

4.22) Θ uniform on $[-\pi, \pi]$

\bar{Y} has mean m and variance σ^2

Θ and \bar{Y} are independent

$$\{\bar{X}(t); t \in \mathbb{R}\} \quad \bar{X}(t) = \bar{Y} \cos(2\pi f_0 t + \Theta)$$

Find mean and auto correlation function.

$$E[\bar{X}(t)] = E[\bar{Y} \cos(2\pi f_0 t + \Theta)] = E[\bar{Y}] E[\cos(2\pi f_0 t + \Theta)] \\ = 0$$

$$R_{\bar{X}}(t, s) = E[\bar{X}_t \bar{X}_s] ; \text{ all } t, s \in \mathbb{T}$$

$$= E[(\bar{Y} \cos(2\pi f_0 t + \Theta))(\bar{Y} \cos(2\pi f_0 s + \Theta))] \quad \text{var}(\bar{Y}) = E[\bar{Y}^2] - (E[\bar{Y}])^2 \\ t[\bar{Y}^2] = \sigma^2 + m^2$$

$$= E[\bar{Y}^2] E[\cos(2\pi f_0 t + \Theta) \cos(2\pi f_0 s + \Theta)]$$

$$= (\sigma^2 + m^2) E[\cos(2\pi f_0 t + \Theta) \cos(2\pi f_0 s + \Theta)]$$

$$= (\sigma^2 + m^2) \int_{-\pi}^{\pi} \frac{1}{2\pi} \cos(2\pi f_0 t + \Theta) \cos(2\pi f_0 s + \Theta) d\Theta$$

$$= \frac{(\sigma^2 + m^2)}{2\pi} \int_{-\pi}^{\pi} [\cos(2\pi f_0 t) \cos(\Theta) - \sin(2\pi f_0 t) \sin(\Theta)][\cos(2\pi f_0 s) \cos(\Theta) - \sin(2\pi f_0 s) \sin(\Theta)] d\Theta$$

$$= \frac{(\sigma^2 + m^2)}{2\pi} \int_{-\pi}^{\pi} [\cos(2\pi f_0 t) \cos(2\pi f_0 s) \cos^2(\Theta) - \sin(2\pi f_0 t) \cos(2\pi f_0 s) \sin(\Theta) \cos(\Theta) - \sin(2\pi f_0 t) \cos(2\pi f_0 s) \sin(\Theta) \cos(\Theta) \\ + \sin(2\pi f_0 t) \sin(2\pi f_0 s) \sin^2(\Theta)] d\Theta$$

$$\left\{ \int_{-\pi}^{\pi} (\sin \Theta)(\cos \Theta) d\Theta = \frac{1}{2} \sin^2 \Theta \Big|_{-\pi}^{\pi} = 0 \right\}$$

$$= \frac{(\sigma^2 + m^2)}{2\pi} \int_{-\pi}^{\pi} [\cos(2\pi f_0 t) \cos(2\pi f_0 s) \cos^2(\Theta) + \sin(2\pi f_0 t) \sin(2\pi f_0 s) \sin^2(\Theta)] d\Theta$$

$$= \frac{(\sigma^2 + m^2)}{2\pi} (\pi) [\cos(2\pi f_0 t) \cos(2\pi f_0 s) + \sin(2\pi f_0 t) \sin(2\pi f_0 s)]$$

$$\int_{-\pi}^{\pi} \cos^2 \Theta = \left[\frac{1}{2} \Theta + \frac{1}{4} \sin 2\Theta \right]_{-\pi}^{\pi} = \frac{2\pi}{2} = \pi$$

$$\int_{-\pi}^{\pi} \sin^2 \Theta = \left[\frac{1}{2} \Theta - \frac{1}{4} \sin 2\Theta \right]_{-\pi}^{\pi} = \frac{2\pi}{2} = \pi$$

$$= \frac{\sigma^2 + m^2}{2} [\cos(2\pi f_0 t - 2\pi f_0 s)]$$

$$= \frac{\sigma^2 + m^2}{2} \cos(2\pi f(t-s))$$

4.22 cont...)

