A Microplasticity Analysis of Micro-Cutting Force Variation in Ultra-Precision Diamond Turning

This paper describes a microplasticity model for analyzing the variation of cutting force in ultra-precision diamond turning. The model takes into account the effect of material anisotropy due to the changing crystallographic orientation of workpieces being cut. A spectrum analysis technique is deployed to extract the features of the cutting force patterns. The model has been verified through a series of cutting experiments conducted on aluminum single crystals with different crystallographic cutting planes. The results indicate that the model can predict well the patterns of the cutting force variation. It is also found that there exists a fundamental cyclic frequency of variation of cutting force per revolution of the workpiece. Such a frequency is shown to be closely related to the crystallographic orientation of the materials being cut. The successful development of the microplasticity model provides a quantitative means for explaining periodic fluctuation of micro-cutting force in diamond turning of crystalline materials. [DOI: 10.1115/1.1454108]

1 Introduction

During the last few decades, a number of research projects have developed accurate tool force models for studying the characteristics of cutting forces in ultra-precision machining [1]. Lo-A-Foe et al. [2] have developed a cutting force model which takes into account the forces on the clearance face of the tool. J. D. Kim and D. S. Kim [3] have proposed a Round-Edge Cutting Model (RECM) for explaining the effect of plowing due to the tool edge radius on the characteristics of micro-cutting forces. Recently, Arcon and Dow [4] developed an empirical tool force model which made use of the shear angle from micrographs of chip cross sections to characterize the material flow and the parameters on the cutting force equations. Many of the studies have been based on the concept of a shear zone in the formation of chip. In the machining of ductile metals, the magnitude of the shear plane angle indicates the machinability of the work materials and the efficiency of the cutting process. The shear angle has been found to vary with the work material, the tool geometry, and the cutting conditions.

However, one of the drawbacks in the existing theories is that they are ex post facto in nature, in which metal cutting experiments are necessary to obtain important material parameters prior to the prediction of the shear angle. Another shortcoming of existing theories is that most studies of the cutting mechanism are performed under the assumption that the material is isotropic and is a homogeneous continuum. The effect of material anisotropy is often not included in the theories of analysis. One important source of material anisotropy lies in the crystallographic orientation of the work material which has drawn a lot of attention from researchers [5].

In ultra-precision diamond turning, the cutting is usually performed with a depth of cut of less than the average grain size of a polycrystalline aggregate. When cutting is performed within a grain, the statistical distribution of crystal defects and their interaction with the stress field of the cutting tip plays a dominant role in the cutting mechanism. As most of the ultra-precision machine is dimensionally stable and mechanically rigid, the fluctuation in the cutting forces could not be due to the machine tool chatter. Sato et al. [6] found that the shear angles and the cutting forces vary with the crystallographic orientation of both single crystal and polycrystalline materials. Black [7] has reported a variation in the shear front-lamellar structure at the top of the chip with the grain orientation. Most researchers imply that the shear planes in metal cutting are slip plane glide planes themselves. In this paper, a microplasticity model is proposed to predict the pattern of the cyclic variation of cutting forces in diamond face turning. The model is based on the previous work of Lee and Zhou [8]. Power spectrum analysis is deployed to extract the features of the cyclic cutting force patterns [9]. The relationships between the cyclic variation of the cutting forces with the crystallographic orientation of the work materials being cut are also explored.

2 Microplasticity Analysis for Micro-Cutting Force Prediction

2.1 Microplasticity Model for Shear Angle Prediction. In the microplasticity model, the basic cutting mechanism that occurs in ultra-precision diamond turning of a polycrystalline aggregate is considered to be similar to that in the machining of a single crystal. During machining, the tool tip acts as a strong source of dislocations. Fine cracks are produced near the vicinity of the tool tip and trigger the primary shearing process. As the tool advances, the material ahead of the tool is compressed in the cutting direction and a shear band joining the top of the tool and the surface of the work material develops. Plane-strain orthogonal cutting is assumed in the model and a cutting tool of zero rake angle is used. The machining process is assumed to be well lubricated. The effect of plowing and sliding due to tool edge radius is ignored. The deformation is considered to be accomplished by the crystallographic slip only. Equal hardening of the slip systems is assumed. A large plastic deformation in the shear zone is treated as a succession of incremental plastic strains and the workpiece material is assumed to be rigidly plastic and incompressible.

