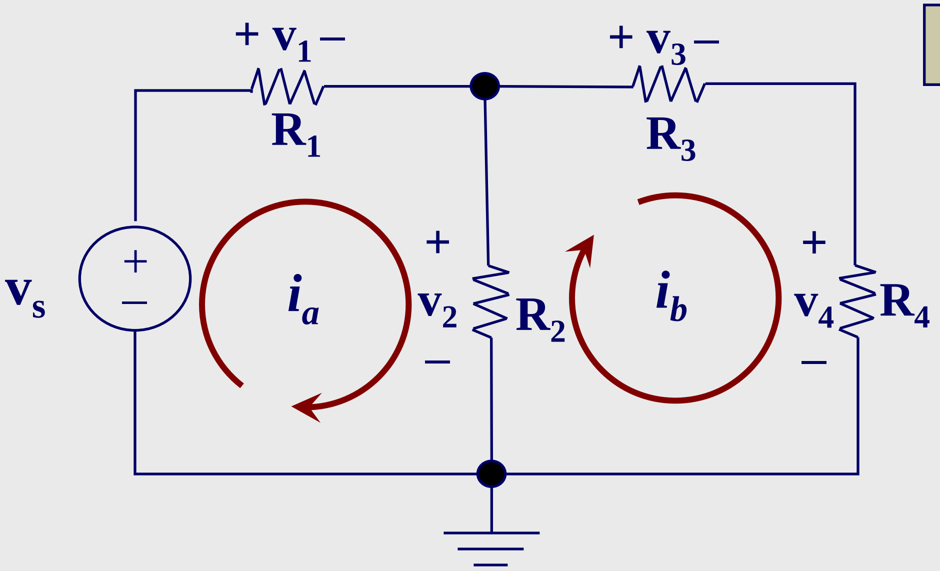
NB: the direction of current defines the **polarity** of the voltage



Negative voltage

Mesh Current Method

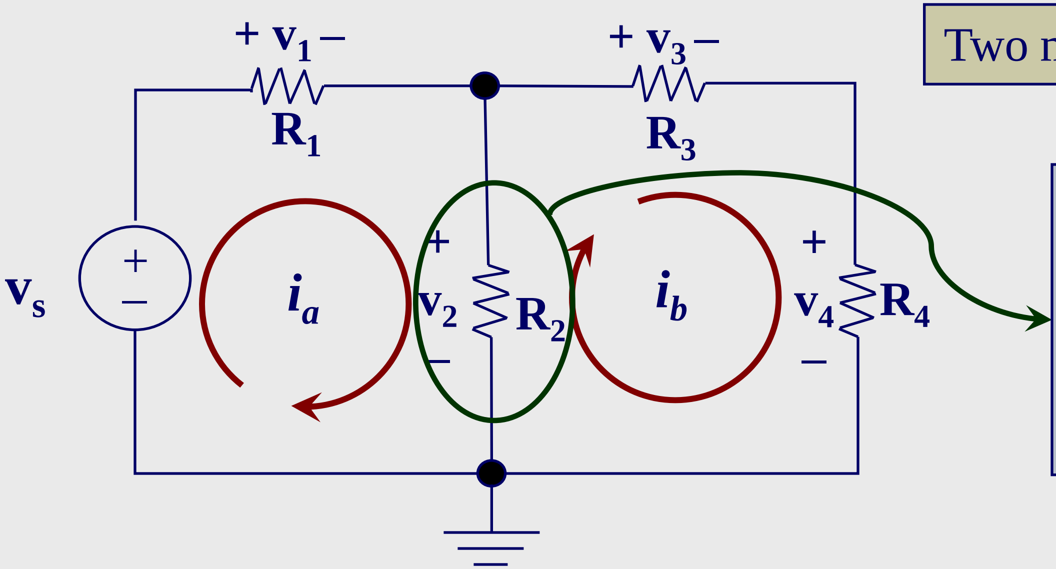
- Write **n** equations of **n** unknowns in terms of mesh currents (where **n** is the number of meshes)
  - Use of KVL to solve unknown currents
  - Important to be **consistent** with current direction



Two meshes ⟶ n = 2

Mesh Current Method

- Write **n** equations of **n** unknowns in terms of mesh currents (where **n** is the number of meshes)
  - Use of KVL to solve unknown currents
  - Important to be **consistent** with current direction

