

Homework Problem Set 5

Due on Wednesday, February 12, 2014.

From Bertsekas and Tsitsiklis, *Introduction to Probability, 2nd Ed.* and Schaum's.

1. (5 pts) Schaum's 3.14.
2. (5 pts) Schaum's 3.26.
3. (5 pts) Chapter 2 Problem 38.
4. (5 pts) Chapter 2 Problem 39.
5. (5 pts) Chapter 2 Problem 40.
6. (5 pts) Chapter 2 Problem 41.
7. (5 pts) Chapter 3 Problem 1.
8. (5 pts) Chapter 3 Problem 2.
9. (5 pts) Schaum's 2.45.
10. (30 pts) MATLAB Problem. Adapted from Kay. Expectation and Conditioning of Discrete Random Variables.

There are many different ways to simulate joint random variables. We will explore two approaches in this problem. Consider that you have the following joint PMF, $p_{X,Y}(x,y)$:

	x=0	x=1
y=0	1/8	1/8
y=1	1/4	1/2

One way to simulate this is to generate a decent sized vector of random variables. Since these variables are between 0 and 1, break up the unit interval as follows (to visualize this, draw a line and label the end points 0 and 1): For those random numbers obtained between 0 and 1/8, you assign $x = 0, y = 0$ (table coordinate point (0,0), note you need two vectors x and y); for values between 1/8 and 2/8, you choose (1,0), for values between 2/8 and 4/8 you choose (0,1), and for values between 4/8 and 1, you choose (1,1). Code this as a big 'if' statement. Pay attention to what is going on here, note from the table that (0,0) has a joint PMF of 1/8 and so does (1,0), while (1,1) has a whole half (the second half in our unit interval). The script "simulate_joint_PMF.m" from HW4 does this for a much more general case.

a) Using this approach, write your own MATLAB script to simulate the joint PMF above. Use 10,000 trials. Now check if your probability of getting a point (x,y) matches those in the table. For example, check the times that both x and y equal zero and divide over your total number of trials. Turn in your script and simulation results.

Another approach to this problem is to use conditioning and the fact that $p_{X,Y}(x,y) = p_{Y|X}(y|x)p_X(x)$. First, determine the marginal PMF, $p_X(x)$. Then determine the conditional PMF, $p_{Y|X}(y|x)$.

To simulate you first determine an x by splitting up the uniform random variable. Then given that x , you determine a y by simulating the conditional PMF. Draw another unit line, what portion of it corresponds to the marginal PMF of $x = 0$? What does the conditional PMF, $p_{Y|X}(y|0)$, look like?

b) Write a MATLAB script that performs this second approach. Simulate it for 10,000 trials and make sure that it matches part "a" and the original joint PMF. Turn in your script and simulation results.

c) Calculate the expectation of X analytically. Then take the average of the results from your marginal PMF that you simulated above. Does the expectation of X match the average of the results from your marginal PMF?

d) Calculate the variance of X analytically. Then calculate the variance of the results from your marginal PMF that you simulated above. Does the analytical variance of X match the simulated variance?