RED- You can write on this exam.

ECEn 370
Introduction to Probability

Section 001

Midterm

Winter, 2012

Instructor

Professor Brian Mazzeo

Closed Book - You can bring one 8.5 X 11 sheet of handwritten notes on both sides.

Graphing or Scientific Calculator Allowed

3-Hour Suggested Time Limit

IMPORTANT!

- WRITE YOUR NAME on every page of the exam.
- Answer questions 1-27 on the provided bubble sheet.
- Questions 1-27 are worth 1 point each.
- Do not discuss the exam with other students.
- NOTE: Use all of the digits on your calculator, or fractions, before at the end rounding to the number of significant digits used in the problem.

1. A batch of fifty items is inspected by testing three randomly selected items. If one of the three is defective, the batch is rejected. What is the probability that the batch is accepted if it contains seven defectives?

A) 0.003
B) 0.140
C) 0.381
D) 0.429
E) 0.493
F) 0.630
G) 0.636
H) 0.676
I) 0.812
J) None of the Above

2. Ninety students, including Joe and Jane, are to be split into three classes of equal size, and this is to be done at random. What is the probability that Joe and Jane end up in the same class?

A) 0.106
B) 0.107
C) 0.322
D) 0.326
E) 0.328
F) 0.333
G) 0.339
H) 0.966
I) 0.983
J) None of the Above

3. If at first you don't succeed, try, try, try again. A computer will successfully send a message across a network with probability 0.6. The computer will retry sending the message until it is successfully sent. Given that we know that the computer will successfully transmit the message on or before the fourth attempt, what is the probability that the computer successfully sends the message on the first attempt?

A) 0.124
B) 0.240
C) 0.326
D) 0.375
E) 0.616
F) 0.625
G) 0.775
H) 0.938
I) 0.946
J) None of the Above

4. A random variable X has a probability density function given by

$$f_X(x) = \begin{cases} \frac{(x-2)^3}{4}, & \text{if } 2 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the cumulative distribution function, $F_X(x)$, and compute the value of the following expression:

$$F_X(1) + F_X(3) + F_X(9)$$

A) 0
B) 1/16
C) 1/4
D) 17/16
E) 2
F) 33/16
G) 4
H) 54
I) 1297/16
J) None of the Above

5. You have the following joint PMF of random variables X and Y:

y \ x	1	2	3	4	5
4	0	1/10	0	0	0
3	0	0	0	0	3/10
2	0	0	3/10	0	0
1	3/10	0	0	0	0

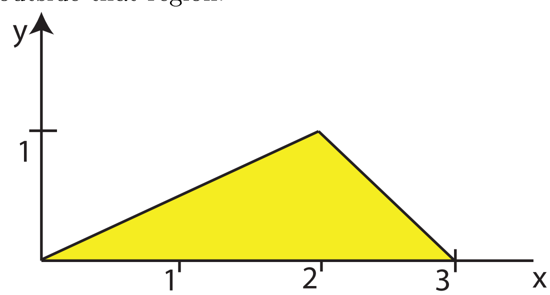
Find E[Z] where $Z = 2XY$

A) 0.58
B) 0.96
C) 2.20
D) 2.90
E) 6.38
F) 7.40
G) 12.76
H) 14.8
I) 24.9
J) None of the Above

6. You have three ten-sided dice numbered from 1 to 10. Each face has an equal probabilitiy of appearing during a roll (uniformly distributed). The outcomes, representing each die, are the random variables X, Y, and Z. Find $\mathrm{P}\left(min(X,Y,Z)=8\right)$.

A) 0.005
B) 0.019
C) 0.027
D) 0.037
E) 0.064
F) 0.091
G) 0.125
H) 0.216
I) 0.400
J) None of the Above

7. You have the following joint PDF with a constant density, $f_{X,Y}(x,y)=c$, in the shaded region and is zero outside that region:



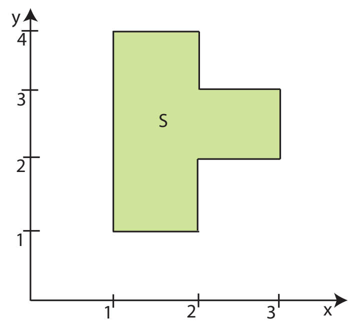
Find $\mathrm{E}[2X-3]$

A) 1/3
B) 4/3
C) 3/2
D) 5/3
E) 11/6
F) 2
G) 13/6
H) 7/3
I) 5/2
J) None of the Above

8. A binary signal S is transmitted, and we are given that $\mathrm{P}(S=1)=0.7$ and $\mathrm{P}(S=-1)=0.3$. The received signal is $Y=N+S$, where N is normal noise, with zero mean and unit variance, independent of S. What is the probability that $S=1$, as a function of the observed value of -0.1 for Y?

A) 0.141
B) 0.260
C) 0.344
D) 0.656
E) 0.677
F) 0.700
G) 0.740
H) 0.802
I) 0.859
J) None of the Above

9. We are told that the joint PDF of the random variables X and Y is a constant c on the set S shown in the figure below and is zero outside. We wish to determine the value of c and the marginal PDFs of X and Y.