Referring to the workpiece coordinate system (CD-CP-OD) as shown in Fig. 1, the symmetric strain tensor in the shear band, \( \varepsilon_{w} \), is given by:

\[
\varepsilon_{w} = d\tau/2 = \begin{bmatrix}
\sin 2\phi & 0 & \cos 2\phi \\
0 & 0 & 0 \\
\cos 2\phi & 0 & -\sin 2\phi
\end{bmatrix}
\]  

where \( d\tau \) is the shear strain in the shear band.
The crystallographic orientation of the crystal is represented by the Miller indices such that (hk I) is parallel to the cutting direction and [uvw] is parallel to the normal to the cutting plane. The imposed strain tensor, $e_w$, is transformed from the workpiece coordinate system (CD-CP-OD) to the crystallographic axis of the crystal where the crystallographic slip system is based, i.e.,

$$e_c = P e_w P^T$$

where $e_c$ is the strain tensor referred to the cube axes of the crystal, $P$ is the transformation matrix and $P^T$ is its transpose.

$$P = \begin{bmatrix}
    r_1 & u_1 & n_1 \\
    r_2 & u_2 & n_2 \\
    r_3 & u_3 & n_3
\end{bmatrix}$$

where $r_1 = \frac{u}{\sqrt{u^2 + v^2 + w^2}}, r_2 = \frac{v}{\sqrt{u^2 + v^2 + w^2}}, r_3 = \frac{w}{\sqrt{u^2 + v^2 + w^2}}$

$$n_1 = \frac{h}{\sqrt{h^2 + k^2 + l^2}}, n_2 = \frac{k}{\sqrt{h^2 + k^2 + l^2}}, n_3 = \frac{l}{\sqrt{h^2 + k^2 + l^2}}$$

$$u_1 = n_2 r_3 - n_3 r_2, u_2 = n_3 r_1 - n_1 r_3, u_3 = n_1 r_2 - n_2 r_1$$

The increment of plastic work done during deformation $dW$ is given by:

$$dW = \sigma d\epsilon_w$$

where $\sigma$ is the equivalent stress or the plastic work per unit volume and strain, and $d\epsilon_w$ is the macroscopic effective strain. $\phi$ coincides with the direction of the maximum shear stress and makes an angle of 45 deg with the cutting direction. If the shear angle deviates from 45 deg by an angle of $\psi$, the shear strain in the shear band will be increased by a factor of $1/\cos 2\psi$ in order to produce the same amount of macroscopic deformation. The shear band will occur at an angle $\phi$ such that the plastic work done in deforming the metal will be the minimum. It must be noted that the shear band is macroscopic in nature and may not be parallel to a particular crystallographic slip plane of the crystal. However, the shear in the band has to be accomplished by a homogeneously distributed slip, i.e., all alternative slip systems cooperate in the shear band development. Hence the Taylor model of polycrystalline plasticity can be applied for the analysis of the shear band formation.

The virtual work equation for deforming a single crystal can be written as:

$$\sigma d\epsilon_w = \tau_c d\Gamma$$

where $d\Gamma$ is the total dislocation shear strain accumulated in the crystal and $\tau_c$ is the critical resolved shear stress on the active slip systems.

The effective strain $d\epsilon_w$ is related to the total dislocation shear strain by the Taylor factor $M$, i.e.,

$$M = \frac{d\Gamma}{d\epsilon_w}$$

and

$$\sigma = M \tau_c$$

The Taylor factor $M$ is a dimensionless number that is sensitive to the crystallographic orientation of the material being cut. It is often used as an index of plastic anisotropy—the extent to which the strength $\sigma$ of a crystal varies with crystallographic orientation for a given critical shear stress $\tau_c$. A large value of $M$ indicates a large shear strength of crystal being cut and hence the cutting force. Any variation in shear strength will cause the fluctuation in cutting force. $M$ is calculated according to the maximum work principle of Bishop and Hill [10] which states that the state of actual stress $\sigma_{ij}$ required to cause a given increment of strain $d\epsilon_{ij}$ is the one that maximizes the work done during deformation $dW$. Referring the stress and the strain to the cubic axes, the plastic work done, $dW$, during deformation is given by:

$$dW = -B d\epsilon_{11(c)} + 2F d\epsilon_{22(c)} + 2G d\epsilon_{13(c)} + 2H d\epsilon_{12(c)}$$

where

$$A = (\sigma_{22(c)} - \sigma_{33(c)})/\sqrt{\tau_c}, B = (\sigma_{33(c)} - \sigma_{11(c)})/\sqrt{\tau_c}, C = \sigma_{11(c)}/\sqrt{\tau_c}$$

