

Homework Problem Set 13 (THE LAST ONE!)

Due on Friday, April 6, 2011.

Note: There will be no class on Wednesday, April 11, due to ECEn on Display.

From Bertsekas and Tsitsiklis, *Introduction to Probability, 2nd Ed.*

1. (5 pts) Chapter 7 Problem 1.
2. (5 pts) Chapter 7 Problem 3.
3. (5 pts) Chapter 7 Problem 4.
4. (5 pts) Chapter 7 Problem 10.
5. (5 pts) Chapter 7 Problem 11.
6. (5 pts) Chapter 7 Problem 13.

7. MATLAB Problem (25 points)

Here we will employ some of your linear algebra skills to be able to find steady-state probabilities.

a) Find the Markov probability matrix \mathbf{M} associated with Problem 7.13 above.

b) Perform the following decomposition in MATLAB.

$$[\mathbf{V}, \mathbf{D}] = \text{eig}(\mathbf{M})$$

\mathbf{V} is your matrix of eigenvectors. \mathbf{D} is your matrix of eigenvalues. What you will notice is that you will have eigenvalues of 1 (steady-state solution) and eigenvalues of <1 which correspond to the transient.

c) Find the steady-state values by the following code:

$$\text{SS_matrix} = \mathbf{V} * [0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 1] / \mathbf{V}$$

You should then see that you have a steady-state solution which corresponds to the steady-state values that you calculated in Problem 13.

d) Notice that you can achieve the same by executing the code:

$$\mathbf{V} * \mathbf{D}^{100} / \mathbf{V}$$

This is basically just showing how the transients decay.

e) Then perform the following:

$$\mathbf{M}^{1000}$$

Why does this give you the same result as part d and c?

f) What happens if you take SS_matrix to the n th power? Why do you get the result that you do?

g) Consider the matrix, $\mathbf{M} = \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$. What happens to \mathbf{M}^n for large n ?

h) Consider the chain given by $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$. What happens to \mathbf{M}^n for large n ? Why?