1. (Lecture 34) Let $Y_1, Y_2, \ldots$ be a sequence of random variables.

A. (T/F) If the sequence $Y_n$ “converges with probability 1” then it also “converges in probability”.

B. (T/F) If the sequence $Y_n$ “converges in probability” then it also “converges with probability 1”.

Now suppose $Y_1, Y_2, \ldots$ are independent identically distributed random variables with common mean $\mu$ and variance $\sigma^2$.

C. (T/F) Let $S_n = Y_1 + Y_2 + \cdots + Y_n$. The CDF of $S_n$ converges to the standard normal CDF.

D. (T/F) Let $M_n = \frac{Y_1 + Y_2 + \cdots + Y_n}{n}$. The CDF of $M_n$ converges to the standard normal CDF.

E. (T/F) Let $Z_n = \frac{Y_1 + Y_2 + \cdots + Y_n - n\mu}{\sigma\sqrt{n}}$. The CDF of $Z_n$ converges to the standard normal CDF.

2. (Lecture 33) You decide your student housing needs a make-over and decide to put in hardwood flooring (wow!). You go to your building products company and they tell you that their boards have a width of 3 inches with a tolerance of +/- .2 inch, which you assume gives you a uniform distribution for a single board from 2.8 to 3.2 inches. The width of your room is 120 inches. If you buy 41 boards and plan to lay them side by side, what is the probability that you will be able to cover the width of your room? Use $\Phi(z)$ to denote the CDF of the standard normal random variable.

Hint: If $X$ is a uniform random variable over $[a, b]$, $\text{var}(X) = \frac{(b-a)^2}{12}$.

Solution:
We can say that $S_n = X_1 + \cdots + X_{41}$.

$\text{E}[X_i] = 3$

$\text{var}(X) = \frac{(3.2 - 2.8)^2}{12} = 0.0133$

$\sigma = \sqrt{\text{var}(X)} = 0.1155$

Use the normal approximation based on the central limit theorem.

$\mathbf{P}(S_n \leq c) \approx \Phi \left( \frac{c - n\mu}{\sigma\sqrt{n}} \right)$

We are trying to find:

$\mathbf{P}(S_n > 120) = 1 - \mathbf{P}(S_n \leq c) \approx 1 - \Phi \left( \frac{120 - \mu}{\sigma\sqrt{n}} \right) = 1 - \Phi \left( \frac{120 - 41(3)}{0.1155\sqrt{41}} \right) = 1 - \Phi \left( \frac{-3}{0.7396} \right) = 1 - \Phi(-4.06) = \Phi(4.06) \approx 1$

It is interesting to note that if we only bought 40 boards, then we would have a probability of .5 that we would have enough for the room. That is because $\Phi(0) = .5$. Having a little bit of extra material is a good thing.