The work done is calculated for all fifty-six possible stress states and the one whose work value is the highest becomes the yielding stress state [11]. The Taylor factor $M$ can be determined by substituting the calculated value of maximum work done $dW$ into the following equation:

$$M = \frac{dW}{(\tau_c d\epsilon_{ij(c)})}$$

The resolved shear stress on the inclined plane varies as $1/\cos 2\psi$ from a 45 deg plane, an effective Taylor factor $M'$ is defined as $M/\cos 2\psi$. Based on the minimum work principle, a shear band will form at such a direction along which the effective Taylor factor gets its minimal value. Very often the variation of the effective Taylor factor $M'$ with shear angle is associated with a plateau and a range of shear angles is then possible based on the principle of minimum work alone. This uncertainty can be removed if the load instability criterion is imposed. A shear band will form as:

Table 1 Labels for the slip systems

<table>
<thead>
<tr>
<th>Slip plane, $n$</th>
<th>$\langle 11 \rangle$</th>
<th>$\langle 11 \rangle$</th>
<th>$\langle 11 \rangle$</th>
<th>$\langle 11 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slip direction, $b$</td>
<td>$\langle 10 \rangle$</td>
<td>$\langle 10 \rangle$</td>
<td>$\langle 10 \rangle$</td>
<td>$\langle 10 \rangle$</td>
</tr>
<tr>
<td>Slip system</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
<td>$b_1$</td>
</tr>
</tbody>
</table>
\[
\frac{1}{\sigma} \frac{ds}{de_w} = \frac{1}{M} \frac{dM}{de_w} + \frac{M}{\tau_e} \frac{d\tau_e}{d\Omega} \leq 0
\] (13)

\((1/M)(dM/de_w)\) depends on the rate of change of the crystallographic orientation of the material with strain, and is called the texture softening factor \(S\) if it is negative or the texture hardening factor if it is positive. The second term \((M/\tau_e)(d\tau_e/d\Omega)\) represents the slip plane hardening contribution which is usually positive. Hence, a shear band will develop when \((1/M)(dM/de_w)\) is the most negative.

### 2.2 Texture Softening Factor

#### 2.2.1 Selection of Active Set of Slip Systems

In Section 2.1, it is mentioned that the shear in the band has to be accomplished by the slip systems cooperating in the development of the shear band. For a crystal to undergo an arbitrary plastic strain or deformation by slip, five independent slip systems (represented in terms of the slip direction \(b\) and a slip plane normal \(n\)) are needed. In face-centered cubic (F.C.C.) crystals, the \(\{111\}\{110\}\) family of slip systems is dominant. The labels of the slip systems for the \(\{111\}\{110\}\) family of slip systems are tabulated in Table 1. Taking

\[
\begin{pmatrix}
E_{22(c)} \\
E_{33(c)} \\
2E_{23(c)} \\
2E_{13(c)} \\
2E_{12(c)}
\end{pmatrix} = \begin{pmatrix}
(n_1^2 b_2^2) & (n_2^2 b_2^2) & (n_3^2 b_2^2) & (n_1^2 b_2^2) & (n_3^2 b_2^2) \\
(n_1^2 b_3^2) & (n_2^2 b_3^2) & (n_3^2 b_3^2) & (n_1^2 b_3^2) & (n_3^2 b_3^2) \\
(n_1^2 b_1^2 + n_3^2 b_1^2) & (n_2^2 b_1^2 + n_3^2 b_1^2) & (n_3^2 b_1^2) & (n_1^2 b_1^2 + n_3^2 b_1^2) & (n_3^2 b_1^2) \\
(n_1^2 b_1^2 + n_3^2 b_1^2) & (n_2^2 b_1^2 + n_3^2 b_1^2) & (n_3^2 b_1^2) & (n_1^2 b_1^2 + n_3^2 b_1^2) & (n_3^2 b_1^2) \\
(n_1^2 b_2^2 + n_3^2 b_2^2) & (n_2^2 b_2^2 + n_3^2 b_2^2) & (n_3^2 b_2^2) & (n_1^2 b_2^2 + n_3^2 b_2^2) & (n_3^2 b_2^2)
\end{pmatrix}
\]

(15)

According to the Taylor criterion [11], the work done \(\delta w\) in activating the preferred set of slip systems is less than that of all other sets of systems that could geometrically accomplish the strain, i.e.,

\[
\delta w = \tau_e \sum \delta \gamma
\] (17)

where the summation sign denotes the sum of the incremental shears, on each of the five independent systems of a set. In other words, the sum of the total shears \(\Sigma \gamma\) as determined by Eq. (16) should be the minimum. The Taylor criterion provides one possible way to predict the set of slip systems which will actually operate while the strain is being imposed.

