Schedule…

Date	Day	Class No.	Title	Chapters	HW Due date	Lab Due date	Exam
1 Oct	Wed	9	Equivalent Circuits	3.6			
2 Oct	Thu						
3 Oct	Fri		Recitation		HW 4		
4 Oct	Sat						
5 Oct	Sun						
6 Oct	Mon	10	Energy Storage	3.7, 4.1		NO LAB	
7 Oct	Tue					NO LAB	
8 Oct	Wed	11	Dynamic Circuits	4.2 – 4.4			

Equivalence - Equality

Mosiah 29: 38

38 Therefore they relinquished their desires for a king, and became exceedingly anxious that every man should have an **equal** chance throughout all the land; yea, and every man expressed a willingness to answer for his own sins.

Current Sources

- All current sources can be modeled as voltage sources (and vice-versa)
  - Many sources are best modeled as voltage sources (batteries, electric outlets etc.)
  - There are some things that are best modeled as current sources:
    - **Van de Graaff generator**

Behaves as a current source because of its very high output voltage coupled with its very high output resistance and so it supplies the same few microamps at any output voltage up to hundreds of thousands of volts

Lecture 9 – Equivalent Circuits

Thévenin Equivalent

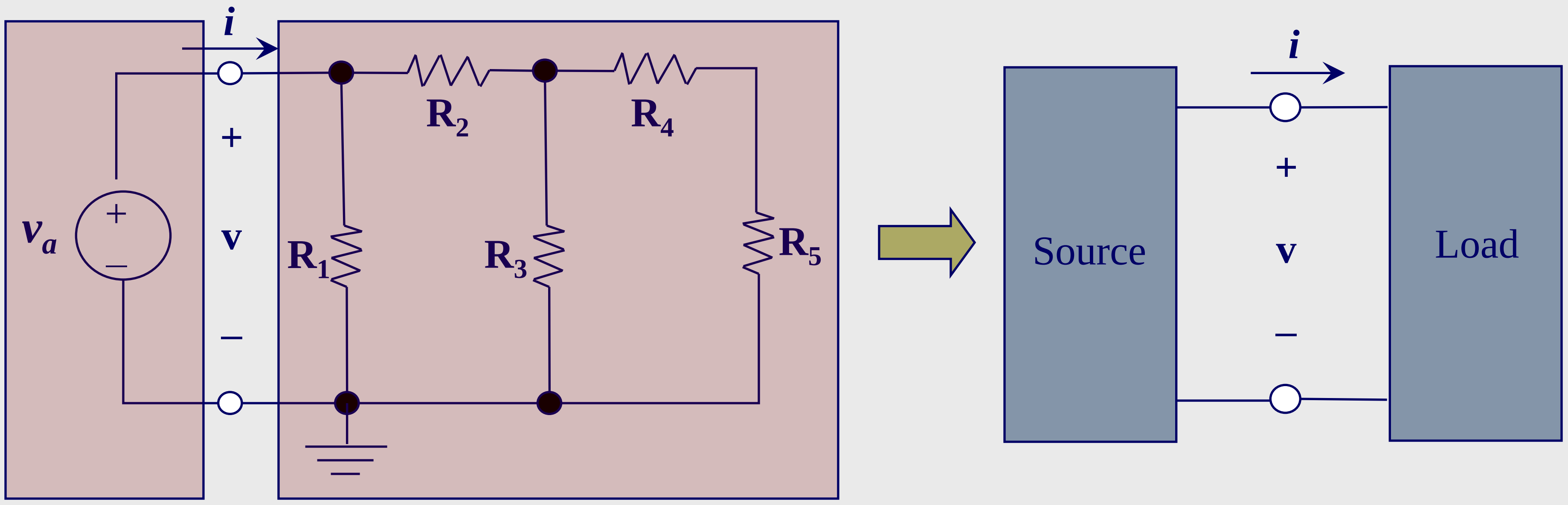
Norton Equivalent

Network Analysis

- Network Analysis Methods:
  - ✓ Node voltage method
  - ✓ Mesh current method
  - ✓ Superposition
  - ➲ Equivalent circuits
    - ✓ Source transformation
    - ➲ Thévenin equivalent
    - ➲ Norton equivalent

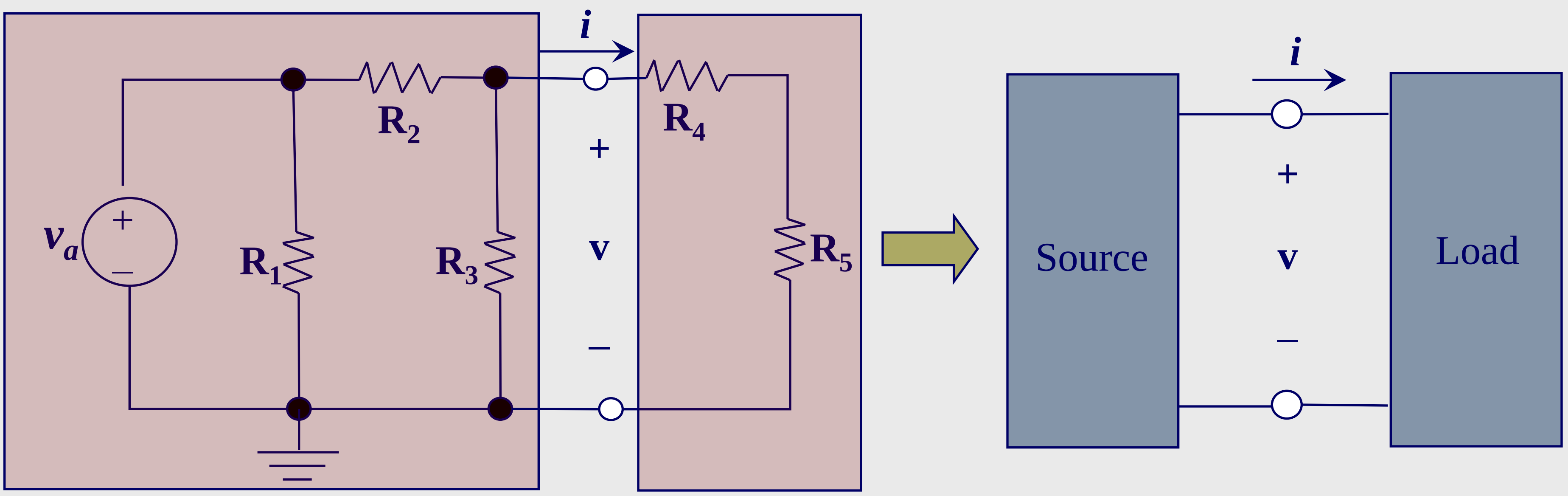
Equivalent Circuits

- It is always possible to view a complicated circuit in terms of a much simpler **equivalent source** and **equivalent load** circuit.



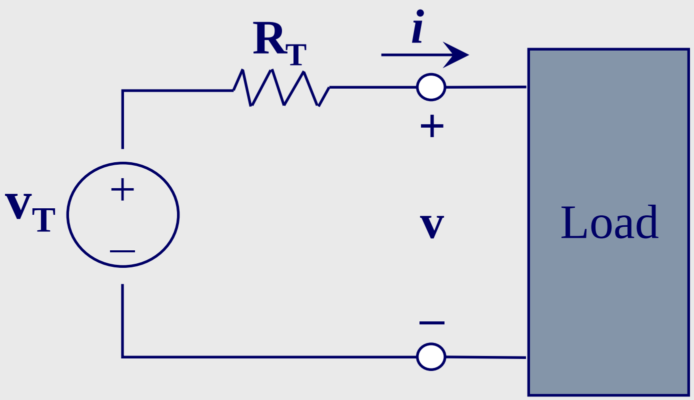
Equivalent Circuits

- It is always possible to view a complicated circuit in terms of a much simpler **equivalent source** and **equivalent load** circuit.

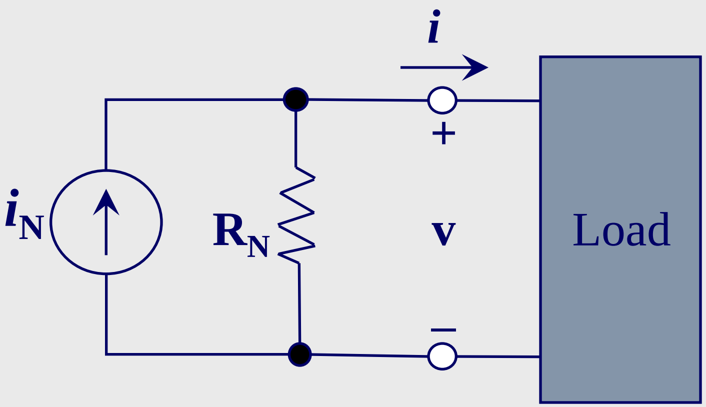


Equivalent Circuits

- Equivalent circuits fall into one of two classes:
  - **<u>Thévenin</u>**: voltage source $\mathbf{v_T}$ and series resistor $\mathbf{R_T}$
  - **<u>Norton</u>**: current source $\boldsymbol{i_N}$ and parallel resistor $\mathbf{R_N}$



Thévenin Equivalent



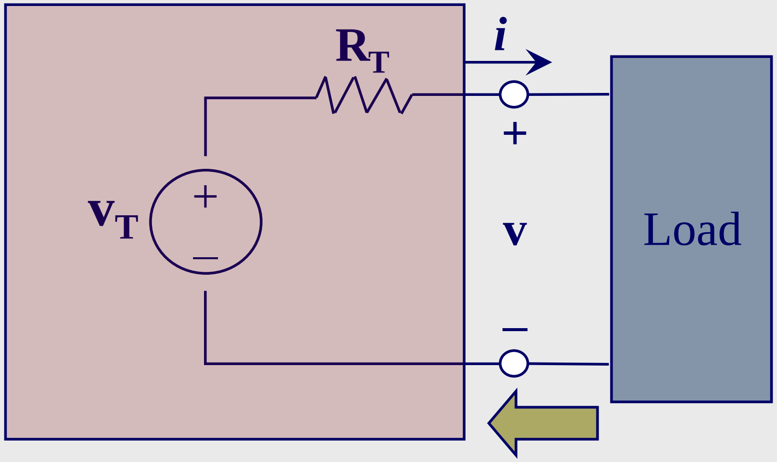
Norton Equivalent

NB: $\mathbf{R_T = R_N}$

Equivalent Circuits

<u>Thévenin Theorem</u>: when *viewed from the load*, any network comprised of independent sources and linear elements (resistors), may be represented by an equivalent circuit.

