Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
29 Sept	Mon	8	Network Analysis	3.4 – 3.5		LAB 3	
30 Sep	Tue						
1 Oct	Wed	9	Equivalent Circuits	3.6			
2 Oct	Thu						
3 Oct	Fri		Recitation		HW 4		
4 Oct	Sat						
5 Oct	Sun						
6 Oct	Mon	10	Energy Storage	3.7, 4.1 – 4.2		NO LAB	
7 Oct	Tue					NO LAB	

Dependence

Mosiah 4: 19, 21

19 For behold, are we not all beggars? Do we not all **depend** upon the same Being, even God, for all the substance which we have, for both food and raiment, and for gold, and for silver, and for all the riches which we have of every kind?

• • •

21 And now, if God, who has created you, on whom you are **dependent** for your lives and for all that ye have and are, doth grant unto you whatsoever ye ask that is right, in faith, believing that ye shall receive, O then, how ye ought to impart of the substance that ye have one to another.

Lecture 8 – Network Analysis

Controlled Sources

Superposition

Source Transformations

Network Analysis

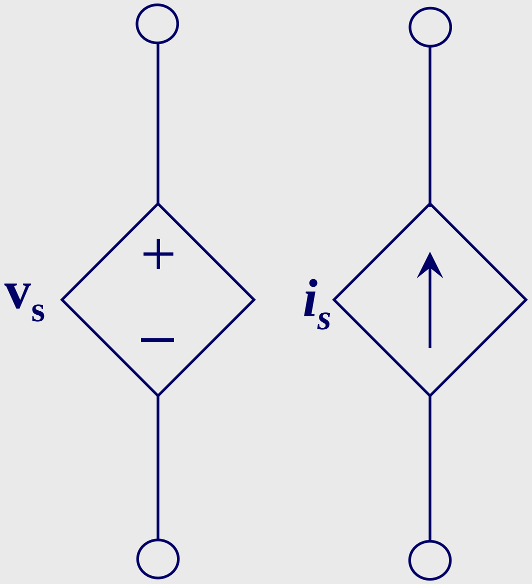
- Network Analysis Methods:
  - Node voltage method
  - Mesh current method
  - Superposition
  - Equivalent circuits
    - Source transformation
    - Thévenin equivalent
    - Norton equivalent

Controlled (Dependent) Sources

Node and Mesh Analysis

Dependent (Controlled) Sources

- Diamond shaped source indicates dependent source
- Dependent sources are an important part of amplifiers



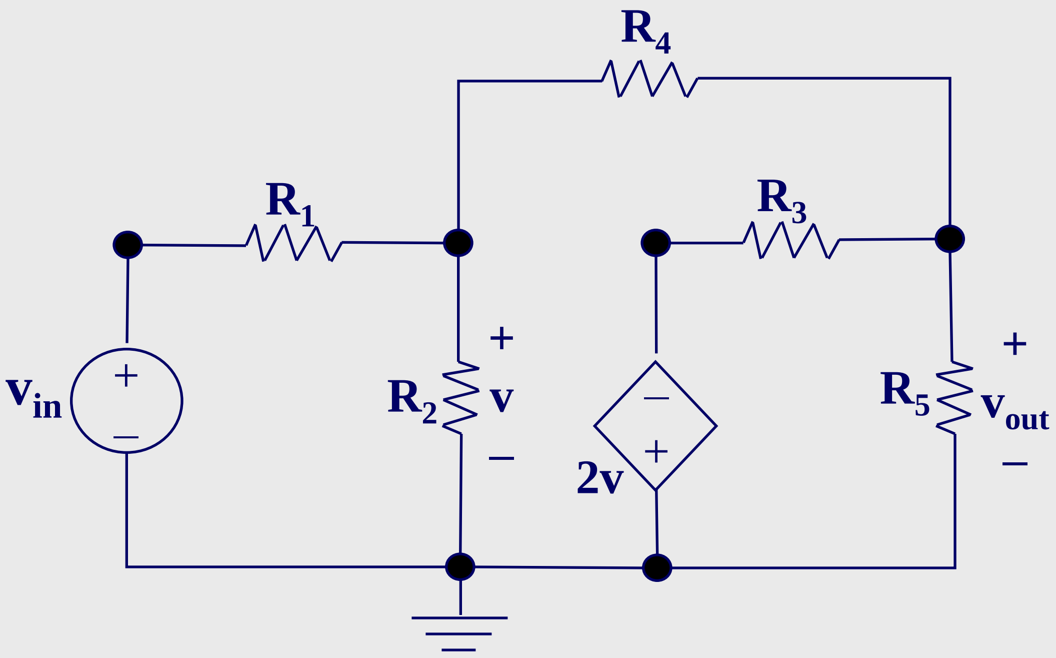
Source Type	Relationship
Voltage controlled voltage source (**VCVS**)	$\mathbf{v}_s = a\mathbf{v}_x$
Current controlled voltage source (**CCVS**)	$\mathbf{v}_s = a\boldsymbol{i}_x$
Voltage controlled current source (**VCCS**)	$\boldsymbol{i}_s = a\mathbf{v}_x$
Current controlled current source (**CCCS**)	$\boldsymbol{i}_s = a\boldsymbol{i}_x$

Controlled Sources

- Network analysis with controlled sources:
  - Initially treat controlled sources as ideal sources
  - In addition to equations obtained by node/mesh analysis there will be the **constraint equation** (the controlled source equation)
  - Substitute constraint equation into node/mesh equations

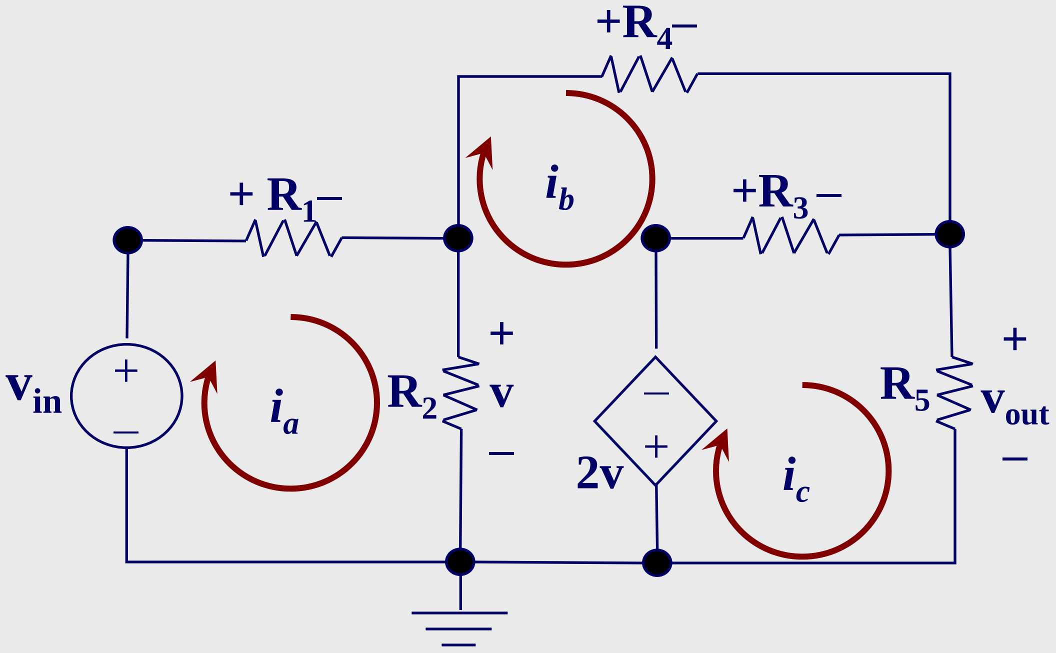
Controlled Sources

- **Example1**: find the **gain** ($A_v = v_{out}/v_{in}$)
  - $R_1 = 1\Omega$, $R_2 = 0.5\Omega$, $R_3 = 0.25\Omega$, $R_4 = 0.25\Omega$, $R_5 = 0.25\Omega$



Controlled Sources

- **Example1**: find the **gain** ($\mathbf{A_v = v_{out}/v_{in}}$)
  - $\mathbf{R_1} = 1\Omega$, $\mathbf{R_2} = 0.5\Omega$, $\mathbf{R_3} = 0.25\Omega$, $\mathbf{R_4} = 0.25\Omega$, $\mathbf{R_5} = 0.25\Omega$

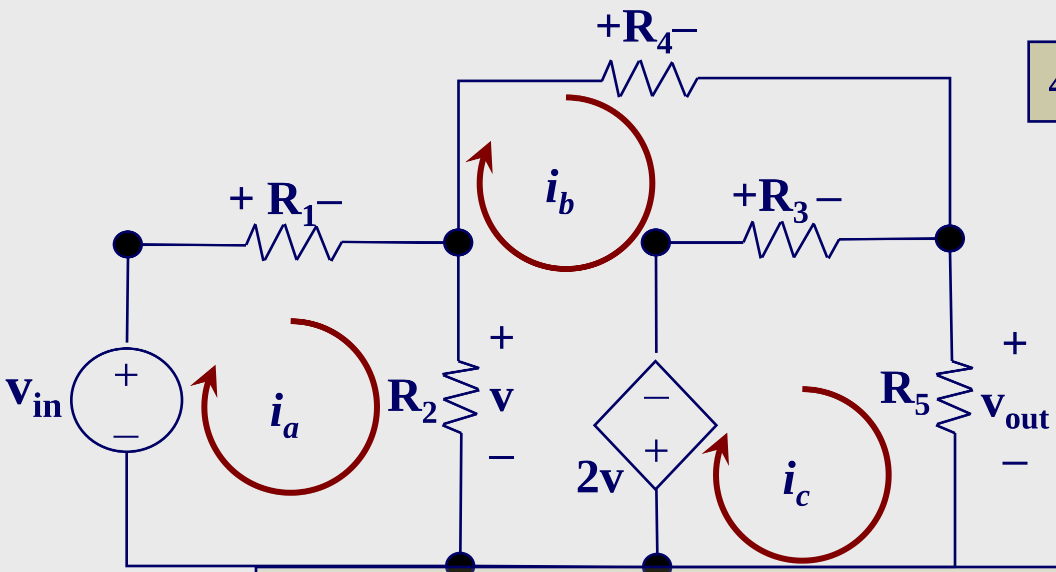


Choose mesh analysis – simpler than node analysis

1. Mesh current directions chosen
2. Voltage polarities chosen and labeled
3. Identify n – m (3) mesh currents
   - ➢ $\boldsymbol{i_a}$ is **independent**
   - ➢ $\boldsymbol{i_a}$ is **independent**
   - ➢ $\boldsymbol{i_c}$ is **independent**
4. Apply KVL around meshes **a**, **b,** and **c**