Bishop and Hill [11] have proposed another principle for the prediction of active slip systems. It is stated that in the deformation of a single crystal, the actual stress corresponding to a given strain is not less than any other stress that satisfies the yielding conditions. In other words, the state of stress giving the maximum value of \(dW\) in Eq. (11) will actually motivate the strain. The Bishop and Hill’s Principle of Maximum Work criterion has been reported to give the same results as the Taylor criterion in the prediction of active set of slip systems [11]. When considering \(\{111\}\{110\}\) slip, the Bishop and Hill criterion has the advantage that the maximum of only fifty-six stress states is sought whereas the minimum of 384 stress states is required in the Taylor criterion.

However, ambiguities still exist in the selection of active slip systems based on the Bishop and Hill criterion since the selected stress states give either five out of six or five out of eight possible combinations of slip systems for accomplishing a given strain. The ambiguity in identifying the set of five slip systems is resolved in the present study by selecting the set of active slip systems which minimizes the second order plastic work as proposed [11]. In this method, an infinitesimal strain is applied to each possible set of the slip systems. This leads to a small crystal rotation. For a given imposed strain state, the second order plastic work \(dW^*\) for each possible combination of slip system can be determined from:

\[
dW^* = \sigma_{ij}^* d\epsilon_{ij}^*
\] (18)

where \(\sigma_{ij}^*\) and \(d\epsilon_{ij}^*\) are the new stress and strain components respectively due to the rotation and the strain hardening of the crystal. The set of active slip systems which corresponds to the minimum second order plastic work will then be selected.

#### 2.2.2 Determination of Texture Softening Factor

The texture softening factor \(S\) is computed numerically by determining the rate of lattice rotation \((d\Omega/d\epsilon_{ij})\) and the associated change in Taylor factor \((dM/d\Omega)\) as follows:

\[
S = \frac{1}{M} \frac{d\Omega}{d\epsilon_{ij}} \frac{dM}{d\Omega}
\] (19)

Suppose an infinitesimal strain \([\epsilon^*]\) is applied to the active set of slip systems determined in Section 2.2.1. The strain components of \([\epsilon^*]\) are related to the active set of five independent slip systems by

\[
[\epsilon^*] = [E^*][\gamma]
\] (20)
where \( E' \) is a square matrix that denotes the direction cosines of the selected active slip systems with respect to the cube axes of the crystal. The value of the shears on each slip system can be obtained by rewriting Eq. (20) as:

\[
\begin{align*}
W &= \frac{1}{2} \sum_{i=1}^{5} \\
&= \begin{pmatrix}
0 & (b_1' n_2^0 - b_2' n_1^0) \gamma & (b_1' n_3^0 - b_3' n_1^0) \gamma' \\
(b_1' n_1^0 - b_2' n_2^0) \gamma & 0 & (b_2' n_3^0 - b_3' n_2^0) \gamma' \\
(b_1' n_2^0 - b_2' n_1^0) \gamma' & (b_2' n_3^0 - b_3' n_2^0) \gamma' & 0
\end{pmatrix}
\end{align*}
\]

(22)

Assume the initial orientation of the lattice to be \( P \) as given in Eq. (3) and the shears will cause the crystal to rotate to a new orientation \( P' \) given by:

\[
P' = (I - W)P
\]

(23)

where \( I \) is the unit matrix.

