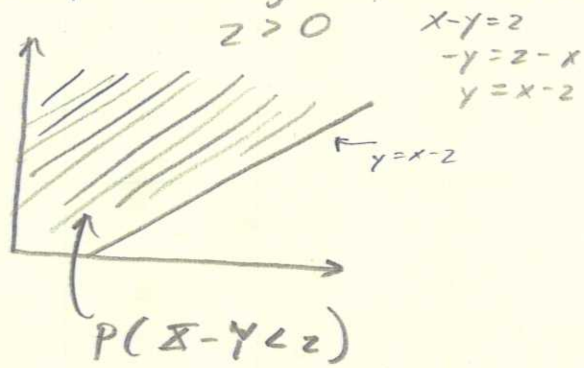
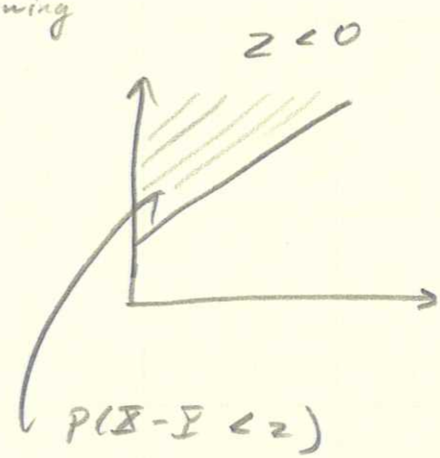
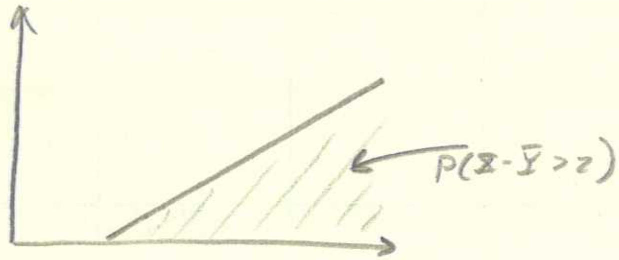
Problem 4.9

The important thing to consider is the following





This would be a difficult piecewise integral so, we use $1 - P(X - Y > z)$



For $z > 0$,

$$P(X - Y < z)$$
$$= 1 - P(X - Y > z)$$
$$= 1 - \int_0^\infty \int_{z+y}^\infty f_{X,Y}(x,y)\, dx\, dy$$

For $z < 0$,

$$P(X - Y < z)$$
$$= \int_0^\infty \int_{x-z}^\infty f_{X,Y}(x,y)\, \underbrace{dy\, dx}_{\text{Notice, this is different}}$$

If you solve these integrals, you will get the CDFs you need.

Problem 4.17

In the statement that we can assume X and Y have zero mean, they mean to say that because

$$\mathrm{cov}(X-Y,\ X+Y) = \mathrm{cov}(X-Y+b,\ X+Y+c)$$ where b and c are scalars.

then we can simplify our math by assuming that
$E[X]=0$, $E[Y]=0$

$$\mathrm{cov}(X-Y, X+Y) = E[(X-Y)(X+Y)] - E[X-Y]\,E[X+Y]$$

$$= E[(X-Y)(X+Y)] - \underbrace{(E[X]-E[Y])(E[X]+E[Y])}_{\text{This can then be eliminated}}$$

$$= E[(X-Y)(X+Y)]$$

$$= E[X^2] - E[Y^2]$$

Because $E[X]=0$ and $E[Y]=0$

$$= \mathrm{var}(X) - \mathrm{var}(Y)$$

$= 0$ because $\mathrm{var}(X) = \mathrm{var}(Y)$ by assumption.