Controlled Sources

- **Example1**: find the **gain** ($A_v = v_{out}/v_{in}$)
  - $R_1 = 1\Omega$, $R_2 = 0.5\Omega$, $R_3 = 0.25\Omega$, $R_4 = 0.25\Omega$, $R_5 = 0.25\Omega$



4. Apply KVL at nodes **a, b**, and **c**

KVL around Mesh a :

$$-v_{in} + v_1 + v_2 = 0$$

$$v_{in} - i_a R_1 - (i_a - i_b)R_2 = 0$$

$$i_a(R_1 + R_2) - i_b R_2 = v_{in}$$

KVL around Mesh b :

$$-v_2 + v_4 - v_3 - 2v = 0$$

$$-(i_a - i_b)R_2 + i_b R_4 - (i_c - i_b)R_3 = 2(i_a - i_b)R_2$$

$$-3i_a R_2 + i_b(3R_2 + R_3 + R_4) - i_c R_3 = 0$$

KVL around Mesh c :

$$2v + v_3 + v_5 = 0$$

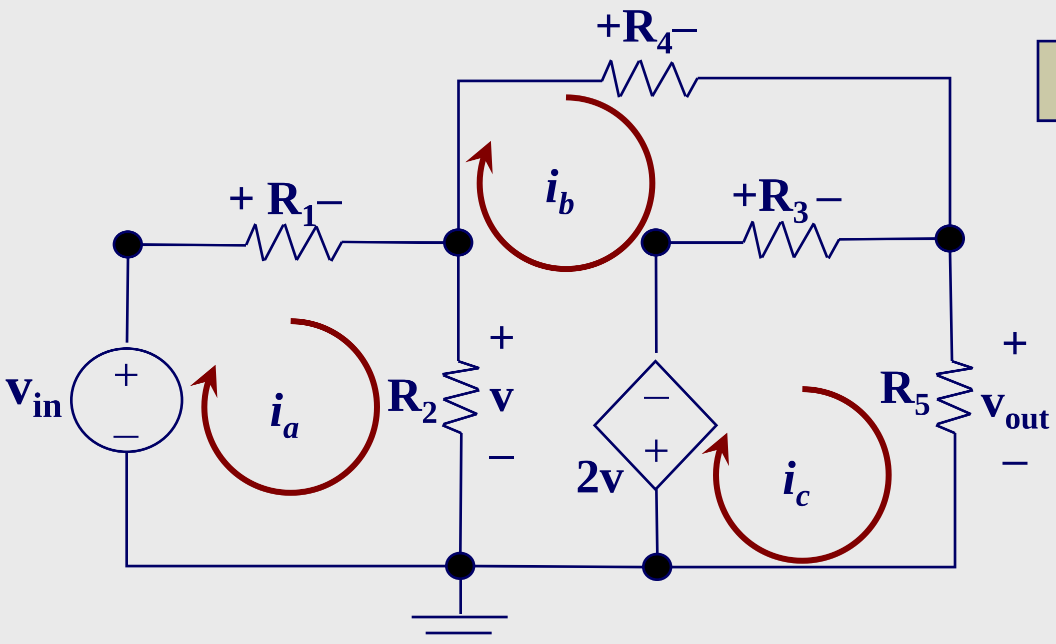
$$2v_2 + (i_c - i_b)R_3 + i_c R_5 = 0$$

$$-i_b R_3 + i_c(R_3 + R_5) = -2(i_a - i_b)R_2$$

$$2i_a R_2 - i_b(2R_2 + R_3) + i_c(R_3 + R_5) = 0$$

Controlled Sources

- **<u>Example1</u>**: find the **gain** ($A_v = v_{out}/v_{in}$)
  - $R_1 = 1\Omega$, $R_2 = 0.5\Omega$, $R_3 = 0.25\Omega$, $R_4 = 0.25\Omega$, $R_5 = 0.25\Omega$



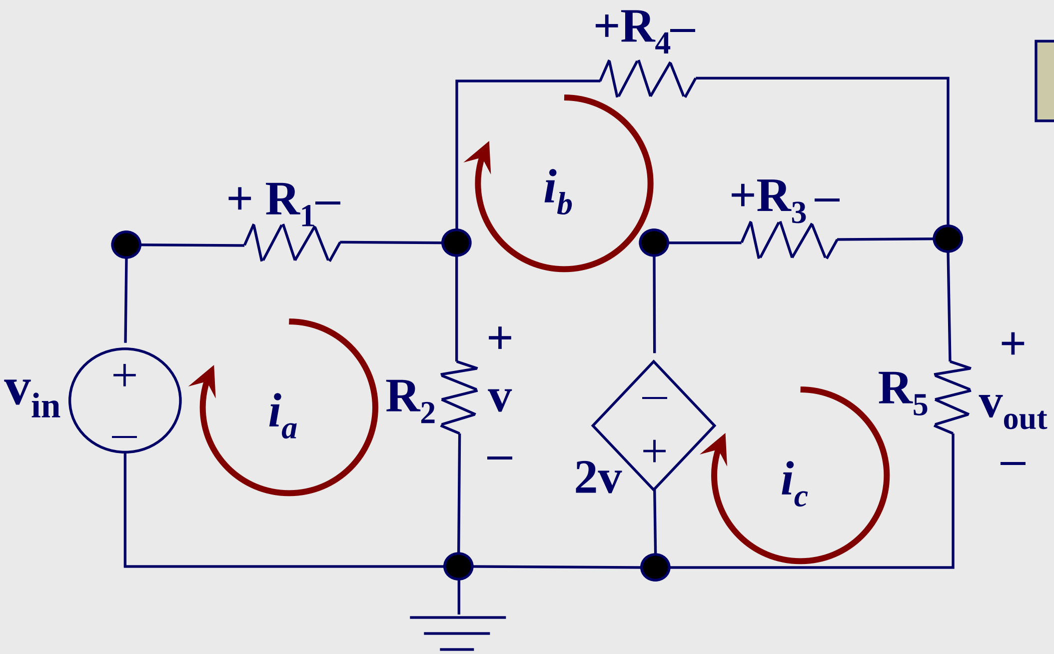
5. Solve the **n – m** equations

$$1.5i_a - 0.5i_b = v_{in}$$
$$-1.5i_a + 2i_b - 0.25i_c = 0$$
$$i_a - 1.25i_b + 0.5i_c = 0$$

$$i_a = 0.88v_{in}A$$
$$i_b = 0.64v_{in}A$$
$$i_c = -0.16v_{in}A$$

Controlled Sources

- **<u>Example1</u>**: find the **gain** ($\mathbf{A_v = v_{out}/v_{in}}$)
  - $\mathbf{R_1} = 1\Omega$, $\mathbf{R_2} = 0.5\Omega$, $\mathbf{R_3} = 0.25\Omega$, $\mathbf{R_4} = 0.25\Omega$, $\mathbf{R_5} = 0.25\Omega$



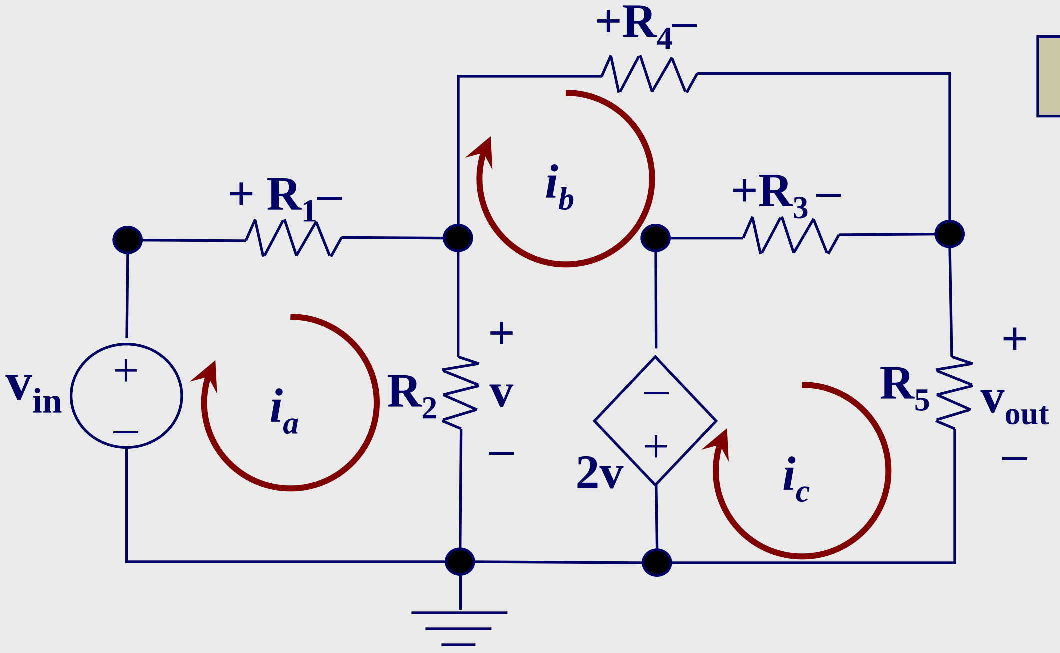
5. Solve the **n – m** equations (Matrices)

$$\begin{bmatrix} 1.5 & -0.5 & 0 \\ -1.5 & 2 & -0.25 \\ 1 & -1.25 & 0.5 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} v_{in} \\ 0 \\ 0 \end{bmatrix}$$

$$AX = B$$
$$A^{-1}AX = A^{-1}B$$
$$X = A^{-1}B$$

Controlled Sources

- **Example1**: find the **gain** ($A_v = v_{out}/v_{in}$)
  - R_1 = 1Ω, R_2 = 0.5Ω, R_3 = 0.25Ω, R_4 = 0.25Ω , R_5 = 0.25Ω



