Chapter 11
EXTERNAL FLOW: DRAG AND LIFT

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Chapter 11 External Flow: Drag and Lift

Drag, Lift, and Drag Coefficients

11-1C Solution We are to compare the speed of two bicyclists.

Analysis The bicyclist who leans down and brings his body closer to his knees goes faster since the frontal area and thus the drag force is less in that position. The drag coefficient also goes down somewhat, but this is a secondary effect.

Discussion This is easily experienced when riding a bicycle down a long hill.

11-2C Solution We are to discuss how the local skin friction coefficient changes with position along a flat plate in laminar flow.

Analysis The local friction coefficient decreases with downstream distance in laminar flow over a flat plate.

Discussion At the front of the plate, the boundary layer is very thin, and thus the shear stress at the wall is large. As the boundary layer grows downstream, however, the boundary layer grows in size, decreasing the wall shear stress.

11-3C Solution We are to define the frontal area of a body and discuss its applications.

Analysis The frontal area of a body is the area seen by a person when looking from upstream (the area projected on a plane normal to the direction of flow of the body). The frontal area is appropriate to use in drag and lift calculations for blunt bodies such as cars, cylinders, and spheres.

Discussion The drag force on a body is proportional to both the drag coefficient and the frontal area. Thus, one is able to reduce drag by reducing the drag coefficient or the frontal area (or both).

11-4C Solution We are to define the planform area of a body and discuss its applications.

Analysis The planform area of a body is the area that would be seen by a person looking at the body from above in a direction normal to flow. The planform area is the area projected on a plane parallel to the direction of flow and normal to the lift force. The planform area is appropriate to use in drag and lift calculations for slender bodies such as flat plate and airfoils when the frontal area is very small.

Discussion Consider for example an extremely thin flat plate aligned with the flow. The frontal area is nearly zero, and is therefore not appropriate to use for calculation of drag or lift coefficient.

11-5C Solution We are to explain when a flow is 2-D, 3-D, and axisymmetric.

Analysis The flow over a body is said to be two-dimensional when the body is very long and of constant cross-section, and the flow is normal to the body (such as the wind blowing over a long pipe perpendicular to its axis). There is no significant flow along the axis of the body. The flow along a body that possesses symmetry along an axis in the flow direction is said to be axisymmetric (such as a bullet piercing through air). Flow over a body that cannot be modeled as two-dimensional or axisymmetric is three-dimensional. The flow over a car is three-dimensional.

Discussion As you might expect, 3-D flows are much more difficult to analyze than 2-D or axisymmetric flows.
11-6C Solution We are to discuss the difference between upstream and free-stream velocity.

Analysis The velocity of the fluid relative to the immersed solid body sufficiently far away from a body is called the free-stream velocity, $V$. The upstream (or approach) velocity $V$ is the velocity of the approaching fluid far ahead of the body. These two velocities are equal if the flow is uniform and the body is small relative to the scale of the free-stream flow.

Discussion This is a subtle difference, and the two terms are often used interchangeably.

11-7C Solution We are to discuss the difference between streamlined and blunt bodies.

Analysis A body is said to be streamlined if a conscious effort is made to align its shape with the anticipated streamlines in the flow. Otherwise, a body tends to block the flow, and is said to be blunt. A tennis ball is a blunt body (unless the velocity is very low and we have “creeping flow”).

Discussion In creeping flow, the streamlines align themselves with the shape of any body – this is a much different regime than our normal experiences with flows in air and water. A low-drag body shape in creeping flow looks much different than a low-drag shape in high Reynolds number flow.

11-8C Solution We are to discuss applications in which a large drag is desired.

Analysis Some applications in which a large drag is desirable: parachuting, sailing, and the transport of pollens.

Discussion When sailing efficiently, however, the lift force on the sail is more important than the drag force in propelling the boat.

11-9C Solution We are to define drag and discuss why we usually try to minimize it.

Analysis The force a flowing fluid exerts on a body in the flow direction is called drag. Drag is caused by friction between the fluid and the solid surface, and the pressure difference between the front and back of the body. We try to minimize drag in order to reduce fuel consumption in vehicles, improve safety and durability of structures subjected to high winds, and to reduce noise and vibration.

Discussion In some applications, such as parachuting, high drag rather than low drag is desired.

11-10C Solution We are to define lift, and discuss its cause and the contribution of wall shear to lift.

Analysis The force a flowing fluid exerts on a body in the normal direction to flow that tends to move the body in that direction is called lift. It is caused by the components of the pressure and wall shear forces in the direction normal to the flow. The wall shear contributes to lift (unless the body is very slim), but its contribution is usually small.

Discussion Typically the nonsymmetrical shape of the body is what causes the lift force to be produced.
Chapter 11 External Flow: Drag and Lift

11-11C Solution We are to explain how to calculate the drag coefficient, and discuss the appropriate area.

Analysis When the drag force $F_D$, the upstream velocity $V$, and the fluid density $\rho$ are measured during flow over a body, the drag coefficient is determined from

$$C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A}$$

where $A$ is ordinarily the **frontal area** (the area projected on a plane normal to the direction of flow) of the body.

Discussion In some cases, however, such as flat plates aligned with the flow or airplane wings, the planform area is used instead of the frontal area. Planform area is the area projected on a plane parallel to the direction of flow and normal to the lift force.

11-12C Solution We are to explain how to calculate the lift coefficient, and discuss the appropriate area.

Analysis When the lift force $F_L$, the upstream velocity $V$, and the fluid density $\rho$ are measured during flow over a body, the lift coefficient can be determined from

$$C_L = \frac{F_L}{\frac{1}{2} \rho V^2 A}$$

where $A$ is ordinarily the **planform area**, which is the area that would be seen by a person looking at the body from above in a direction normal to the body.

Discussion In some cases, however, such as flat plates aligned with the flow or airplane wings, the planform area is used instead of the frontal area. Planform area is the area projected on a plane parallel to the direction of flow and normal to the lift force.

11-13C Solution We are to define and discuss terminal velocity.

Analysis The **maximum velocity a free falling body can attain** is called the **terminal velocity**. It is determined by setting the weight of the body equal to the drag and buoyancy forces, $W = F_D + F_B$.

Discussion When discussing the settling of small dust particles, terminal velocity is also called **terminal settling speed** or **settling velocity**.

11-14C Solution We are to discuss the difference between skin friction drag and pressure drag, and which is more significant for slender bodies.

Analysis The **part of drag that is due directly to wall shear stress** $\tau_w$ is called the **skin friction drag** $F_D, \text{friction}$ since it is caused by frictional effects, and the **part that is due directly to pressure $P$ and depends strongly on the shape of the body** is called the **pressure drag** $F_D, \text{pressure}$. For slender bodies such as airfoils, the friction drag is usually more significant.

Discussion For blunt bodies, on the other hand, pressure drag is usually more significant than skin friction drag.
11-15C
Solution We are to discuss the effect of surface roughness on drag coefficient.

Analysis The friction drag coefficient is independent of surface roughness in laminar flow, but is a strong function of surface roughness in turbulent flow due to surface roughness elements protruding farther into the viscous sublayer.

Discussion If the roughness is very large, however, the drag on bodies is increased even for laminar flow, due to pressure effects on the roughness elements.

11-16C
Solution We are to discuss the effect of streamlining, and its effect on friction drag and pressure drag.

Analysis As a result of streamlining, (a) friction drag increases, (b) pressure drag decreases, and (c) total drag decreases at high Reynolds numbers (the general case), but increases at very low Reynolds numbers (creeping flow) since the friction drag dominates at low Reynolds numbers.

Discussion Streamlining can significantly reduce the overall drag on a body at high Reynolds number.

11-17C
Solution We are to define and discuss flow separation.

Analysis At sufficiently high velocities, the fluid stream detaches itself from the surface of the body. This is called separation. It is caused by a fluid flowing over a curved surface at a high velocity (or technically, by adverse pressure gradient). Separation increases the drag coefficient drastically.

Discussion A boundary layer has a hard time resisting an adverse pressure gradient, and is likely to separate. A turbulent boundary layer is in general more resilient to flow separation than a laminar flow.

11-18C
Solution We are to define and discuss drafting.

Analysis Drafting is when a moving body follows another moving body by staying close behind in order to reduce drag. It reduces the pressure drag and thus the drag coefficient for the drafted body by taking advantage of the low pressure wake region of the moving body in front.

Discussion We often see drafting in automobile and bicycle racing.

11-19C
Solution We are to discuss how drag coefficient varies with Reynolds number.

Analysis (a) In general, the drag coefficient decreases with Reynolds number at low and moderate Reynolds numbers. (b) The drag coefficient is nearly independent of Reynolds number at high Reynolds numbers (Re > 10^5).

Discussion When the drag coefficient is independent of Re at high values of Re, we call this Reynolds number independence.
11-20C  
**Solution**  
We are to discuss the effect of adding a fairing to a circular cylinder.

**Analysis**  
As a result of attaching fairings to the front and back of a cylindrical body at high Reynolds numbers, (a) friction drag increases, (b) pressure drag decreases, and (c) total drag decreases.

**Discussion**  
In creeping flow (very low Reynolds numbers), however, adding a fairing like this would actually increase the overall drag, since the surface area and therefore the skin friction drag would increase significantly.

11-21  
**Solution**  
The drag force acting on a car is measured in a wind tunnel. The drag coefficient of the car at the test conditions is to be determined.