$$\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \bar{X}(t) dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \bar{Y} \cos(2\pi f_0 t + \theta) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \left[\frac{\bar{Y} \sin(2\pi f_0 T + \theta)}{2\pi f_0} \right]_0^T$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \left(\frac{\bar{Y}}{2\pi f_0} \right) [\sin(2\pi f_0 T + \theta) - \sin(\theta)]$$

$$\downarrow \left| \left| \frac{\bar{Y}}{2\pi f_0} [\sin(2\pi f_0 T + \theta) - \sin(\theta)] \right| \right| \leq \left| \left| \frac{2\bar{Y}}{2\pi f_0} \right| \right|$$

$$\left| \left| \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \bar{X}(t) dt \right| \right| \leq \lim_{T \rightarrow \infty} \frac{1}{T} \left| \left| \frac{2\bar{Y}}{2\pi f_0} \right| \right| = 0 \quad \text{because } \left| \left| \frac{2\bar{Y}}{2\pi f_0} \right| \right| \text{ is finite}$$

∴ $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \bar{X}(t) dt = 0.$

$$\begin{aligned}
 4.26) \quad \hat{\bar{X}}_n &= a \bar{X}_{n-1} + b \\
 \epsilon &\triangleq E[(\bar{X}_n - \hat{\bar{X}}_n)^2] \\
 &= E[\bar{X}_n^2] - 2E\bar{X}_n \hat{\bar{X}}_n + E[\hat{\bar{X}}_n^2] \\
 &= R_{\bar{X}}(n, n) - 2a E[\bar{X}_n \bar{X}_{n-1}] - 2b E[\bar{X}_n] + a^2 E[\bar{X}_{n-1}^2] + 2ab E[\bar{X}_{n-1}] + b^2 \\
 &= R_{\bar{X}}(n, n) - 2a R_{\bar{X}}(n, n-1) - 2bm_n + a^2 R_{\bar{X}}(n-1, n-1) + 2abm_{n-1} + b^2
 \end{aligned}$$

We want the derivative to be zero:

$$\begin{aligned}
 0 = \frac{d\epsilon}{db} &= -2m_n + 2am_{n-1} + 2b \\
 -m_n + am_{n-1} + b &= 0 \\
 b &= m_n - am_{n-1}
 \end{aligned}$$

$$\begin{aligned}
 \text{Notice } \epsilon &= E[(\bar{X}_n - m_n) - a(\bar{X}_{n-1} - m_{n-1})]^2 \\
 &= k_{\bar{X}}(n, n) - 2a k_{\bar{X}}(n-1, n-1) \\
 &\quad + a^2 k_{\bar{X}}(n-1, n-1)
 \end{aligned}$$

$$\begin{aligned}
 \epsilon &= R_{\bar{X}}(n, n) - 2a R_{\bar{X}}(n, n-1) - 2(m_n - am_{n-1})m_n + a^2 R_{\bar{X}}(n-1, n-1) + 2a(m_n - am_{n-1})m_{n-1} \\
 &\quad + (m_n - am_{n-1})^2 \\
 &= R_{\bar{X}}(n, n) + a^2 R_{\bar{X}}(n-1, n-1) - 2a R_{\bar{X}}(n, n-1) \\
 &\quad - 2m_n^2 + 2am_n m_{n-1} + 2am_n m_{n-1} - 2a^2 m_{n-1}^2 + m_n^2 - 2am_n m_{n-1} + a^2 m_{n-1}^2 \\
 &= K_{\bar{X}}(n, n) + a^2 K_{\bar{X}}(n-1, n-1) - 2a k_{\bar{X}}(n, n-1)
 \end{aligned}$$

$$0 = \frac{d\epsilon}{da} = -2K_{\bar{X}}(n, n-1) + 2a k_{\bar{X}}(n-1, n-1)$$

$$a = \frac{k_{\bar{X}}(n, n-1)}{k_{\bar{X}}(n-1, n-1)}$$

$$b = m_n - \frac{k_{\bar{X}}(n, n-1)}{k_{\bar{X}}(n-1, n-1)} m_{n-1}$$

$$4.26 \text{ cont...}) \quad \hat{\underline{X}}_n(\underline{X}_{n-1}, \underline{X}_{n-m}) = a_1 \underline{X}_{n-1} + a_m \underline{X}_{n-m} + b.$$

$$\epsilon \stackrel{\Delta}{=} E[(\underline{X}_n - \hat{\underline{X}}_n)^2]$$

$$= E[\underline{X}_n^2] - 2E[\underline{X}_n \hat{\underline{X}}_n] + E[\hat{\underline{X}}_n^2]$$