Compute the following:
$c+f_Y(2.5)+f_Y(1.5)+f_Y(3.5)+f_X(0.5)+f_X(1.5)+f_X(2.5)$

A) 2
B) 9/4
C) 5/2
D) 11/4
E) 3
F) 13/4
G) 7/2
H) 15/4
I) 4
J) None of the Above

10. From the figure above in problem 9, compute the following where $F_{X,Y}$ is the CDF of the joint PDF.
$F_{X,Y}(-2,-1) + F_{X,Y}(1.5,1.5) + F_{X,Y}(3.5,1.5) + F_{X,Y}(2,2.5) + F_{X,Y}(2.5,4.5) + F_{X,Y}(4.5,6)$

A) 31/16
B) 32/16
C) 33/16
D) 34/16
E) 35/16
F) 36/16
G) 37/16
H) 38/16
I) 39/16
J) None of the Above

11. Each morning, Hungry Hungry Horace eats some sausages. On any given morning, the number of sausages he eats is equally likely to be 2, 3, 8, or 9, independent of what he has done in the past. Let X be the number of sausages that Harry eats in 10 days. Compute $q = \mathrm{E}[X] + \mathrm{var}(X)$.

A) $0 < q \leq 10$
B) $10 < q \leq 30$
C) $30 < q \leq 50$
D) $50 < q \leq 70$
E) $70 < q \leq 90$
F) $90 < q \leq 110$
G) $110 < q \leq 130$
H) $130 < q \leq 150$
I) $150 < q \leq 170$
J) None of the Above

12. A stock market trader buys 2 shares of stock A and 3 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers x and y satisfying

$$-1 \leq x \leq 2, \qquad 0 \leq x - y \leq 1$$

Find the mean of the trader's profits (or losses if negative).

A) -1
B) -1/2
C) 0
D) 1/2
E) 1
F) 3/2
G) 2
H) 5/2
I) 3
J) None of the Above

13. Let X be a random variable that takes values from 0 to 9 with equal probability 1/10. Find the PMF of the random variable $Y = \min(|X-3|, |X-4|, |X-5|)$.

Calculate $p_Y(0) + p_Y(1)$

A) 1/10
B) 2/10
C) 3/10
D) 4/10
E) 5/10
F) 6/10
G) 7/10
H) 8/10
I) 9/10
J) None of the Above

14. Al thows darts at a circular target of radius 2 m and is equally likely to hit any point on the target. Let X be the distance of Al's hit from the center. Find $\mathrm{E}[X] + \mathrm{var}(X)$.

A) 1.801 m
B) 1.803 m
C) 1.805 m
D) 1.807 m
E) 1.809 m
F) 1.811 m
G) 1.813 m
H) 1.815 m
I) 1.817 m
J) None of the Above

15. Let A, B, and C be three events in Ω. If $\mathrm{P}(A) = \frac{1}{3}$, $\mathrm{P}(B) = \frac{1}{4}$, $\mathrm{P}(C) = \frac{1}{2}$, $\mathrm{P}(A \cap B) = \frac{1}{8}$, $\mathrm{P}(A \cap C) = \frac{1}{6}$, and $\mathrm{P}(B \cap C) = 0$, find $\mathrm{P}(A \cup B \cup C)$.

A) 16/24
B) 17/24
C) 18/24
D) 19/24
E) 20/24
F) 21/24
G) 22/24
H) 23/24
I) 1
J) None of the Above

16. Suppose you are in a Family Home Evening Group with ten men and ten women. On any given Monday night, the probability of a boy showing up is $8/10$ and the probability of a girl showing up is $9/10$, independently of any one else. What is the probability, P, that less than two people show up?

A) $10^{-12} < P < 10^{-11}$
B) $10^{-13} < P < 10^{-12}$
C) $10^{-14} < P < 10^{-13}$
D) $10^{-15} < P < 10^{-14}$
E) $10^{-16} < P < 10^{-15}$
F) $10^{-17} < P < 10^{-16}$
G) $10^{-18} < P < 10^{-17}$
H) $10^{-19} < P < 10^{-18}$
I) $10^{-20} < P < 10^{-19}$
J) None of the Above

17. Suppose I have a uniform random variable, X, with the following PMF:

$$p_X(x) = \begin{cases} 1/5, & \text{if } x = 3,\ 4,\ 5,\ 6,\ 7 \\ 0, & \text{otherwise.} \end{cases}$$

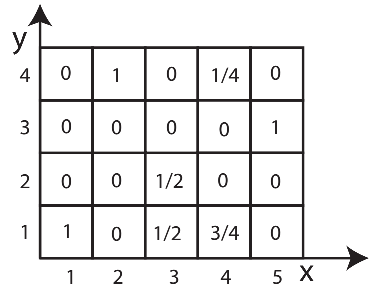
Suppose I have a random variable, $Z = (X-4)^2$. What is $\mathrm{E}[Z]$?

