S. Okabe

Assoc. Professor.

Y. Yokoyama

Professor.

Faculty of Engineering, Kanazawa University, Kanazawa, Japan

Study on Vibratory Feeders: Calculation of Natural Frequency of Bowl-Type Vibratory Feeders

This paper treats a method of calculating natural frequency of vibratory feeders. In a bowl-type feeder, the deformation of the spring is complicated and the exact calculation of its constant is difficult. Therefore an approximate calculation is presented under some assumptions. The relations between spring constant and spring setting condition are clarified and shown in various diagrams. The equations of natural frequency for the fixed type and the semi-floating type feeder are represented briefly. The vibration direction of bowl-type feeder is also discussed. The theoretical results are confirmed by experimental studies.

1. Introduction

Vibratory feeders are a very useful method for conveying or feeding various parts and materials in automatic asssembly system [1]. Much theoretical and experimental researchers related to the conveying mechanism of feeders has been reported [2-7].

As seen in previous studies, the feeding or conveying velocity of parts is influenced by the vibration amplitude of the trough or bowl. Generally, most vibratory feeders are used at the resonant or near-resonant frequency of the mechanical system to improve feeding efficiency. Therefore, it is very desirable to predict the natural frequency of the feeder, although until now feeders have been designed experimentally.

This paper treats the method of calculating natural frequency of vibratory feeders. In a linear type vibratory feeder, the spring constant and inertia term can be calculated easily. But in bowl-type feeder, the deformation of the spring is complicated and the exact calculation of spring constant is difficult. In this paper, an approximate calculation is made and some relations between natural frequency and the setting condition of the spring are shown. These results will be useful for design, development and practical use of bowl-type feeders.

2. Equivalent Model of Bowl-Type Vibratory Feeder

The bowl-type vibratory feeder is made up of four main parts, that is, bowl, springs, base and exciter. The bowl is usually supported on three or four sets of inclinded leaf springs fixed to the base, and is vibrated by an electromagnetic exciter mounted on the base.

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Fig. 1 Equivalent model of bowi-type vibratory feeder

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Fig. 2 Deflection of leaf spring in bowl-type vibratory feeder

The vibratory feeder is often mounted on vibration isolators, for example rubber feet, as shown schematically in Fig. 1(a), (b), to minimize the force transmission to the foundation. The relations between mounting conditions and dynamic characteristics of the feeder have already been clarified [8]. According to this previous study, if the stiffness of the vibration isolator is less than about one-fifth of that of the lead spring, the vibratory characteristics can be approximated by those of a floating type feeder which is supported at the nodal point of spring, as shown in Fig. 1(c).

If the feeder is mounted on a foundation without a vibration isolator, the equivalent model of the feeder is presented in Fig. 1(d).

3. Deformation of Leaf Spring and Some Assumptions for Analyses

In a bowl-type feeder, three or four sets of inclined leaf springs are arranged along a circumference. Then the movement of the bowl has an angular vibration about its vertical axis together with a vertical vibration.

- Nomenclature .

- b = width of leaf spring
- E = Young's modulus
- f_{n1} = natural frequency of fixed type vibratory feeder
- f_{n2} = natural frequency of floating and semi-floating type vibratory feeder
- G = shear modulus
- h = thickness of leaf spring
- I_z = geometrical moment of inertia
- \tilde{J} = inertia moment about vertical axis of bowl
- k_e = equivalent spring constant
- k_1 = numerical factor
- K.E. = kinetic energy of bowl
 - l = length of leaf spring
 - M = mass of bowl
- M_e = equivalent inertia mass
- M_s = bending moment at end of spring (in width direction)
- M_t = torsional moment
- n = number of leaf springs
- R_s = shearing force at end of spring (in width direction)

r = radius of base circle

point A, as shown in Fig. 2.

 r_o = radius of setting circle on bowl

following assumptions are presented:

Calculation of Spring Constant

each deformation.

line of the spring.

4.