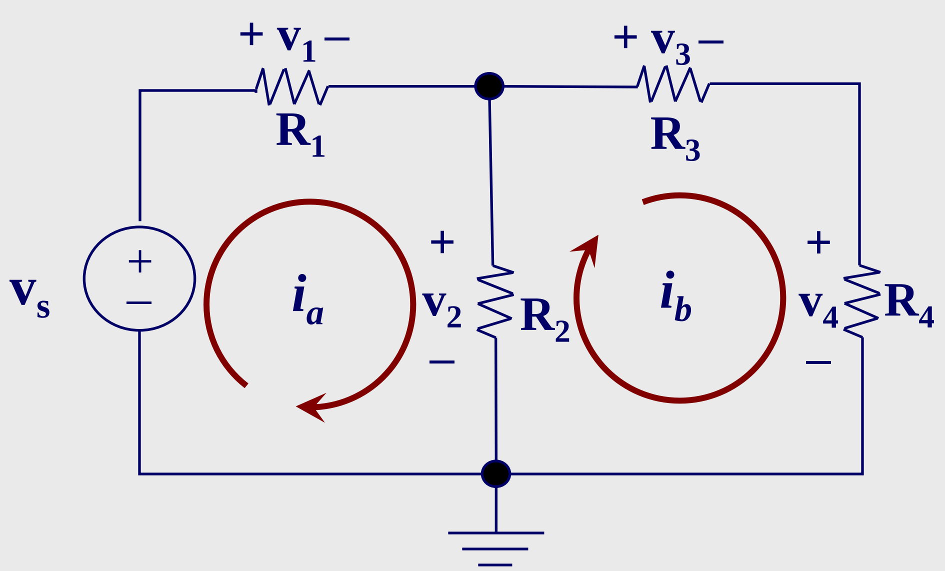


Two meshes ⟶ n = 2

$\boldsymbol{i_1}$ (current through $\mathbf{R_1}$) = $\boldsymbol{i_a}$ but what about $\boldsymbol{i_2}$?
According to the current directions of $\boldsymbol{i_a}$ and $\boldsymbol{i_b}$, and the polarity of $\mathbf{v_2}$:
$\boldsymbol{i_2}$ (current through $\mathbf{R_2}$) = $\boldsymbol{i_a} - \boldsymbol{i_b}$

Mesh Current Method

- Write **n** equations of **n** unknowns in terms of mesh currents (where **n** is the number of meshes)
  - Use of KVL to solve unknown currents
  - Important to be **consistent** with current direction



KVL around Mesh a :

$$-v_s + v_1 + v_2 = 0$$

$$v_s - i_a R_1 - (i_a - i_b) R_2 = 0$$

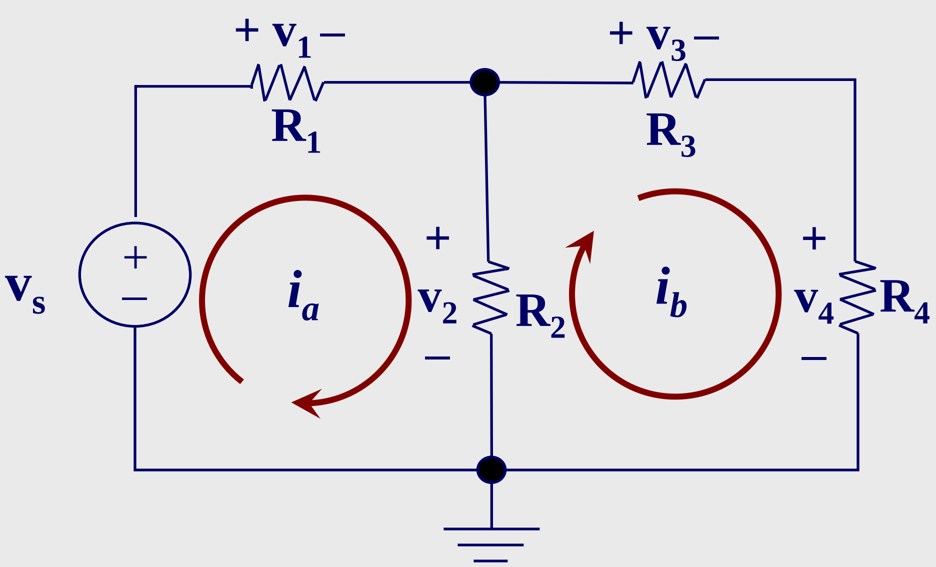
KVL around Mesh b :

$$-v_2 + v_3 + v_4 = 0$$

$$-(i_a - i_b) R_2 + i_b R_3 + i_b R_4 = 0$$

Mesh Current Method

- Write **n** equations of **n** unknowns in terms of mesh currents (where **n** is the number of meshes)
  - Use of KVL to solve unknown currents
  - Important to be **consistent** with current direction