The net rotation \( d\Omega \) can be determined by:

\[
d\Omega = \sqrt{\omega_{12}^{2} + \omega_{13}^{2} + \omega_{23}^{2}}
\]

(24)

Referring the stress and the strain to the cubic axes, the plastic work done, \( dW' \), during this deformation is given by:

\[
dW' = -Bde'_{1h(c)} + Ade'_{2(c)} + 2Fde'_{3h(c)} + 2Gde'_{1h(c)} + 2Hde'_{2(c)}
\]

(25)

The corresponding change in Taylor factor \( dM \) can be determined by substituting the calculated value of maximum work \( dW' \) into the following equation:

\[
dM = \frac{dW'}{(\tau, de'_{1(c)})}
\]

(26)

Hence, the Texture Softening Factor \( S \) can be determined by Eqs. (12), (19), (24) and (26).

2.3 Criterion for Shear Angle Prediction. The criterion is based on the combination of the effective Taylor factor \( M' \), the number of slip systems and the texture softening factor in sequence until a unique solution is obtained [12]. In order to predict the most likely shear angles, the minimum \( M' \) is calculated first. Should a range of shear angles all possess the same minimum \( M' \), then the one with both minimum \( M' \) and the smallest number of slip systems will be selected. If there is still no unique shear angle,

\[
[\gamma] = [E^{-1}][e']
\]

(21)

The shear \( [\gamma] \) will cause the lattice rotation and the lattice rotation tensor \( W \) is given by:

![Fig. 2 A schematic diagram of the signal flow in cutting force measurement](image)

### Table 2 Specifications of work materials

<table>
<thead>
<tr>
<th>Specimen no.</th>
<th>Specifications of materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aluminium single crystal with (001) as the cutting plane</td>
</tr>
<tr>
<td>2</td>
<td>Aluminium single crystal with (111) as the cutting plane</td>
</tr>
</tbody>
</table>

### Table 3 Cutting conditions for the cutting tests

<table>
<thead>
<tr>
<th>Group no.</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spindle speed (rpm)</td>
<td>3000</td>
<td>3000</td>
</tr>
<tr>
<td>Feed (mm min(^{-1}))</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Depth of cut (µm)</td>
<td>15</td>
<td>10, 20, 30</td>
</tr>
<tr>
<td>Tool rake angle</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>Front clearance angle</td>
<td>10°</td>
<td>10°</td>
</tr>
<tr>
<td>Tool nose radius (mm)</td>
<td>0.7740</td>
<td>0.7740</td>
</tr>
</tbody>
</table>

![Image of experimental setup](image)
of the single crystal, $\beta$ is the friction angle between the tool and the chip material. The friction angle and the shear angle play an essential role in determining the cutting forces. The friction angle is related to the shear angle by the Merchant equation based on the principle of minimum work:

$$\phi = 45^\circ - \beta/2 + \alpha/2$$  \hspace{1cm} (28)

The change in friction is embedded in the change of shear angle. In conventional machining theory, the shear angle is to be determined from experiment whereas, in the present study, the shear angle is deduced from the microplasticity model. As the friction angle can be negative under dynamic cutting condition, the shear angle at which the shear band will form can be larger than $45^\circ$ and is consistent with the shear angle predicted from the microplasticity consideration in diamond turning highly anisotropic materials. When a diamond tool with zero rake angle is used, the main cutting force can be derived by substituting Eq. (27) into Eq. (28) as:

$$F_c(t) = \tau bd \cot \phi(t)$$  \hspace{1cm} (29)

The general applicability of the microplasticity approach does not depend on the values of the friction angle $\beta$ or the rake angle $\alpha$. However, with a non-zero rake angle, the calculation for the formation of the shear band is more complicated and the deformation can no longer be assumed to be simple plane-strain compression. The introduction of a more complex plasticity analysis would not alter the basic principle and the new approach introduced in the present study which can be used to deduce theoretically the shear angle from crystal plasticity theory. The outcome can be checked by the variation of the cutting forces which is analyzed by the power spectrum method.

3 Experimental Procedures

The model was verified through a series of cutting experiments which was divided into two groups, i.e. Group A and Group B. Group A includes the cutting tests for studying the patterns of the variation of the cutting force. In Group B, the effect of depth of cut on the variation of the cutting force was investigated. Table 2 and Table 3 tabulate the specifications of the workpiece and the cutting conditions being used respectively. The diameter of the workpiece was 12.7 mm. The initial crystallographic orientation of the specimens was checked by a standard reflection technique on an X-ray diffractometer.