**Assumptions**  
1. The flow of air is steady and incompressible.  
2. The cross-section of the tunnel is large enough to simulate free flow over the car.  
3. The bottom of the tunnel is also moving at the speed of air to approximate actual driving conditions or this effect is negligible.  
4. Air is an ideal gas.

**Properties**  
The density of air at 1 atm and 25°C is \( \rho = 1.164 \text{ kg/m}^3 \).

**Analysis**  
The drag force acting on a body and the drag coefficient are given by

\[
F_D = C_D A \rho V^2 \quad \text{and} \quad C_D = \frac{2F_D}{\rho AV^2}
\]

where \( A \) is the frontal area. Substituting and noting that 1 m/s = 3.6 km/h, the drag coefficient of the car is determined to be

\[
C_D = \frac{2 \times (220 \text{ N})}{(1.164 \text{ kg/m}^3)(1.25 \times 1.65 \text{ m}^2)(90/3.6 \text{ m/s})^2 \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right)} = 0.29
\]

**Discussion**  
Note that the drag coefficient depends on the design conditions, and its value will be different at different conditions. Therefore, the published drag coefficients of different vehicles can be compared meaningfully only if they are determined under identical conditions. This shows the importance of developing standard testing procedures in industry.

11-22  
**Solution**  
The resultant of the pressure and wall shear forces acting on a body is given. The drag and the lift forces acting on the body are to be determined.

**Analysis**  
The drag and lift forces are determined by decomposing the resultant force into its components in the flow direction and the normal direction to flow,

- **Drag force:** \( F_D = F_R \cos \theta = (580 \text{ N}) \cos 35^\circ = 475 \text{ N} \)
- **Lift force:** \( F_L = F_R \sin \theta = (580 \text{ N}) \sin 35^\circ = 333 \text{ N} \)

**Discussion**  
Note that the greater the angle between the resultant force and the flow direction, the greater the lift.
Chapter 11 External Flow: Drag and Lift

11-23 Solution The total drag force acting on a spherical body is measured, and the pressure drag acting on the body is calculated by integrating the pressure distribution. The friction drag coefficient is to be determined.

Assumptions 1 The flow of air is steady and incompressible. 2 The surface of the sphere is smooth. 3 The flow over the sphere is turbulent (to be verified).

Properties The density and kinematic viscosity of air at 1 atm and 5°C are \( \rho = 1.269 \text{ kg/m}^3 \) and \( \nu = 1.382 \times 10^{-5} \text{ m}^2/\text{s} \). The drag coefficient of sphere in turbulent flow is \( C_D = 0.2 \), and its frontal area is \( A = \pi D^2/4 \) (Table 11-2).

Analysis The total drag force is the sum of the friction and pressure drag forces. Therefore,

\[ F_D = F_{D,\text{friction}} + F_{D,\text{pressure}} = 5.2 - 4.9 = 0.3 \text{ N} \]

where \( F_D = C_D A \frac{\rho V^2}{2} \) and \( F_{D,\text{friction}} = C_{D,\text{friction}} A \frac{\rho V^2}{2} \)

Taking the ratio of the two relations above gives

\[ C_{D,\text{friction}} = \frac{F_{D,\text{friction}}}{F_D} C_D = \frac{0.3 \text{ N}}{5.2 \text{ N}} (0.2) = 0.0115 \]

Now we need to verify that the flow is turbulent. This is done by calculating the flow velocity from the drag force relation, and then the Reynolds number:

\[ Re = \frac{V D}{\nu} = \frac{(60.2 \text{ m/s})(0.12 \text{ m})}{1.382 \times 10^{-5} \text{ m}^2/\text{s}} = 5.23 \times 10^5 \]

which is greater than \( 2 \times 10^5 \). Therefore, the flow is turbulent as assumed.

Discussion Note that knowing the flow regime is important in the solution of this problem since the total drag coefficient for a sphere is 0.5 in laminar flow and 0.2 in turbulent flow.

11-24 Solution A car is moving at a constant velocity. The upstream velocity to be used in fluid flow analysis is to be determined for the cases of calm air, wind blowing against the direction of motion of the car, and wind blowing in the same direction of motion of the car.

Analysis In fluid flow analysis, the velocity used is the relative velocity between the fluid and the solid body. Therefore:

(a) Calm air: \( V = V_{\text{car}} = 110 \text{ km/h} \)

(b) Wind blowing against the direction of motion:

\[ V = V_{\text{car}} + V_{\text{wind}} = 110 + 30 = 140 \text{ km/h} \]

(c) Wind blowing in the same direction of motion:

\[ V = V_{\text{car}} - V_{\text{wind}} = 110 - 30 = 80 \text{ km/h} \]

Discussion Note that the wind and car velocities are added when they are in opposite directions, and subtracted when they are in the same direction.
Chapter 11 External Flow: Drag and Lift

**11-25E**

**Solution**  The frontal area of a car is reduced by redesigning. The amount of fuel and money saved per year as a result are to be determined.

**Assumptions**  
1. The car is driven 12,000 miles a year at an average speed of 55 km/h.  
2. The effect of reduction of the frontal area on the drag coefficient is negligible.

**Properties**  
The densities of air and gasoline are given to be 0.075 lbm/ft$^3$ and 50 lbm/ft$^3$, respectively. The heating value of gasoline is given to be 20,000 Btu/lbm. The drag coefficient is $C_D = 0.3$ for a passenger car (Table 11-2).

**Analysis**  
The drag force acting on a body is determined from

$$F_D = C_D A \frac{\rho V^2}{2}$$

where $A$ is the frontal area of the body. The drag force acting on the car before redesigning is

$$F_D = 0.3 (18 \, \text{ft}^2) \frac{(0.075 \, \text{lbm/ft}^3)(55 \, \text{mph})^2}{2} \left(1.4667 \, \text{ft/s} / 1 \, \text{mph}\right) \left(1 \, \text{lbf} / 32.2 \, \text{lbm} \cdot \text{ft/s}^2\right) = 40.9 \, \text{lbf}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 12,000 miles are

$$W_{\text{drag}} = F_D L = (40.9 \, \text{lbf})(12,000 \, \text{miles/yr}) \left(\frac{5280 \, \text{ft}}{1 \, \text{mile}}\right) \left(\frac{1 \, \text{Btu}}{778.169 \, \text{lbm} \cdot \text{ft}}\right) = 3.330 \times 10^6 \, \text{Btu/yr}$$

$$E_{\text{in}} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{3.330 \times 10^6 \, \text{Btu/yr}}{0.30} = 1.110 \times 10^7 \, \text{Btu/yr}$$

Then the amount and costs of the fuel that supplies this much energy are

$$\text{Amount of fuel} = \frac{E_{\text{in}}}{\rho_{\text{fuel}}} = \frac{(1.110 \times 10^7 \, \text{Btu/yr})/(20,000 \, \text{Btu/lbm})}{50 \, \text{lbm/ft}^3} = 11.10 \, \text{ft}^3/\text{yr}$$

$$\text{Cost} = \text{(Amount of fuel})(\text{Unit cost}) = (11.10 \, \text{ft}^3/\text{yr})(3.10/\text{gal})(7.4804 \, \text{gal/ft}^3) = 257.4 \, \text{$/yr}$$

That is, the car uses 11.10 ft$^3 = 83.03$ gallons of gasoline at a cost of $257.4$ per year to overcome the drag.

The drag force and the work done to overcome it are directly proportional to the frontal area. Then the percent reduction in the fuel consumption due to reducing frontal area is equal to the percent reduction in the frontal area:

$$\text{Reduction ratio} = \frac{A - A_{\text{new}}}{A} = \frac{18 - 15}{18} = 0.1667$$

Amount reduction = (Reduction ratio)(Amount)

$$= 0.1667(83.03 \, \text{gal/year}) = 13.8 \, \text{gal/year}$$

Cost reduction = (Reduction ratio)(Cost) = 0.1667($257.4/\text{yr}) = $42.9/\text{yr}

Therefore, reducing the frontal area reduces the fuel consumption due to drag by 16.7%.

**Discussion**  
Note from this example that significant reductions in drag and fuel consumption can be achieved by reducing the frontal area of a vehicle.
**Solution**  The previous problem is reconsidered. The effect of frontal area on the annual fuel consumption of the car as the frontal area varied from 10 to 30 ft² in increments of 2 ft² are to be investigated.

**Analysis**  The EES Equations window is printed below, along with the tabulated and plotted results.

\[
\begin{align*}
CD &= 0.3 \\
rho &= 0.075 \text{ "lbm/ft}^3\" \\
V &= 55 \times 1.4667 \text{ "ft/s"} \\
Eff &= 0.32 \\
Price &= 2.20 \text{ "$/gal"} \\
efuel &= 20000 \text{ "Btu/lbm"} \\
rho_{\text{gas}} &= 50 \text{ "lbm/ft}^3\" \\
L &= 12000 \times 5280 \text{ "ft"} \\
FD &= CD \times A \times (\rho \times V^2)/2/32.2 \text{ "lbf"} \\
W_{\text{drag}} &= FD \times L/778.169 \text{ "Btu"} \\
Ein &= W_{\text{drag}} / Eff \\
m &= Ein/efuel \text{ "lbm"} \\
Vol &= (m/rho_{\text{gas}}) \times 7.4804 \text{ "gal"} \\
Cost &= Vol \times Price
\end{align*}
\]

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<th>A, \text{ ft}^2</th>
<th>F_{\text{drag}} \text{ lbf}</th>
<th>Amount, \text{ gal}</th>
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**Discussion**  As you might expect, the cost goes up linearly with area, since drag force goes up linearly with area.
Solution  A circular sign is subjected to high winds. The drag force acting on the sign and the bending moment at the bottom of its pole are to be determined.