2 equations, 2 unknowns :

$$i_a(R_1 + R_2) - i_b R_2 = v_s$$

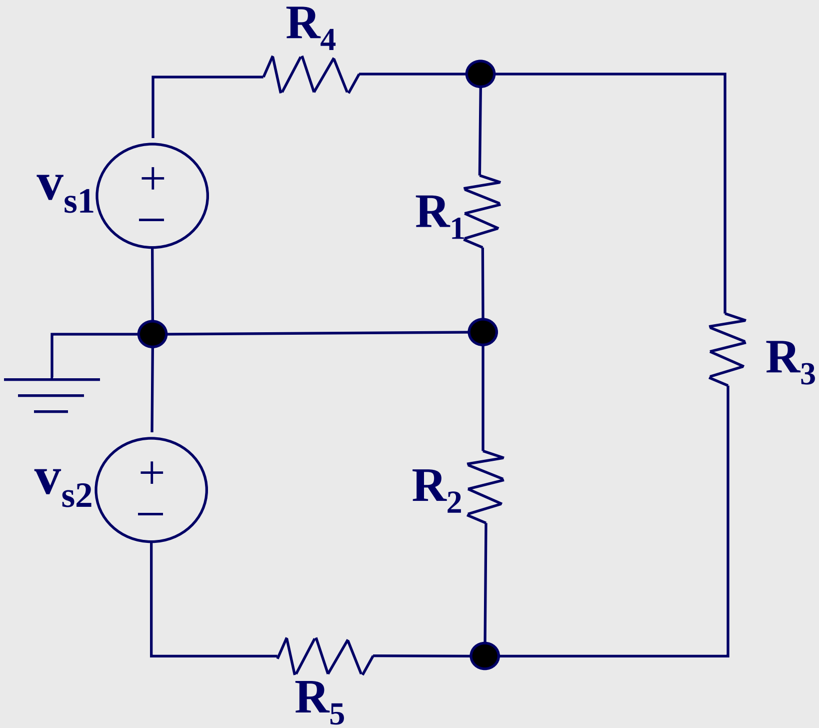
$$-i_a R_2 + i_b(R_2 + R_3 + R_4) = 0$$

Mesh Current Method

1. Label each mesh current consistently
   - Current directions are chosen **arbitrarily** unless given
2. Label the voltage polarity of each circuit element
   - Strategically (based on current direction) choose polarity unless already given
3. In a circuit with **n** meshes and **m** current sources **n – m** independent equations result
   - Each of the **m** current sources is associated with a dependent variable
   - If a mesh is not connected to a source then its voltage is treated as an independent variable
4. Apply KVL at each mesh associated with **independent** variables
   - Express voltages in terms of mesh currents
5. Solve the linear system of **n – m** unknowns

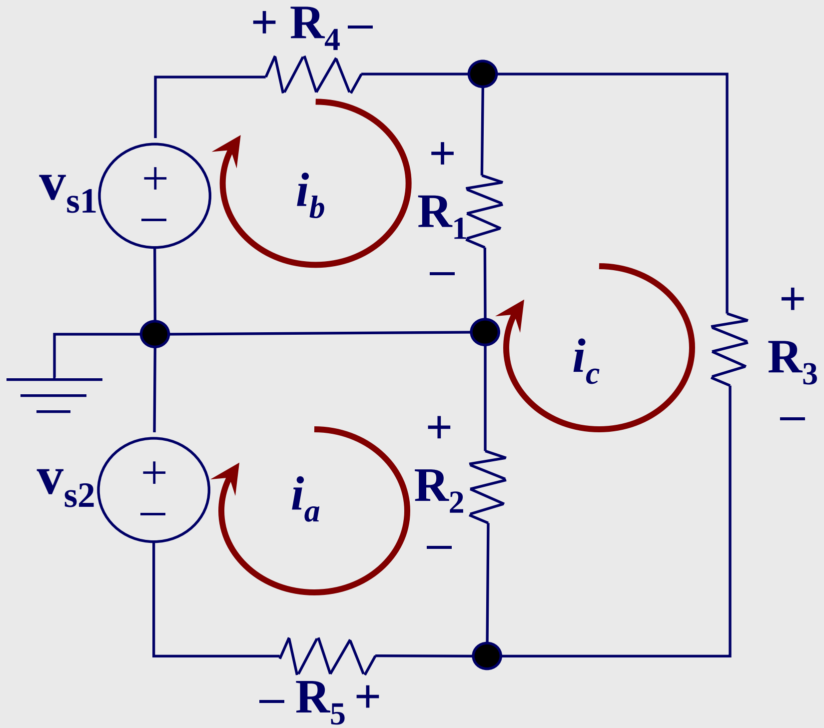
Mesh Current Method

- **Example8**: find the voltages across the loads
  - $\mathbf{V_{s1}} = \mathbf{V_{s2}} = 110V$, $\mathbf{R_1} = 15\Omega$, $\mathbf{R_2} = 40\Omega$, $\mathbf{R_3} = 16\Omega$, $\mathbf{R_4} = \mathbf{R_5} = 1.3\Omega$