- Equivalent circuit consists of an ideal voltage source $\mathbf{v_T}$ in series with an **equivalent resistance** $\mathbf{R_T}$



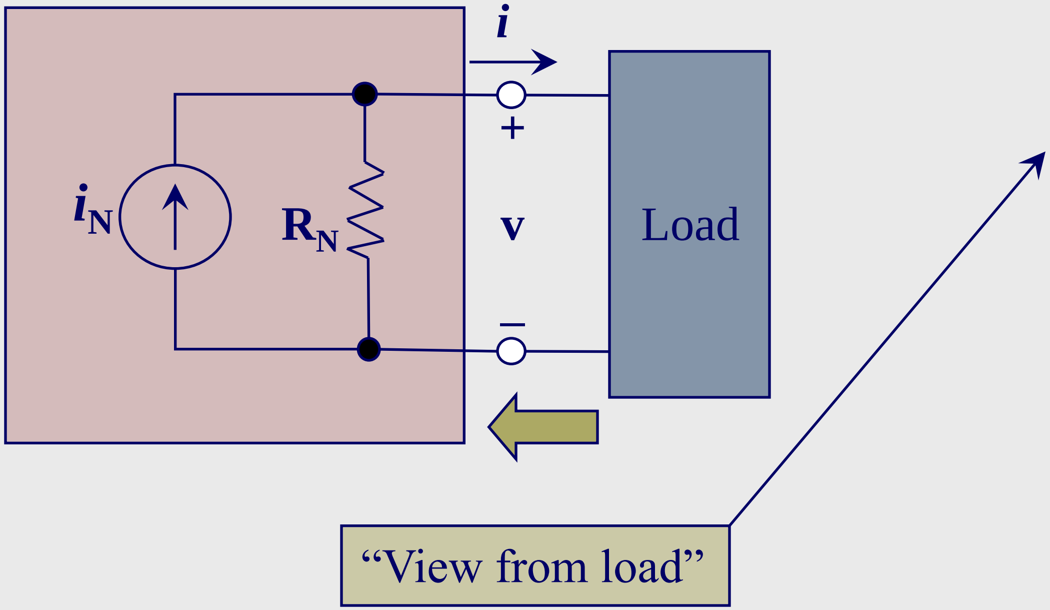
"View from load"

A fancy way of saying: "The circuit that includes everything except for the load"

Equivalent Circuits

<u>Norton Theorem</u>: when *viewed from the load*, any network comprised of independent sources and linear elements (resistors), may be represented by an equivalent circuit.

- Equivalent circuit consists of an ideal current source $\boldsymbol{i}_N$ in parallel with an **equivalent resistance R_N**



A fancy way of saying: "The circuit that includes everything except for the load"

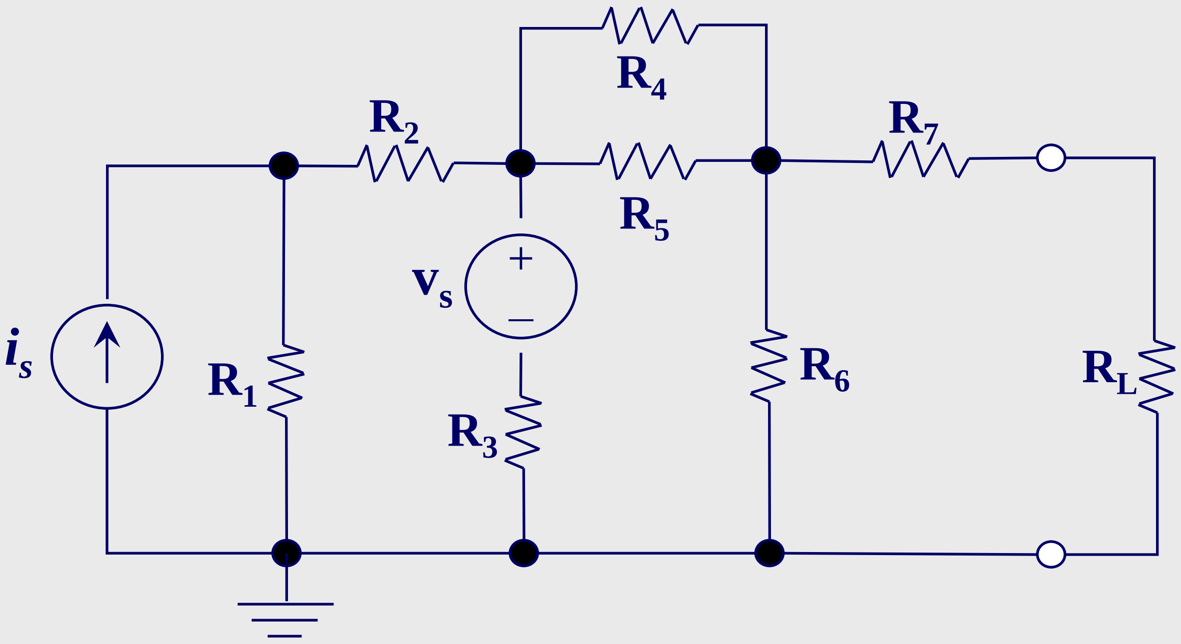
Thévenin and Norton Resistances

<u>Computation of Thévenin and Norton Resistances</u>:

1. Remove the load (open circuit at load terminal)
2. Zero all independent sources
   - Voltage sources ⟶ short circuit (**v** = 0)
   - Current sources ⟶ open circuit (***i*** = 0)
3. Compute equivalent resistance (with load removed)

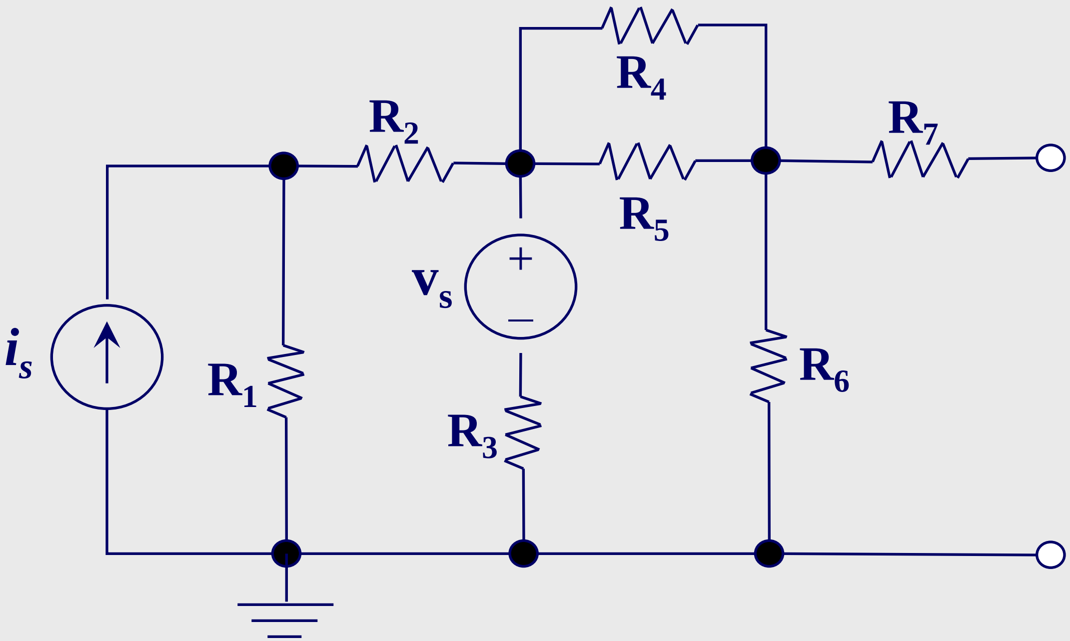
Thévenin and Norton Resistances

- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω



Thévenin and Norton Resistances

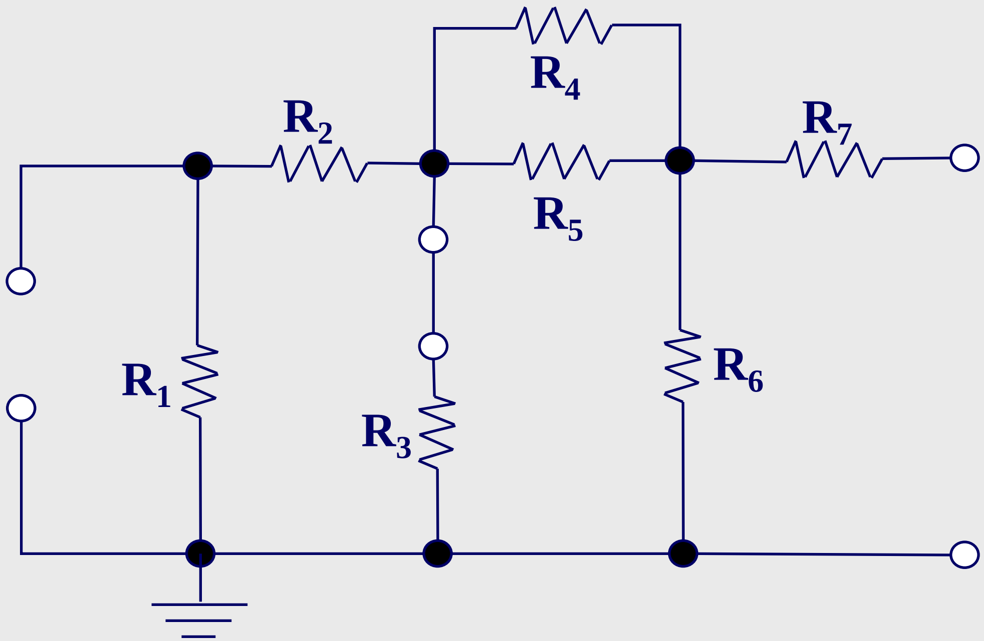
- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω



1. Remove the load

Thévenin and Norton Resistances

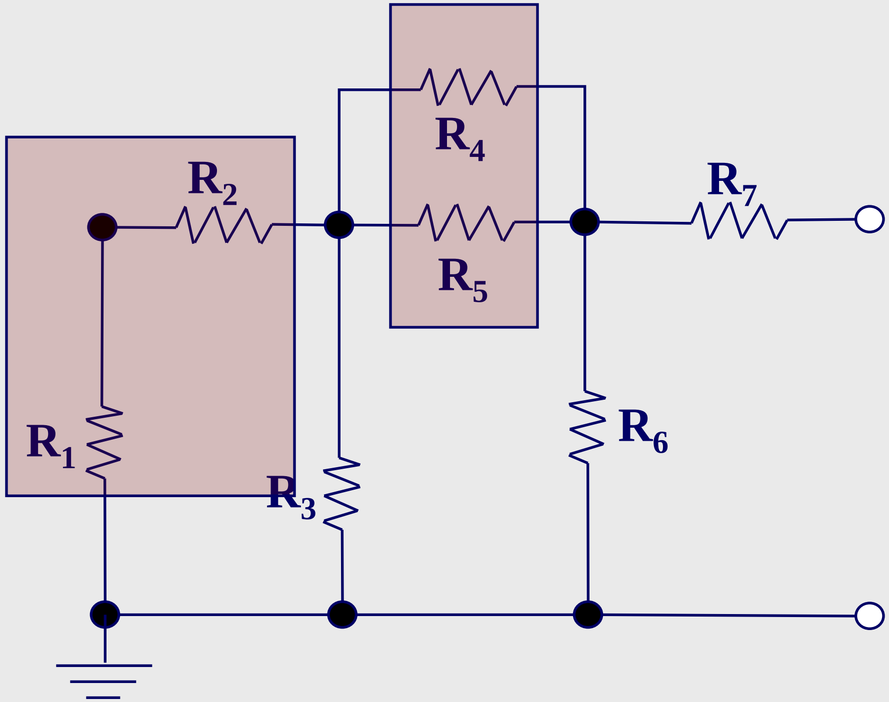
- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω



2. Zero all independent sources
   - short circuit voltage sources
   - Open circuit current sources

Thévenin and Norton Resistances

- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω



3. Compute equivalent resistance

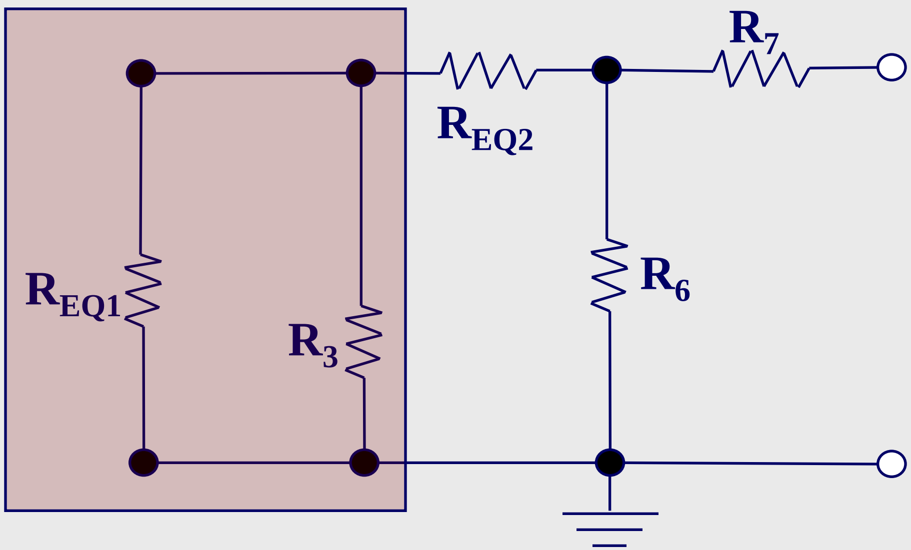
$$R_{EQ1} = R_1 + R_2 = 4 + 6 = 10\Omega$$

$$R_{EQ2} = \frac{R_4 R_5}{R_4 + R_5} = \frac{(2)(2)}{(2)+(2)} = \frac{4}{4} = 1\Omega$$

Thévenin and Norton Resistances

- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω

3. Compute equivalent resistance



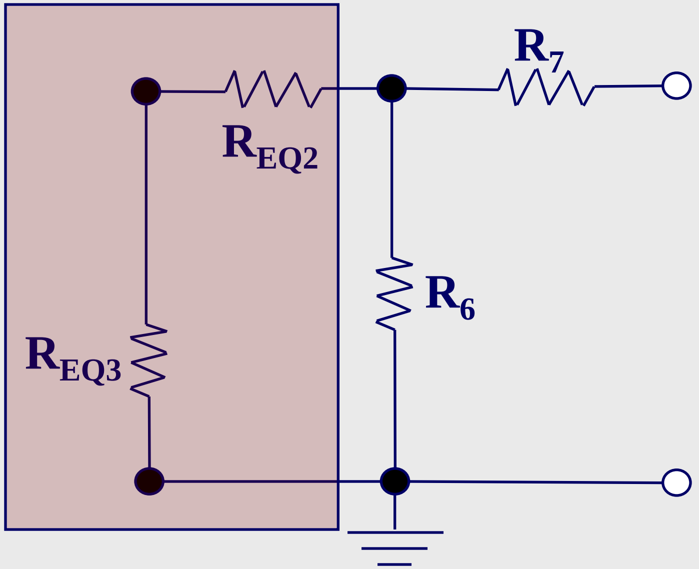
$$R_{EQ1} = 10\Omega$$
$$R_{EQ2} = 1\Omega$$

$$R_{EQ3} = \frac{R_{EQ1} R_3}{R_{EQ1} + R_3} = \frac{(10)(10)}{(10)+(10)} = \frac{100}{20} = 5\Omega$$

Thévenin and Norton Resistances

- **<u>Example1</u>**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω

3. Compute equivalent resistance

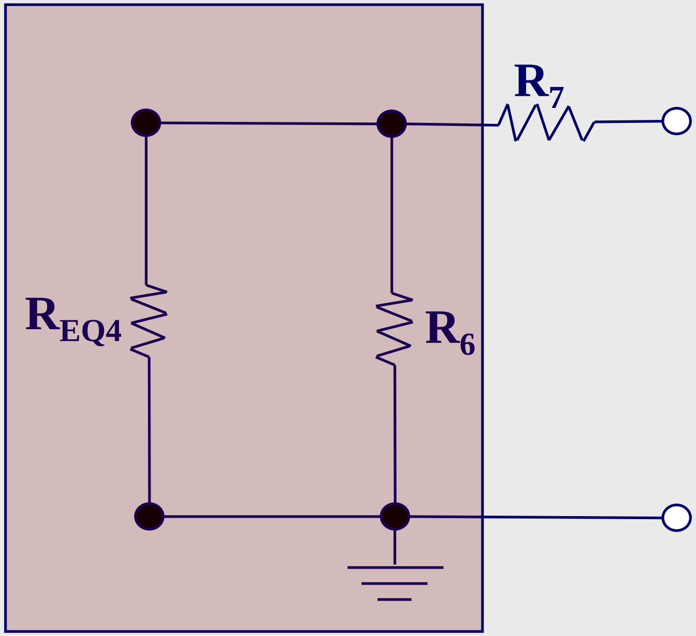


$$R_{EQ2} = 1\Omega$$
$$R_{EQ3} = 5\Omega$$

$$R_{EQ4} = R_{EQ3} + R_{EQ2}$$
$$= 5 + 1$$
$$= 6\Omega$$

Thévenin and Norton Resistances

- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω



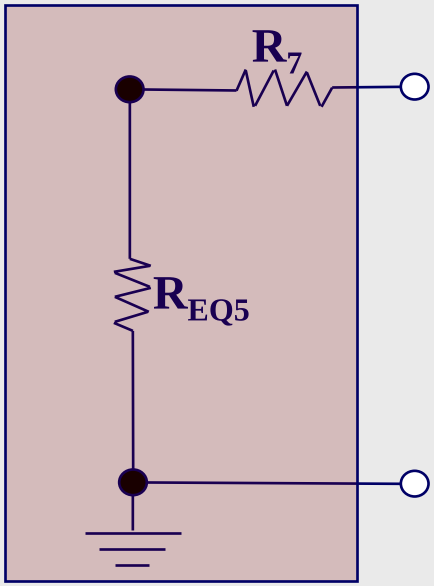
$R_{EQ4} = 6\Omega$

3. Compute equivalent resistance

$$R_{EQ5} = \frac{R_{EQ4}R_6}{R_{EQ4} + R_6} = \frac{(6)(3)}{(6)+(3)} = \frac{18}{9} = 2\Omega$$

Thévenin and Norton Resistances

- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω



3. Compute equivalent resistance

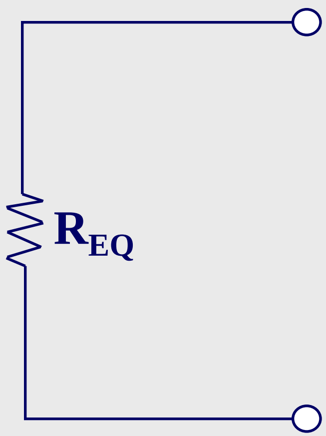
$$R_{EQ} = R_{EQ5} + R_7$$
$$= 2 + 5$$
$$= 7\Omega$$

$$R_{EQ5} = 2\Omega$$

Thévenin and Norton Resistances

- **Example1**: find the equivalent resistance as seen by the load $\mathbf{R_L}$
  - i_s =0.5A, v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω , $\mathbf{R_4}$ = 2Ω , $\mathbf{R_5}$ = 2Ω , $\mathbf{R_6}$ = 3Ω, $\mathbf{R_7}$ = 5Ω

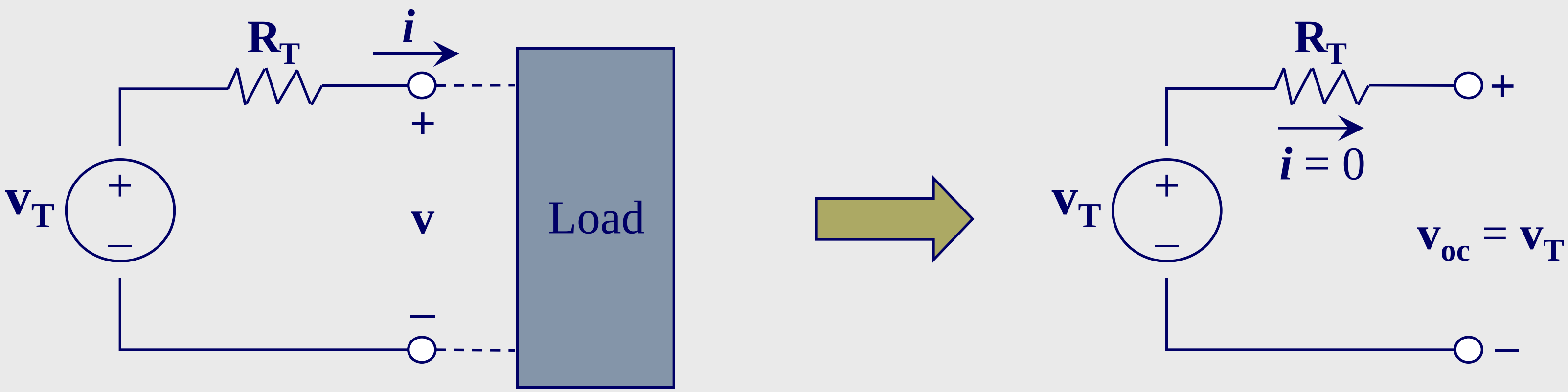
3. Compute equivalent resistance



$$R_{EQ} = 7\Omega$$

Thévenin Voltage

Thévenin equivalent voltage: equal to the **open-circuit voltage** (v_{oc}) present at the load terminals (load removed)



