Prediction of snow permittivity and air temperature using received snow backscatter values over Greenland

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***Abstract*— The dry snow zone is the largest component of the Greenland ice sheet and is identified as the region that experiences no annual melt. Therefore, radar backscatter (σ^0) is expected to be relatively constant over time within the dry snow. However, annual variation was discovered in QuikSCAT data. This paper tests the hypothesis that annual variation in backscatter is caused by changes in permittivity. To do this, a model is provided relating permittivity to backscatter. Using this model, we test if received backscatter values can predict the permittivity and temperature of the corresponding snow. Both ML and MAP estimators are employed, and MAP is shown to have superior performance for the selected values of SNR. However, neither estimator is shown to consistently predict permittivity.**

I. Introduction

Satellite-borne scatterometers are radars designed for a variety of purposes. One of these purposes is monitoring important indicators of the global climate such as the Greenland and Antarctic ice sheets. The accuracy of these applications requires accurate calibration of the scatterometer. Although scatterometers are calibrated prior to launch, system degradation requires the scatterometer to be calibrated after launch as well. Accurate post-launch calibration can be achieved by using radar backscatter data from natural land targets with temporally constant and isotropic backscatter. Currently, post-launch scatterometer calibration is performed using data collected from the Greenland and Antarctic ice sheets [1].

The Greenland ice sheet is divided into zones or facies which are distinguished by their melting characteristics. The dry snow region is the largest part of the ice sheet and is characterized by no melt throughout the year. Thus, σ^0 in the dry snow zone should be relatively constant. Recently, an anomaly in QuikSCAT data was discovered in the dry snow region. This anomaly is characterized by a slight decrease in σ_0 during the summer months followed by a return to the winter backscatter. An example of this cyclical variation is shown in Fig. 1.

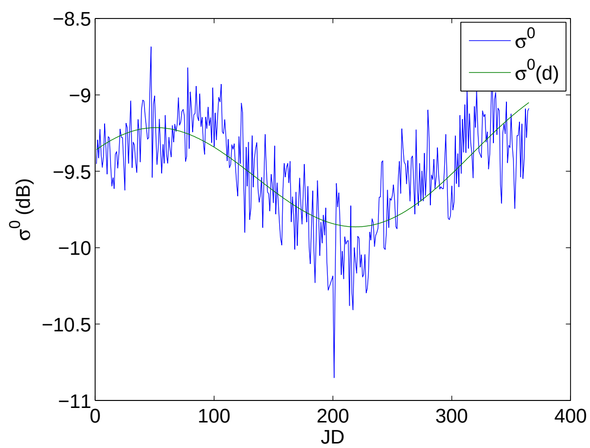

Fig. 1. σ^0 at $-59.2229°$ longitude and $77.1670°$ latitude in 2006.

This cyclical variation has been shown to not be caused by instrumentation of QuikSCAT.

In this paper, we examine the feasibility of predicting changes in permittivity based on received backscatter values in the dry snow zone. A model is created which relates temperature to backscatter by using known relations between temperature and snow density, snow density and permittivity, and permittivity and backscatter. The sensitivity of the model is tested by estimating the expected permittivity of a backscatter value corrupted by additive noise. The permittivity is estimated using both maximum-likelihood (ML) and maximum a-posteriori (MAP) estimation, where the temperature is used as prior data for MAP estimation.

By testing the sensitivity of the model, we determine if the received backscatter can be used to reliably predict the permittivity. This aids in our exploration of causes for annual variation. If typical amounts of atmospheric and thermal noise distort the backscatter values such that incorrect permittivity values are estimated, then backscatter cannot be used to predict permittivity.

This paper is organized as follows. In Section II, we provide analytical models relating temperature to permittivity and snow density, which in turn are related to the received backscatter. In Section III, we test the sensitivity of the model by introducing additive noise into the backscatter model and estimating the change in permittivity using ML and MAP estimation. In Section IV the results of the estimations are given. In Section V these results are discussed with possible explanations for their behavior.

II. Analytical Model

In this section, a model is created which relates permittivity to backscatter σ^0. Because MAP estimation is conditioned on known temperature values, the relationship between temperature and backscatter is also modeled. This model is created by deriving the relationship between temperature and snow density and the relationship between snow density and permittivity.