Mesh Current Method

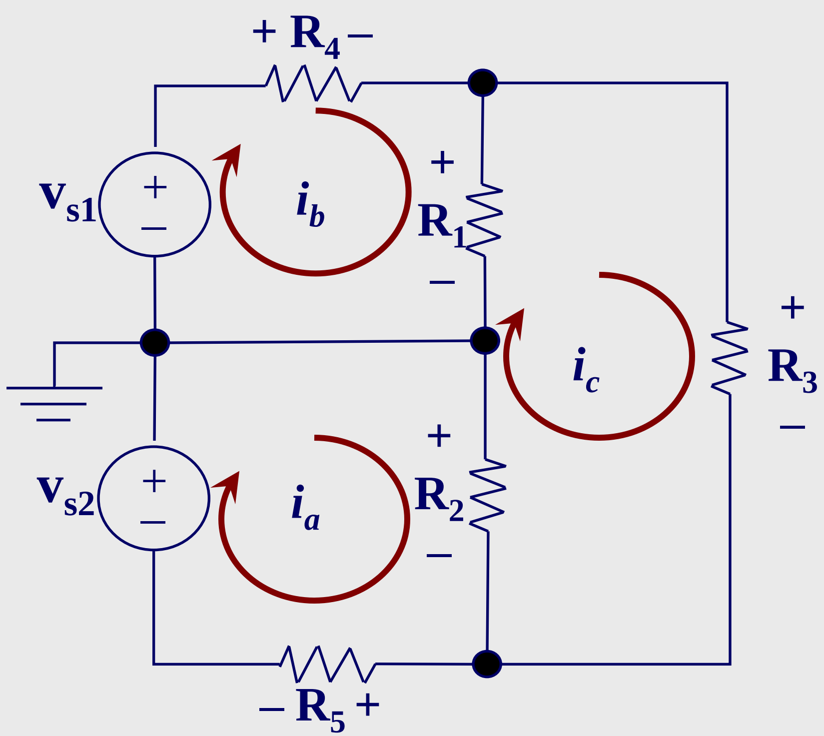
- **<u>Example8</u>**: find the voltages across the loads
  - $\mathbf{V_{s1}} = \mathbf{V_{s2}} = 110V$, $\mathbf{R_1} = 15\Omega$, $\mathbf{R_2} = 40\Omega$, $\mathbf{R_3} = 16\Omega$, $\mathbf{R_4} = \mathbf{R_5} = 1.3\Omega$



1. Mesh current directions given
2. Voltage polarities chosen and labeled
3. Identify n – m (3) mesh currents
   - i_a is **independent**
   - i_a is **independent**
   - i_c is **independent**
4. Apply KVL around meshes **a**, **b**, and **c**

Mesh Current Method

- **Example8**: find the voltages across the loads
  - $\mathbf{V_{s1}} = \mathbf{V_{s2}} = 110\text{V}$, $\mathbf{R_1} = 15\Omega$, $\mathbf{R_2} = 40\Omega$, $\mathbf{R_3} = 16\Omega$, $\mathbf{R_4} = \mathbf{R_5} = 1.3\Omega$



4. Apply KVL at nodes **a, b,** and **c**

KVL around Mesh a:

$$-v_{s2} + v_2 + v_5 = 0$$

$$v_{s2} - (i_a - i_c)R_2 - i_a R_5 = 0$$

KVL around Mesh b:

$$-v_{s1} + v_4 + v_1 = 0$$

$$v_{s1} - i_b R_4 - (i_b - i_c)R_1 = 0$$

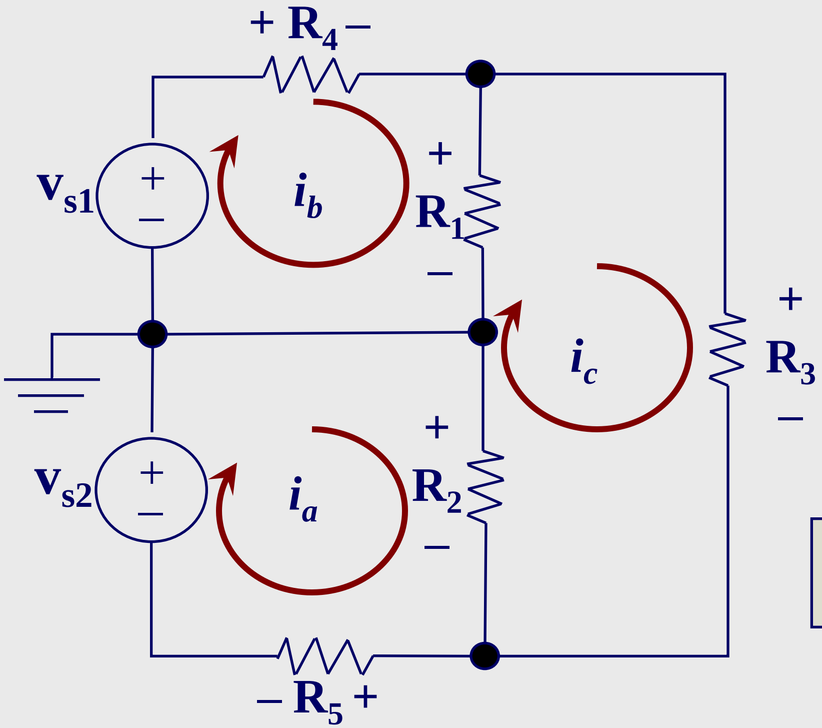
KVL around Mesh c:

$$v_3 - v_2 - v_1 = 0$$

$$i_c R_3 - (i_a - i_c)R_2 - (i_b - i_c)R_1 = 0$$

Mesh Current Method

- **Example8**: find the voltages across the loads
  - $\mathbf{V_{s1}} = \mathbf{V_{s2}} = 110V$, $\mathbf{R_1} = 15\Omega$, $\mathbf{R_2} = 40\Omega$, $\mathbf{R_3} = 16\Omega$, $\mathbf{R_4} = \mathbf{R_5} = 1.3\Omega$