Remove load

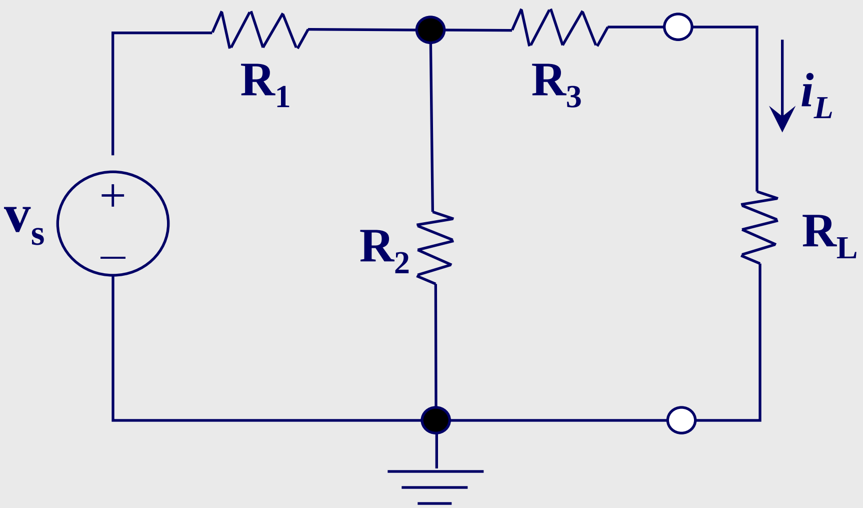
Thévenin Voltage

<u>Computing Thévenin voltage</u>:

1. Remove the load (open circuit at load terminals)
2. Define the open-circuit voltage ($\mathbf{v_{oc}}$) across the load terminals
3. Chose a network analysis method to find $\mathbf{v_{oc}}$
   - node, mesh, superposition, etc.
4. Thévenin voltage $\mathbf{v_T} = \mathbf{v_{oc}}$

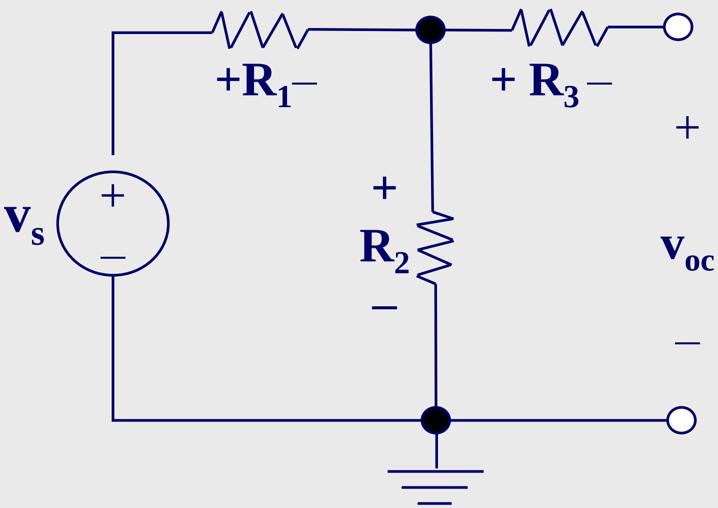
Thévenin Voltage

- **<u>Example2</u>**: find the Thévenin voltage
  - $v_s = 10V$, $\mathbf{R_1} = 4\Omega$, $\mathbf{R_2} = 6\Omega$, $\mathbf{R_3} = 10\Omega$



Thévenin Voltage

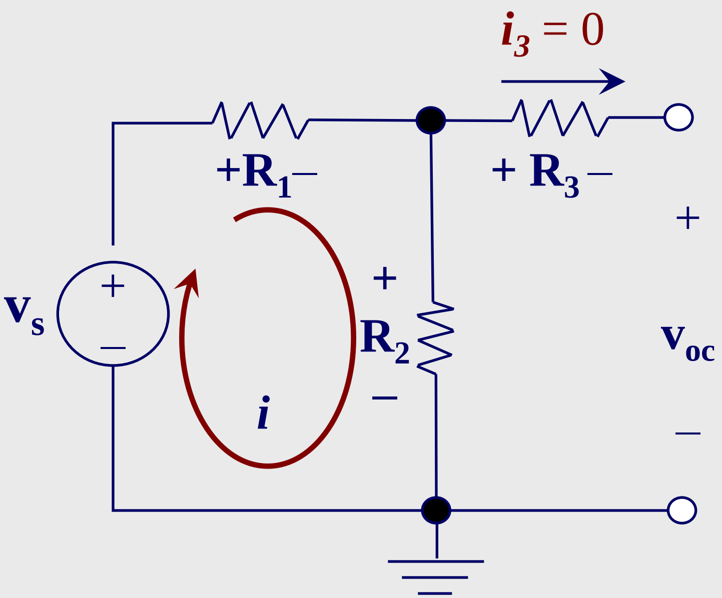
- **Example2**: find the Thévenin voltage
  - v_s = 10V, R_1 = 4Ω, R_2 = 6Ω , R_3 = 10Ω



1. Remove the load
2. Define v_{oc}

Thévenin Voltage

- **Example2**: find the Thévenin voltage
  - v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω

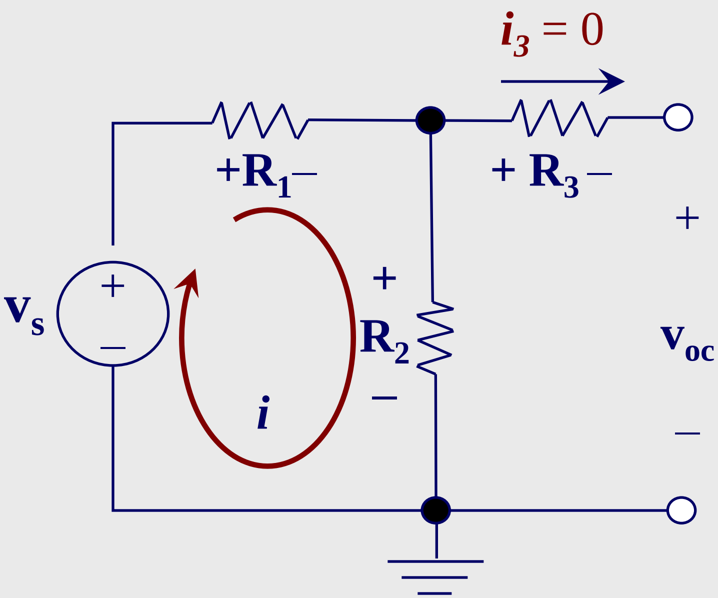


3. Choose a network analysis method
   - Voltage divider

$$v_{oc} = \frac{R_2}{R_1 + R_2} v_s$$

Thévenin Voltage

- **Example2**: find the Thévenin voltage
  - v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω , $\mathbf{R_3}$ = 10Ω



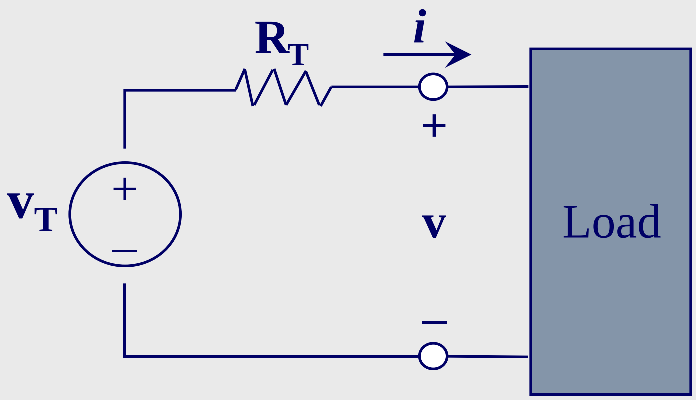
4. $v_T = v_{oc}$

$$v_T = \frac{R_2}{R_1 + R_2} v_s$$

Thévenin Equivalent Circuit

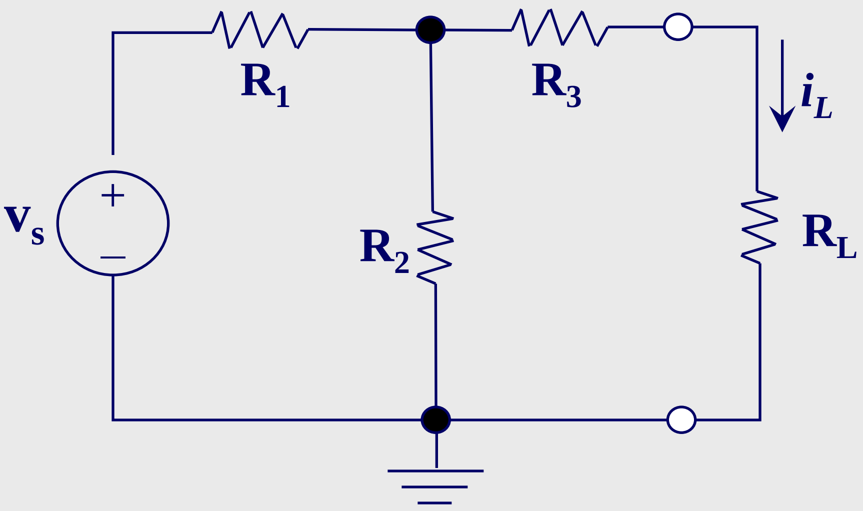
Computing Thévenin Equivalent Circuit:

1. Compute the Thévenin resistance $\mathbf{R_T}$
2. Compute the Thévenin voltage $\mathbf{v_T}$



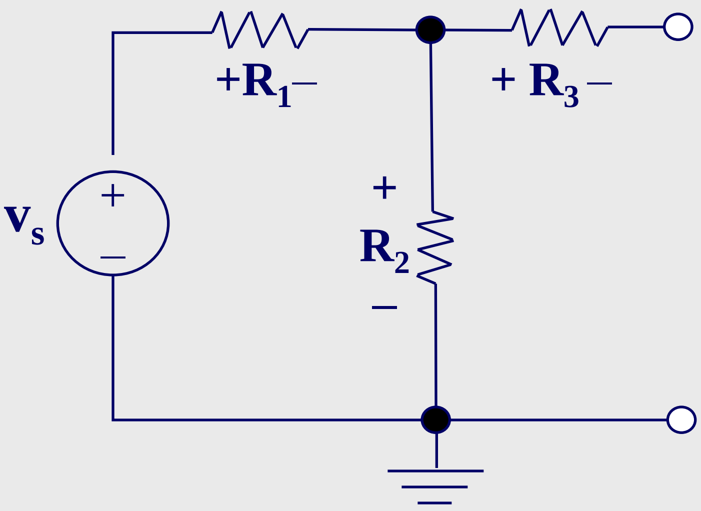
Thévenin Equivalent Circuit

- **Example3**: find i_L by finding the Thévenin equivalent circuit
  - $v_s = 10V$, $R_1 = 4\Omega$, $R_2 = 6\Omega$, $R_3 = 10\Omega$, $R_L = 10\Omega$