A number of assumptions are made in this model. It is assumed that snow crystals stay the same size at all subfreezing temperatures (an assumption borrowed from [2]). It is assumed that there is an infinite layer of snow below the surface. This assumption is reasonable because the Greenland ice sheet is several kilometers thick. Because snow density data was only available down to $-20°C$, this model is only valid between $-20°$ and $0°C$.

A. Relation of Temperature to Snow Density

Snow density is defined as the mass of the snow to the volume of the snow, given in g/cm^3. Data relating air temperature to snow density is found in [3] and [4]. Given a temperature T, the density of snow ρ_s is roughly given by

$$\rho_s = \frac{11}{1400}T + \frac{109}{700}. \tag{1}$$

B. Relation of Snow Density to Permittivity

In [2], snow is modeled as spherically shaped ice crystals in some background medium. In the case of dry snow, the background is simply air. The real part of the permittivity of dry snow, ϵ'_{ds}, is found to be related to the snow density by

$$\epsilon'_{ds} = (1 + 0.51\rho_s)^3. \tag{2}$$

The imaginary part ϵ''_{ds} depends on the real part of the permittivity of dry snow and the permittivity of ice ϵ_i [2]. It is given as

$$\epsilon''_{ds} = 3v_i\epsilon''_i \frac{(\epsilon'_{ds})^2(2\epsilon'_{ds}+1)}{(\epsilon'_i + 2\epsilon'_{ds})(\epsilon'_i + 2(\epsilon'_{ds})^2)} \tag{3}$$

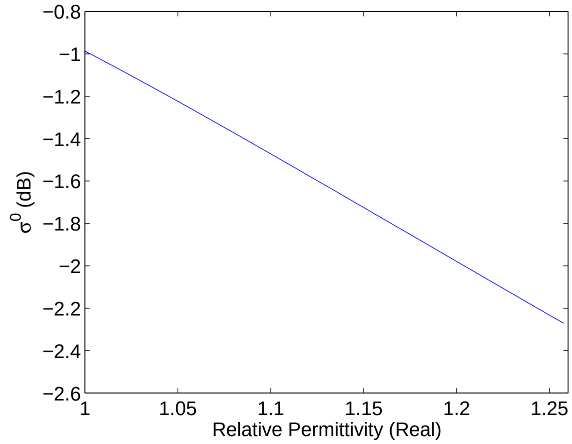

Fig. 2. σ^0 versus permittivity according to Eq. (4).

where v_i is the volume fraction of ice in the snow given by $v_i = \rho_s/0.916$ where 0.916 is the density of pure ice. The parameter ϵ'_i is assumed to have a constant value of 3.15. The imaginary part ϵ''_i depends on the operating frequency of the scatterometer and the conductivity of ice which does vary slightly with temperature [5].

C. Relation of Permittivity and Snow Density to Received Backscatter

The backscatter of the scatterometer in an infinite layer of dry snow is given by [6] as

$$\sigma^0(\theta) = \Upsilon^2(\theta)\frac{\sigma_v cos(\theta')}{2\kappa_e} \tag{4}$$

where $\Upsilon(\theta)$ is the transmissivity of the air-snow surface, θ is the angle of incidence, σ_v is the volume scattering coefficient, θ' is the transmitted angle, and κ_e is the extinction coefficient. We use a fixed θ of $54°$. The parameters $\Upsilon(\theta)$, σ_v, θ', and κ_e all depend on the permittivity of the snow. The equations used to calculate these parameters are derived from [2], [6], [7]. Figure 2 shows the relationship between σ^0 and the real part of permittivity which is the dominating part.

III. Simulation

A simulation can be performed to test the sensitivity of the model. The purpose of this simulation is to test whether the permittivity could be estimated from received backscatter values.

To generate the received backscatter values, a series of steps are taken. First, a temperature is randomly generated. It is not assumed that all temperatures between $-20°$ C and $0°$ C were equally likely. Rather, the

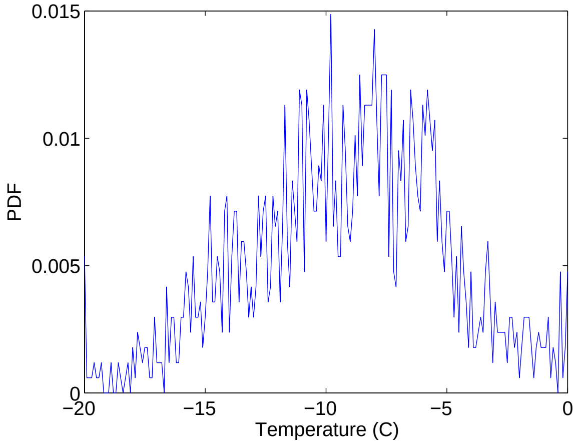

Fig. 3. Normalized histogram of temperature data collected on the Greenland ice sheet in 2006.

likelihood of a given temperature is weighted according to a histogram formed from 70 days of empirical data collected from June 11-August 20, 2006 and divided into $L = 200$ bins (see Fig. 3). Using a given temperature, the snow density was calculated using Eq. (2), the permittivity is calculated using Eq. (3), and the true backscatter value is calculated using Eq. (4).