$$= R_{\underline{X}}(n, n) - 2E[\underline{X}_n(a_1 \underline{X}_{n-1} + a_m \underline{X}_{n-m} + b)] + E[(a_1 \underline{X}_{n-1} + a_m \underline{X}_{n-m} + b)^2]$$

$$= R_{\underline{X}}(n, n) - 2a_1 R_{\underline{X}}(n, n-1) - 2a_m R_{\underline{X}}(n, n-m) - 2b m_n \\ + a_1^2 R_{\underline{X}}(n-1, n-1) + a_m^2 R_{\underline{X}}(n-m, n-m) + b^2$$

$$+ 2a_1 a_m R_{\underline{X}}(n-1, n-m) + 2b(a_1 m_{n-1} + a_m m_{n-m})$$

$$0 = \frac{d\epsilon}{db} = -2m_n + 2b + 2(a_1 m_{n-1} + a_m m_{n-m})$$

$$0 = -m_n + b + a_1 m_{n-1} + a_m m_{n-m}$$

$$b = m_n - a_1 m_{n-1} - a_m m_{n-m}$$

$$\begin{aligned} \epsilon &= R_{\underline{X}}(n, n) - 2a_1 R_{\underline{X}}(n, n-1) - 2a_m R_{\underline{X}}(n, n-m) - 2(m_n - a_1 m_{n-1} - a_m m_{n-m}) m_n \\ &\quad + a_1^2 R_{\underline{X}}(n-1, n-1) + a_m^2 R_{\underline{X}}(n-m, n-m) + (m_n - a_1 m_{n-1} - a_m m_{n-m})^2 \\ &\quad + 2a_1 a_m R_{\underline{X}}(n-1, n-m) + 2(m_n - a_1 m_{n-1} - a_m m_{n-m})(a_1 m_{n-1} + a_m m_{n-m}) \\ &= K_{\underline{X}}(n, n) - 2a_1 K_{\underline{X}}(n, n-1) - 2a_m K_{\underline{X}}(n, n-m) + a_1^2 K_{\underline{X}}(n-1, n-1) \\ &\quad + a_m^2 K_{\underline{X}}(n-m, n-m) + 2a_1 a_m K_{\underline{X}}(n-1, n-m) \end{aligned}$$

$$0 = \frac{d\epsilon}{da_1} = -2K_{\underline{X}}(n, n-1) + 2a_1 K_{\underline{X}}(n-1, n-1) + 2a_m K_{\underline{X}}(n-1, n-m)$$

$$0 = \frac{d\epsilon}{da_m} = -2K_{\underline{X}}(n, n-m) + 2a_m K_{\underline{X}}(n-m, n-m) + 2a_1 K_{\underline{X}}(n-1, n-m)$$

$$K_{\underline{X}}(n, n-1) = a_1 K_{\underline{X}}(n-1, n-1) + a_m K_{\underline{X}}(n-1, n-m)$$

$$K_{\underline{X}}(n, n-m) = a_1 K_{\underline{X}}(n-1, n-m) + a_m K_{\underline{X}}(n-m, n-m)$$

$$a_m = \frac{K_{\underline{X}}(n, n-m) - a_1 K_{\underline{X}}(n-1, n-m)}{K_{\underline{X}}(n-m, n-m)}$$

$$a_1 = \frac{K_{\underline{X}}(n-1, n-m) K_{\underline{X}}(n, n-m) - K_{\underline{X}}(n, n-1) K_{\underline{X}}(n-m, n-m)}{K_{\underline{X}}(n-1, n-m)^2 - K_{\underline{X}}(n-1, n-1) K_{\underline{X}}(n-m, n-m)}$$

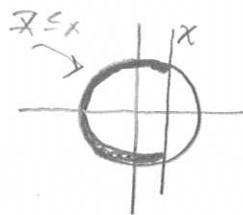
$$a_m = \frac{K_{\underline{X}}(n-1, n-m) K_{\underline{X}}(n, n-1) - K_{\underline{X}}(n, n-m) K_{\underline{X}}(n-1, n-1)}{K_{\underline{X}}(n-1, n-m)^2 - K_{\underline{X}}(n-1, n-1) K_{\underline{X}}(n-m, n-m)}$$

4.26 cont...)