- U_1 = strain energy in width direction
- U_2 = strain energy in thickness direction
- $U_3 =$ strain energy in torsion
- x = distance from cramping part on leaf spring
- α = slope of deflection at upper end of spring (in width direction)

In this case, the deformation of each spring is very com-

The deformation of the leaf spring is influenced by the

plicated. Therefore, in order to simplify the discussion, the

deformations in (i) thickness direction, (ii) width direction

and (iii) torsion. These deformations are independent of one

another without any geometrical constraint, so that the total

deformation can be calculated by means of vector addition of

Consider a leaf spring inclinded at an angle γ to the

Let O be the center of the circle (named base circle) which is

horizontal and fixed to a base at point D and to a bowl at

inscribed tangent to the center lines of leaf springs, as shown

in Fig. 2(a). Let r be the radius of this base circle and ϕ be the

angle between \overline{OA} and \overline{OH} . \overline{OH} is perpendicular to the center

If the bowl is rotated by an angle θ , the upper end of the leaf

- β = angular displacement at upper end of spring
- β_m = ratio of equivalent inertia mass of bowl to that of base
- γ = inclination of leaf spring
- $\gamma' = vibration direction angle$
- δ = deflection at upper end of leaf spring
- δ_1 = width direction component of deflection at upper end of leaf spring
- θ = rotation of bowl
- θ' = torsional displacement of leaf spring
- $\kappa = offset factor$
- ϕ = offset angle

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spring is moved from A to C'. Denoting the displacement \overrightarrow{AC}' by δ , and the angle $\langle C'AB'$ in Fig. 2(b) by β , there exists the following relation:

$$\theta = \frac{o}{r} \cos \beta \sin \gamma \tag{1}$$

As mentioned above, assume that the deflections of the spring in each direction are independent of one another and calculate the strain energy of each deformation.

Let δ_1 be the deflection in the width direction (corresponds to the displacement $\overline{B'C'}$ in Fig. 2(b)) and α be the slope of the deflection at the upper end of the spring. Then these quantities can be calculated geometrically by the following equations:

$$\delta_1 = r - \frac{r}{\cos \phi} \cos(\phi + \theta)$$
$$\approx r \frac{\theta^2}{2} + r\theta \tan \phi$$
(2)

 $\alpha = \theta \cos \gamma$ (3)

The following relations also exist:

$$\frac{P_s l^3}{3EI_z} + \frac{M_s l^2}{2EI_z} = \delta_1 \tag{4}$$

$$\frac{P_s l^2}{2EI_z} + \frac{M_s l}{EI_z} = \alpha \tag{5}$$

Where P_s : shearing force at the end of spring,

- M_s : bending moment at the end of spring,
- E: Young's modulus,
- I_z : geometrical moment of inertia $\left(=\frac{b^2h}{12}\right)$,
- *l* : length of leaf spring,
- b : width of leaf spring,
- h: thickness of leaf spring.

Hence

$$P_s = \frac{Eb^3 b}{l^3} \left(\delta_1 - \frac{\alpha l}{2} \right) \tag{6}$$

$$M_s = \frac{Eb^3 h}{l^2} \left(\frac{\alpha l}{3} - \frac{\delta_1}{2}\right) \tag{7}$$

Substituting equations (1-3) in equations (6) and (7), and neglecting small terms because of small θ , we obtain

$$P_{s} = \frac{Eb^{3}h}{2rl^{2}}\delta\cos\beta\sin2\gamma\left(\kappa - \frac{1}{2}\right)$$
(8)

$$M_{s} = \frac{Eb^{3}h}{2rl}\delta\cos\beta\sin 2\gamma\left(\frac{1}{3} - \frac{\kappa}{2}\right)$$
(9)

where κ is the offset factor, defined by

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$$\kappa = \frac{r \tan \phi}{l \cos \gamma} \tag{10}$$