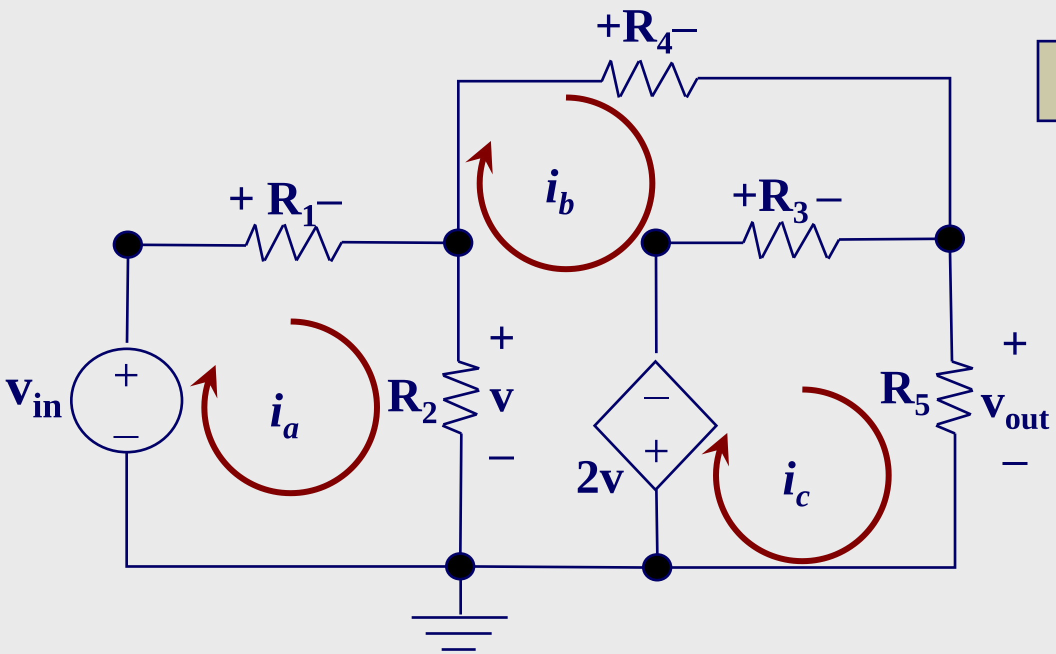
5. Solve the **n – m** equations (Matrices)

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 0.88 & 0.32 & 0.16 \\ 0.64 & 0.96 & 0.48 \\ -0.16 & 1.76 & 2.88 \end{bmatrix} \begin{bmatrix} v_{in} \\ 0 \\ 0 \end{bmatrix}$$

$$i_a = 0.88 v_{in} A$$
$$i_b = 0.64 v_{in} A$$
$$i_c = -0.16 v_{in} A$$

Controlled Sources

- **<u>Example1</u>**: find the **gain** ($A_v = v_{out}/v_{in}$)
  - $R_1 = 1\Omega$, $R_2 = 0.5\Omega$, $R_3 = 0.25\Omega$, $R_4 = 0.25\Omega$, $R_5 = 0.25\Omega$

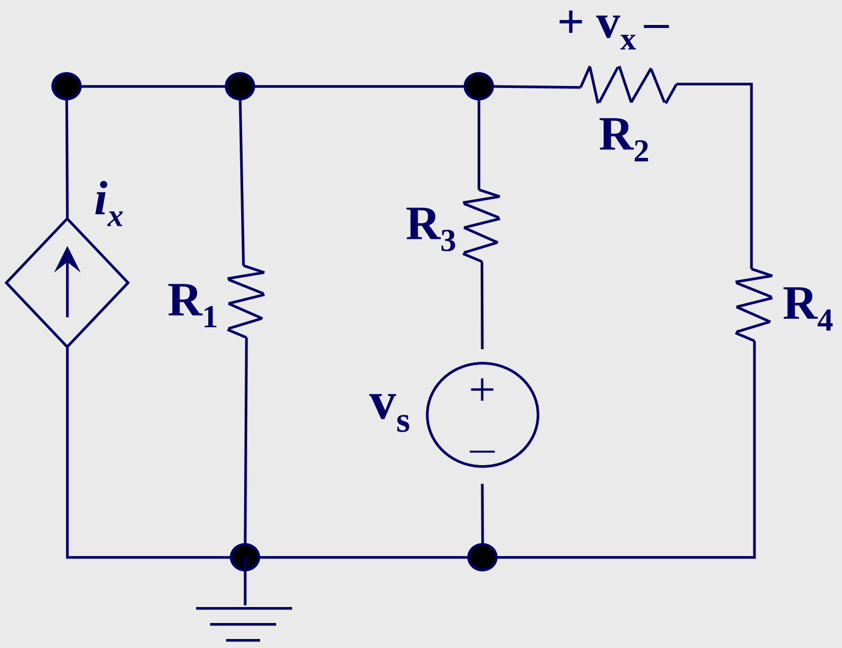


➢ Find the gain

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_5 i_c}{v_{in}} = \frac{0.25(-0.16 v_{in})}{v_{in}} = -0.04$$

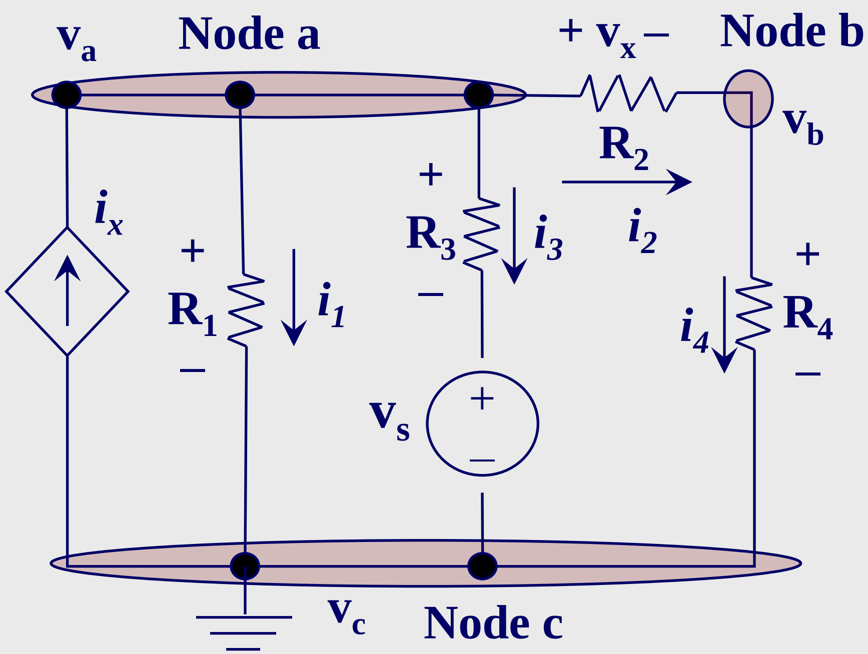
Controlled Sources

- **<u>Example2</u>**: Find $\mathbf{v_1}$
  - $\mathbf{v_s} = 15\text{V}$, $\mathbf{R_1} = 8\Omega$, $\mathbf{R_2} = 6\Omega$, $\mathbf{R_3} = 6\Omega$, $\mathbf{R_4} = 6\Omega$, $\boldsymbol{i_x} = \mathbf{v_x}/3$



Controlled Sources

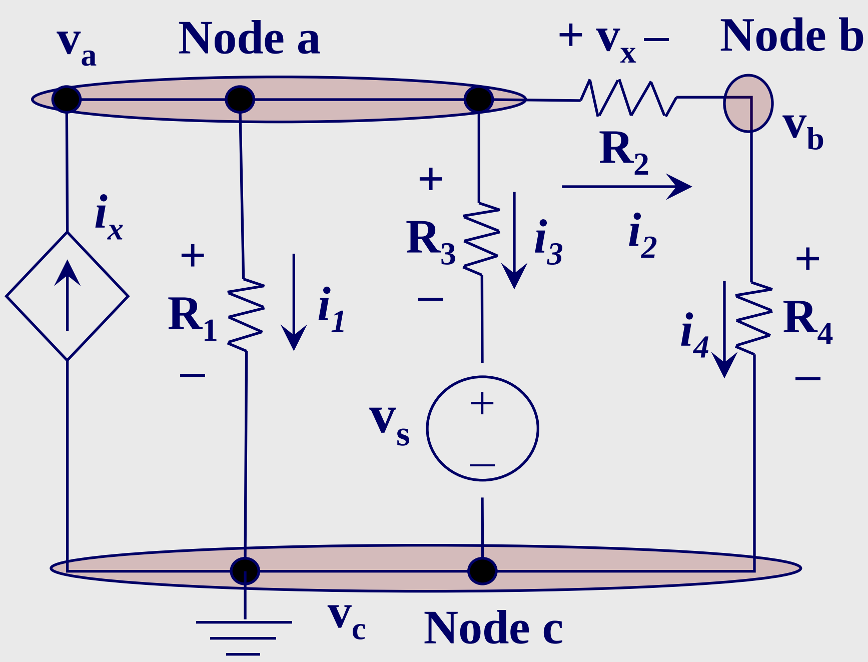
- **<u>Example2</u>**: Find $\mathbf{v_1}$
  - $\mathbf{v_s}$ = 15V, $\mathbf{R_1}$ = 8Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 6Ω, $\mathbf{R_4}$ = 6Ω, $\boldsymbol{i_x} = \mathbf{v_x}/3$



1. Label currents and voltages (polarities "arbitrarily" chosen)
2. Choose **Node c** ($\mathbf{v_c}$) as the reference node ($\mathbf{v_c}$ = 0)
3. Define remaining n – 1 (2) voltages
   - $\mathbf{v_a}$ is **independent**
   - $\mathbf{v_b}$ is **independent**
4. Apply KCL at nodes **a** and **b**

Controlled Sources

- **<u>Example2</u>**: Find $\mathbf{v_1}$
  - $\mathbf{v_s}$ = 15V, $\mathbf{R_1}$ = 8Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 6Ω, $\mathbf{R_4}$ = 6Ω, $\boldsymbol{i_x} = \mathbf{v_x}/3$



4. Apply KCL at nodes **a** and **b**

KCL at Node a :

$$i_x - i_1 - i_2 - i_3 = 0$$

$$\frac{v_x}{3} - \frac{v_a}{R_1} - \frac{v_{ab}}{R_2} - \frac{v_{as}}{R_3} = 0$$

$$v_a\left(\frac{1}{3} - \frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3}\right) + v_b\left(\frac{1}{R_2} - \frac{1}{3}\right) = -\frac{v_s}{R_3}$$