$$a_m = 0 \text{ when } k_{\bar{x}}^{(n-1, n-m)} K_{\bar{x}}^{(n, n-1)} - k_{\bar{x}}^{(n, n-m)} k_{\bar{x}}^{(n-1, n-1)} = 0$$

This is true if the process is stationary or it is first-order Markov.

$$4.29) \quad a) F_{X(\theta)}(x) = \Pr(X(\theta) \leq x) = \Pr(\cos \theta \leq x)$$



$$\text{For } x < -1, \quad F_{X(\theta)}(x) = 0$$

$$\text{For } x > 1, \quad F_{X(\theta)}(x) = 1$$

$$F_{X(\theta)}(x) = \Pr(\theta \in [-\pi, -\cos^{-1}(x)] \cup [\cos^{-1}(x), \pi])$$

$$= 1 - \Pr(\theta \in (-\cos^{-1}(x), \cos^{-1}(x)))$$

$$= 1 - \int_{-\cos^{-1}(x)}^{\cos^{-1}(x)} \frac{d\theta}{2\pi}$$

$$= 1 - \frac{\cos^{-1}(x)}{\pi}$$

$$F_{X(\theta)}(x) = \begin{cases} 0, & x < -1 \\ 1 - \frac{\cos^{-1}(x)}{\pi}, & -1 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$

$$b) E[X(t)] = E[\cos(t + \theta)] = \int_{-\pi}^{\pi} \cos(t + \theta) d\theta \frac{1}{2\pi} = 0$$

$$c) K_X(t, s) = E[X(t)X(s)] = E[\cos(t + \theta) \cos(s + \theta)]$$

$$= \int_{-\pi}^{\pi} \cos(t + \theta) \cos(s + \theta) \frac{d\theta}{2\pi}$$

$$= \cancel{\int_{-\pi}^{\pi}} \left(\frac{1}{2} \cos(t+s+2\theta) + \frac{1}{2} \cos(t+s) \right) \frac{d\theta}{2\pi}$$

$$= \underline{\frac{1}{2} \cos(t+s)}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} [\cos(t)\cos\theta - \sin(t)\sin\theta][\cos(s)\cos\theta - \sin(s)\sin\theta]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (\cos(s)\cos(t)\cos^2\theta - \sin(s)\cos(t)\sin\theta\cos\theta - \sin(s)\cos(t)\sin\theta\cos\theta + \sin(s)\sin(t)\sin^2\theta)$$

$$= \frac{1}{2\pi} (\pi(\cos(s)\cos(t) - \sin(s)\sin(t))) = \frac{1}{2} \cos(s-t)$$

$$4.30) \text{ a) } E[\bar{Y}_n] = E[\bar{X}_n + W_n] = E\bar{X}_n + EW_n = m$$

$$\begin{aligned} R_{\bar{Y}}(n, k) &= E[(\bar{X}_n + W_n)(\bar{X}_k + W_k)] \\ &= E[\bar{X}_n \bar{X}_k] + E[\cancel{\bar{X}_n W_k}]^0 + E[\cancel{W_n \bar{X}_k}]^0 + E[W_n W_k] \\ &= R_{\bar{X}}(n, k) + R_W(n, k) \\ &= R_{\bar{X}}(n, k) + \sigma_w^2 \delta_{n-k} \end{aligned}$$

$$\begin{aligned} K_{\bar{Y}}(n, k) &= R_{\bar{Y}}(n, k) - E[\bar{Y}_n]E[\bar{Y}_k] = R_{\bar{X}}(n, k) + \sigma_w^2 \delta_{n-k} - m^2 \\ &= K_{\bar{X}}(n, k) + \sigma_w^2 \delta_{n-k} \end{aligned}$$

$$\begin{aligned} \text{b) } R_{\bar{X}\bar{Y}}(k, j) &= E[\bar{X}_k \bar{Y}_j] \\ &= E[\bar{X}_k (\bar{X}_j + W_j)] \\ &= E[\bar{X}_k \bar{X}_j] + E[\bar{X}_k W_j] \\ &= R_{\bar{X}}(k, j) \end{aligned}$$