Thévenin Equivalent Circuit

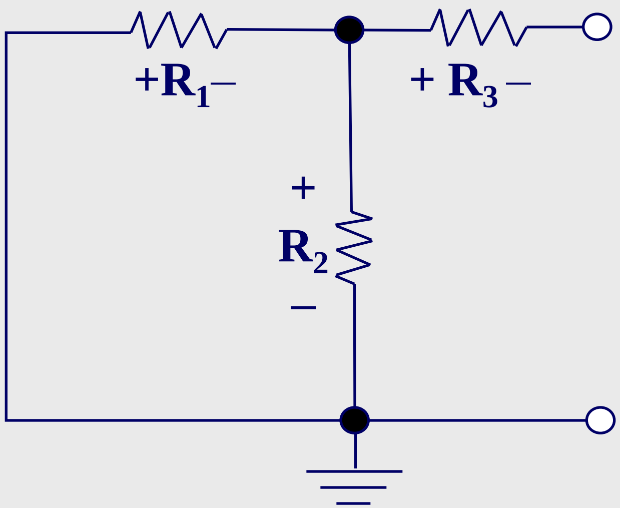
- **Example3**: find i_L by finding the Thévenin equivalent circuit
  - v_s = 10V, R_1 = 4Ω, R_2 = 6Ω, R_3 = 10Ω, R_L = 10Ω



1. Compute $\mathbf{R_T}$
   - Remove $\mathbf{R_L}$

Thévenin Equivalent Circuit

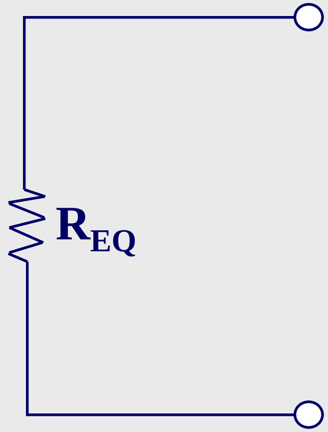
- **Example3**: find i_L by finding the Thévenin equivalent circuit
  - v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 10Ω, $\mathbf{R_L}$ = 10Ω



1. Compute $\mathbf{R_T}$
   - Remove $\mathbf{R_L}$
   - Zero sources

Thévenin Equivalent Circuit

- **Example3**: find i_L by finding the Thévenin equivalent circuit
  - $v_s = 10V$, $\mathbf{R_1} = 4\Omega$, $\mathbf{R_2} = 6\Omega$, $\mathbf{R_3} = 10\Omega$, $\mathbf{R_L} = 10\Omega$

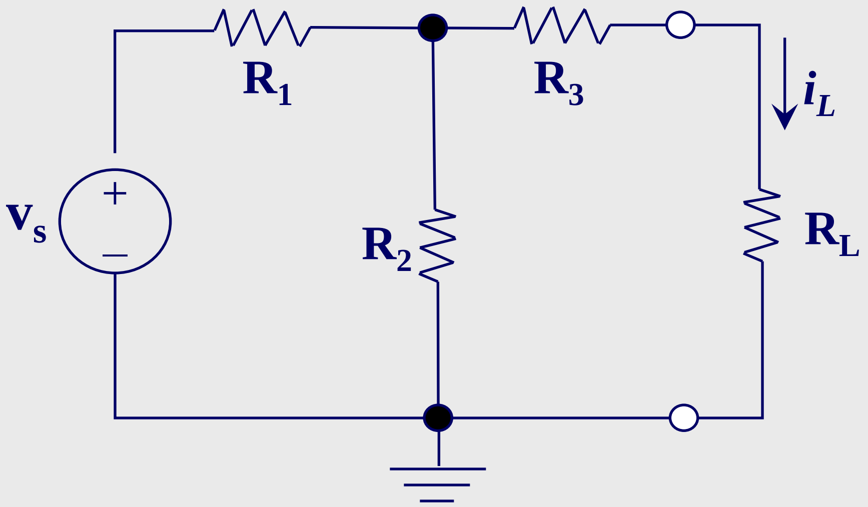


1. Compute $\mathbf{R_T}$
   - Remove $\mathbf{R_L}$
   - Zero sources
   - Compute $\mathbf{R_T} = \mathbf{R_{EQ}}$

$$R_T = R_3 + R_1 \| R_2$$

Thévenin Equivalent Circuit

- **Example3**: find $\boldsymbol{i_L}$ by finding the Thévenin equivalent circuit
  - v_s = 10V, $\mathbf{R_1}$ = 4Ω, $\mathbf{R_2}$ = 6Ω, $\mathbf{R_3}$ = 10Ω, $\mathbf{R_L}$ = 10Ω

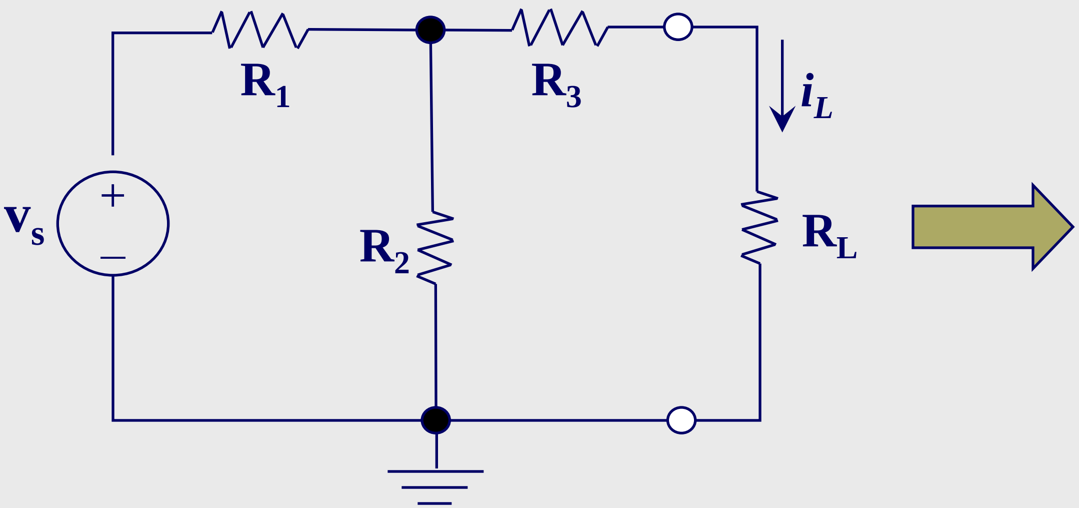


1. Compute $\mathbf{R_T}$
2. Compute $\mathbf{v_T}$
   - (previously computed)

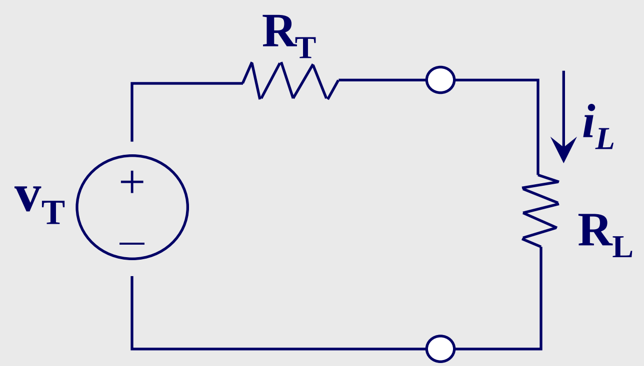
$$v_T = \frac{R_2}{R_1 + R_2} v_s$$

Thévenin Equivalent Circuit

- **Example3**: find i_L by finding the Thévenin equivalent circuit
  - $v_s = 10V$, $\mathbf{R_1} = 4\Omega$, $\mathbf{R_2} = 6\Omega$, $\mathbf{R_3} = 10\Omega$, $\mathbf{R_L} = 10\Omega$



$$R_T = R_3 + R_1 \| R_2$$



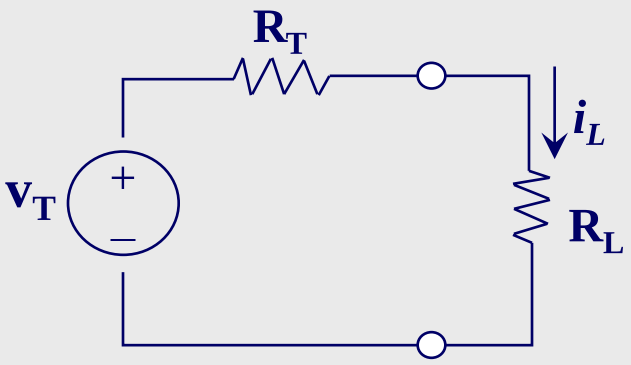
$$v_T = \frac{R_2}{R_1 + R_2} v_s$$

NB: i_L is the same in both circuits

Thévenin Equivalent Circuit

- **Example3**: find i_L by finding the Thévenin equivalent circuit
  - v_s = 10V, R_1 = 4Ω, R_2 = 6Ω, R_3 = 10Ω, R_L = 10Ω