Then the strain energy in the width direction U_1 is represented by

$$U_{1} = \frac{1}{2EI_{z}} \int_{1}^{t} (P_{s}x + M_{s})^{2} dx$$
$$= \frac{Eb^{2}h}{24lr} \delta^{2} \cos^{2}\beta \sin^{2}(2\gamma) (3\kappa^{2} - 3\kappa + 1)$$
(11)

Next, consider the strain energy for the thickness direction. The deformation in the thickness direction can be approximated as that of parallel leaf springs and the strain energy U_2 can be represented by the following equation:

$$U_2 = \frac{Ebh^3}{2l^3} \,\delta^2 \,\cos^2\beta \tag{13}$$

The strain energy for torsion U_1 is calculated by

$$U_3 = \frac{1}{2k_1 b h^3 G} \int_0^t M_t^2 dx$$
(13)

where k_1 is a numerical factor depending on the ratio b/h, G is the shear modulus and M_t is the torsional moment of the spring. The angle of torsion at the upper end of the spring is geometrically given by $\theta' \doteq \theta \sin \gamma$, hence

$$\theta' \div \frac{\theta \cos \beta \sin^2 \gamma}{r} \tag{14}$$

and the torsional moment M_t is expressed as:

$$M_l = \frac{k_1 b h^3 G}{l} \theta' \tag{15}$$

Substituting equations (14) and (15) in equation (13) gives

$$U_3 = \frac{k_1 b h^3 G \delta^2 \cos^2 \beta \sin^4 \gamma}{2 l r^2} \tag{16}$$

The equivalent spring constant of bowl-type feeder k_e can be calculated from

$$\frac{k_e}{2}\delta^2 = n(U_1 + U_2 + U_3) \tag{17}$$

where *n* is a number of leaf springs. Then k_e is written as:

$$k_e = \frac{nEbh^3}{l^3} \left\{ 1 + \frac{1}{12} \left(\frac{b}{h}\right)^2 \left(\frac{l}{r}\right)^2 \sin^2(2\gamma) \left(3\kappa^2 - 3\kappa + 1\right) + k \left(\frac{G}{E}\right) \left(\frac{l}{r}\right)^2 \sin^4\gamma \right\} \cos^2\beta$$
(18)

Generally, the third term is much smaller than the first and second terms and Eq. (18) becomes approximately

$$k_e = \frac{nEbh^3}{l^3} \left\{ 1 + \frac{1}{12} \left(\frac{b}{h}\right)^2 \left(\frac{l}{r}\right)^2 \sin^2(2\gamma) \times (3\kappa^2 - 3\kappa + 1) \right\} \cos^2\beta$$
(19)

If it is assumed that the angle θ is small, then

$$\tan\beta \doteqdot \tan\phi \sin\gamma \tag{20}$$

and $\cos^2\beta$ in equation (19) becomes

$$\cos^2\beta = \frac{1}{1 + \left\{\kappa \frac{l}{2r}\sin(2\gamma)\right\}^2}$$
(21)

Substituting equation (21) in equation (19), the approximate spring constant can be calculated numerically for any given condition of the leaf spring.

5. Calculation of Equivalent Inertia Term

Since the bowl has an angular vibration about its vertical axis together with a vertical vibration, the inertia term is related to both the mass of bowl M and the inertia moment J. The kinematic energy of the bowl is represented by

$$K.E. = \frac{1}{2}M\dot{\delta}^2 \cos^2\beta \cos^2\gamma + \frac{1}{2}J\dot{\theta}^2$$
$$= \frac{1}{2}\left(M\cos^2\gamma + \frac{J\sin^2\gamma}{r^2}\right)\cos^2\beta \cdot \dot{\delta}^2$$
(22)

Hence the equivalent inertia M_e is given by

$$M_e = \left(M\cos^2\gamma + \frac{J\sin^2\gamma}{r^2}\right)\cos^2\beta \tag{23}$$

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Fig. 3 Effect of offset factor on equivalent spring constant