The true backscatter value σ^0_{true} is then corrupted by additive white Gaussian noise. Typical noise values corresponding to noise in the electronics, satellite, atmosphere, and other random sources correspond to SNR values ranging from 11 dB to 20 dB. The corrupted backscatter is denoted σ^0_{rec} where

$$\sigma^0_{\text{rec}} = \sigma^0_{\text{true}} + \eta \tag{5}$$

where $\eta \sim \mathcal{N}(0, \sigma^2)$ with $\sigma^2 = \frac{N_0}{2}$. $\frac{N_0}{2}$ is the power spectral density, and N_0 is the transmitted power divided by the linear SNR.

Using the simulated received backscatter σ^0_{rec}, the permittivity of the snow can be estimated using both ML and MAP estimation.

A. ML Estimation

To estimate the actual permittivity value ϵ_{true} using the received backscatter σ^0_r, the maximum-likelihood test can be employed. We define $P(\epsilon_{ds})$ as the probability that ϵ_{ds} occurs.

The decision rule that maximizes the probability of choosing $\epsilon_{ds} = \epsilon_{\text{true}}$ is [8]

$$\hat{\epsilon}_{ds} = \arg\max_{\epsilon_{ds}} P(\epsilon_{ds}|\sigma^0_{\text{rec}}). \tag{6}$$

Using Bayes' rule, Eq. (6) can be written as

$$\begin{aligned} \hat{\epsilon}_{ds} &= \arg\max_{\epsilon_{ds}} P(\epsilon_{ds}|\sigma^0_{\text{rec}}) \\ &= \arg\max_{\epsilon_{ds}} \frac{p(\sigma^0_{\text{rec}}|\epsilon_{ds})P(\epsilon_{ds})}{p(\sigma^0_{\text{rec}})} \end{aligned} \tag{7}$$

where $p(\sigma^0_{\text{rec}})$ is the probability of σ^0_{rec} being received. Since the denominator of Eq. (7) does not depend on ϵ_{ds},

$$\hat{\epsilon}_{ds} = \arg\max_{\epsilon_{ds}} p(\sigma^0_{\text{rec}}|\epsilon_{ds})P(\epsilon_{ds}). \tag{8}$$

Since ML estimation assumes that all prior probabilities are equal, it means all values of $P(\epsilon_{ds})$ are equal, so $P(\epsilon_{ds})$ can be factored out of the equation to obtain

$$\hat{\epsilon_{ds}} = \arg\max_{\epsilon_{ds}} p(\sigma^0_{rec}|\epsilon_{ds}). \tag{9}$$

To calculate $p(\sigma^0_r|\epsilon_{ds})$, we assume that it is equivalent to calculating $p(\sigma^0_r|\sigma^0_{ds})$, where σ^0_{ds} is the backscatter value corresponding to ϵ_{ds}. Since there is a function relating ϵ_{ds} to σ^0_{ds}, $p(\sigma^0_{ds}|\epsilon_{ds}) = 1$.

Note that $\sigma^0_{rec} = \sigma^0_{true} + \eta$. Thus, $p(\sigma^0_{rec}|\sigma^0_{ds})$ can be shown to have a Gaussian distribution with mean 0 and variance σ^2. In the simulation, there were $L = 200$ discrete choices for σ^0 based on the L possible temperature values. Thus, the likelihood function corresponding to $\sigma^0_{ds} = \sigma^0_i$, $i = 1...L$ is

$$p(\sigma^0_{\text{rec}}|\sigma^0_{ds}) = \sigma^0_i) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2\sigma^2}(\sigma^0_{\text{rec}} - \sigma^0_i)^2. \tag{10}$$

This probability is maximized when ${\sigma^0}_{ds}$ is chosen to be the σ^0_i which minimizes the Euclidean distance between σ^0_{rec} and σ^0_i, that is,

$$\hat{\sigma}^0_{ds} = \arg\min_{\sigma^0_i} |\sigma^0 - \sigma^0_i|. \tag{11}$$

The ϵ_i corresponding to σ^0_i is then estimated to be the correct value. If $\sigma^0_i = \sigma^0_{\text{true}}$, then the estimator is correct and there is no error.