$$R_T = R_3 + R_1 \parallel R_2$$

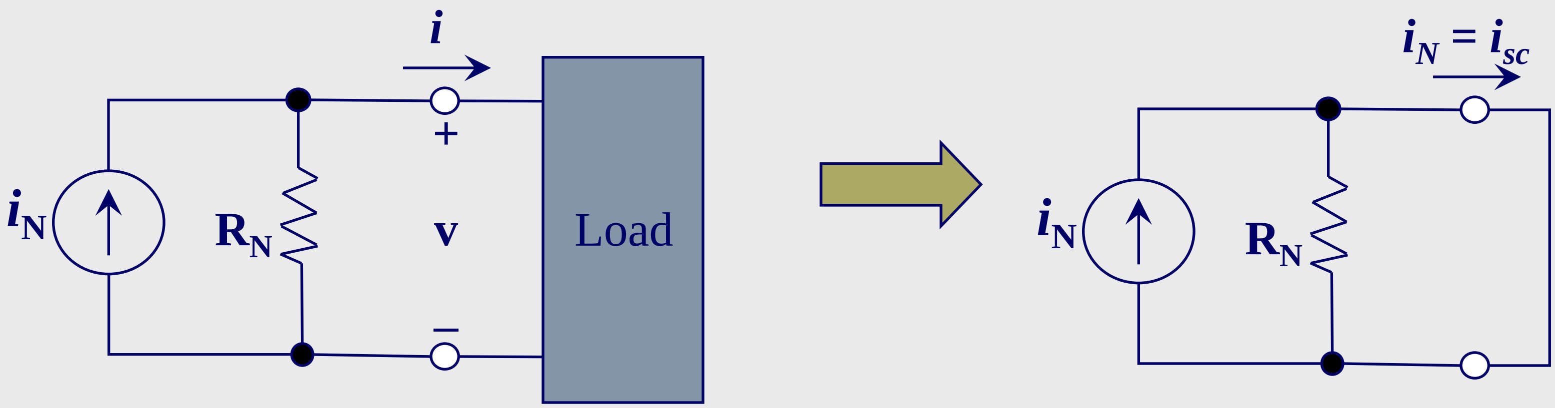


$$v_T = \frac{R_2}{R_1 + R_2} v_s$$

$$\begin{aligned}
i_L &= \frac{v_T}{R_T + R_L} \\
&= \left(\frac{R_2}{R_1 + R_2} v_s\right) \cdot \frac{1}{\left(R_3 + R_1 \parallel R_2\right) + R_L} \\
&= \left[\frac{(6)(10)}{(4)+(6)}\right] \cdot \frac{1}{\left[(10)+(4) \parallel (6)+(10)\right]} \\
&= \left(\frac{60}{10}\right) \cdot \frac{1}{20 + 2.4} \\
&= \frac{6}{22.4} \\
&= 0.27\,A
\end{aligned}$$

Norton Current

<u>Current equivalent current</u>: equal to the **short-circuit current** (i_{sc}) present at the load terminals (load replaced with short circuit)



Short circuit load

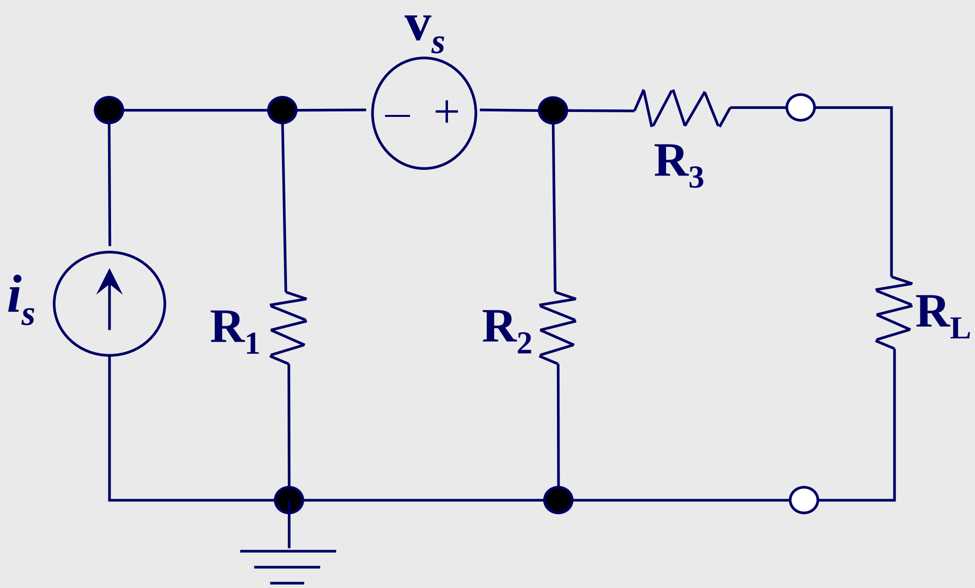
Norton Current

Computing Norton current:

1. Replace the load with a short circuit
2. Define the short-circuit current ($\boldsymbol{i_{sc}}$) across the load terminals
3. Chose a network analysis method to find $\boldsymbol{i_{sc}}$
   - node, mesh, superposition, etc.
4. Norton current $\boldsymbol{i_N} = \boldsymbol{i_{sc}}$

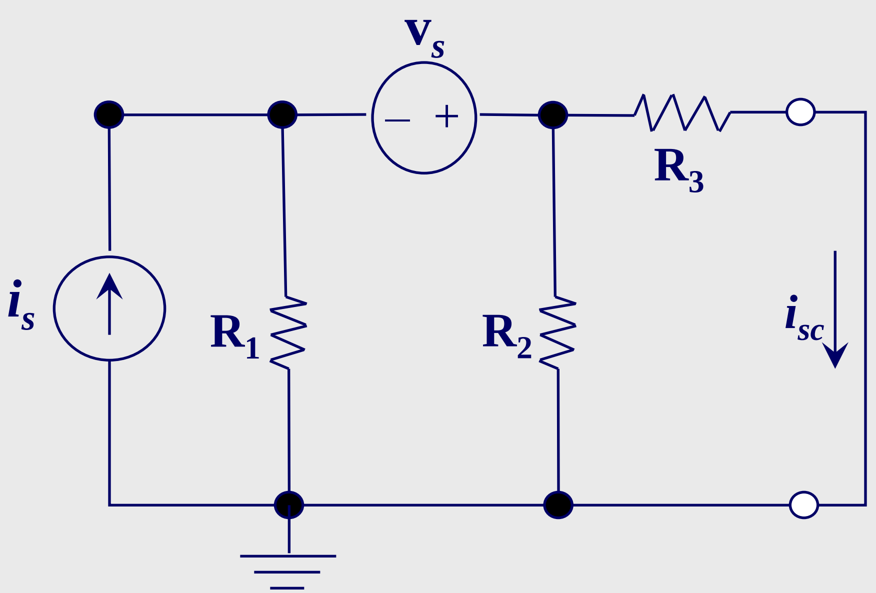
Norton Current

- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, $\boldsymbol{i_s}$ = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



Norton Current

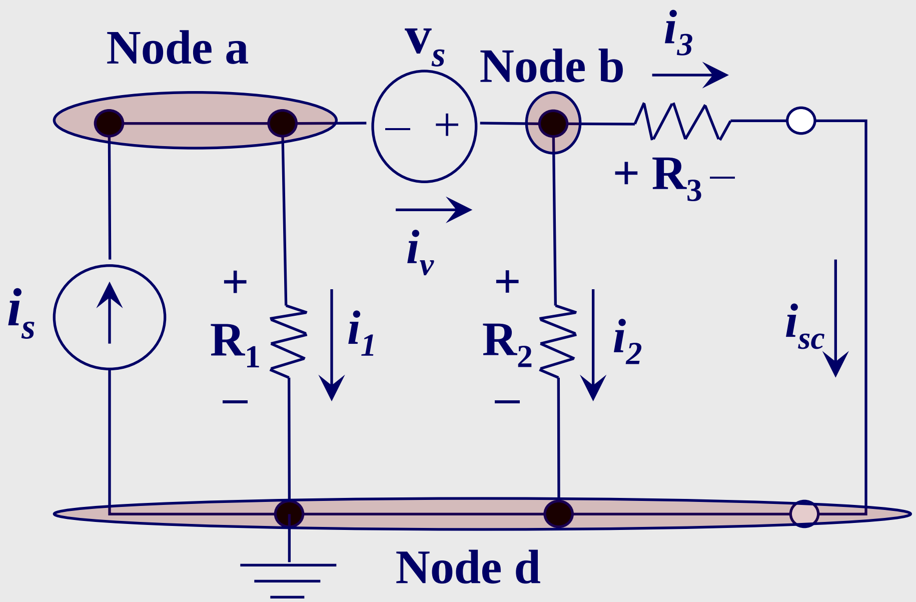
- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, $\boldsymbol{i_s}$ = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



1. Short-circuit the load
2. Define $\boldsymbol{i_{sc}}$

Norton Current

- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, $\boldsymbol{i_s}$ = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω

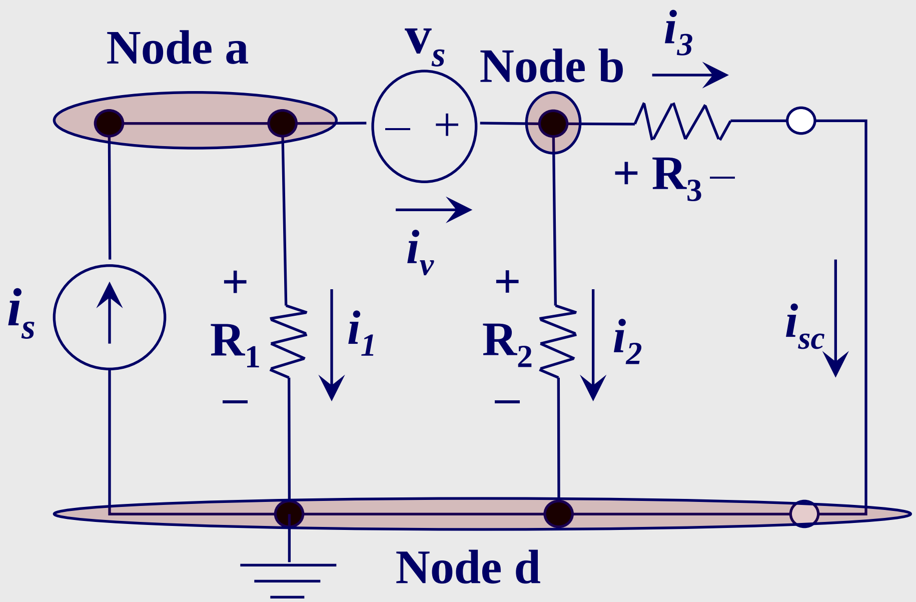


3. Choose a network analysis method
   - Node voltage

1. **v_a is independent**
2. **v_b is dependent** (actually $\mathbf{v_a}$ and $\mathbf{v_b}$ are dependent on each other but choose $\mathbf{v_b}$) $\mathbf{v_b} = \mathbf{v_a} + \mathbf{v_s}$

Norton Current

- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, i_s = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