Assumptions  1 The flow of air is steady and incompressible. 2 The drag force on the pole is negligible. 3 The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties  The drag coefficient for a thin circular disk is $C_D = 1.1$ (Table 11-2). The density of air at 100 kPa and $10^\circ C = 283 \, K$ is

$$\rho = \frac{P}{RT} = \frac{100 \, kPa}{(0.287 \, kPa \cdot m^3/kg \cdot K)(283 \, K)} = 1.231 \, kg/m^3$$

Analysis  The frontal area of a circular plate subjected to normal flow is $A = \pi D^2/4$. Then the drag force acting on the sign is

$$F_D = C_D A \frac{\rho V^2}{2}$$

$$= (1.1)[\pi (0.5 \, m)^2/4](1.231 \, kg/m^3)(150 / 3.6 \, m/s)^2 \left( \frac{1 \, N}{1 \, kg \cdot m/s^2} \right) = 231 \, N$$

Noting that the resultant force passes through the center of the stop sign, the bending moment at the bottom of the pole becomes

$$M_{bottom} = F_D \times L = (231 \, N)(1.5 + 0.25) \, m = 404 \, Nm$$

Discussion  Note that the drag force is equivalent to the weight of over 23 kg of mass. Therefore, the pole must be strong enough to withstand the weight of 23 kg hanged at one of its end when it is held from the other end horizontally.
11-28E

Solution We are to estimate how much money is wasted by driving with a pizza sign on a car roof.

Properties \( \rho_{\text{fuel}} = 50.2 \text{ lbm/ft}^3 \), \( H_{\text{fuel}} = 1.53 \times 10^7 \text{ ft-lbf/lbm} \), \( \rho_{\text{air}} = 0.07518 \text{ lbm/ft}^3 \), \( \mu_{\text{air}} = 1.227 \times 10^{-5} \text{ lbm/ft-s} \)

Analysis First some conversions: \( V = 45 \text{ mph} = 66.0 \text{ ft/s} \) and the total distance traveled in one year = \( L = 10,000 \text{ miles} = 5.280 \times 10^7 \text{ ft} \). The additional drag force due to the sign is

\[
F_D = \frac{1}{2} \rho_{\text{air}} V^2 C_D A
\]

where \( A \) is the frontal area. The work required to overcome this additional drag is force times distance. So, letting \( L \) be the total distance driven in a year,

\[
\text{Work}_{\text{drag}} = F_D L = \frac{1}{2} \rho_{\text{air}} V^2 C_D A L
\]

The energy required to perform this work is much greater than this due to overall efficiency of the car engine, transmission, etc. Thus,

\[
\text{Energy}_{\text{required}} = \frac{\text{Work}_{\text{drag}}}{\eta_{\text{overall}}} = \frac{1}{2} \rho_{\text{air}} V^2 C_D A L \eta_{\text{overall}}
\]

But the required energy is also equal to the heating value of the fuel \( H_{\text{fuel}} \) times the mass of fuel required. In terms of required fuel volume, volume = mass/density. Thus,

\[
\text{Volume}_{\text{fuel required}} = \frac{m_{\text{fuel required}}}{\rho_{\text{fuel}}} = \frac{\text{Energy}_{\text{required}}}{H_{\text{fuel}}} = \frac{1}{2} \rho_{\text{air}} V^2 C_D A L \frac{\rho_{\text{fuel}}}{\eta_{\text{overall}} H_{\text{fuel}}}
\]

The above is our answer in variable form. Finally, we plug in the given values and properties to obtain the numerical answer.

\[
\text{Volume}_{\text{fuel required}} = \frac{\frac{1}{2} \left( 0.07518 \text{ lbm/ft}^3 \right) \left( 66.0 \text{ ft/s} \right)^2 \left( 0.94 \right) \left( 0.612 \text{ ft}^2 \right) \left( 5.280 \times 10^7 \text{ ft} \right)}{\left( 50.2 \text{ lbm/ft}^3 \right) \left( 0.332 \right) \left( 1.53 \times 10^7 \text{ ft-lbf/lbm} \right) \left( 32.174 \text{ lbf-ft/s}^2 \right)}
\]

which yields \( V_{\text{fuel required}} = 0.6062 \text{ ft}^3 \), which is equivalent to 4.535 gallons per year. At $3.50 per gallon, the total cost is about $15.87 per year, or rounding to two significant digits, the total cost is about $16 per year.

Discussion Any more than 2 significant digits in the final answer cannot be justified. Bill would be wise to lobby for a more aerodynamic pizza sign.
Solution

An advertisement sign in the form of a rectangular block that has the same frontal area from all four sides is mounted on top of a taxicab. The increase in the annual fuel cost due to this sign is to be determined.

Assumptions

1. The flow of air is steady and incompressible.
2. The car is driven 60,000 km a year at an average speed of 50 km/h.
3. The overall efficiency of the engine is 28%.
4. The effect of the sign and the taxicab on the drag coefficient of each other is negligible (no interference), and the edge effects of the sign are negligible (a crude approximation).
5. The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties

The densities of air and gasoline are given to be 1.25 kg/m³ and 0.72 kg/L, respectively. The heating value of gasoline is given to be 42,000 kJ/kg. The drag coefficient for a square rod for normal flow is $C_D = 2.2$ (Table 11-1).

Analysis

Noting that 1 m/s = 3.6 km/h, the drag force acting on the sign is

$$F_D = C_D A \frac{\rho V^2}{2} = (2.2)(0.9 \times 0.3 \text{ m}^2) \left( \frac{1.25 \text{ kg/m}^3}{1 \text{ kg} \cdot \text{m/s}^2} \right) \left( \frac{50}{3.6 \text{ m/s}} \right)^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 71.61 \text{ N}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 60,000 km are

$$W_{\text{drag}} = F_D L = (71.61 \text{ N})(60,000 \text{ km/year}) = 4.30 \times 10^6 \text{ kJ/year}$$

$$E_{\text{in}} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{4.30 \times 10^6 \text{ kJ/year}}{0.28} = 1.54 \times 10^7 \text{ kJ/year}$$

Then the amount and cost of the fuel that supplies this much energy are

$$\text{Amount of fuel} = \frac{E_{\text{in}}}{HV_{\text{fuel}}} = \frac{(1.54 \times 10^7 \text{ kJ/year})/(42,000 \text{ kJ/kg})}{0.72 \text{ kg/L}} = 509 \text{ L/year}$$

Cost = (Amount of fuel)(Unit cost) = (509 L/year)($1.10/L) = $560/year

That is, the taxicab will use 509 L of gasoline at a cost of $560 per year to overcome the drag generated by the advertisement sign.

Discussion

Note that the advertisement sign increases the fuel cost of the taxicab significantly. The taxicab operator may end up losing money by installing the sign if he/she is not aware of the major increase in the fuel cost, and negotiate accordingly.
11-30E

Solution A person who normally drives at 55 mph now starts driving at 75 mph. The percentage increase in fuel consumption of the car is to be determined.

Assumptions 1 The fuel consumption is approximately proportional to the drag force on a level road (as stated). 2 The drag coefficient remains the same.

Analysis The drag force is proportional to the square of the velocity, and power is force times velocity. Therefore,

\[ F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad W_{\text{drag}} = F_D V = C_D A \frac{\rho V^3}{2} \]

Then the ratio of the power used to overcome drag force at \( V_2 = 75 \) mph to that at \( V_1 = 55 \) mph becomes

\[ \frac{W_{\text{drag2}}}{W_{\text{drag1}}} = \frac{V_2^3}{V_1^3} = \frac{75^3}{55^3} = 2.54 \]

Therefore, the power to overcome the drag force and thus fuel consumption per unit time more than doubles as a result of increasing the velocity from 55 to 75 mph.

Discussion This increase appears to be large. This is because all the engine power is assumed to be used entirely to overcome drag. Still, the simple analysis above shows the strong dependence of the fuel consumption on the cruising speed of a vehicle. A better measure of fuel consumption is the amount of fuel used per unit distance (rather than per unit time). A car cruising at 55 mph will travel a distance of 55 miles in 1 hour. But a car cruising at 75 mph will travel the same distance at \( 55/75 = 0.733 \) h or 73.3% of the time. Therefore, for a given distance, the increase in fuel consumption is \( 2.54 \times 0.733 = 1.86 \) an increase of 86%. This is large, especially with the high cost of gasoline these days.

11-31

Solution A submarine is treated as an ellipsoid at a specified length and diameter. The powers required for this submarine to cruise horizontally in seawater and to tow it in air are to be determined.

Assumptions 1 The submarine can be treated as an ellipsoid. 2 The flow is turbulent. 3 The drag of the towing rope is negligible. 4 The motion of submarine is steady and horizontal.

Properties The drag coefficient for an ellipsoid with \( L/D = 25/5 = 5 \) is \( C_D = 0.1 \) in turbulent flow (Table 11-2). The density of sea water is given to be 1025 kg/m\(^3\). The density of air is given to be 1.30 kg/m\(^3\).