KCL at Node b :

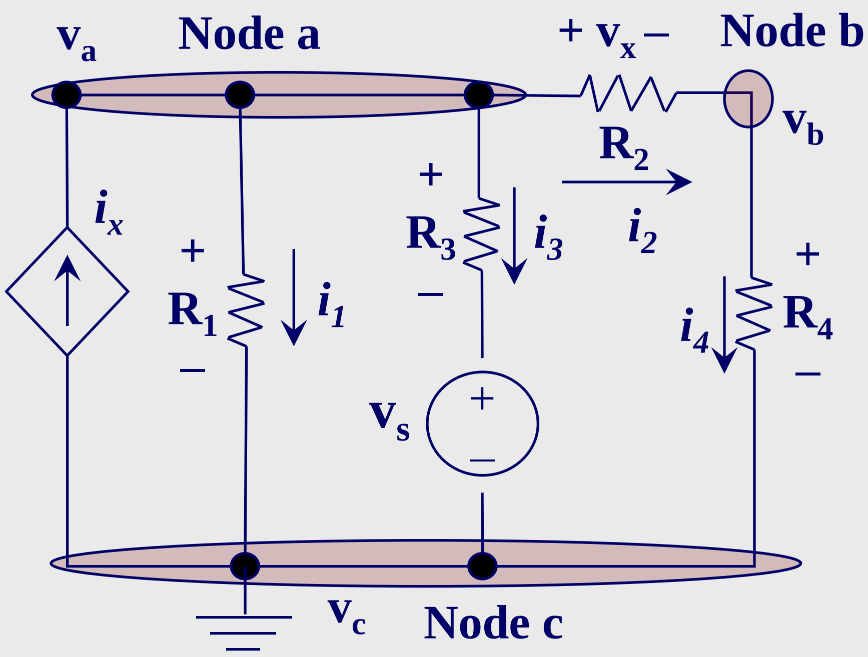
$$i_2 - i_4 = 0$$

$$\frac{v_{ab}}{R_2} - \frac{v_b}{R_4} = 0$$

$$v_a\left(\frac{1}{R_2}\right) - v_b\left(\frac{1}{R_2} + \frac{1}{R_4}\right) = 0$$

Controlled Sources

- **<u>Example2</u>**: Find $\mathbf{v_1}$
  - $\mathbf{v_s}$ = 15V, $\mathbf{R_1}$ = 8Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 6Ω, $\mathbf{R_4}$ = 6Ω, $\boldsymbol{i_x} = \mathbf{v_x}/3$



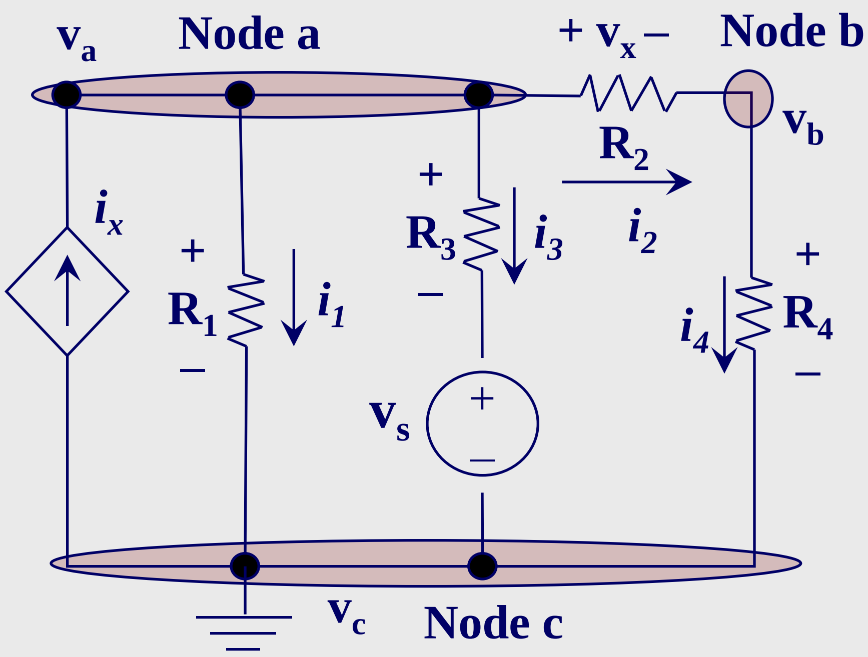
5. Solve the n – 1 – m equations

$$3v_a + 4v_b = 60$$
$$v_a - 2v_b = 0$$

$$v_a = 12V$$
$$v_b = 6V$$

Controlled Sources

- **<u>Example2</u>**: Find $\mathbf{v_1}$
  - $\mathbf{v_s} = 15V$, $\mathbf{R_1} = 8\Omega$, $\mathbf{R_2} = 6\Omega$, $\mathbf{R_3} = 6\Omega$, $\mathbf{R_4} = 6\Omega$, $\boldsymbol{i_x} = \mathbf{v_x}/3$



5. Solve the n – 1 – m equations

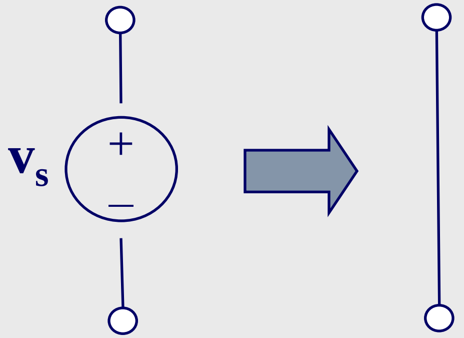
$$v_1 = v_a$$
$$= 12V$$

The Principle of Superposition

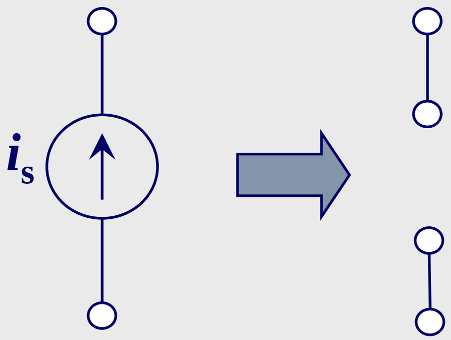
Superposition

<u>Superposition</u>: in a linear circuit containing N sources, each branch voltage and current is the sum of N voltages and currents

- Each of which can be found by setting **all but one** source equal to zero and solving the circuit containing that single source



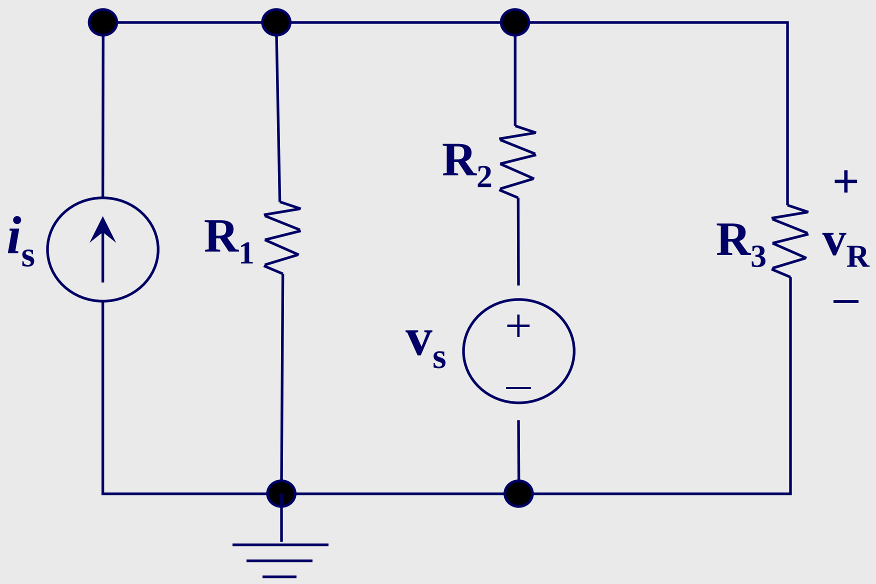
When setting voltage sources to zero they become **short circuits** (v = 0)



When setting current sources to zero they become **open circuits** (i = 0)

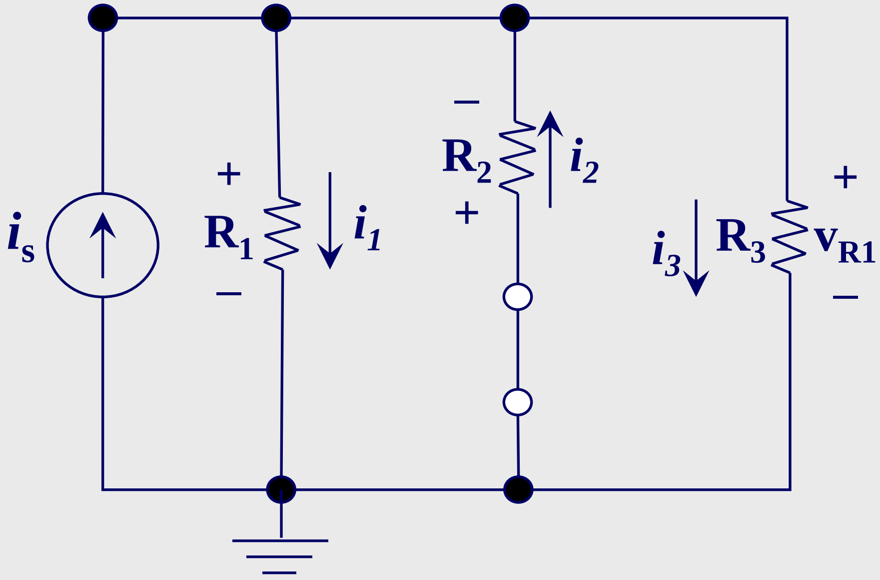
Superposition

- **<u>Example3</u>**: use superposition to find $\mathbf{v_R}$
  - i_s = 12A, $\mathbf{v_s}$ = 12V, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 0.3Ω, $\mathbf{R_3}$ = 0.23Ω



