

Homework Problem Set 13 (THE LAST ONE!)

Due on Wednesday, April 9, 2014.

From Bertsekas and Tsitsiklis, *Introduction to Probability, 2nd Ed.*

1. (5 pts) Chapter 7 Problem 1.
2. (5 pts) Chapter 7 Problem 3.
3. (5 pts) Chapter 7 Problem 4.
4. (5 pts) Chapter 7 Problem 10.
5. (5 pts) Chapter 7 Problem 11.
6. (5 pts) Chapter 7 Problem 13.

7. MATLAB Problem (25 points)

Here we will employ some of your linear algebra skills to be able to find steady-state probabilities.

a) Find the Markov probability matrix  $\mathbf{M}$  associated with Problem 7.13 above.

b) Perform the following decomposition in MATLAB.

$$[V, D] = \text{eig}(M)$$

$V$  is your matrix of eigenvectors.  $D$  is your matrix of eigenvalues. What you will notice is that you will have eigenvalues of 1 (steady-state solution) and eigenvalues of  $<1$  which correspond to the transient.

c) Find the steady-state values by the following code:

$$\text{SS\_matrix} = V * [0 \ 0 \ 0; 0 \ 0 \ 0; 0 \ 0 \ 1] / V$$

You should then see that you have a steady-state solution which corresponds to the steady-state values that you calculated in Problem 13.

d) Notice that you can achieve the same by executing the code:

$$V * D^{100} / V$$

This is basically just showing how the transients decay.

e) Then perform the following:

$$M^{1000}$$

Why does this give you the same result as part d and c?

f) What happens if you take  $\text{SS\_matrix}$  to the  $n$ th power? Why do you get the result that you do?

g) Consider the matrix,  $\mathbf{M} = \begin{bmatrix} 0.2 & 0.8 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$ . What happens to  $\mathbf{M}^n$  for large  $n$ ?

h) Consider the chain given by  $\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$ . What happens to  $\mathbf{M}^n$  for large  $n$ ? Why?