6. Natural Frequency of a Bowl-Type Vibratory Feeder

6.1 Fixed type vibratory feeder. In this type, the base of the feeder is attached to a foundation without a vibration isolator as shown in Fig. 1(d), so the vibratory system can have one-degree of freedom. The natural frequency of this type is expressed by

$$f_{n1} = \frac{1}{2\pi} \sqrt{\frac{nEbh^{3} \left\{1 + \frac{1}{12} \left(\frac{b}{h}\right)^{2} \left(\frac{l}{r}\right)^{2} \sin^{2}(2\gamma) \left(3\kappa^{2} - 3\kappa + 1\right)}{l^{3} \left(M\cos^{2}\gamma + \frac{J\sin^{2}\gamma}{r^{2}}\right)}} - \frac{1}{16}}$$

(24)

In an ordinary feeder, $b/h \neq 10 \sim 20$, $l/r \neq l$, $\kappa = 0$ and $\gamma = 40-70$, then equation (24) becomes approximately

$$f_{n1} = 0.046 \sqrt{\frac{nEb^3 h \sin^2(2\gamma)}{l(Mr^2 \cos^2 \gamma + J \sin^2 \gamma)}}$$
(25)

6.2 Floating type and semi-floating type feeders. When the feeder is mounted on a vibration isolator, the vibratory system has two degrees of freedom, as seen in Figs. 1(a) and 1(b). However, as discussed in the previous report [8], if the stiffness of a vibration isolator is less than about one-fifth of that of the leaf spring, the resonant frequency of a semi-floating type feeder can be identical to that of the floating type. Generally, in an ordinary vibratory feeder, the vibration isolator would satisfy the condition mentioned above. Therefore, the resonant frequency of the floating type or semi-floating type feeder is expressed as:

$$f_{n2} = \sqrt{1 + \beta_m} \bullet f_{n1} \tag{26}$$

Where α_m is the ratio of the equivalent mass of bowl to that of



Fig. 4 Effect of inclination angle of leaf spring on equivalent spring constant

the base and f_{n1} is the resonant frequency of a fixed type feeder which is expressed as equation (24).

7. Relations between Equivalent Spring Constant and Setting Conditions of Leaf Spring

Fig. 3 shows an example of the magnification factor versus the offset factor for various l/r. The magnification factor is the spring constant ratio of a bowl-type feeder spring to that of a linear-type feeder when the same leaf springs are used. It

is seen from this diagram that the magnification factor increases as l/r increases, and at $\kappa = 0$ and 1.0, it has a very large value for large l/4. When l/r = 0, it is equivalent to $r \rightarrow \infty$, then the feeding direction becomes linear and the magnification factor is unity.

Fig. 4 shows an example of the magnification factor versus the inclination angle γ for various b/h. It is seen from this diagram that the magnification factor is equal to unity at $\gamma = 0^{\circ}$ and 90°, and it becomes very large value at $\gamma = 45^{\circ}$.

8. Relations between Vibration Direction and Inclination Angle of Leaf Spring

Gnerally speaking, in a vibratory feeder, the vibration direction is one of the most important and effective parameters for conveying velocity. The conveying velocity is sensitive to the vibration direction γ' . Therefore, it is desirable that the optimum vibration direction should be chosen practically for given conditions.

In a linear-type vibratory feeder, the vibration direction angle γ' , measured from a normal direction to the conveying surface, is coincident with the inclination angle of the leaf springs, γ . Thus γ should be chosen equal to the optimum vibration direction.

However, in a bowl-type feeder, γ' is not always equal to γ ,

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Fig. 5 Various setting positions of leaf spring



Fig. 6 Effect of setting positions of leaf springs on vibration direction (when radius of base circle is constant)

but it is a function of γ , κ and l/r. This angle γ' at the upper end of the leaf spring is geometrically given by the following equation:

$$\sin \gamma' = \frac{\cos \beta \sin \gamma}{\cos \phi} \tag{27}$$

or

$$\tan \gamma' = \tan \gamma \sqrt{1 + (\kappa \frac{l}{r} \cos \gamma)^2}$$
(28)