Analysis Noting that 1 m/s = 3.6 km/h, the velocity of the submarine is equivalent to \( V = 40/3.6 = 11.11 \) m/s. The frontal area of an ellipsoid is \( A = \pi D^2/4 \). Then the drag force acting on the submarine becomes

In water: \( F_D = C_D A \frac{\rho V^2}{2} = (0.1)[\pi(5 \text{ m})^2 / 4](1025 \text{ kg/m}^3)(11.11 \text{ m/s})^2 \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 124.2 \text{ kN} \)

In air: \( F_D = C_D A \frac{\rho V^2}{2} = (0.1)[\pi(5 \text{ m})^2 / 4](1.30 \text{ kg/m}^3)(11.11 \text{ m/s})^2 \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 0.1575 \text{ kN} \)

Noting that power is force times velocity, the power needed to overcome this drag force is

In water: \( W_{\text{drag}} = F_D V = (124.2 \text{ kN})(11.11 \text{ m/s}) \left( \frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}} \right) = 1380 \text{ kW} \)

In air: \( W_{\text{drag}} = F_D V = (0.1575 \text{ kN})(11.11 \text{ m/s}) \left( \frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}} \right) = 1.75 \text{ kW} \)

Therefore, the power required for this submarine to cruise horizontally in seawater is 1380 kW and the power required to tow this submarine in air at the same velocity is 1.75 kW.

Discussion Note that the power required to move the submarine in water is about 800 times the power required to move it in air. This is due to the higher density of water compared to air (sea water is about 800 times denser than air).
Solution
A billboard is subjected to high winds. The drag force acting on the billboard is to be determined.

Assumptions 1 The flow of air is steady and incompressible. 2 The drag force on the supporting poles are negligible. 3 The flow is turbulent so that the tabulated value of the drag coefficient can be used. 4 The 3-D effects are negligible, and thus the billboard is approximated as a very long flat plate (two-dimensional).

Properties If the plate were two dimensional, the drag coefficient would be $C_D = 1.9$ (Table 11-1). However, for this 3-D case, Table 11-2 yields $C_D = 1.10 + 0.02(20/8 + 8/20) = 1.158$, which is a better approximation of the drag coefficient. We calculate the gas constant for air (Table A-1E) as

$$R_{\text{air}} = \frac{R}{M} = \frac{10.732 \text{ psia} \cdot \text{ft}^3/\text{lbmol} \cdot \text{R}}{28.97 \text{ lbm/lbmol}} = 0.37045 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R}$$

The density of air at 14.3 psia and 40°F = 40 + 459.67 = 499.67 R is thus

$$\rho = \frac{P}{RT} = \frac{14.3 \text{ psia}}{(0.37045 \text{ psia} \cdot \text{ft}^3/\text{lbm} \cdot \text{R})(499.67 \text{ R})} = 0.077254 \text{ lbm/ft}^3$$

Analysis The drag force acting on the billboard is determined from

$$F_D = C_D A \frac{\rho V^2}{2} = (1.158)(12 \times 20 \text{ ft}^2) \left(0.077254 \text{ lbm/ft}^3\right)(55 \text{ mi/h})^2 \left(\frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2}\right) \left(\frac{1.4667 \text{ ft/s}}{1 \text{ mi/h}}\right)^2$$

$$= 2171 \text{ lbf} \approx 2170 \text{ lbf}$$

Discussion Note that the drag force is equivalent to the weight of 2170 lbm of mass. Therefore, the support bars must be strong enough to withstand the weight of 2170 lbm hanging on one of their ends when they are held from the other end horizontally (which causes a fairly large moment or torque on the support bars).
11-33

Solution  A semi truck is exposed to winds from its side surface. The wind velocity that will tip the truck over to its side is to be determined.

Assumptions  1 The flow of air in the wind is steady and incompressible.  2 The edge effects on the semi truck are negligible (a crude approximation), and the resultant drag force acts through the center of the side surface.  3 The flow is turbulent so that the tabulated value of the drag coefficient can be used.  4 The distance between the wheels on the same axle is also 2 m.  5 The semi truck is loaded uniformly so that its weight acts through its center.

Properties  The density of air is given to be $\rho = 1.10 \text{ kg/m}^3$. The drag coefficient for a body of rectangular cross-section corresponding to $L/D = 2/2 = 1$ is $C_D = 2.2$ when the wind is normal to the side surface (Table 11-2).

Analysis  When the truck is first tipped, the wheels on the wind-loaded side of the truck will be off the ground, and thus all the reaction forces from the ground will act on wheels on the other side. Taking the moment about an axis passing through these wheels and setting it equal to zero gives the required drag force to be

$$\sum M_{\text{wheels}} = 0 \rightarrow F_D \times (2 \text{ m}) - W \times (1 \text{ m}) = 0 \rightarrow F_D = \frac{W}{2}$$

Substituting, the required drag force is determined to be

$$F_D = \frac{mg}{2} = \frac{(5000 \text{ kg})(9.81 \text{ m/s}^2)}{2} = 24,525 \text{ N}$$

The wind velocity that will cause this drag force is determined to be

$$F_D = C_D A \frac{\rho V^2}{2} \rightarrow 24,525 \text{ N} = (2.2)(2.5 \times 9 \text{ m}^2) \frac{(1.10 \text{ kg/m}^3) V^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) \rightarrow V = 31.48 \text{ m/s}$$

which is equivalent to a wind velocity of $V = 31.48 \times 3.6 = 113 \text{ km/h}$.

Discussion  This is very high velocity, and it can be verified easily by calculating the Reynolds number that the flow is turbulent as assumed.
11-34
Solution  A bicyclist is riding his bicycle downhill on a road with a specified slope without pedaling or breaking. The terminal velocity of the bicyclist is to be determined for the upright and racing positions.

Assumptions  1 The rolling resistance and bearing friction are negligible. 2 The drag coefficient remains constant. 3 The buoyancy of air is negligible.

Properties  The density of air is given to be $\rho = 1.25 \text{ kg/m}^3$. The frontal area and the drag coefficient are given to be 0.45 m$^2$ and 1.1 in the upright position, and 0.4 m$^2$ and 0.9 on the racing position.

Analysis  The terminal velocity of a free falling object is reached when the drag force equals the component of the total weight (bicyclist + bicycle) in the flow direction,

$$ F_D = W_{\text{total}} \sin \theta \quad \rightarrow \quad C_D A \frac{\rho V^2}{2} = m_{\text{total}} g \sin \theta $$

Solving for $V$ gives

$$ V = \sqrt{\frac{2 gm_{\text{total}} \sin \theta}{C_D A \rho}} $$

The terminal velocities for both positions are obtained by substituting the given values:

Upright position: $ V = \sqrt{\frac{2(9.81 \text{ m/s}^2)(70 + 15 \text{ kg}) \sin 8^\circ}{1.09 \text{ m}^2(1.25 \text{ kg/m}^3)}} = 19.4 \text{ m/s} = 69.7 \text{ km/h} \approx 70 \text{ km/h} $

Racing position: $ V = \sqrt{\frac{2(9.81 \text{ m/s}^2)(70 + 15 \text{ kg}) \sin 8^\circ}{0.9 \text{ m}^2(1.25 \text{ kg/m}^3)}} = 22.7 \text{ m/s} = 81.8 \text{ km/h} \approx 82 \text{ km/h} $

Discussion  Note that the position of the bicyclist has a significant effect on the drag force, and thus the terminal velocity. So it is no surprise that the bicyclists maintain the racing position during a race.

11-35
Solution  The pivot of a wind turbine with two hollow hemispherical cups is stuck as a result of some malfunction. For a given wind speed, the maximum torque applied on the pivot is to be determined.

Assumptions  1 The flow of air in the wind is steady and incompressible. 2 The air flow is turbulent so that the tabulated values of the drag coefficients can be used.

Properties  The density of air is given to be $\rho = 1.25 \text{ kg/m}^3$. The drag coefficient for a hemispherical cup is 0.4 and 1.2 when the hemispherical and plain surfaces are exposed to wind flow, respectively.

Analysis  The maximum torque occurs when the cups are normal to the wind since the length of the moment arm is maximum in this case. Noting that the frontal area is $\pi D^2/4$ for both cups, the drag force acting on each cup is determined to be

Convex side: $ F_{D1} = C_{D1} A \frac{\rho V^2}{2} = (0.4)[\pi(0.08 \text{ m})^2 / 4] \frac{(1.25 \text{ kg/m}^3)(15 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 0.283 \text{ N} $  

Concave side: $ F_{D2} = C_{D2} A \frac{\rho V^2}{2} = (1.2)[\pi(0.08 \text{ m})^2 / 4] \frac{(1.25 \text{ kg/m}^3)(15 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 0.848 \text{ N} $  

The moment arm for both forces is 20 cm since the distance between the centers of the two cups is given to be 25 cm. Taking the moment about the pivot, the net torque applied on the pivot is determined to be

$$ M_{\text{max}} = F_{D2} L - F_{D1} L = (F_{D2} - F_{D1}) L = (0.848 - 0.283 \text{ N})(0.20 \text{ m}) = 0.113 \text{ N} \cdot \text{m} $$

Discussion  Note that the torque varies between zero when both cups are aligned with the wind to the maximum value calculated above.
Solution  The previous problem is reconsidered. The effect of wind speed on the torque applied on the pivot as the wind speed varies from 0 to 50 m/s in increments of 5 m/s is to be investigated.