Superposition

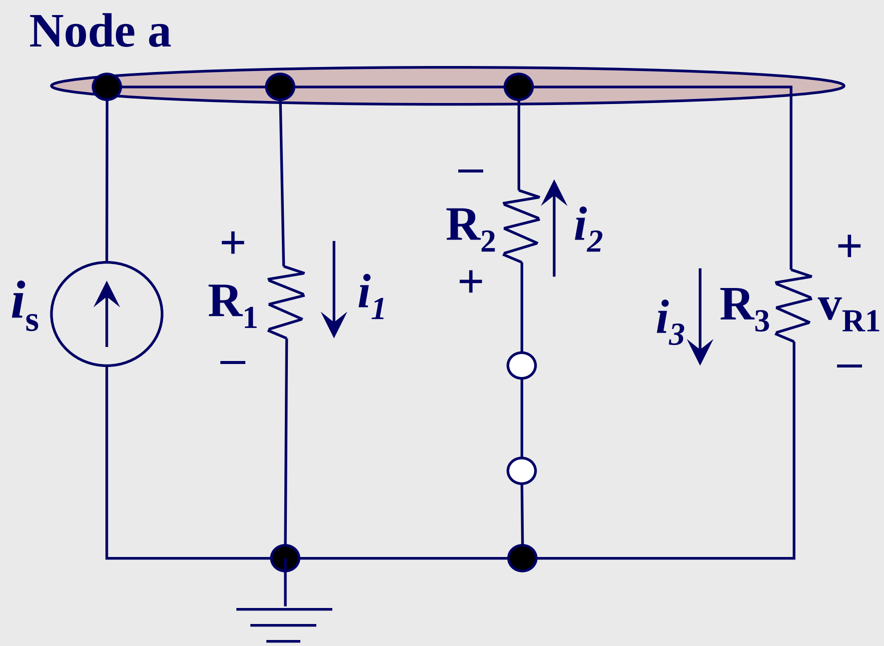
- **Example3**: use superposition to find $\mathbf{v_R}$
  - i_s = 12A, $\mathbf{v_s}$ = 12V, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 0.3Ω, $\mathbf{R_3}$ = 0.23Ω



1. Remove all sources except i_s
   - Source $\mathbf{v_s}$ is replaced with short circuit

Superposition

- **Example3**: use superposition to find $\mathbf{v_R}$
  - $i_s = 12\text{A}$, $\mathbf{v_s} = 12\text{V}$, $\mathbf{R_1} = 1\Omega$, $\mathbf{R_2} = 0.3\Omega$, $\mathbf{R_3} = 0.23\Omega$



KCL at Node a :

$$i_s - i_1 + i_2 - i_3 = 0$$

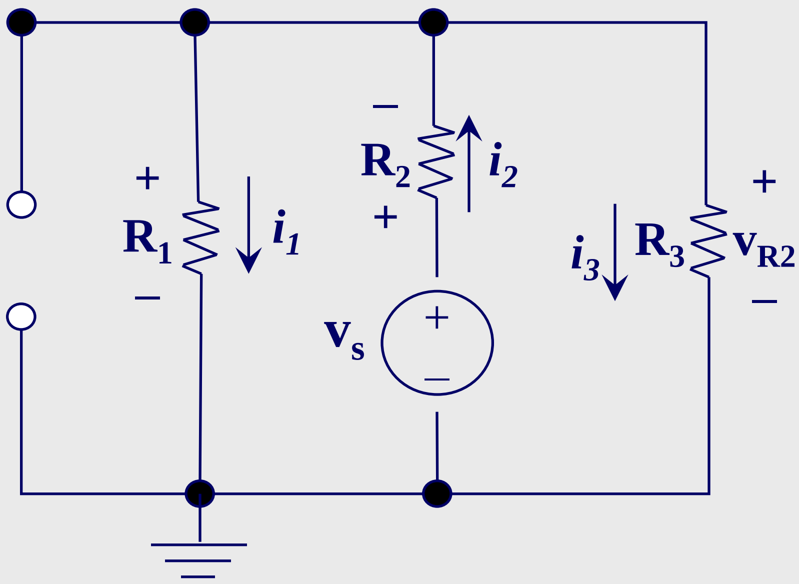
$$\frac{v_{R_1}}{R_1} - \frac{(0 - v_{R_1})}{R_2} + \frac{v_{R_1}}{R_3} = i_s$$

$$v_{R_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = i_s$$

$$v_{R_1} = \frac{12}{8.68} = 1.38V$$

Superposition

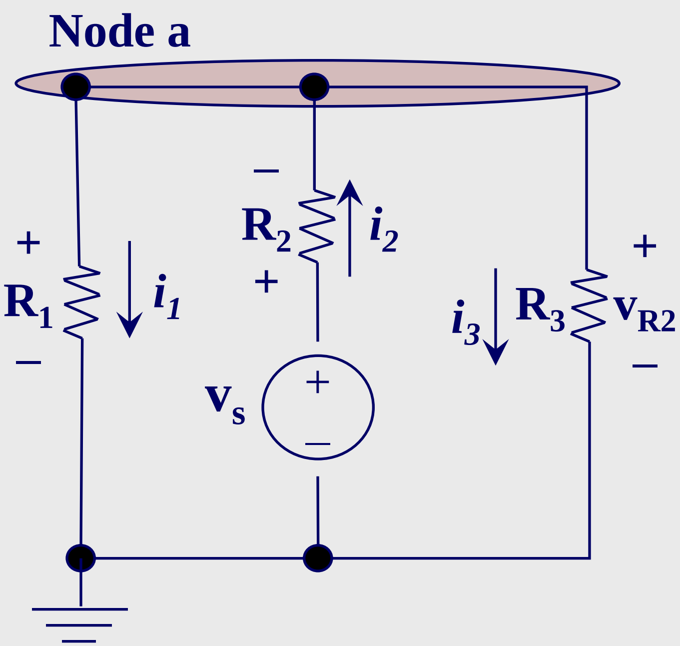
- **Example3**: use superposition to find $\mathbf{v_R}$
  - i_s = 12A, $\mathbf{v_s}$ = 12V, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 0.3Ω, $\mathbf{R_3}$ = 0.23Ω



2. Remove all sources except $\mathbf{v}_s$
   - Source i_s is replaced with open circuit

Superposition

- **<u>Example3</u>**: use superposition to find $\mathbf{v_R}$
  - i_s = 12A, $\mathbf{v_s}$ = 12V, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 0.3Ω, $\mathbf{R_3}$ = 0.23Ω



KCL at Node a :

$$-i_1 + i_2 - i_3 = 0$$

$$\frac{v_{R_2}}{R_1} - \frac{v_{sR_2}}{R_2} + \frac{v_{R_2}}{R_3} = 0$$

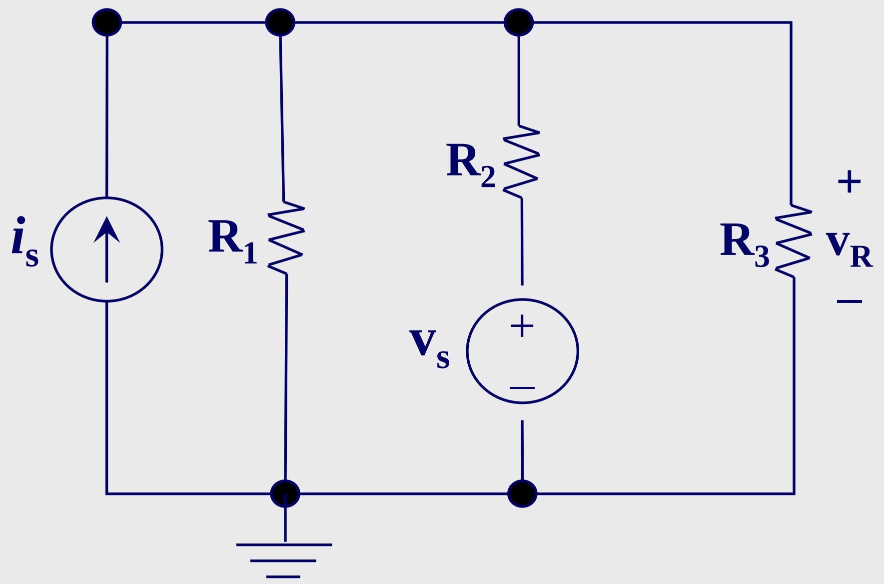
$$v_{R_2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{v_s}{R_2}$$

$$v_{R_2} = \frac{1}{8.68}\left(\frac{12}{0.3}\right)$$

$$= 4.61V$$

Superposition

- **Example3**: use superposition to find $\mathbf{v_R}$
  - i_s = 12A, $\mathbf{v_s}$ = 12V, $\mathbf{R_1}$ = 1Ω, $\mathbf{R_2}$ = 0.3Ω, $\mathbf{R_3}$ = 0.23Ω