c) This is the same as Problem 4.26

$$\begin{aligned} \hat{\bar{X}}_n &= a \bar{Y}_n + b \\ &\quad \uparrow \\ &\quad \bar{Y}_{n-1} \text{ in 4.26} \\ a &= \frac{k_{\bar{X}}(n, n-1)}{k_{\bar{X}}(n-1, n-1)} = \frac{\stackrel{4.30}{E[\bar{X}_n \bar{Y}_n]} - E[\bar{X}_n]E[\bar{Y}_n]}{\stackrel{4.30}{E[\bar{Y}_n \bar{Y}_n]} - E[\bar{Y}_n]E[\bar{Y}_n]} = \frac{E[\bar{X}_n \bar{Y}_n]}{E[\bar{Y}_n^2]} = \boxed{\frac{R_{\bar{X}}(n, n)}{R_{\bar{X}}(n, n) + \sigma_w^2}} \end{aligned}$$

$$\begin{aligned} b &= m_n - \frac{k_{\bar{X}}(n, n-1)}{k_{\bar{X}}(n-1, n-1)} m_{n-1} = m_{\bar{X}_n} - \frac{R_{\bar{X}}(n, n)}{R_{\bar{X}}(n, n) + \sigma_w^2} m_{\bar{Y}_n} = \boxed{m - \frac{R_{\bar{X}}(n, n)}{R_{\bar{X}}(n, n) + \sigma_w^2} m} \end{aligned}$$

see next page

$$4.30) c) E[\underline{X}_n] = m$$

$$\underline{Y}_n = \underline{X}_n + W_n$$

$$R_{\underline{X}}(n, k)$$

$$E[W_n] = 0$$

$$\text{Var}(W_n) = \sigma_w^2$$

$$\hat{\underline{X}}(\underline{Y}_n) = a\underline{Y}_n + b \quad - \text{task find MMSE}$$

\uparrow
 $\underline{X}_{n-1} = \alpha$

$$\begin{aligned} a &= \frac{K_{\underline{X}}(n, n-1)}{\underbrace{K_{\underline{X}}(n-1, n-1)}_{4.26}} = \frac{E[\underline{X}_n \underline{Y}_n] - E[\underline{X}_n]E[\underline{Y}_n]}{E[\underline{Y}_n \underline{Y}_n] - E[\underline{Y}_n]E[\underline{Y}_n]} \\ &= \frac{R_{\underline{X}}(n, n) - m^2}{R_{\underline{X}}(n, n) + \sigma_w^2 - m^2} \end{aligned}$$

From 4.26

$$\underline{E}\underline{Y}_n \text{ w/ } K_{\underline{Z}}(t, s)$$

$$E\underline{X}_n = m_n$$

$$\underline{X}_{n-1} = \alpha$$

$$\hat{\underline{X}}_n(\alpha) = a\alpha + b$$

$$a = \frac{K_{\underline{X}}(n, n-1)}{K_{\underline{X}}(n-1, n-1)}$$

$$b = m_n - \frac{K_{\underline{X}}(n, n-1)}{K_{\underline{X}}(n-1, n-1)} m_{n-1}$$

$$b = m_n - \underbrace{\frac{K_{\underline{X}}(n, n-1)}{K_{\underline{X}}(n-1, n-1)} m_{n-1}}_{4.26} = m - \frac{R_{\underline{X}}(n, n) - m^2}{R_{\underline{X}}(n, n) + \sigma_w^2 - m^2} m$$

4.30d) Using 4.26

$$4.26: \hat{\underline{X}}_n(\underline{X}_{n-1}, \underline{X}_{n-m}) = q_1 \underline{X}_{n-1} + q_m \underline{X}_{n-m} + b$$

$$4.30: \hat{\underline{X}}_n(\underline{Y}_n, \underline{Y}_{n-1}) = q_1 \underline{Y}_n + q_2 \underline{Y}_{n-1} + b$$

$$4.26: q_1 = \frac{K_{\underline{X}}(n-1, n-m) K_{\underline{X}}(n, n-m) - K_{\underline{X}}(n, n-1) K_{\underline{X}}(n-m, n-m)}{K_{\underline{X}}(n-1, n-m)^2 - K_{\underline{X}}(n-1, n-1) K_{\underline{X}}(n-m, n-m)}$$