All cutting tests were performed on a two-axis CNC ultra-precision lathe (Optoform 30 from Taylor Hobson Pneumo Co.). The cutting force was captured approximately at the midway of the tool travel between the periphery and the center of the workpiece. Since the captured length (0.2 second) was relatively small as compared with the overall cutting cycle (19 seconds), it was assumed that the mean cutting force was quasi-static in the measuring range. Figure 2 shows a schematic diagram of the signal flow in the analysis of the micro-cutting force. The cutting force was measured by a Kistler 9252A piezoelectric force transducer mounted directly under the tool post. The force signal captured from the transducer was first pre-amplified by a charge amplifier.
and the analogue voltage output was recorded and digitized by a digitizing oscilloscope (Tektronix TDS744A). Then, the digitized signal was passed to a personal computer for analyzing. The variation of the cutting forces was analyzed by a power spectrum analysis software package exclusively built for the study. In the spectrum analysis, the PSD at the zero frequency, which stands for the averages of the cutting forces, was filtered out for ease of analysis.

4 Results and Discussion

4.1 Variation of Cutting Force With Crystallographic Orientation of the Workpiece. Figure 3 shows the model predicted shear angle variation in diamond turning aluminum single crystals under conditions in Group A. The cutting was done on (001) and (111) crystallographic planes, respectively. The corresponding variation of the cutting force is shown in Fig. 4. It is observed that the shear angle varies with the crystallographic orientation of the materials being cut. The patterns of shear angle variation are found to be distinctive for different crystals. There seems to exist a fundamental cyclic frequency of variation of the cutting force for each workpiece revolution. As observed in Fig. 4, the fundamental cyclic frequencies are found to be four for (001) crystal and three for the (111) crystal respectively. To verify these findings, the spectrum analysis method was employed to extract the features of the cutting force patterns as discussed below.

The background spectrum for the cutting force signals was obtained in air cutting. The result is depicted in Fig. 5. It is noticed that the background spectrum is composed of random frequency components with a low power spectral density (PSD) which would be attributed to the spray of coolant and fine vibration of the machine.

Figures 6 and 7 show the predicted and the measured spectral plots for the variation of cutting force in diamond turning of (001) and (111) crystals respectively. The predicted spectra are found to agree well with the measured spectra. Remarkable frequency components \( f_{p,1}, f_{p,2}, f_{p,3}, f_{p,4}, f_{p,5}, f_{p,6} \) are observed in the predicted spectral plots for cutting force and they are also reflected in the measured spectral plots (i.e., \( f_{m,1}, f_{m,2}, f_{m,3}, f_{m,4}, f_{m,5}, f_{m,6} \)). The first frequencies, \( f_{p,1} \) and \( f_{m,1} \), are found to be close to the fundamental rotational frequency of the spindle (i.e., 50 Hz). Comparing Fig. 6 to Fig. 7, distinctive patterns of frequency distributions are observed for different crystals. As shown in Table 4, both the predicted and the measured spectral plots exhibit dominant frequency components (i.e., \( f_{p,2} \) and \( f_{m,2} \)) which can be correlated to the spindle rotational frequency. A parameter named fundamental cyclic frequency of variation of cutting force is defined as the ratio between the dominant frequency components and the spindle rotational frequency, in cycles per workpiece revolution. As shown in Table 4, the predicted fundamental cyclic frequency of the variation of cutting force is found to agree well with the measured one for each of the crystals except for the (111) crystal. From Fig. 7, the measured dominant frequency component \( f_{m,2} = 300 \text{ Hz} \) for the (111) crystal appears at almost double of the predicted dominant frequency component \( f_{p,3} = 150 \text{ Hz} \) and is almost identical to its first harmonic \( f_{p,3} = 297 \text{ Hz} \). In Fig. 4(b), the cyclic cutting force is shown to be made up of two patterns hereby referred to as the major pattern with a higher amplitude and a minor pattern with a lower amplitude. The presence of six peaks per cycle...
of workpiece revolution is well predicted except that there is a discrepancy in the amplitude of the peak. This implies that the (111) single crystal might possess two fundamental cyclic frequencies which are three for the major patterns as well as six for both of the major and the minor patterns of the cutting force variation.

4.2 Influence of Depth of Cut on the Variation of Cutting Forces. Figures 8 and 9 show the variation of power spectral densities (PSD) with depth of cut in diamond turning of (001) and (111) aluminum single crystals under conditions in Group B. As depth of cut increases, the predicted power spectral densities (PSD) are found to increase accordingly. The findings agree well with the measured PSD for all materials being investigated. The remarkable increase in the dominant frequency component with depth of cut illustrates that the influence of the crystallographic orientation of single crystal materials could be pronounced at a large depth of cut.