A) -1
B) -1/2
C) 0
D) 1/2
E) 1
F) 3/2
G) 2
H) 5/2
I) 3
J) None of the Above

18. Messages transmitted by a computer in Provo through a data network are destined for Salt Lake City with probability 0.5, Las Vegas, with probability 0.3, and for Denver with probability 0.2. The transit time X of a message is random. Its mean is 0.05 seconds if it is destined for Salt Lake City, 0.1 seconds if it is destined for Las Vegas, and 0.3 seconds if it is destined for Denver. Calculate $\mathbf{E}[X]$.

A) 0.000045
B) 0.00045
C) 0.0045
D) 0.045
E) 0.115
F) 0.125
G) 0.45
H) 4.5
I) 45
J) None of the Above

The following conditional PMF is used for problems 19 and 20. You have the following conditional PMF of random variable Y conditioned on random variable X:



and marginal PMF of X given by:

$$p_X(x) = \begin{cases} 1/6, & \text{if } x = 1,2,3,4 \\ 1/3, & \text{if } x = 5 \\ 0, & \text{otherwise.} \end{cases}$$

The above conditional PMF is used for problems 19 and 20.

19. For the conditions above, compute $\mathbf{E}[Z]$, where $Z = 12XY$.

A) 101
B) 102
C) 103
D) 104
E) 105
F) 106
G) 107
H) 108
I) 109
J) None of the Above

20. Find the probability $\mathrm{P}(\min(X,Y) = 3)$.

A) 1/24
B) 1/8
C) 1/6
D) 1/3
E) 1/2
F) 7/12
G) 2/3
H) 3/4
I) 1
J) None of the Above

21. A proportion, p, of BYU students have served missions. I question N BYU students and I sum up the number that have served missions. Since I can't talk to all of the students, to estimate the proportion I then take my sum and divide by N to form a sample average to estimate p. Mark on your answer sheet all of the statements that are true (you can have multiple bubbles filled in on this one, if necessary).

A) The expected value of the sample average decreases as N increases.
B) The expected value of the sample average is the same as N increases.
C) The expected value of the sample average increases as N increases.
D) The variance of the sample average decreases as N increases.
E) The variance of the sample average stays the same as N increases.
F) The variance of the sample average increases as N increases.
G) If N is 1, then the expected value of the sample average is 1 or 0, depending on p.
H) If N is 1, then the expected value of the sample average is p.
I) If I ask a different set of N BYU students, I will always get the same sample average.
J) None of the above are true.

22. Let X be the roll of a fair six-sided die and let A be the event that the roll is an even number. Find $p_{X|A}(4)$.

A) 1/12
B) 2/12
C) 3/12
D) 4/12
E) 5/12
F) 6/12
G) 7/12
H) 8/12
I) 9/12
J) None of the Above

23. The PMF for X is given by

$$p_X(x) = \begin{cases} 1/9, & \text{if } x \text{ is an integer in the range } [-4, 4], \\ 0, & \text{otherwise} \end{cases}$$

Let $Y = |X|$.
What is $p_Y(-1) + p_Y(0) + p_Y(1) + p_Y(2)$?

A) 1/9
B) 2/9
C) 3/9
D) 4/9
E) 5/9
F) 6/9
G) 7/9
H) 8/9
I) 1
J) None of the Above

24. A parking lot contains 100 cars, 10 of which happen to be lemons. We select 4 of these cars at random and take them for a test drive. Find the probability that 2 of the cars tested turn out to be lemons.

A) 5.00×10^{-5}
B) 0.24×10^{-2}
C) 3.24×10^{-2}
D) 4.00×10^{-2}
E) 4.60×10^{-2}
F) 4.86×10^{-2}
G) 5.00×10^{-2}
H) 8.100×10^{-2}
I) 40.0×10^{-2}
J) None of the Above

For problems 25 and 26,

A source transmits a message (a string of symbols) through a noisy communication channel. Each sysmbol is 0 or 1 with probability 0.4 and 0.6, respectively, and is received incorrectly with probability 0.3 and 0.7, respectively. Errors in different symbol transmissions are independent.

25. What is the probability that the string of symbols 1011 is received correctly?

A) 0.0081
B) 0.0189
C) 0.0256
D) 0.0384
E) 0.0441
F) 0.0864
G) 0.1029
H) 0.2401
I) 0. 2535
J) None of the Above

26. In an effort to improve reliability, each symbol is transmitted three times and the received string is decoded by majority rule. In orther words, a 0 (or 1) is trasmitted as 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a 0 is correctly decoded?

A) 0.441
B) 0.443
C) 0.445
D) 0.447
E) 0.449
F) 0.451
G) 0.453
H) 0.455
I) 0.457
J) None of the Above

27. This is a difficult problem for the last one. The joint pdf of random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{y}e^{-x/y}e^{-y} & x > 0,\, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find $\mathrm{P}\left(X > 1 | Y = 2\right)$.

A) 0.45
B) 0.47
C) 0.49
D) 0.51
E) 0.53
F) 0.55
G) 0.57
H) 0.59
I) 0.61
J) None of the Above