Control Volume Summary

The control volume (C.V.) approach provides an easy means by which "ballpark" engineering answers can be obtained. Underlying the C.V. forms of the fundamental physical laws is the Reynolds Transport Theorem (RTT).

$$\frac{DB}{Dt} = \frac{\partial}{\partial t}\int_{c.v} \rho\beta d\forall + \int_{c.s.} \rho\beta\left(\vec{V}\cdot\vec{n}\right)dA$$

$\frac{\partial}{\partial t}\int_{c.v} \rho\beta d\forall$ -- rate of change with time in C.V.

$\int_{c.s.} \rho\beta\left(\vec{V}\cdot\vec{n}\right)dA$--net rates of inflow and outflow

B=extensive property (e.g. mas, momentum, energy)

β=intensive property dB/dm

The RTT allows the time-rate-of-change of a property associated with a system (a fixed collection of mass particles) to be written in terms of a fixed volume in space (i.e. C.V.). This is significant because the fundamental physical laws are written for a system.

Mass: (A scalar Eq.) $\frac{DM_{sys}}{Dt} = 0 = \frac{\partial}{\partial t}\int_{c.v} \rho d\forall + \int_{c.s.} \rho\left(\vec{V}\cdot\vec{n}\right)dA$ [β=1]

Momentum: (A vector Eq.) $\frac{DMom_{sys}}{Dt} = \sum\vec{F} = \frac{\partial}{\partial t}\int_{c.v} \rho\vec{V} d\forall + \int_{c.s.} \rho\vec{V}\left(\vec{V}\cdot\vec{n}\right)dA$ [$\beta=\vec{V}$]

Energy: (A scalar Eq.) $\frac{DE_{sys}}{Dt} = \dot{Q} - \dot{W} = \frac{\partial}{\partial t}\int_{c.v} \rho e d\forall + \int_{c.s.} \rho e\left(\vec{V}\cdot\vec{n}\right)dA$ [β=e]

Entropy: (A scalar Eq.)

$$\frac{1}{T_{surr}}\frac{dQ}{dt} \le \frac{DS_{sys}}{DT} = \frac{\partial}{\partial t}\int_{c.v.} \rho s d\forall + \int_{c.s.} \rho s\left(\vec{V}\cdot\vec{n}\right)dA$$

[β=s]

Steady flow: $\frac{\partial}{\partial t} \to 0$

1-D flow: $\int$ expressions become algebraic expressions

Velocity is the velocity relative to the control surfaces for the flux terms and is the velocity at points in the control volume for the unsteady term.