Overall, the microplasticity model is capable of explaining the periodic variation of micro-cutting force in diamond turning crystalline materials. The main features of the cutting force patterns are well predicted and confirmed by the cutting tests. There is good agreement between the experimental findings and the predicted results. The discrepancy in the power spectral densities (PSD) between the predicted and the measured spectra could have been due to the following reasons:

<table>
<thead>
<tr>
<th>Table 4 A comparison between the predicted and measured dominant frequency component and fundamental cyclic frequency of the variation of the cutting force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen no.</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>2*</td>
</tr>
</tbody>
</table>

Note: * For frequency component $f_{p,3}$, $f_{p,3}$. 

Fig. 8 Effect of depth of cut on the power spectral densities (PSD) of the cutting force in diamond turning aluminum single crystal with (001) cutting plane: (a) is the predicted and (b) is the measured

Fig. 9 Effect of depth of cut on the power spectral densities (PSD) of the cutting force in diamond turning aluminum single crystal with (111) cutting plane: (a) is the predicted and (b) is the measured
(i) The spray of coolant which will introduce disturbances to the cutting force signals and hence the frequency spectrum at the high frequency range;
(ii) The progress of tool wear during machining;
(iii) Fine vibration between the tool and workpiece which might affect the uncut chip thickness and hence the cutting forces;
(iv) In the present study, the effect of friction between the tool and the workpiece is not taken into consideration for ease of analysis. Some preliminary experimental and simulation work has been conducted by the authors to study the effect of cutting friction on the variation of the cutting forces. It is predicted that the periodicity of the fluctuation of cutting forces is affected by the change of the frictional condition. The (111) crystal appears to possess two periodicity of fluctuation of the cutting forces which is six for low friction condition and three for medium and high friction conditions. However, the in-situ situations among the cutting friction, the crystallographic orientation of the workpiece as well as the periodic fluctuation of the cutting forces have not yet been explored. There is a need for further study of the phenomenon of friction-induced fluctuation of cutting forces in diamond turning.

5 Conclusion

The variation of micro-cutting forces in diamond turning of crystalline materials has been analyzed based on a microplasticity model and spectrum analysis technique. The model takes into account the effect of crystallographic orientation of work materials being cut. The spectrum analysis technique was used to extract the features of the cutting force variation. A series of cutting tests was conducted to evaluate the performance of the model. It is found that the model predicts well the variation of micro-cutting forces in diamond turning of crystalline materials. Experimental results also indicate that the variation of the cutting forces is related closely to the crystallographic orientation of the cutting force variation. A series of cutting tests was conducted to evaluate the performance of the model. It is found that the model predicts well the variation of micro-cutting forces in diamond turning of crystalline materials. Experimental results also indicate that the variation of the cutting forces is related closely to the crystallographic orientation of the cutting force variation.

Acknowledgments

The authors would like to express their sincere thanks to the Research Committee of The Hong Kong Polytechnic University for the financial support of the research work.

Nomenclature

\[
\begin{align*}
A &= \text{area of the undeformed chip section} \\
b &= \text{width of cut} \\
d &= \text{depth of cut} \\
\alpha &= \text{rake angle of the diamond tool} \\
\beta &= \text{friction angle between the tool and the chip material} \\
\beta_w &= \text{macroscopic effective strain} \\
\Omega &= \text{net rotation of crystal lattice} \\
\Gamma &= \text{total dislocation shear strain accumulated in a crystal} \\
W &= \text{increment of plastic work done during deformation} \\
\varepsilon &= \text{strain tensor referred to the cube axes of a crystal} \\
\varepsilon_w &= \text{symmetric strain tensor in the shear band} \\
F_c &= \text{cutting force} \\
\sigma &= \text{equivalent stress or the plastic work per unit volume and strain} \\
\delta &= \text{work done in activating the preferred set of slip systems} \\
\phi &= \text{shear angle} \\
M &= \text{Taylor factor} \\
M' &= \text{effective Taylor factor} \\
S &= \text{texture softening factor} \\
\tau &= \text{shear stress of the single crystal} \\
\tau_c &= \text{critical resolved shear stress on the active slip systems of a crystal} \\
W &= \text{lattice rotation tensor}
\end{align*}
\]

References