Analysis  The EES Equations window is printed below, along with the tabulated and plotted results.

\[
\begin{align*}
CD1 &= 0.40 \text{ "Curved bottom"} \\
CD2 &= 1.2 \text{ "Plain frontal area"} \\
\rho &= 1.25 \text{ "kg/m}^3" \\
D &= 0.08 \text{ "m"} \\
L &= 0.25 \text{ "m"} \\
A &= \pi D^2/4 \text{ "m}^2" \\
F_{D1} &= CD1 \cdot A \cdot (\rho \cdot V^2)/2 \text{ "N"} \\
F_{D2} &= CD2 \cdot A \cdot (\rho \cdot V^2)/2 \text{ "N"} \\
F_{D\text{net}} &= F_{D2} - F_{D1} \\
\text{Torque} &= (F_{D2} - F_{D1}) \cdot L/2
\end{align*}
\]

<table>
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<th>V, m/s</th>
<th>( F_{D\text{net}}, \text{ N} )</th>
<th>Torque, Nm</th>
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Discussion  Since drag force grows as velocity squared, the torque also grows as velocity squared.
### 11-37E

**Solution** A spherical tank completely submerged in fresh water is being towed by a ship at a specified velocity. The required towing power is to be determined.

**Assumptions** 1 The flow is turbulent so that the tabulated value of the drag coefficient can be used. 2 The drag of the towing bar is negligible.

**Properties** The drag coefficient for a sphere is \( C_D = 0.2 \) in turbulent flow (it is 0.5 for laminar flow). We take the density of water to be 62.4 lbm/ft\(^3\).

**Analysis** The frontal area of a sphere is \( A = \pi D^2/4 \). Then the drag force acting on the spherical tank is

\[
F_D = C_D A \frac{D V^2}{2} = (0.2) [\pi (5 \text{ ft})^2 / 4] \left( \frac{62.4 \text{ lbm/ft}^3}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 548 \text{ lbf}
\]

Since power is force times velocity, the power needed to overcome this drag force during towing is

\[
\dot{W}_{\text{towing}} = \dot{W}_{\text{drag}} = F_D V = (548 \text{ lbf})(12 \text{ ft/s}) \left( \frac{1 \text{ kW}}{737.56 \text{ lbf} \cdot \text{ft/s}} \right) = 8.92 \text{ kW} = 12.0 \text{ hp}
\]

Therefore, the additional power needed to tow the tank is **12.0 hp**.

**Discussion** Note that the towing power is proportional to the cube of the velocity. Therefore, the towing power can be reduced to one-eight (which is 1.5 hp) by reducing the towing velocity by half to 6 ft/s. But the towing time will double this time for a given distance.

### 11-38

**Solution** The power delivered to the wheels of a car is used to overcome aerodynamic drag and rolling resistance. For a given power, the speed at which the rolling resistance is equal to the aerodynamic drag and the maximum speed of the car are to be determined.

**Assumptions** 1 The air flow is steady and incompressible. 2 The bearing friction is negligible. 3 The drag and rolling resistance coefficients of the car are constant. 4 The car moves horizontally on a level road.

**Properties** The density of air is given to be \( \rho = 1.20 \text{ kg/m}^3 \). The drag and rolling resistance coefficients are given to be \( C_D = 0.32 \) and \( C_{RR} = 0.04 \), respectively.

**Analysis** (a) The rolling resistance of the car is

\[
F_{RR} = C_{RR} W = 0.04(950 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 372.8 \text{ N}
\]

The velocity at which the rolling resistance equals the aerodynamic drag force is determined by setting these two forces equal to each other,

\[
F_D = C_D A \frac{D V^2}{2} \quad \Rightarrow \quad 372.8 \text{ N} = (0.32)(1.8 \text{ m}^2) \left( \frac{(1.20 \text{ kg/m}^3)V^2}{2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) \quad \Rightarrow \quad V = 32.8 \text{ m/s} \quad \text{(or 118 km/h)}
\]

(b) Power is force times speed, and thus the power needed to overcome drag and rolling resistance is the product of the sum of the drag force and the rolling resistance and the velocity of the car,

\[
\dot{W}_{\text{total}} = \dot{W}_{\text{drag}} + \dot{W}_{\text{RR}} = (F_D + F_{RR}) V = C_D A \frac{D V^3}{2} + F_{RR} V
\]

Substituting the known quantities, the maximum speed corresponding to a wheel power of 80 kW is determined to be

\[
(0.32)(1.8 \text{ m}^2) \left( \frac{(1.20 \text{ kg/m}^3)V^3}{2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) + 372.8 V = 80,000 \text{ W} \left( \frac{1 \text{ N} \cdot \text{m/s}}{1 \text{ W}} \right) \quad \text{or} \quad 0.3456 V^3 + 372.8 V = 80,000
\]

whose solution is \( V = 55.56 \text{ m/s} = 200 \text{ km/h} \).

**Discussion** A net power input of 80 kW is needed to overcome rolling resistance and aerodynamic drag at a velocity of 200 km/h. About 75% of this power is used to overcome drag and the remaining 25% to overcome the rolling resistance. At much higher velocities, the fraction of drag becomes even higher as it is proportional to the cube of car velocity.
**Solution**  The previous problem is reconsidered. The effect of car speed on the required power to overcome (a) rolling resistance, (b) the aerodynamic drag, and (c) their combined effect as the car speed varies from 0 to 150 km/h in increments of 15 km/h is to be investigated.

**Analysis**  The EES *Equations* window is printed below, along with the tabulated and plotted results.

\[
\begin{align*}
\rho &= 1.20 \text{ kg/m}^3 \\
C_{roll} &= 0.04 \\
m &= 950 \text{ kg} \\
g &= 9.81 \text{ m/s}^2 \\
V &= \frac{Vel}{3.6} \text{ m/s} \\
W &= m \times g \\
F_{roll} &= C_{roll} \times W \\
A &= 1.8 \text{ m}^2 \\
C_D &= 0.32 \\
F_D &= C_D \times A \times \frac{\rho \times V^2}{2} \text{ N} \\
\text{Power}_{RR} &= F_{roll} \times \frac{V}{1000} \text{ kW} \\
\text{Power}_{Drag} &= F_D \times \frac{V}{1000} \text{ kW} \\
\text{Power}_{Total} &= \text{Power}_{RR} + \text{Power}_{Drag}
\end{align*}
\]

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<tr>
<th>V, m/s</th>
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<th>W(_{\text{rolling}}) kW</th>
<th>W(_{\text{total}}) kW</th>
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**Discussion**  Notice that the rolling power curve and drag power curve intersect at about 118 km/h (73.3 mph). So, near highway speeds, the overall power is split nearly 50% between rolling drag and aerodynamic drag.
We are to estimate how many additional liters of fuel are wasted per year by driving with a ball on a car antenna.

Properties

\[ \rho_{\text{fuel}} = 0.802 \, \text{kg/L}, \, H_{V \text{fuel}} = 44,020 \, \text{kJ/kg}, \, \rho_{\text{air}} = 1.204 \, \text{kg/m}^3, \, \mu_{\text{air}} = 1.825 \times 10^{-5} \, \text{kg/m} \cdot \text{s} \]

Analysis

The additional drag force due to the sun ball is

\[ F_D = \frac{1}{2} \rho_{\text{air}} V^2 C_D A \]

where \( A \) is the frontal area of the sphere. The work required to overcome this additional drag is force times distance. So, letting \( L \) be the total distance driven in a year,

\[ \text{Work}_{\text{drag}} = F_D L = \frac{1}{2} \rho_{\text{air}} V^2 C_D AL \]

The energy required to perform this work is much greater than this due to overall efficiency of the car engine, transmission, etc. Thus,

\[ E_{\text{required}} = \frac{\text{Work}_{\text{drag}}}{\eta_{\text{overall}}} = \frac{1}{2} \rho_{\text{air}} V^2 C_D AL \]

But the required energy is also equal to the heating value of the fuel \( H_{V} \) times the mass of fuel required. In terms of required fuel volume, volume = mass/density. Thus,

\[ V_{\text{fuel required}} = \frac{m_{\text{fuel required}}}{\rho_{\text{fuel}}} = \frac{E_{\text{required}}}{H_{V}} = \frac{\frac{1}{2} \rho_{\text{air}} V^2 C_D AL}{\rho_{\text{fuel}} \eta_{\text{overall}} H_{V}} \]

The above is our answer in variable form. Finally, we plug in the given values and properties to obtain the numerical answer,

\[ V_{\text{fuel required}} = \frac{0.5(1.204 \, \text{kg/m}^3)(20.8 \, \text{m/s})^2(0.87)(2.08 \times 10^{-3} \, \text{m}^2)(15,000,000 \, \text{m})}{(0.802 \, \text{kg/L})(0.312)(44,020 \, \text{kJ/kg})} = 0.6418 \, \text{L} \approx 0.64 \, \text{L} \]

Discussion

If fuel costs around $1 per liter, the total cost is around 64 cents per year to drive with the ball compared to without the ball – Suzy should not worry about wasting this negligible amount of money if the sun ball on her antenna gives her some pleasure when she drives. We give the answer to two significant digits – more than that is unwarranted.
A garbage can is found in the morning tipped over due to high winds the night before. The wind velocity during the night when the can was tipped over is to be determined.

**Assumptions**
1. The flow of air in the wind is steady and incompressible.
2. The ground effect on the wind and the drag coefficient is negligible (a crude approximation) so that the resultant drag force acts through the center of the side surface.
3. The garbage can is loaded uniformly so that its weight acts through its center.

**Properties**
- The density of air is given to be \( \rho = 1.25 \text{ kg/m}^3 \), and the average density of the garbage inside the can is given to be 150 kg/m\(^3\). The drag coefficient of the garbage can is given to be 0.7.