$$4.30: q_1 = \frac{K_{\underline{X}}(n, n-1) K_{\underline{X}\underline{Y}}(n, n-1) - K_{\underline{X}\underline{Y}}(n, n) K_{\underline{Y}}(n-1, n-1)}{K_{\underline{Y}}(n, n-1)^2 - K_{\underline{Y}}(n, n) K_{\underline{Y}}(n-1, n-1)} = \frac{(R_{\underline{X}}(n, n-1)-m^2) R_{\underline{X}}(n, n-1) - R_{\underline{X}}(n, n) [R_{\underline{X}}(n, n) + \delta_w^2]}{(R_{\underline{X}}(n, n-1)-m^2)^2 - (R_{\underline{X}}(n, n) + \delta_w^2)(R_{\underline{X}}(n-1, n-1) + \delta_w^2)}$$

$$4.26: q_m = \frac{K_{\underline{X}}(n-1, n-m) K_{\underline{X}}(n, n-1) - K_{\underline{X}}(n, n-m) K_{\underline{X}}(n-1, n-1)}{K_{\underline{X}}(n-1, n-m)^2 - K_{\underline{X}}(n-1, n-1) K_{\underline{X}}(n-m, n-m)}$$

$$4.30: q_2 = \frac{K_{\underline{Y}}(n, n-1) K_{\underline{X}\underline{Y}}(n, n) - K_{\underline{X}\underline{Y}}(n, n-1) K_{\underline{X}}(n, n)}{K_{\underline{Y}}(n, n-1)^2 - K_{\underline{Y}}(n, n) K_{\underline{Y}}(n-1, n-1)} = \frac{(R_{\underline{X}}(n, n-1)-m^2) R_{\underline{Y}}(n, n) - R_{\underline{X}}(n, n-1) [R_{\underline{X}}(n, n) + \delta_w^2]}{(R_{\underline{X}}(n, n-1)-m^2)^2 - (R_{\underline{X}}(n, n) + \delta_w^2)(R_{\underline{X}}(n-1, n-1) + \delta_w^2)}$$

$$4.26: b = m_n - q_1 m_{n-1} - q_m m_{n-m}$$

$$4.30: b = m_{\underline{X}_n} - q_1 m_{\underline{X}_n} - q_2 m_{\underline{Y}_{n-1}}$$

$$\boxed{b = m(1 - q_1 - q_2)}$$

similar changes need to be made here because
 $E[\underline{X}_n] E[\underline{Y}_n] = m^2$

4.31)

$$\underline{Y}_1 = \underline{W}_1$$

$$\underline{Y}_2 = \underline{W}_2$$

$$\underline{Y}_3 = \underline{W}_1 \oplus \underline{W}_2$$

$$P_{\underline{Y}_1}(x) = P_{\underline{X}_i}(x) = P_{\underline{Y}_3}(x) = \frac{1}{2} \quad ; \quad x = 0, 1$$

$$P_{\underline{X}_i}(x) = \frac{1}{2} \quad ; \quad x = 0, 1, \text{ identically distributed}$$

$P_{\underline{Y}_1, \underline{Y}_2}(a, b)$ has one of the following three forms, $P_{\underline{Y}_1, \underline{Y}_2}, P_{\underline{Y}_1, \underline{Y}_3}, P_{\underline{Y}_2, \underline{Y}_3}$

$P_{\underline{Y}_1, \underline{Y}_2} = P_{\underline{Y}_1} P_{\underline{Y}_2}$ because \underline{W}_1 and \underline{W}_2 are independent.

$$\begin{aligned} P_{\underline{Y}_1, \underline{Y}_3}(a, b) &= \Pr(\underline{W}_1(n) = a, W_1(n) \oplus W_2(n) = b) \\ &= \underbrace{\Pr(W_1(n) \oplus W_2(n) = b \mid \underline{W}_1(n) = a)}_{\Pr(W_2(n) = a \oplus b) = \frac{1}{2}} \underbrace{\Pr(W_1(n) = a)}_{\frac{1}{2}} \\ &= \frac{1}{4} \quad \forall a, b \end{aligned}$$

$$P_{\underline{Y}_2, \underline{Y}_3} = P_{\underline{Y}_2} P_{\underline{Y}_3}$$

The process is not iid. If \underline{X}_0 and \underline{X}_1 are known, then $\underline{X}_2 = \underline{X}_0 \oplus \underline{X}_1$.

$$P_{\underline{X}_0, \underline{X}_1, \underline{X}_2} \neq \prod_{i=0}^2 P_{\underline{X}_i}$$