4. Express voltages in terms of currents

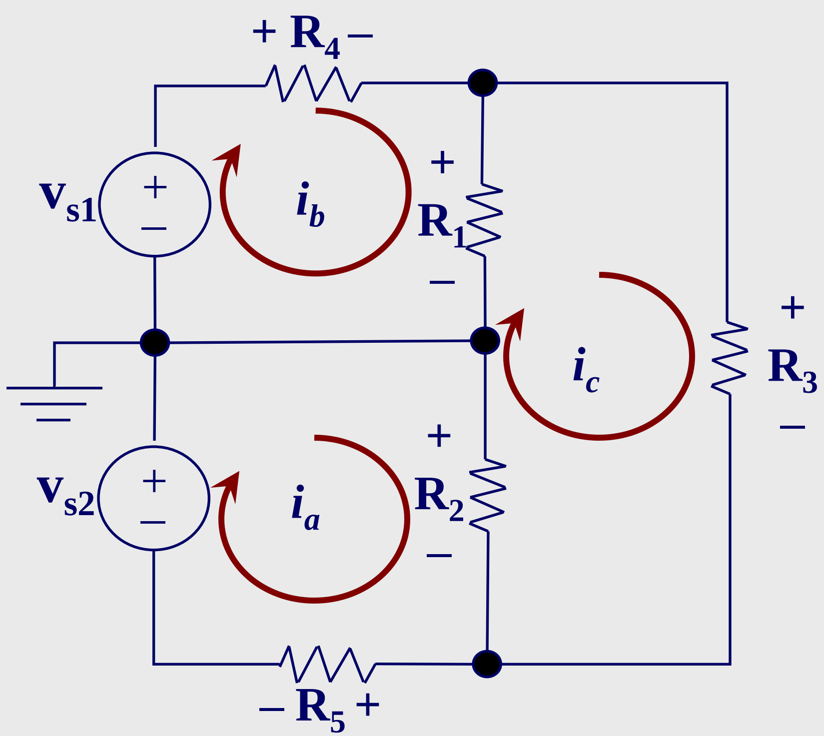
$$i_a(R_2+R_5)-i_cR_2=v_{s2}$$

$$i_b(R_1+R_4)-i_cR_1=v_{s1}$$

$$-i_aR_2-i_bR_1+i_c(R_1+R_2+R_3)=0$$

Mesh Current Method

- **Example8**: find the voltages across the loads
  - $\mathbf{V_{s1}} = \mathbf{V_{s2}} = 110V$, $\mathbf{R_1} = 15\Omega$, $\mathbf{R_2} = 40\Omega$, $\mathbf{R_3} = 16\Omega$, $\mathbf{R_4} = \mathbf{R_5} = 1.3\Omega$



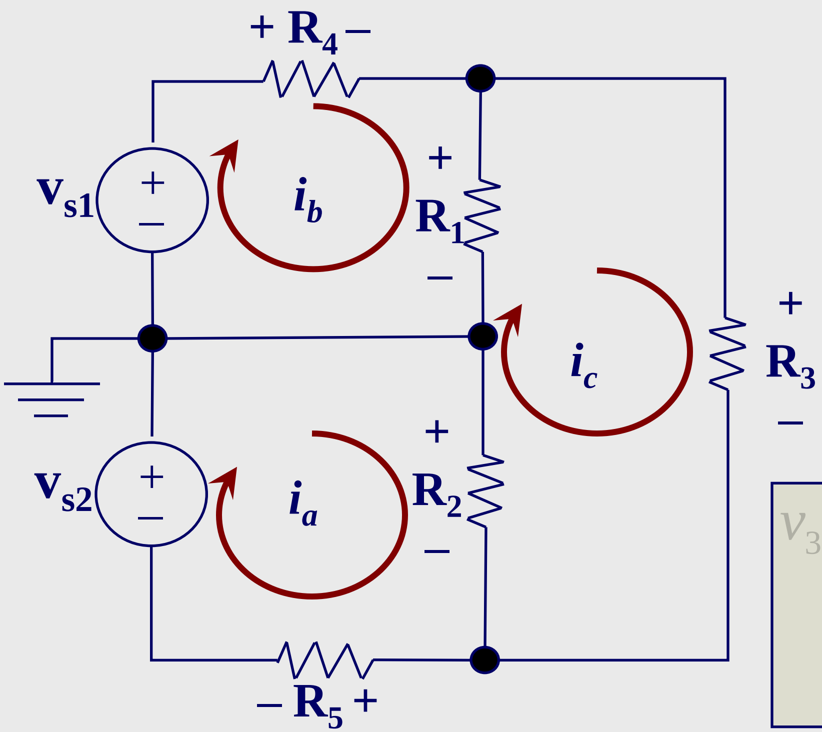
5. Solve the **n – m** equations

$$41.3i_a - 40i_c = 110$$
$$16.3i_b - 15i_c = 110$$
$$-40i_a - 15i_b + 71i_c = 0$$