3. Choose a network analysis method
   - Node voltage

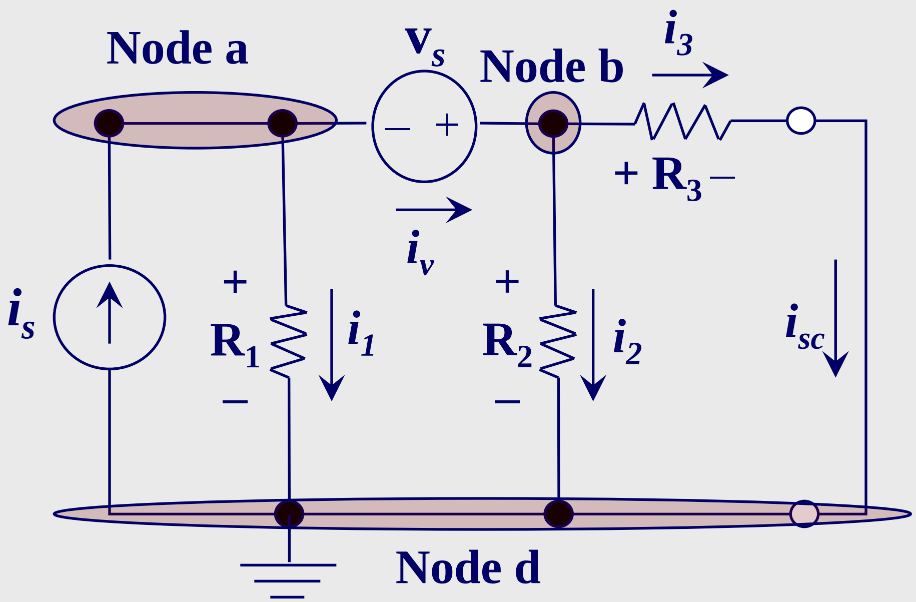
KCL at node a :

$$i_s - i_1 - i_v = 0$$

$$\frac{v_a}{R_1} + i_v = i_s$$

Norton Current

- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, i_s = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



3. Choose a network analysis method
   - Node voltage

KCL at node b:

$$i_v - i_2 - i_3 = 0$$

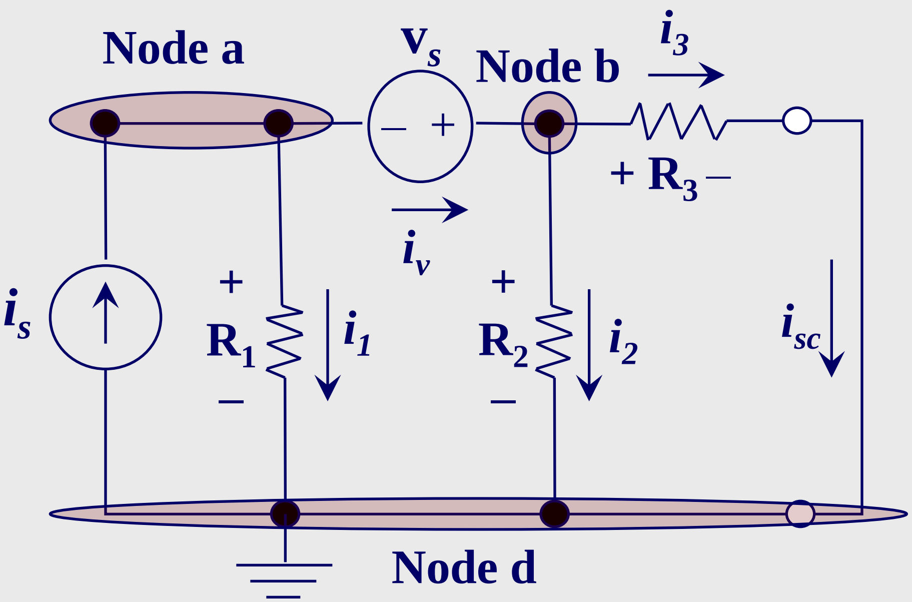
$$i_v - \frac{v_b}{R_2} - \frac{v_b}{R_3} = 0$$

$$i_v - \frac{(v_a + v_s)}{R_2} - \frac{(v_a + v_s)}{R_3} = 0$$

$$i_v - v_a\left(\frac{1}{R_2} + \frac{1}{R_3}\right) = v_s\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

Norton Current

- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, i_s = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



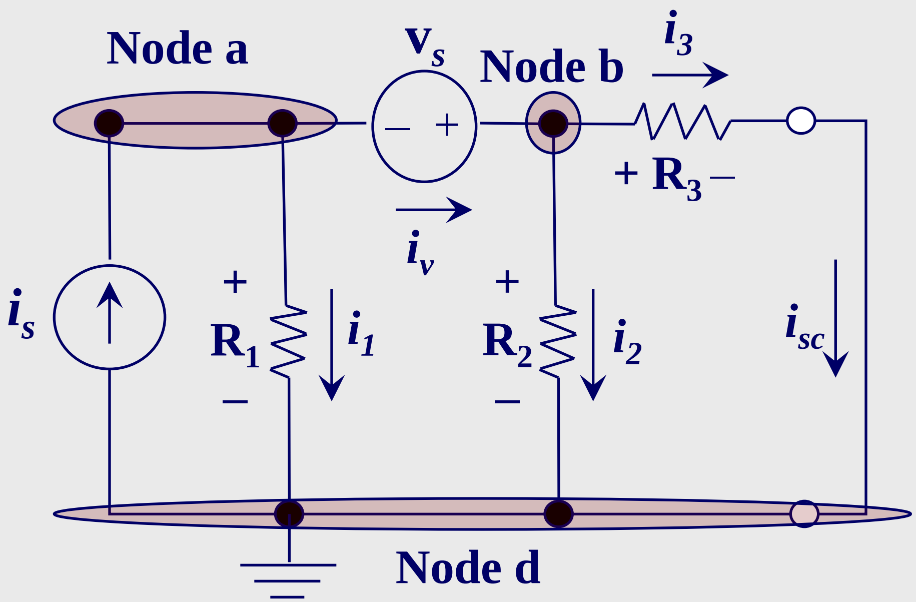
3. Choose a network analysis method
   - Node voltage

$$6i_v - 5v_a = 30$$
$$6i_v + v_a = 12$$

$$i_v = 2.5A$$
$$v_a = -3V$$

Norton Current

- **<u>Example4</u>**: find the Norton equivalent current $\boldsymbol{i_N}$
  - $v_s = 6V$, $i_s = 2A$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_L = 10\Omega$

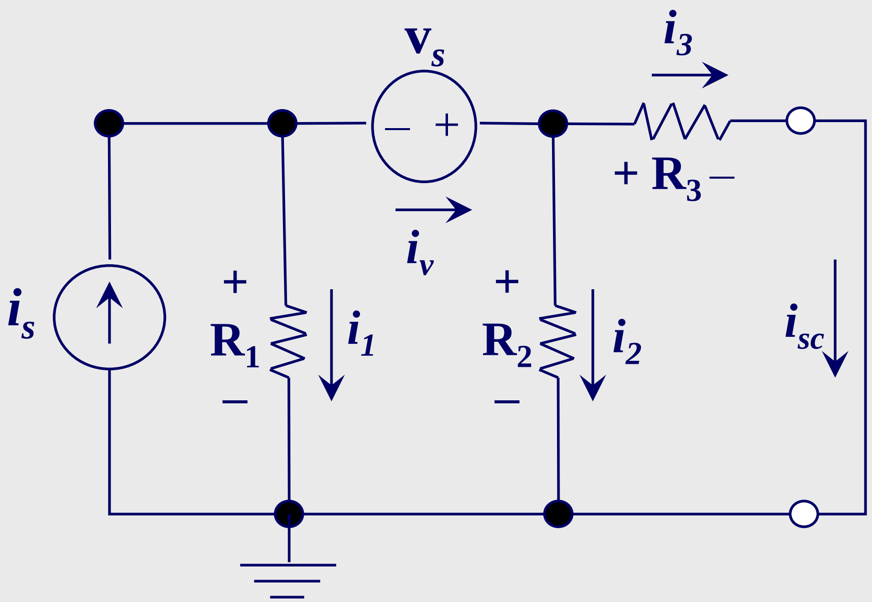


3. Choose a network analysis method
   - Node voltage

$$i_{sc} = \frac{v_b}{R_3} = \frac{(v_s - v_b)}{2} = \frac{3}{2}A$$

Norton Current

- **Example4**: find the Norton equivalent current $\boldsymbol{i_N}$
  - v_s = 6V, i_s = 2A, R_1 = 6Ω, R_2 = 3Ω, R_3 = 2Ω, R_L = 10Ω



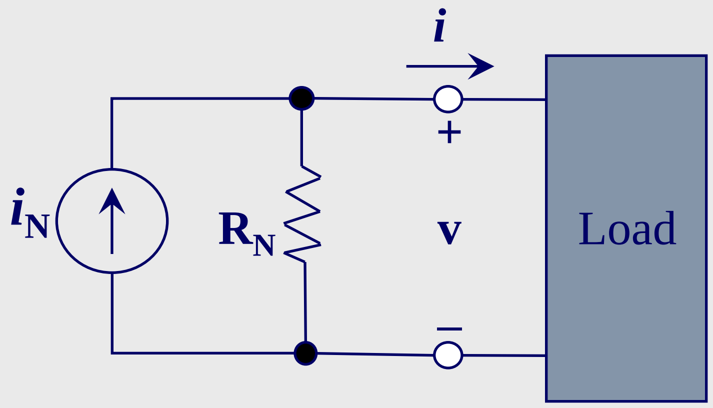
4. $i_N = i_{sc}$

$$i_N = i_{sc} = 1.5A$$

Norton Equivalent Circuit

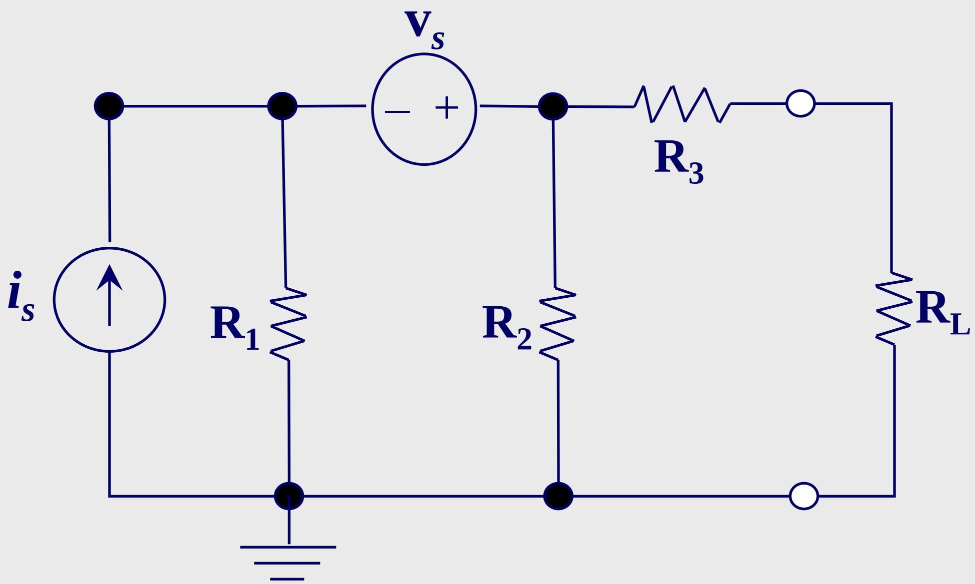
Computing Norton Equivalent Circuit:

1. Compute the Norton resistance $\mathbf{R_N}$
2. Compute the Norton current $\boldsymbol{i_N}$



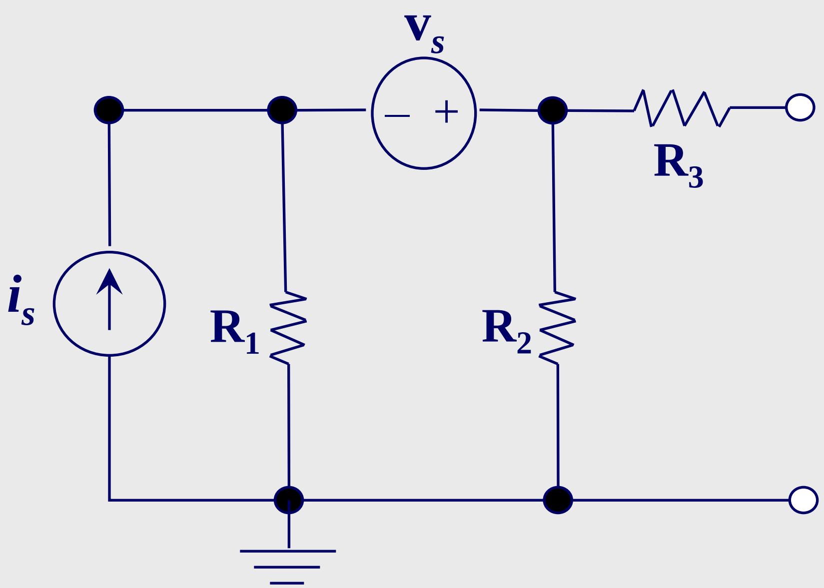
Norton Equivalent Circuit

- **Example5**: find the Norton equivalent circuit
  - v_s = 6V, $\boldsymbol{i_s}$ = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



Norton Equivalent Circuit

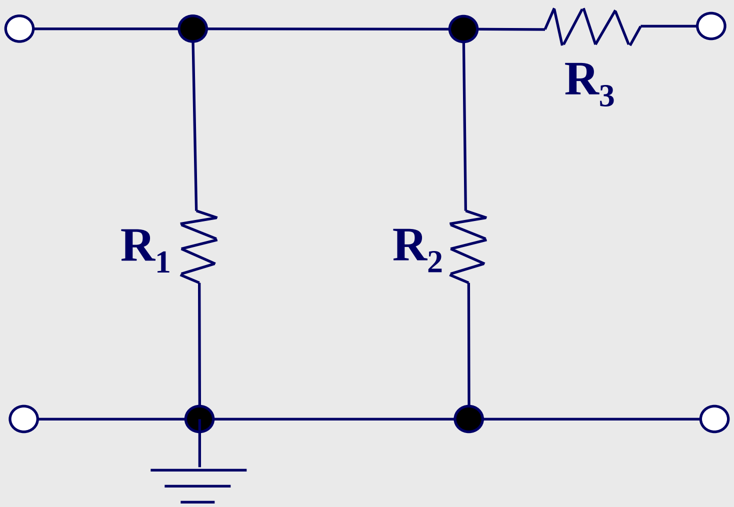
- **Example5**: find the Norton equivalent circuit
  - $v_s = 6V$, $i_s = 2A$, $R_1 = 6\Omega$, $R_2 = 3\Omega$, $R_3 = 2\Omega$, $R_L = 10\Omega$



1. Compute R_N
   - Remove R_L

Norton Equivalent Circuit

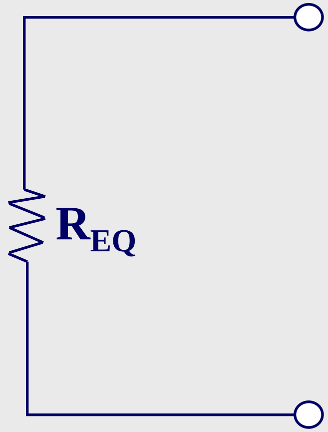
- **<u>Example5</u>**: find the Norton equivalent circuit
  - v_s = 6V, $\boldsymbol{i_s}$ = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



1. Compute $\mathbf{R_N}$
   - Remove $\mathbf{R_L}$
   - Zero sources

Norton Equivalent Circuit

- **Example5**: find the Norton equivalent circuit
  - v_s = 6V, i_s = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω

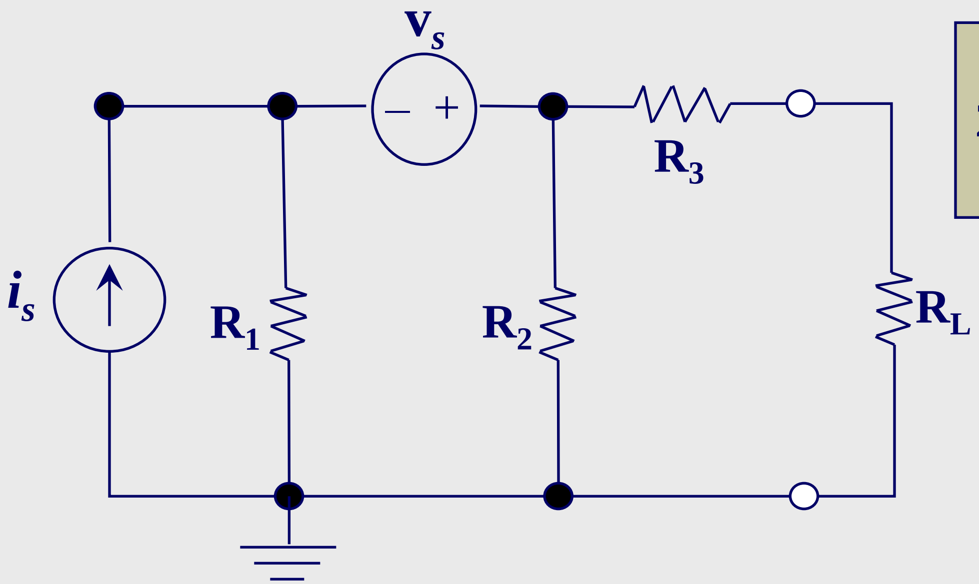


1. Compute $\mathbf{R_N}$
   - Remove $\mathbf{R_L}$
   - Zero sources
   - Compute $\mathbf{R_N} = \mathbf{R_{EQ}}$

$$R_N = R_3 + R_1 \,||\, R_2$$

Norton Equivalent Circuit

- **Example5**: find the Norton equivalent circuit
  - v_s = 6V, $\boldsymbol{i_s}$ = 2A, $\mathbf{R_1}$ = 6Ω, $\mathbf{R_2}$ = 3Ω, $\mathbf{R_3}$ = 2Ω, $\mathbf{R_L}$ = 10Ω



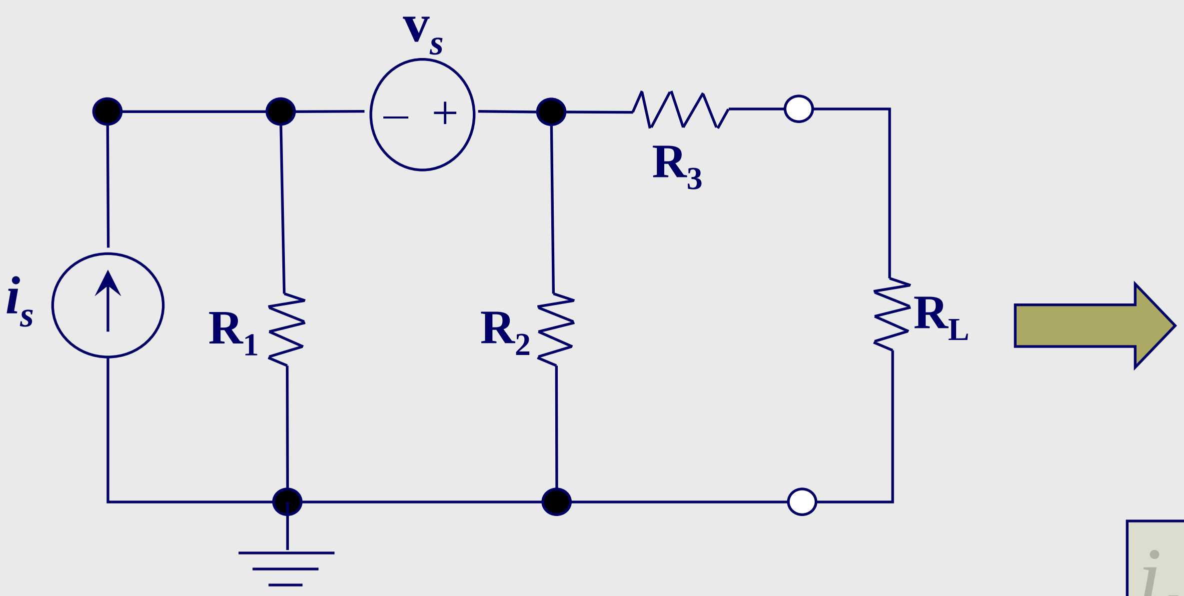
1. Compute $\mathbf{R_N}$
2. Compute $\boldsymbol{i_N}$
   - (previously computed)

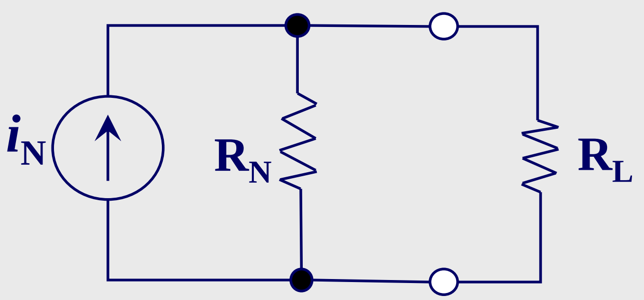
$$i_N = i_{sc} = 1.5A$$

$$R_N = R_3 + R_1 \| R_2 = 4\Omega$$

Norton Equivalent Circuit

- **Example5**: find the Norton equivalent circuit
  - v_s = 6V, i_s = 2A, R_1 = 6Ω, R_2 = 3Ω, R_3 = 2Ω, R_L = 10Ω





$i_N = 1.5A$

$R_N = 4\Omega$