**Analysis**
- The volume of the garbage can and the weight of the garbage are
  \[
  V = \frac{\pi D^2}{4} H = \frac{\pi (0.90 \text{ m})^2}{4}(1.1 \text{ m}) = 0.6998 \text{ m}^2
  \]
  \[
  W = mg = \rho g V = (150 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.6998 \text{ m}^3)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 1029.7 \text{ N}
  \]

When the garbage can is first tipped, the edge on the wind-loaded side of the can will be off the ground, and thus all the reaction forces from the ground will act on the other side. Taking the moment about an axis passing through the contact point and setting it equal to zero gives the required drag force to be

\[
\sum M_{\text{contact}} = 0 \rightarrow F_D \times (H/2) - W \times (D/2) = 0 \rightarrow F_D = \frac{WD}{H} = \frac{(1029.7 \text{ N})(0.90 \text{ m})}{1.1 \text{ m}} = 842.5 \text{ N}
\]

Noting that the frontal area is \( DH \), the wind velocity that will cause this drag force is determined to be

\[
F_D = C_D A \frac{\rho V^2}{2} \rightarrow 842.5 \text{ N} = (0.7)(1.1 \times 0.90 \text{ m}^2) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) \left(\frac{1.25 \text{ kg/m}^3}{1 \text{ kg} \cdot \text{m/s}^2}\right) \rightarrow V = 44.10 \text{ m/s}
\]

which is equivalent to a wind velocity of \( V = 44.10 \times 3.6 = 159 \text{ km/h} \).

**Discussion**
The analysis above shows that under the stated assumptions, the wind velocity at some moment exceeded 159 km/h. But we cannot tell how high the wind velocity has been. Such analysis and predictions are commonly used in forensic engineering.
A plastic sphere is dropped into water. The terminal velocity of the sphere in water is to be determined.

**Assumptions**
1. The fluid flow over the sphere is laminar (to be verified).
2. The drag coefficient remains constant.

**Properties**
- The density of plastic sphere is 1150 kg/m\(^3\).
- The density and dynamic viscosity of water at 20°C are \( \rho = 998 \text{ kg/m}^3 \) and \( \mu = 1.002 \times 10^{-3} \text{ kg/m} \cdot \text{s} \), respectively.
- The drag coefficient for a sphere in laminar flow is \( C_D = 0.5 \).

**Analysis**
The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid,

\[
F_D = W - F_B \quad \text{where} \quad F_D = C_D A \frac{\rho_f V^2}{2}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V
\]

Here \( A = \pi D^2/4 \) is the frontal area and \( V = \pi D^3/6 \) is the volume of the sphere. Substituting and simplifying,

\[
C_D A \frac{\rho_f V^2}{2} = \rho_s g V - \rho_f g V \rightarrow C_D \frac{\pi D^2}{4} \frac{\rho_f V^2}{2} = \left( \rho_s - \rho_f \right) g \frac{\pi D^3}{6} \rightarrow C_D \frac{V^2}{8} = \left( \frac{\rho_s}{\rho_f} - 1 \right) \frac{gD}{6}
\]

Solving for \( V \) and substituting,

\[
V = \sqrt{\frac{4gD\left(\rho_s / \rho_f - 1\right)}{3C_D}} = \sqrt{\frac{4(9.81 \text{ m/s}^2)(0.006 \text{ m})(1150/998 - 1)}{3 \times 0.5}} = 0.155 \text{ m/s}
\]

The Reynolds number is

\[
Re = \frac{\rho VD}{\mu} = \frac{(998 \text{ kg/m}^3)(0.155 \text{ m/s})(6 \times 10^{-3} \text{ m})}{1.002 \times 10^{-3} \text{ kg} \cdot \text{m/s}} = 926
\]

which is less than \( 2 \times 10^5 \). Therefore, the flow is laminar as assumed.

**Discussion**
This problem can also be solved “usually more accurately” using a trial-and-error approach by using \( C_D \) data from Fig. 11-34. The \( C_D \) value corresponding to \( Re = 926 \) is about 0.5, and thus the terminal velocity is the same.
Solution  
A spherical hot air balloon that stands still in the air is subjected to winds. The initial acceleration of the balloon is to be determined.

Assumptions  
1. The entire hot air balloon can be approximated as a sphere. 2. The wind is steady and turbulent, and blows parallel to the ground.

Properties  
The drag coefficient for turbulent flow over a sphere is $C_D = 0.2$ (Table 11-2). We take the density of air to be $1.20 \text{ kg/m}^3$.

Analysis  
The frontal area of a sphere is $A = \pi D^2 / 4$. Noting that $1 \text{ m/s} = 3.6 \text{ km/h}$, the drag force acting on the balloon is

$$ F_D = C_D A \frac{\rho V^2}{2} = (0.2)\left[\pi (7 \text{ m})^2 / 4\right] \left(\frac{1.20 \text{ kg/m}^3 \cdot 40 \text{ km/h}}{3.6 \text{ km/h}}\right)^2 \left(\frac{1 \text{ N}}{1 \text{ kg \cdot m/s}^2}\right) = 570.1 \text{ N} $$

Then from Newton’s 2nd law of motion, the initial acceleration in the direction of the winds becomes

$$ a = \frac{F_D}{m} = \frac{570.1 \text{ N}}{350 \text{ kg}} \left(\frac{1 \text{ kg \cdot m/s}^2}{1 \text{ N}}\right) = 1.63 \text{ m/s}^2 $$

Discussion  
Note that wind-induced drag force is the only mechanism to move a balloon horizontally. The balloon can be moved vertically by dropping weights from the cage of the balloon or by the change of conditions of the light gas in the balloon.
11-44E

Solution  
The drag coefficient of a sports car increases when the sunroof is open, and it requires more power to overcome aerodynamic drag. The additional power consumption of the car when the sunroof is opened is to be determined at two different velocities.

Assumptions  
1. The car moves steadily at a constant velocity on a straight path.
2. The effect of velocity on the drag coefficient is negligible.

Properties  
The density of air is given to be 0.075 lbm/ft³. The drag coefficient of the car is given to be \( C_D = 0.32 \) when the sunroof is closed, and \( C_D = 0.41 \) when it is open.

Analysis  
(a) Noting that 1 mph = 1.4667 ft/s and that power is force times velocity, the drag force acting on the car and the power needed to overcome it at 35 mph are:

Open sunroof: \[ F_{D1} = 0.32(18 \text{ ft}^2) \frac{(0.075 \text{ lbm/ft}^3)(35 \times 1.4667 \text{ ft/s})^2 }{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 17.7 \text{ lbf} \]

\[ \dot{W}_{\text{drag1}} = F_{D1}V = (17.7 \text{ lbf})(35 \times 1.4667 \text{ ft/s}) \left( \frac{1 \text{ kW}}{737.56 \text{ lbm} \cdot \text{ft/s}} \right) = 1.23 \text{ kW} \]

Closed sunroof: \[ F_{D2} = 0.41(18 \text{ ft}^2) \frac{(0.075 \text{ lbm/ft}^3)(35 \times 1.4667 \text{ ft/s})^2 }{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 22.7 \text{ lbf} \]

\[ \dot{W}_{\text{drag2}} = F_{D2}V = (22.7 \text{ lbf})(35 \times 1.4667 \text{ ft/s}) \left( \frac{1 \text{ kW}}{737.56 \text{ lbm} \cdot \text{ft/s}} \right) = 1.58 \text{ kW} \]

Therefore, the additional power required for this car when the sunroof is open is \[ \dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag1}} = 1.58 - 1.23 = 0.350 \text{ kW} \] (at 35 mph)

(b) We now repeat the calculations for 70 mph:

Open sunroof: \[ F_{D1} = 0.32(18 \text{ ft}^2) \frac{(0.075 \text{ lbm/ft}^3)(70 \times 1.4667 \text{ ft/s})^2 }{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 70.7 \text{ lbf} \]

\[ \dot{W}_{\text{drag1}} = F_{D1}V = (70.7 \text{ lbf})(70 \times 1.4667 \text{ ft/s}) \left( \frac{1 \text{ kW}}{737.56 \text{ lbm} \cdot \text{ft/s}} \right) = 9.82 \text{ kW} \]

Closed sunroof: \[ F_{D2} = 0.41(18 \text{ ft}^2) \frac{(0.075 \text{ lbm/ft}^3)(70 \times 1.4667 \text{ ft/s})^2 }{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 90.6 \text{ lbf} \]

\[ \dot{W}_{\text{drag2}} = F_{D2}V = (90.6 \text{ lbf})(70 \times 1.4667 \text{ ft/s}) \left( \frac{1 \text{ kW}}{737.56 \text{ lbm} \cdot \text{ft/s}} \right) = 12.6 \text{ kW} \]

Therefore, the additional power required for this car when the sunroof is open is \[ \dot{W}_{\text{extra}} = \dot{W}_{\text{drag2}} - \dot{W}_{\text{drag1}} = 12.6 - 9.82 = 2.78 \text{ kW} \] (at 70 mph)

Discussion  
Note that the additional drag caused by open sunroof is 0.35 kW at 35 mph, and 2.78 kW at 70 mph, which is an increase of 8 folds when the velocity is doubled. This is expected since the power consumption to overcome drag is proportional to the cube of velocity.
Solution  The flat mirror of a car is replaced by one with hemispherical back. The amount of fuel and money saved per year as a result are to be determined.

Assumptions  1 The car is driven 24,000 km a year at an average speed of 95 km/h. 2 The effect of the car body on the flow around the mirror is negligible (no interference).