$$i_a = 13.57A$$
$$i_b = 17.11A$$
$$i_c = 11.26A$$

Mesh Current Method

- **Example8**: find the voltages across the loads
  - $\mathbf{V_{s1}} = \mathbf{V_{s2}} = 110V$, $\mathbf{R_1} = 15\Omega$, $\mathbf{R_2} = 40\Omega$, $\mathbf{R_3} = 16\Omega$, $\mathbf{R_4} = \mathbf{R_5} = 1.3\Omega$



5. Solve the **n – m** equations

$$v_1 = (i_b - i_c)R_1 = (17.11 - 11.26)15 = 87.75V$$

$$v_2 = -(i_a - i_c)R_2 = -(13.57 - 11.26)40 = -92.40V$$

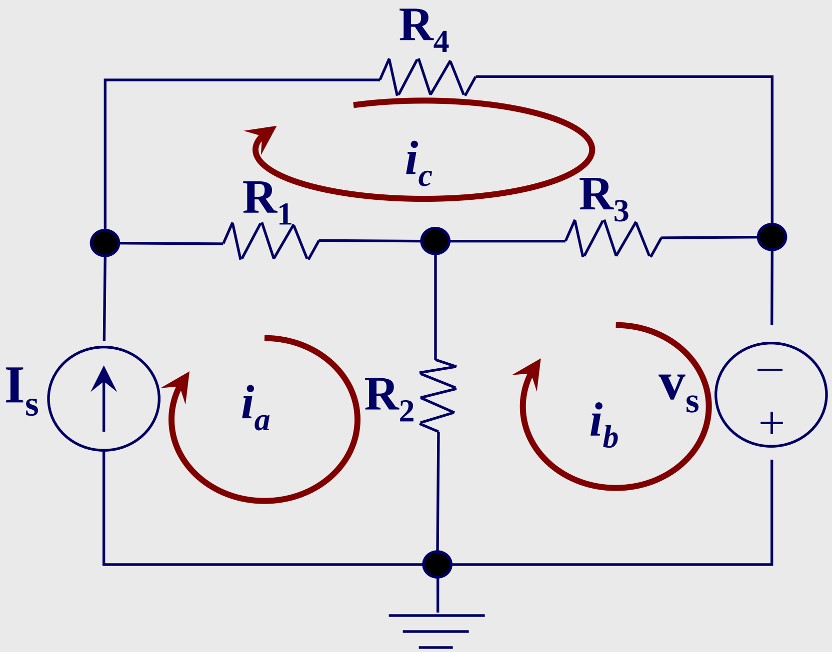
$$v_3 = i_c R_3 = (11.26)16 = 180.16V$$

$$v_4 = i_b R_4 = (17.11)1.3 = 22.24V$$

$$v_5 = -i_a R_5 = -(13.57)1.3 = -17.64V$$

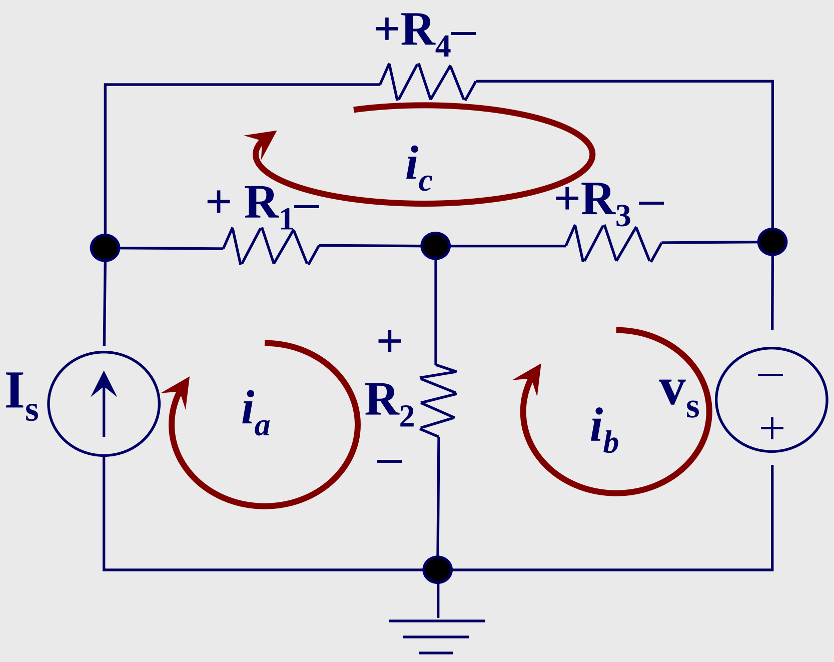
Mesh Current Method

- **Example9**: find the mesh currents
  - $\mathbf{V_s} = 6V$, $I_s = 0.5A$, $\mathbf{R_1} = 3\Omega$, $\mathbf{R_2} = 8\Omega$, $\mathbf{R_3} = 6\Omega$, $\mathbf{R_4} = 4\Omega$