B. MAP Estimation

To perform MAP estimation on the data, many of the equations derived in Section III-A hold. However, MAP estimation no longer assumes equal prior probabilities of $P(\epsilon_{ds})$ in Eq. (8). Thus, the decision rule for MAP is

$$\hat{\epsilon}_{ds} = \arg\max_{\epsilon_{ds}} p(\sigma^0_{\text{rec}}|\epsilon_{ds})P(\epsilon_{ds}). \tag{12}$$

To calculate $P(\epsilon_{ds})$, the weighted temperature data is used. Since there is a function relating temperature to permittivity, the same weighting can be applied to the permittivity as is applied to the temperature. Hence, $\hat{\epsilon}_{ds}$ simultaneously maximizes the Gaussian distribution

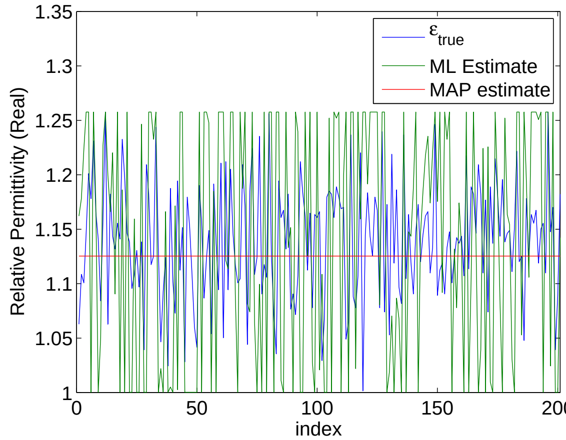

Fig. 4. True permittivity and the estimated values of permittivity using ML and MAP estimation with an SNR of 13 dB.

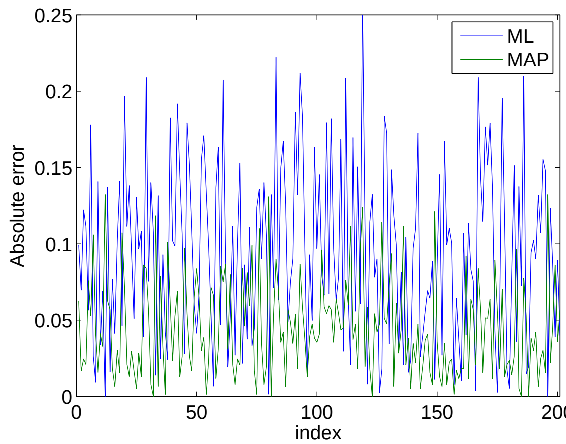

Fig. 5. Absolute error in permittivity for both ML and MAP estimation with an SNR of 13 dB.

given in Eq. 10 and the distribution of prior probabilities given by the histogram shown in Fig. 3.

IV. Results

Figures 4 and 5 give some of the results of the simulation. Figure 4 gives the estimated values of permittivity for both the ML and MAP estimation methods with an SNR of 13 dB. Figure 5 shows the error in permittivity for both methods with the same SNR.

V. Discussion

In general, the error of both the ML and MAP estimators is high compared to the range of possible permittivity values, which range from 1 to 1.25. However, as SNR increases (not shown here), the error of the ML estimator does decrease. Changing the SNR does not affect the MAP estimator. Due to the nature of the prior used in the MAP decision rule, the estimator consistently chooses the permittivity corresponding to the temperature with highest probability independent of the actual permittivity. Since this value is the most likely to start with, the MAP error is smaller than the ML error for the tested values of SNR. Increasing the SNR beyond practical values will give the ML estimator the smaller error.

VI. Conclusion

We found that there is a relationship between permittivity and backscatter in the dry snow zone at temperatures below freezing. However, with our model, the received backscatter values are too noisy to accurately estimate the permittivity of dry snow. This is shown for both the ML and MAP estimators, where temperature data is used as a prior. Future work could include altering the MAP estimator by using different data sets as priors. Additionally, the theoretical model relating permittivity and backscatter could be refined to account for varying snow grain sizes.

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