Properties  The densities of air and gasoline are taken to be 1.20 kg/m$^3$ and 750 kg/m$^3$, respectively. The heating value of gasoline is given to be 44,000 kJ/kg. The drag coefficients $C_D$ are 1.1 for a circular disk, and 0.40 for a hemispherical body.

Analysis  The drag force acting on a body is determined from

$$F_D = C_D A \frac{\rho V^2}{2}$$

where $A$ is the frontal area of the body, which is $A = \pi D^2 / 4$ for both the flat and rounded mirrors. The drag force acting on the flat mirror is

$$F_D = 1.1 \frac{\pi (0.13 \text{ m})^2 (1.20 \text{ kg/m}^3)(95 \text{ km/h})^2}{4} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 6.10 \text{ N}$$

Noting that work is force times distance, the amount of work done to overcome this drag force and the required energy input for a distance of 24,000 km are

$$W_{\text{drag}} = F \times L = (6.10 \text{ N})(24,000 \text{ km/year}) = 146,400 \text{ kJ/year}$$

$$E_{\text{in}} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{146,400 \text{ kJ}}{0.3} = 488,000 \text{ kJ/year}$$

Then the amount and costs of the fuel that supplies this much energy are

$$\text{Amount of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/HV}{\rho_{\text{fuel}}} = \frac{488,000 \text{ kJ/year}}{44,000 \text{ kJ/kg}} = 14.79 \text{ L/year}$$

$$\text{Cost} = \text{Amount of fuel} / \text{Unit cost} = (14.79 \text{ L/year}) / ($0.90/L) = $13.31/\text{year}$$

That is, the car uses 14.79 L of gasoline at a cost of $13.31 per year to overcome the drag generated by a flat mirror extending out from the side of a car. The drag force and the work done to overcome it are directly proportional to the drag coefficient. Then the percent reduction in the fuel consumption due to replacing the mirror is equal to the percent reduction in the drag coefficient:

$$\text{Reduction ratio} = \frac{C_{D,\text{flat}} - C_{D,\text{hemisp}}}{C_{D,\text{flat}}} = \frac{1.1 - 0.4}{1.1} = 0.636$$

$$\text{Amount reduction} = (\text{Reduction ratio})(\text{Amount}) = 0.636(14.79 \text{ L/year}) = 9.60 \text{ L/year}$$

$$\text{Cost reduction} = (\text{Reduction ratio})(\text{Cost}) = 0.636($13.31/\text{year}) = $8.46/\text{year}$$

Since a typical car has two rearview mirrors, the driver saves about $17 per year in gasoline by replacing the flat mirrors by the hemispherical ones.

Discussion  Note from this example that significant reductions in drag and fuel consumption can be achieved by streamlining the shape of various components and the entire car. So it is no surprise that the sharp corners are replaced in late model cars by rounded contours. This also explains why large airplanes retract their wheels after takeoff, and small airplanes use contoured fairings around their wheels.
Flow over Flat Plates

11-46C  Solution  We are to define and discuss the average skin friction coefficient over a flat plate.

Analysis  The *average friction coefficient* in flow over a flat plate is determined by *integrating the local friction coefficient over the entire length of the plate*, and then dividing it by the length of the plate. Or, it can be determined experimentally by measuring the drag force, and dividing it by the dynamic pressure.

Discussion  For the case of a flat plate aligned with the flow, there is no pressure drag, only skin friction drag. Thus, the average friction coefficient is the same as the drag coefficient. This is not true for other body shapes.

11-47C  Solution  We are to discuss the fluid property responsible for the development of a boundary layer.

Analysis  The *fluid viscosity* is responsible for the development of the velocity boundary layer. Velocity forces the boundary layer closer to the wall. Therefore, the higher the velocity (and thus Reynolds number), the lower the thickness of the boundary layer.

Discussion  All fluids have viscosity – a measure of frictional forces in a fluid. There is no such thing as an *inviscid fluid*, although there are *regions*, called *inviscid flow regions*, in which viscous effects are negligible.

11-48C  Solution  We are to define and discuss the friction coefficient for flow over a flat plate.

Analysis  The *friction coefficient* represents the *resistance to fluid flow over a flat plate*. It is proportional to the drag force acting on the plate. The drag coefficient for a flat surface is equivalent to the mean friction coefficient.

Discussion  In flow over a flat plate aligned with the flow, there is no pressure (form) drag – only friction drag.
Solution

Laminar flow of a fluid over a flat plate is considered. The change in the drag force is to be determined when the free-stream velocity of the fluid is tripled.

Analysis

For the laminar flow of a fluid over a flat plate the drag force is given by

$$F_{D1} = C_f A \frac{\rho V^2}{2}$$

where

$$C_f = \frac{1.328}{Re^{0.5}}$$

Therefore

$$F_{D1} = \frac{1.328 A \rho V^2}{Re^{0.5}}$$

Substituting Reynolds number relation, we get

$$F_{D1} = \frac{1.328 A \rho V^2}{(\frac{VL}{V})^{0.5}} = 0.664 \left(\frac{V}{L}\right)^{3/2} A D^{0.5}$$

When the free-stream velocity of the fluid is tripled, the new value of the drag force on the plate becomes

$$F_{D2} = \frac{1.328 A \rho (3V)^2}{(3V)L^{0.5}} = 0.664 (3V)^{3/2} A D^{0.5}$$

The ratio of drag forces corresponding to $V$ and $2V$ is

$$\frac{F_{D2}}{F_{D1}} = \frac{(3V)^{3/2}}{V^{3/2}} = 3^{3/2} = 5.20$$

In other words, there is a **5.20-fold increase** in the drag force on the plate.

Discussion

Note that the drag force increases almost three times in laminar flow when the fluid velocity is doubled.
Solution

Air flows over a plane surface at high elevation. The drag force acting on the top surface of the plate is to be determined for flow along the two sides of the plate.

Assumptions

1. The flow is steady and incompressible.
2. The critical Reynolds number is \( \text{Re}_c = 5 \times 10^5 \).
3. Air is an ideal gas.
4. The surface of the plate is smooth.

Properties

The dynamic viscosity is independent of pressure, and for air at 25°C it is \( \mu = 1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s} \). The air density at 25°C = 298 K and 83.4 kPa is given by:

\[
\rho = \frac{P}{RT} = \frac{83.4 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(298 \text{ K})} = 0.9751 \text{ kg/m}^3
\]

Analysis

(a) If the air flows parallel to the 8 m side, the Reynolds number becomes:

\[
\text{Re}_L = \frac{\rho VL}{\mu} = \frac{(0.9751 \text{ kg/m}^3)(9 \text{ m/s})(5 \text{ m})}{1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 2.373 \times 10^6
\]

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be:

\[
C_f = \frac{0.074}{\left( \text{Re}_L^{1/5} \right)} = \frac{0.074}{(2.373 \times 10^6)^{1/5}} \approx 0.003194
\]

Noting that the pressure drag is zero and thus \( C_D = C_f \) for a flat plate, the drag force acting on the top surface of the plate becomes:

\[
F_D = C_f A \frac{\rho V^2}{2} = 0.003194 \times (5 \times 2.5 \text{ m}^2) \left( 0.9751 \text{ kg/m}^3 \right)(9 \text{ m/s})^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 1.58 \text{ N}
\]

(b) If the air flows parallel to the 2.5 m side, the Reynolds number is:

\[
\text{Re}_L = \frac{\rho VL}{\mu} = \frac{(0.9751 \text{ kg/m}^3)(9 \text{ m/s})(2.5 \text{ m})}{1.849 \times 10^{-5} \text{ kg/m} \cdot \text{s}} = 1.187 \times 10^6
\]

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be:

\[
C_f = \frac{0.074}{\left( \text{Re}_L^{1/5} \right)} = \frac{0.074}{(1.187 \times 10^6)^{1/5}} \approx 0.003044
\]

Then the drag force acting on the top surface of the plate becomes:

\[
F_D = C_f A \frac{\rho V^2}{2} = 0.003044 \times (5 \times 2.5 \text{ m}^2) \left( 0.9751 \text{ kg/m}^3 \right)(9 \text{ m/s})^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 1.50 \text{ N}
\]

Discussion

Note that the drag force is proportional to density, which is proportional to the pressure. Therefore, the altitude has a major influence on the drag force acting on a surface. Commercial airplanes take advantage of this phenomenon and cruise at high altitudes where the air density is much lower to save fuel.
Solution

A train is cruising at a specified velocity. The drag force acting on the top surface of a passenger car of the train is to be determined.

Assumptions
1. The air flow is steady and incompressible.
2. The critical Reynolds number is \( \text{Re}_{cr} = 5 \times 10^5 \).
3. Air is an ideal gas.
4. The top surface of the train is smooth (in reality it can be rough).
5. The air is calm (no significant winds).

Properties
The density and kinematic viscosity of air at 1 atm and 25°C are \( \rho = 1.184 \text{ kg/m}^3 \) and \( \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s} \).

Analysis
The Reynolds number is
\[
\text{Re}_L = \frac{VL}{\nu} = \frac{[95/3.6 \text{ m/s}](8 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 1.352 \times 10^7
\]
which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is determined to be
\[
C_f = \frac{0.074}{\text{Re}_L^{1/5}} = \frac{0.074}{(1.352 \times 10^7)^{1/5}} = 0.002645
\]
Noting that the pressure drag is zero and thus \( C_D = C_f \) for a flat plate, the drag force acting on the surface becomes
\[
F_D = C_f A \left( \frac{\rho V^2}{2} \right) \left( \frac{1.184 \text{ kg/m}^3)(95/3.6 \text{ m/s})^2}{2} \right) = 18.3 \text{ N}
\]

Discussion
Note that we can solve this problem using the turbulent flow relation (instead of the combined laminar-turbulent flow relation) without much loss in accuracy since the Reynolds number is much greater than the critical value. Also, the actual drag force will probably be greater because of the surface roughness effects.
Solution  Air flows over a flat plate. The local friction coefficients at intervals of 1 ft is to be determined and plotted against the distance from the leading edge.