Mesh Current Method

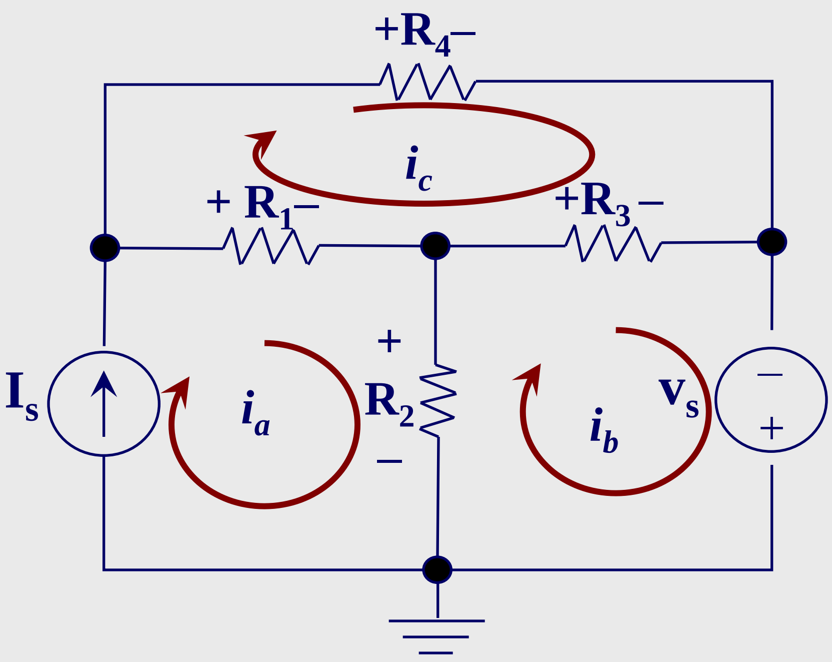
- **Example9**: find the mesh currents
  - $\mathbf{V_s}$ = 6V, I_s = 0.5A, $\mathbf{R_1}$ = 3Ω, $\mathbf{R_2}$ = 8Ω, $\mathbf{R_3}$ = 6Ω, $\mathbf{R_4}$ = 4Ω



1. Mesh current directions given
2. Voltage polarities chosen and labeled
3. Identify n – m (3) mesh currents
   - i_a is **dependent ($i_a = i_s$)**
   - i_a is **independent**
   - i_c is **independent**
4. Apply KVL around meshes **b** and **c**

Mesh Current Method

- **Example9**: find the mesh currents
  - $\mathbf{V_s} = 6V$, $I_s = 0.5A$, $\mathbf{R_1} = 3\Omega$, $\mathbf{R_2} = 8\Omega$, $\mathbf{R_3} = 6\Omega$, $\mathbf{R_4} = 4\Omega$



4. Apply KVL at nodes **b** and **c**

KVL around Mesh b :

$$-v_s - v_2 + v_3 = 0$$

$$v_s + (i_s - i_b)R_2 - (i_b - i_c)R_3 = 0$$

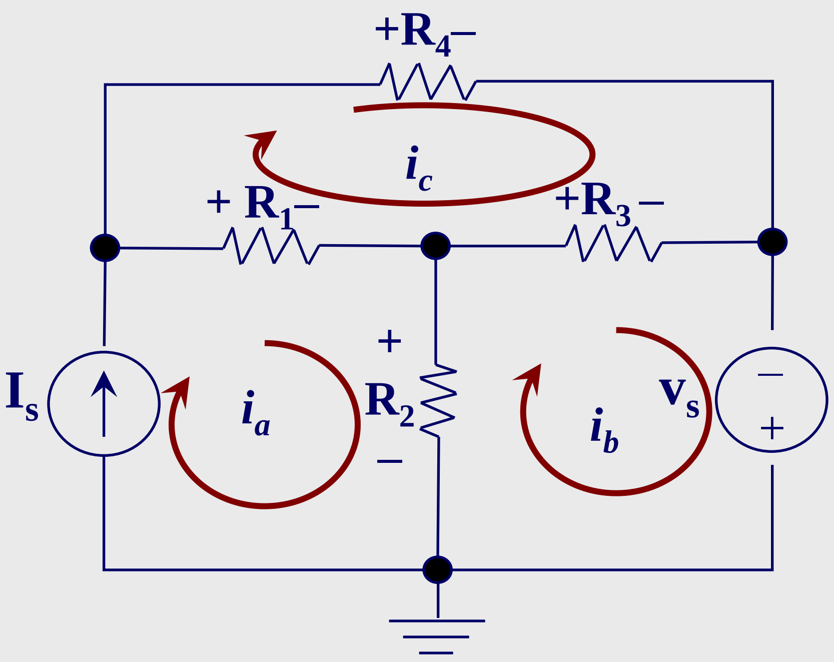
KVL around Mesh c :