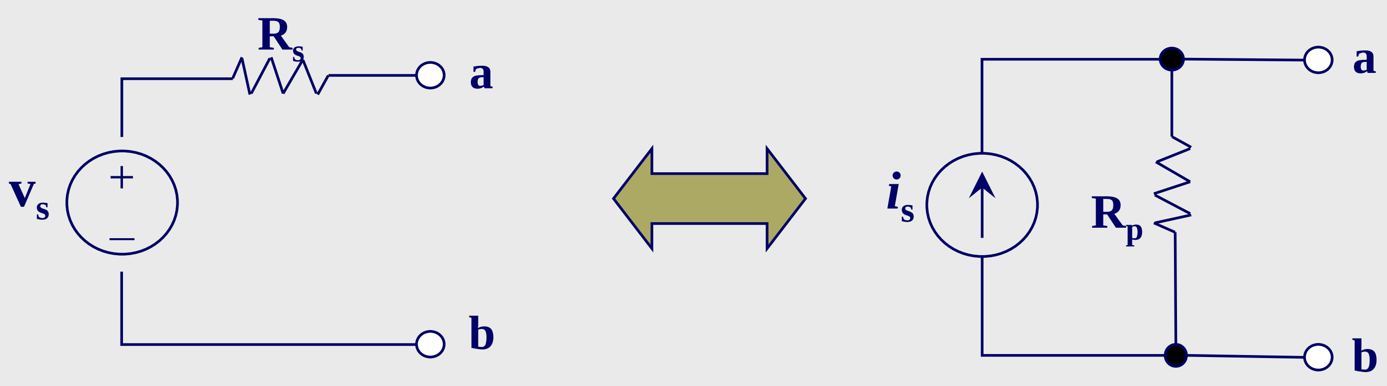


$$v_R = v_{R1} + v_{R2} = 1.38 + 4.61 = 5.99V$$

Source Transformation

Source Transformations

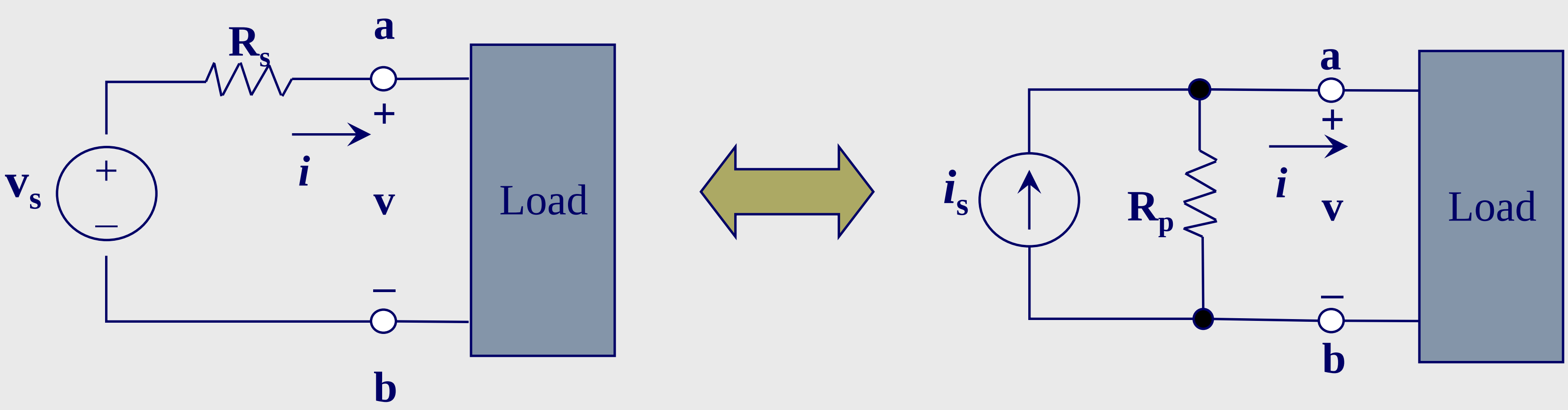
Source transformation: a procedure for transforming one source into another while retaining the terminal characteristics of the original source



Node analysis is easier with **current** sources – **mesh analysis** is easier with **voltage** sources.

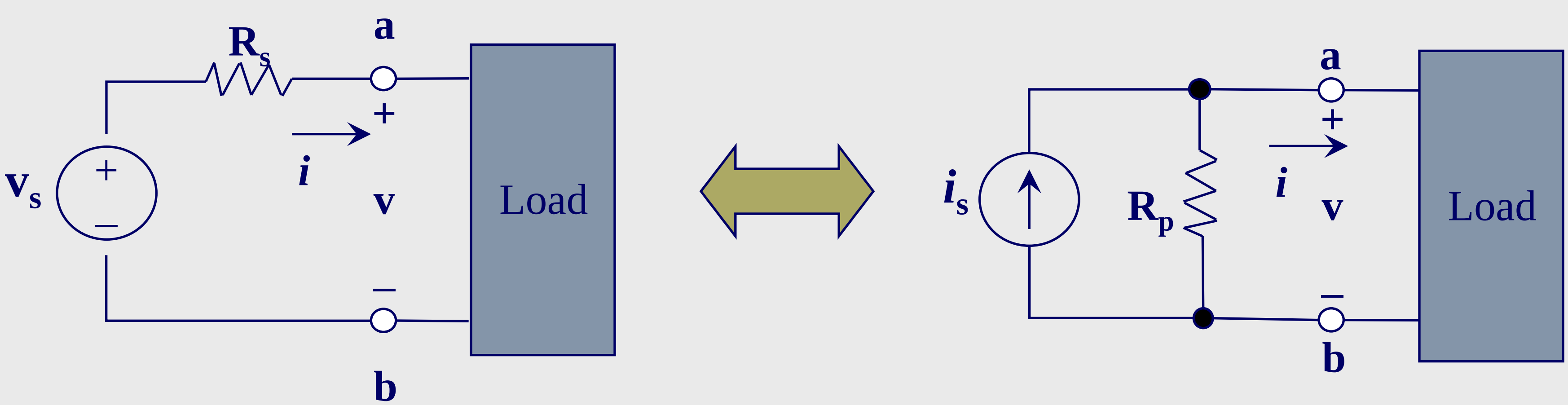
Source Transformations

- How can these circuits be equivalent?



Source Transformations

- How can these circuits be equivalent?



Using KVL:

$$-V_s + v_s + v = 0$$

$$V_s = v + i \cdot R_s$$

$$\frac{V_s}{R_s} = \frac{v}{R_s} + i$$

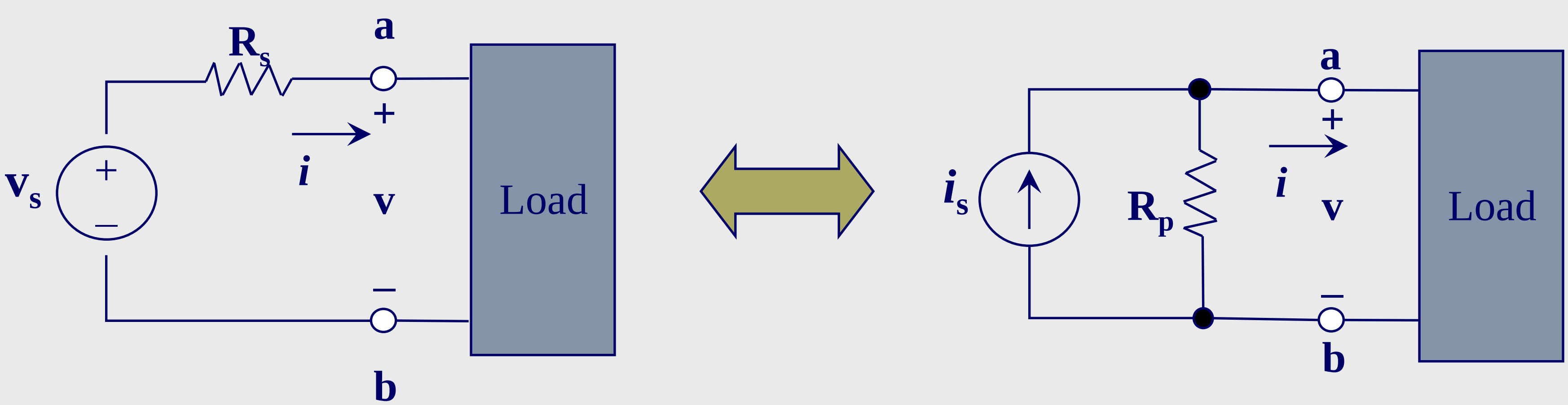
Using KCL:

$$i_s - i_p - i = 0$$

$$i_s = \frac{v}{R_p} + i$$

Source Transformations

- How can these circuits be equivalent?



Using KVL:

$$-V_s + v_s + v = 0$$

$$V_s = v + i \cdot R_s$$

$$\frac{V_s}{R_s} = \frac{v}{R_s} + i$$

$$V_s = R_s i_s$$

$$R_s = R_p$$

Using KCL:

$$i_s - i_p - i = 0$$

$$i_s = \frac{v}{R_p} + i$$

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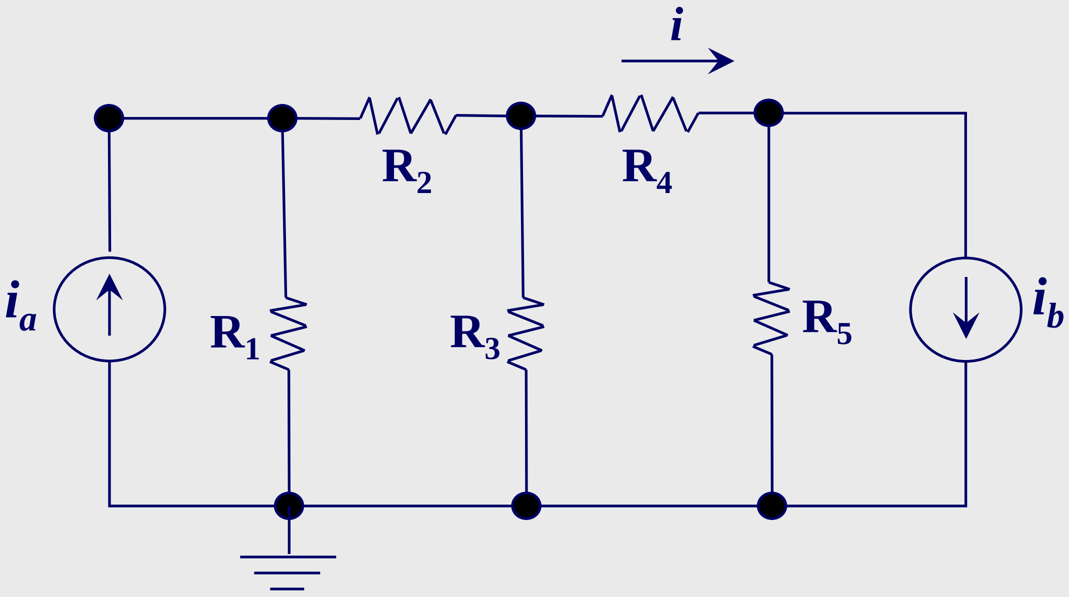
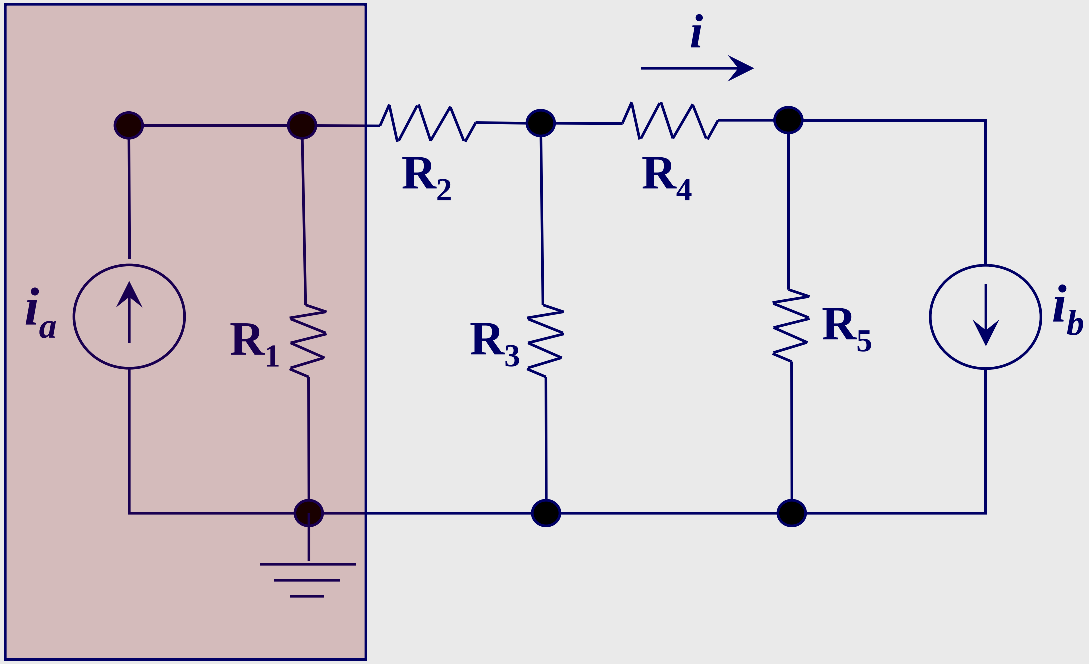
Source Transformations

