1. (Lecture 8) True or False questions:
   - **True** A random variable is a real-valued function of the outcome of an experiment.
   - **True** A discrete random variable has a range that is either finite or countably infinite.
   - **True** A function of a random variable defines another random variable.
   - **True** The probability mass function gives the probability of each numerical value that the random variable can take.

2. (Lecture 9) Suppose that you attempt to graduate every year at the end of summer with a probability of success of 0.4. What is the probability that you graduate at the end of the second or third summer?
   
   This is a geometric random variable with parameter $p = 0.4$.
   
   The probability of first success is thus given by $p_X(x) = (1 - p)^{k-1}p$.
   
   The probability of this happening after the second attempt or third attempt is thus:
   
   $$P(\{\text{success on second or third attempt}\}) = p_X(2) + p_X(3) = (1 - 0.4)^1(0.4) + (1 - 0.4)^2(0.4) = 0.24 + 0.144 = 0.384$$

3. (Lecture 10) A random variable $X$ can take on the values \{-2, -1, 0, 1, 2\}. Its PMF is given by:

   $$p_X(x) = \begin{cases} 
   \frac{1}{10}, & \text{if } x = -2 \\
   \frac{1}{5}, & \text{if } x = -1 \\
   \frac{1}{5}, & \text{if } x = 0 \\
   \frac{1}{5}, & \text{if } x = 1 \\
   \frac{3}{10}, & \text{if } x = 2 \\
   0, & \text{otherwise}
   \end{cases}$$

   What is the PMF of $Y$, given that $Y = 2|X|$?
   
   The PMF of $Y$ is found by:
   
   $$p_Y(y) = \sum_{\{x|g(x) = y\}} p_X(x)$$

   The random variable $Y$ can take on the values \{0, 2, 4\}. Its PMF is thus given by:

   $$p_Y(y) = \begin{cases} 
   \frac{1}{10} + \frac{3}{10}, & \text{if } y = 4 \\
   \frac{1}{5} + \frac{1}{5}, & \text{if } y = 2 \\
   \frac{1}{5}, & \text{if } y = 0 \\
   0, & \text{otherwise}
   \end{cases} = \begin{cases} 
   \frac{2}{5}, & \text{if } y = 4 \\
   \frac{2}{5}, & \text{if } y = 2 \\
   \frac{1}{5}, & \text{if } y = 0 \\
   0, & \text{otherwise}
   \end{cases}$$