1. (2 pts) A discrete-time lowpass filter is to be designed by applying the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function

$$|H_C(j\Omega)^2| = \frac{1}{1 + (\Omega/\Omega_c)^{2N}}$$

The specifications for the discrete-time system are as follows:

$$0.9 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq |\omega| \leq 0.3\pi$$
$$|H(e^{j\omega})| \leq 0.1, \quad 0.5\pi \leq |\omega| \leq \pi$$

Assume that aliasing will not be a problem.

a) (3 pts) Sketch the tolerance bounds of on the magnitude of the frequency response, $|H_c(j\Omega)|$, of the continuous-time Butterworth filter such that after the application of the impulse invariance method ($h[n] = T_d h_c(n T_d)$), the resulting discrete-time filter will satisfy the given design specifications.

b) (7 pts) Determine the integer order $N$ and the quantity $T_d \Omega_c$ such that the continuous-time Butterworth filter exactly meets the specifications determined in part (a) at the passband edge.