- **Example4**: find ***i*** using transformations
  - i_a = 5A, i_b = 2A, R_1 = 5Ω, R_2 = 5Ω , R_3 = 10Ω , R_4 = 10Ω , R_5 = 5Ω



Source Transformations

- **Example4**: find ***i*** using transformations
  - i_a = 5A, i_b = 2A, R_1 = 5Ω, R_2 = 5Ω , R_3 = 10Ω , R_4 = 10Ω , R_5 = 5Ω

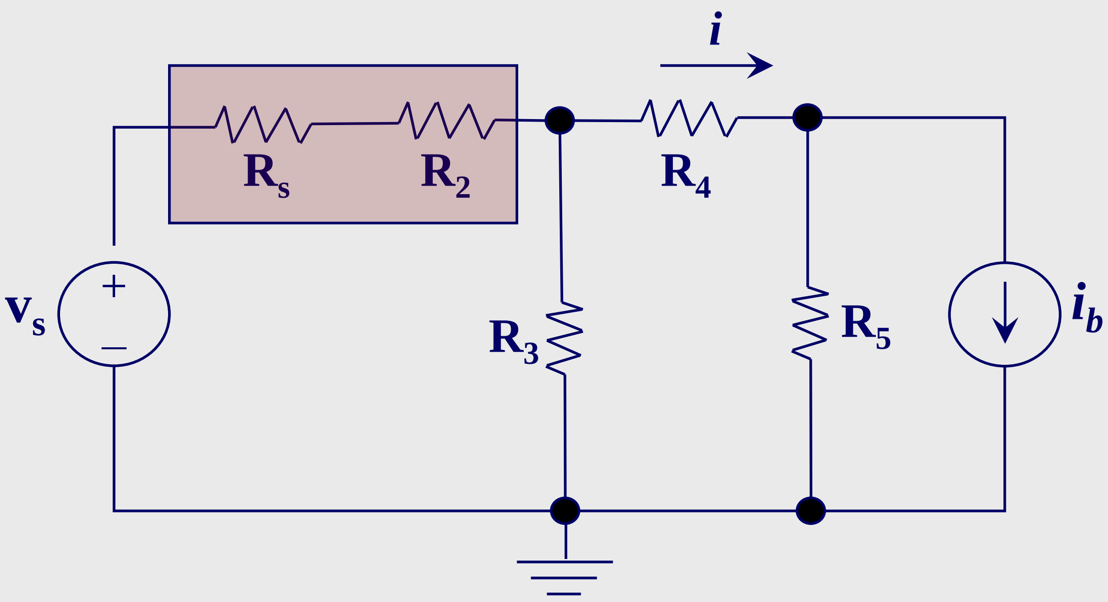


$$v_s = i_s R_p = i_a R_1 = (5)(5) = 25V$$

$$R_s = R_p = 5\Omega$$

Source Transformations

- **Example4**: find ***i*** using transformations
  - i_a = 5A, i_b = 2A, $\mathbf{R_1}$ = 5Ω, $\mathbf{R_2}$ = 5Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 10Ω , $\mathbf{R_5}$ = 5Ω

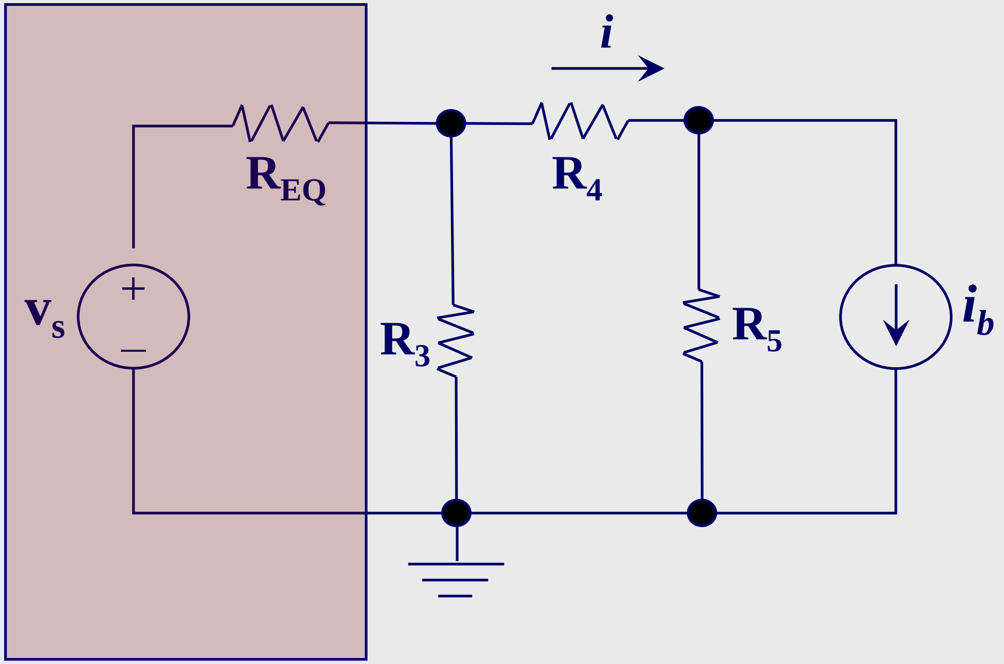


$$v_s = 25V$$
$$R_s = 5\Omega$$

$$R_{EQ} = R_s + R_2$$
$$= 5 + 5$$
$$= 10\Omega$$

Source Transformations

- **Example4**: find ***i*** using transformations
  - i_a = 5A, i_b = 2A, R_1 = 5Ω, R_2 = 5Ω , R_3 = 10Ω , R_4 = 10Ω , R_5 = 5Ω



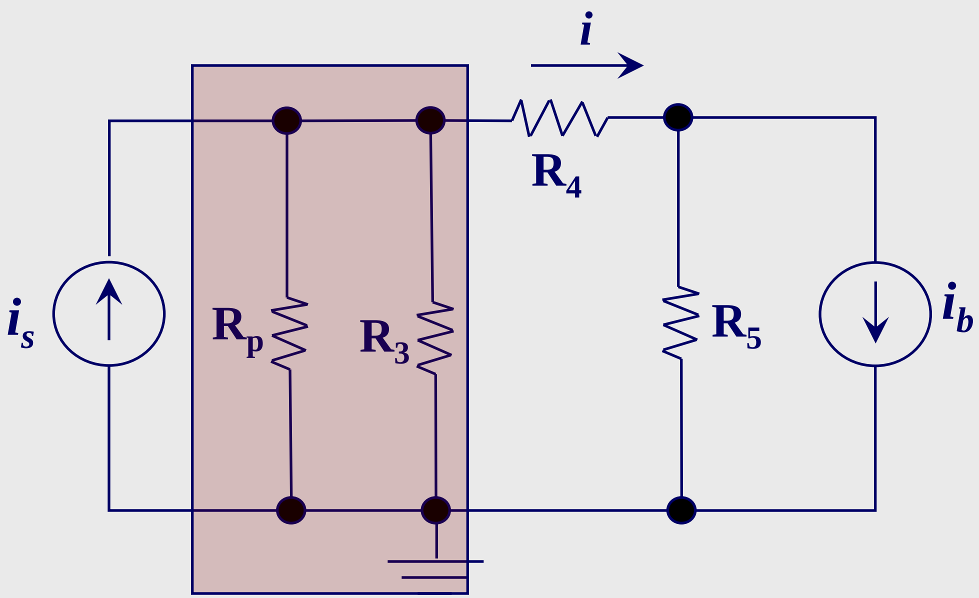
$$i_s = \frac{v_s}{R_s} = \frac{v_s}{R_{EQ}} = \frac{25}{10} = 2.5A$$

$$v_s = 25V$$
$$R_{EQ} = 10\Omega$$

$$R_p = R_s = 10\Omega$$

Source Transformations

- **Example4**: find ***i*** using transformations
  - $i_a = 5A$, $i_b = 2A$, $R_1 = 5\Omega$, $R_2 = 5\Omega$, $R_3 = 10\Omega$, $R_4 = 10\Omega$, $R_5 = 5\Omega$