$$v_4 - v_3 - v_1 = 0$$

$$i_c R_4 - (i_b - i_c)R_3 - (i_s - i_c)R_1 = 0$$

Mesh Current Method

- **Example9**: find the mesh currents
  - $\mathbf{V_s} = 6\text{V}$, $\text{I}_s = 0.5\text{A}$, $\mathbf{R_1} = 3\Omega$, $\mathbf{R_2} = 8\Omega$, $\mathbf{R_3} = 6\Omega$, $\mathbf{R_4} = 4\Omega$



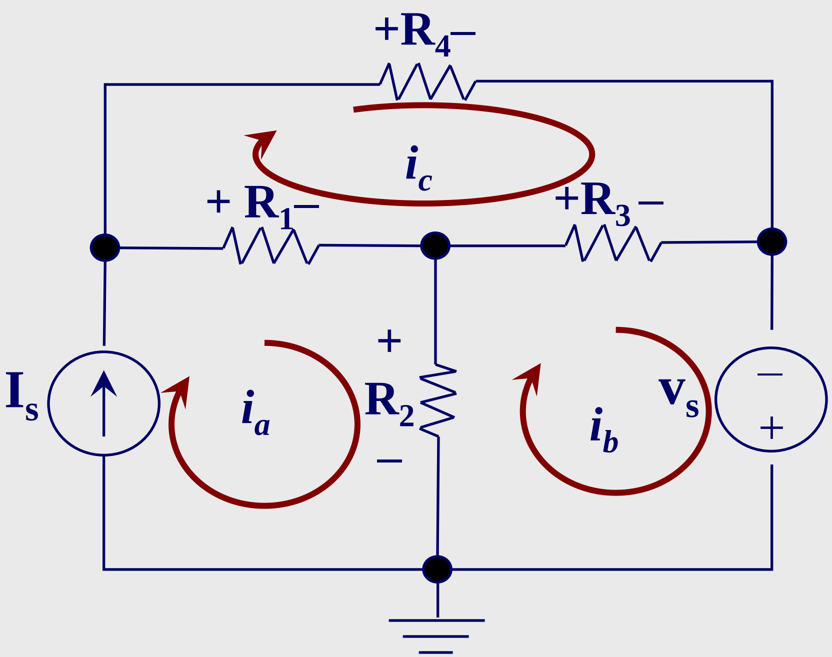
4. Express voltages in terms of currents

$$i_b(R_2 + R_3) - i_c R_3 = v_s + i_s R_2$$

$$-i_b R_3 + i_c(R_1 + R_3 + R_4) = i_s R_1$$

Mesh Current Method

- **Example9**: find the mesh currents
  - $\mathbf{V_s} = 6V$, $I_s = 0.5A$, $\mathbf{R_1} = 3\Omega$, $\mathbf{R_2} = 8\Omega$, $\mathbf{R_3} = 6\Omega$, $\mathbf{R_4} = 4\Omega$



5. Solve the **n – m** equations

$$14i_b - 6i_c = 10$$
$$-6i_b + 13i_c = 1.5$$

$$i_a = 0.50A$$
$$i_b = 0.95A$$
$$i_c = 0.55A$$

Equation Solver Methods

Calculator

Matrix

Cramer’s Rule

Brute Force (Substitution)

TI-89 Equation Solver

1. Press **F1**
   - Select option number **8** (**Clear Home**)
2. Press **APPS**
   - Select option number **1** (**FlashApps**)
   - Select **Simultaneous Eqn Solver**
     - Select **New**…
3. In the new box enter **n** (the number of equations and unknowns)

TI-89 Equation Solver

4. Input the equation coefficients
   - For the set of 3 equations and 3 unknowns from example 8:

$$\begin{aligned} 41.3i_a - 40i_c &= 110 \\ 16.3i_b - 15i_c &= 110 \\ -40i_a - 15i_b + 71i_c &= 0 \end{aligned}$$

     Input the coefficients as follows:

	a1	a2	a3	b1
1	41.3	0	-40	110
2	0	16.3	-15	110
3	-40	-15	71	0

TI-89 Equation Solver

5. Press **F5** (**Solve**)

$$x_1 = 13.57$$
$$x_2 = 17.11$$
$$x_3 = 11.26$$