Consider the case when the setting positions of the springs are varied as seen in Fig. 5(a) while the radius of the base circle r is held constant. The relation between γ' and γ is shown in Fig. 6 for l/r = 1. Note that the vibration direction angle γ' coincides with γ at $\kappa = 0$. It is also seen from this diagram that γ' is slightly different from γ when κ is nearly unity. Similarly, Fig. 7 shows the case when the setting positions of the springs are varied while the radius of the setting circle on bowl r_o (corresponds to the distance \overrightarrow{OA}) is held constant, as seen in Fig. 5(b). As is shown in this diagram, γ' is considerably different from γ when r is small and κ is nearly unity.

Referring to these results, it is convenient to set the offset factor at $\kappa = 0$ for selecting any vibration direction.

9. Experiment

In Fig. 8, a photographic view of the experimental apparatus and its main is shown. A vibrating table (1), on which



Fig. 7 Effect of setting positions of leaf springs on vibration direction (when radius of setting circle on bowl is constant)

a bowl should be fixed, is supported on three sets of inclined leaf springs (2). The displacement of the table is detected through a differential transformer (3). By controlling the screw (4), a desirable static load is applied to the table. The applied load is detected through strain gauges which are mounted on the load detecter ring (5). The displacement of the table and the applied load are recorded simultaneously with an X-Y recorder (6).

Fig. 9 shows an example of a load-displacement diagram for various cramping torques of leaf springs. As seen in this diagram, the equivalent spring characteristic of this system exhibits a hysteresis loop. At the same time, the softening tendency of the spring stiffness is large when the cramping torque is small. From these results, it may be concluded that the mirlo-slip occurs at the cramping parts when the displacement becomes large and, therefore, a large resultant moment is applied.

If the displacement range is small and the cramping torque is large, the spring characteristic can be considered linear. The experimental spring constant in this report is obtained in this linear range.

Figs. 10 and 11 show the experimental results of the spring constant compared with the theoretical values. It is seen from these results that the theoretical values are in good agreement with experimental results when the width of the spring is relatively small. If width becomes large, however, the experimental values are smaller than the theoretical values because of micro-slip and insufficient rigidity of the cramping parts.

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(a) VIEW OF APPARATUS



(b) MAIN PART

Fig. 8 Experimental apparatus and its main part

10. Conclusions

1. Equations for the equivalent spring constant and natural frequency of bowl-type feeders can be derived analytically.

2. In an ordinary bowl-type feeder, the deformation of the spring is complicated and the equivalent spring constant is about $2 \sim 40$ times as large as that in the case where parallel leaf springs are used.

3. The equivalent spring constant is influenced by the offset factor which is related to the setting position of the leaf spring. When the offset factor κ is varied from zero to unity while the rest of the parameters are held constant, the equivalent spring constant takes a minimum value at $\kappa = 0.5$.

4. The equivalent spring constant is influenced by the inclination of the leaf spring. When the inclination of the leaf spring is varied, the spring constant takes a maximum at $\gamma = 45^{\circ}$.

5. The vibration direction angle is not equal to the inclination of leaf spring, except in the case when $\kappa = 0$.

6. The natural frequency of the bowl-type feeder is approximately proportional to the square root of the thickness of the leaf spring and to the two thirds power of its width, while in the linear-type feeder it is proportional to the two thirds power of thickness and the square root of width.

7. The inertia term is related to the mass of the bowl and the inertia moment about its vertical axis.

8. If the cramping torque of spring is small, the equivalent spring characteristic exhibits a hysteresis loop for large amplitude of vibration.

9. It has been confirmed that the theoretical results agree well with the experimental results in the range of practical use.

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Fig. 9 Load-displacement diagram for $h = 1 \text{ mm}, b = 25 \text{ mm}, \kappa = 0$



Fig. 10 Effect of offset factor on equivalent spring constant (h = 1 mm, l = 132 mm, $\gamma = 60^\circ$, r = 100 mm, $E = 96100 \text{ N/mm}^2$)

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