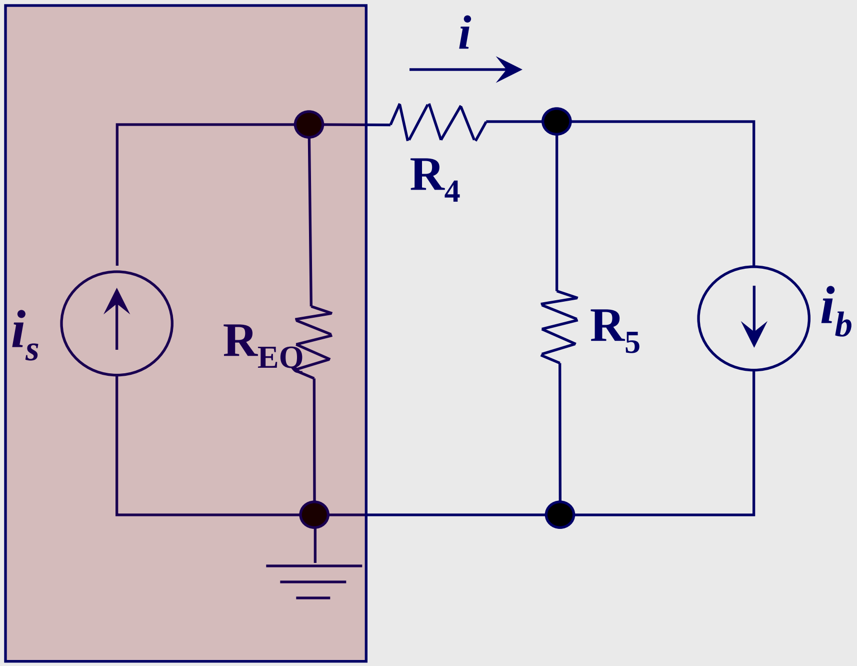


$$i_s = 2.5A$$
$$R_p = 10\Omega$$

$$R_{EQ} = \frac{R_p R_3}{R_p + R_3} = \frac{(10)(10)}{(10)+(10)} = \frac{100}{20} = 5\Omega$$

Source Transformations

- **<u>Example4</u>**: find $\boldsymbol{i}$ using transformations
  - i_a = 5A, i_b = 2A, R_1 = 5Ω, R_2 = 5Ω , R_3 = 10Ω , R_4 = 10Ω , R_5 = 5Ω



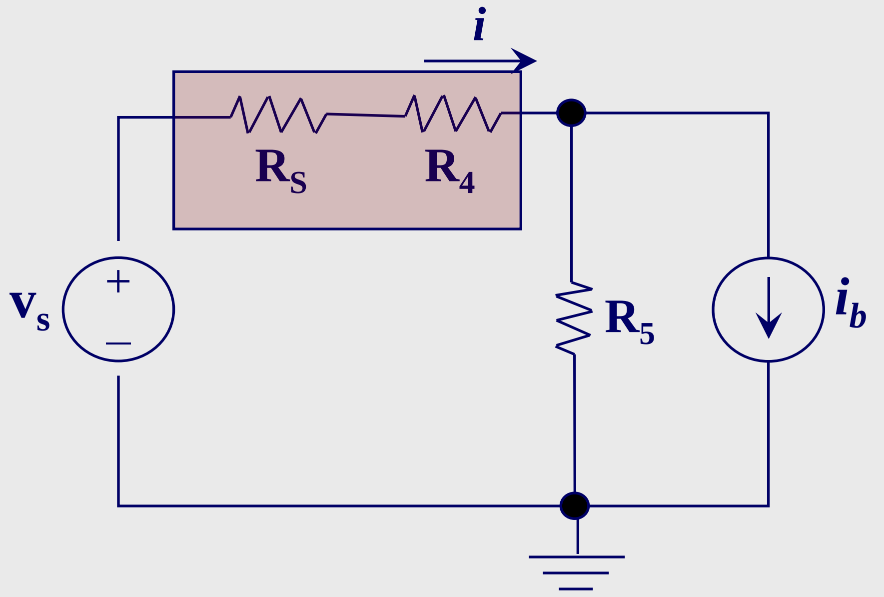
$$i_s = 2.5A$$
$$R_{EQ} = 5\Omega$$

$$v_s = i_s R_p$$
$$= i_s R_{EQ}$$
$$= (2.5)(5)$$
$$= 12.5V$$

$$R_s = R_p$$
$$= 5\Omega$$

Source Transformations

- **<u>Example4</u>**: find ***i*** using transformations
  - i_a = 5A, i_b = 2A, R_1 = 5Ω, R_2 = 5Ω , R_3 = 10Ω , R_4 = 10Ω , R_5 = 5Ω

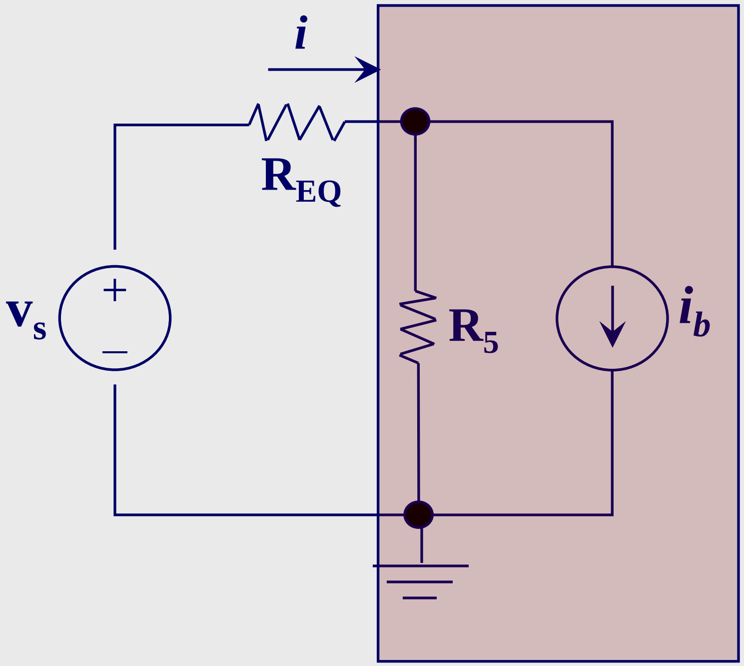


$$v_s = 12.5V$$
$$R_S = 5\Omega$$

$$R_{EQ} = R_s + R_4$$
$$= 5 + 10$$
$$= 15\Omega$$

Source Transformations

- **Example4**: find ***i*** using transformations
  - $i_a = 5A$, $i_b = 2A$, $R_1 = 5\Omega$, $R_2 = 5\Omega$, $R_3 = 10\Omega$, $R_4 = 10\Omega$, $R_5 = 5\Omega$



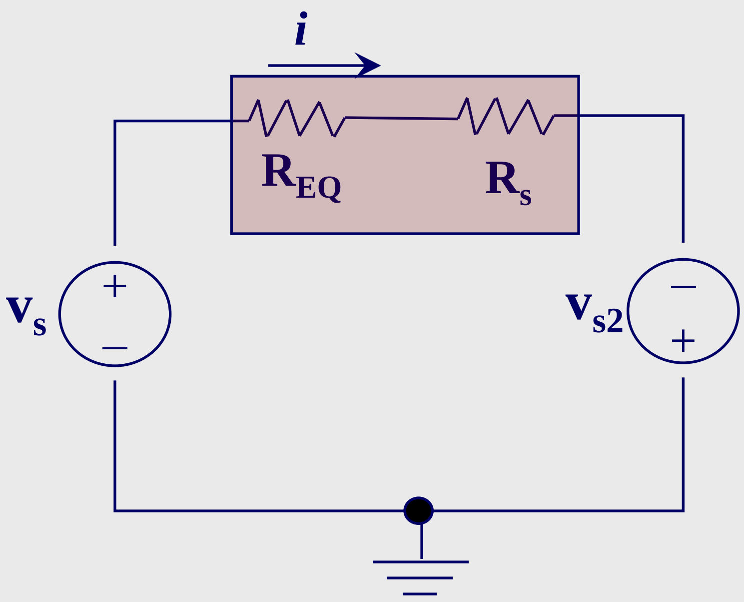
$$v_s = 12.5V$$
$$R_{EQ} = 15\Omega$$

$$v_{s2} = i_s R_p = i_b R_5 = (2)(5) = 10V$$

$$R_s = R_p = 5\Omega$$

Source Transformations

- **Example4**: find ***i*** using transformations
  - $\boldsymbol{i_a}$ = 5A, $\boldsymbol{i_b}$ = 2A, $\mathbf{R_1}$ = 5Ω, $\mathbf{R_2}$ = 5Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 10Ω , $\mathbf{R_5}$ = 5Ω

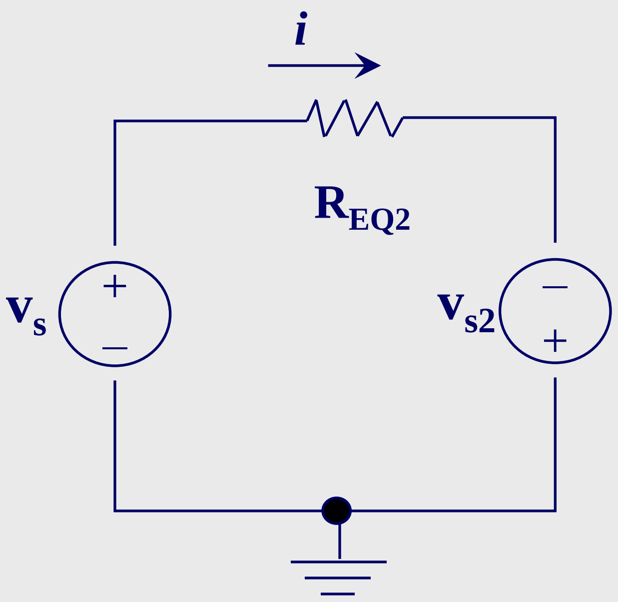


$$R_{EQ2} = R_{EQ} + R_s = 15 + 5 = 20\Omega$$

$$v_{s1} = 12.5V$$
$$v_{s2} = 10V$$
$$R_{EQ} = 15\Omega$$
$$R_s = 5\Omega$$

Source Transformations

- **<u>Example4</u>**: find ***i*** using transformations
  - i_a = 5A, i_b = 2A, R_1 = 5Ω, R_2 = 5Ω , R_3 = 10Ω , R_4 = 10Ω , R_5 = 5Ω



$$v_{s1} = 12.5V$$
$$v_{s2} = 10V$$
$$R_{EQ2} = 20\Omega$$

Using KVL:

$$-v_s + v_{EQ2} - v_{s2} = 0$$
$$i \cdot R_{EQ2} = 12.5 + 10$$
$$i = \frac{22.5}{20\Omega}$$
$$= 1.125A$$