Assumptions  1. The flow is steady and incompressible. 2. The critical Reynolds number is \( Re_{cr} = 5 \times 10^5 \). 3. Air is an ideal gas. 4. The surface of the plate is smooth.

Properties  The density and kinematic viscosity of air at 1 atm and 70°F are \( \rho = 0.07489 \text{ lbm/ft}^3 \) and \( \nu = 0.5913 \text{ ft}^2/\text{h} = 1.643 \times 10^{-4} \text{ ft}^2/\text{s} \).

Analysis  For the first 1 ft interval, the Reynolds number is
\[
Re_L = \frac{VL}{\nu} = \frac{(25 \text{ ft/s})(1 \text{ ft})}{1.643 \times 10^{-4} \text{ ft}^2/\text{s}} = 1.522 \times 10^5
\]
which is less than the critical value of \( 5 \times 10^5 \). Therefore, the flow is laminar. The local friction coefficient is
\[
C_{f,x} = \frac{0.664}{Re^{0.5}} = \frac{0.664}{(1.522 \times 10^5)^{0.5}} = 0.001702
\]
We repeat calculations for all 1-ft intervals. The EES Equations window is printed below, along with the tabulated and plotted results.

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<th>( x ), ft</th>
<th>Re</th>
<th>( C_f )</th>
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</tr>
<tr>
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</table>

Discussion  Note that the Reynolds number exceeds the critical value for \( x > 3 \) ft, and thus the flow is turbulent over most of the plate. For \( x > 3 \) ft, we used \( C_f = 0.074/Re_L^{1/5} - 1742/Re_L \) for friction coefficient. Note that \( C_f \) decreases with \( Re \) in both laminar and turbulent flows.
Solution  Air flows on both sides of a continuous sheet of plastic. The drag force air exerts on the plastic sheet in the direction of flow is to be determined.

Assumptions  1 The flow is steady and incompressible. 2 The critical Reynolds number is $Re_c = 5 \times 10^5$. 3 Air is an ideal gas. 4 Both surfaces of the plastic sheet are smooth. 5 The plastic sheet does not vibrate and thus it does not induce turbulence in air flow.

Properties  The density and kinematic viscosity of air at 1 atm and 60°C are $\rho = 1.059 \text{ kg/m}^3$ and $v = 1.896 \times 10^{-5} \text{ m}^2/\text{s}$.

Analysis  The length of the cooling section is

$$L = V_{\text{sheet}} \Delta t = \left(\frac{18}{60} \text{ m/s}\right)(2 \text{ s}) = 0.6 \text{ m}$$

The Reynolds number is

$$Re_L = \frac{VL}{v} = \frac{(4 \text{ m/s})(1.2 \text{ m})}{1.896 \times 10^{-5} \text{ m}^2/\text{s}} = 2.532 \times 10^5$$

which is less than the critical Reynolds number. Thus the flow is laminar. The area on both sides of the sheet exposed to air flow is

$$A = 2wL = 2(1.2 \text{ m})(0.6 \text{ m}) = 1.44 \text{ m}^2$$

Then the friction coefficient and the drag force become

$$C_f = \frac{1.328}{(2.532 \times 10^5)^{0.5}} = 0.002639$$

$$F_D = C_f A \frac{\rho V^2}{2} = (0.002639)(1.2 \text{ m}^2) \frac{(1.059 \text{ kg/m}^3)(4 \text{ m/s})^2}{2} = 0.0268 \text{ N}$$

Discussion  Note that the Reynolds number remains under the critical value, and thus the flow remains laminar over the entire plate. In reality, the flow may be turbulent because of the motion of the plastic sheet.
Light oil flows over a flat plate. The total drag force per unit width of the plate is to be determined.

**Assumptions**
1. The flow is steady and incompressible.
2. The critical Reynolds number is \( \text{Re}_c = 5 \times 10^5 \).
3. The surface of the plate is smooth.

**Properties**
The density and kinematic viscosity of light oil at 75°F are \( \rho = 55.3 \text{ lbm/ft}^3 \) and \( \nu = 7.751 \times 10^{-3} \text{ ft}^2/\text{s} \).

**Analysis**
Noting that \( L = 22 \text{ ft} \), the Reynolds number at the end of the plate is
\[
\text{Re}_L = \frac{VL}{\nu} = \frac{(6 \text{ ft/s})(22 \text{ ft})}{7.751 \times 10^{-3} \text{ ft}^2/\text{s}} = 1.703 \times 10^4
\]
which is less than the critical Reynolds number. Thus we have laminar flow over the entire plate, and the average friction coefficient is determined from
\[
C_f = 1.328 \text{Re}_{L}^{0.5} = 1.328 \times (1.703 \times 10^4)^{-0.5} = 0.01018
\]
Noting that the pressure drag is zero and thus \( C_D = C_f \) for a flat plate, the drag force acting on the top surface of the plate per unit width becomes
\[
F_D = C_f A \frac{\rho V^2}{2} = 0.01018 \times (22 \times 1 \text{ ft}^2) \left( \frac{55.3 \text{ lbm/ft}^3)(6 \text{ ft/s})^2}{2} \right) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft}^2/\text{s}^2} \right) = 6.92 \text{ lbf}
\]
The total drag force acting on the entire plate can be determined by multiplying the value obtained above by the width of the plate.

**Discussion**
The force per unit width corresponds to the weight of a mass of 6.92 lbm. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 6.92 lbm mass from dropping.
A refrigeration truck is traveling at a specified velocity. The drag force acting on the top and side surfaces of the truck and the power needed to overcome it are to be determined.

**Assumptions**
1. The process is steady and incompressible.
2. The airflow over the entire outer surface is turbulent because of constant agitation.
3. Air is an ideal gas.
4. The top and side surfaces of the truck are smooth (in reality they can be rough).
5. The air is calm (no significant winds).

**Properties**
The density and kinematic viscosity of air at 1 atm and 80°F are \( \rho = 0.07350 \text{ lbm/ft}^3 \) and \( \nu = 0.6110 \text{ ft}^2/\text{s} = 1.697 \times 10^{-4} \text{ ft}^2/\text{s} \).

**Analysis**

The Reynolds number is

\[
Re_L = \frac{VL}{\nu} = \frac{[70 \times 1.4667 \text{ ft/s}](20 \text{ ft})}{1.697 \times 10^{-4} \text{ ft}^2/\text{s}} = 1.210 \times 10^7
\]

The area of the top and side surfaces of the truck is

\[
A = A_{\text{top}} + 2A_{\text{side}} = 9 \times 20 + 2 \times 8 \times 20 = 500 \text{ ft}^2
\]

Noting that the pressure drag is zero and thus \( C_D = C_f \) for a plane surface, the drag force acting on these surfaces becomes

\[
F_D = C_f A \frac{\rho V^2}{2} = 0.002836 \times (500 \text{ ft}^2) \frac{(0.07350 \text{ lbm/ft}^3)(70 \times 1.4667 \text{ ft/s})^2}{2} = 17.1 \text{ lbf}
\]

Noting that power is force times velocity, the power needed to overcome this drag force is

\[
\dot{W}_{\text{drag}} = F_D V = (17.1 \text{ lbf})(70 \times 1.4667 \text{ ft/s}) \frac{1 \text{ kW}}{737.56 \text{ lbf \cdot ft/s}} = 2.37 \text{ kW}
\]

**Discussion**
Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.
**Solution**  The previous problem is reconsidered. The effect of truck speed on the total drag force acting on the top and side surfaces, and the power required to overcome as the truck speed varies from 0 to 100 mph in increments of 10 mph is to be investigated.

**Analysis**  The EES *Equations* window is printed below, along with the tabulated and plotted results.

\[
\begin{align*}
\rho &= 0.07350 \text{ "lbf/ft}^3" \\
\nu &= 0.6110/360 \text{ "ft}^2/\text{s}" \\
V &= \text{Vel} \times 1.4667 \text{ "ft/s"} \\
L &= 20 \text{ "ft"} \\
W &= 2 \times 8 + 9 \\
A &= L \times W \\
Re &= L \times V / \nu \\
C_f &= 0.074 / Re^{0.2} \\
g &= 32.2 \text{ "ft/s}^2" \\
F &= C_f A (\rho V^2) / 2 / 32.2 \text{ "lbf"} \\
P_{\text{drag}} &= F V / 737.56 \text{ "kW"}
\end{align*}
\]

<table>
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<tr>
<th>V, mph</th>
<th>Re</th>
<th>( F_{\text{drag}} ), lbf</th>
<th>( P_{\text{drag}} ), kW</th>
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**Discussion**  The required power increases rapidly with velocity – in fact, as velocity cubed.
11-57

**Solution** Air is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

**Assumptions** 1 The flow is steady and incompressible. 2 The critical Reynolds number is \( \text{Re}_{cr} = 5 \times 10^5 \). 3 Air is an ideal gas. 4 The surface of the plate is smooth.

**Properties** The density and kinematic viscosity of air at 1 atm and 25°C are \( \rho = 1.184 \text{ kg/m}^3 \) and \( \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s} \).

**Analysis** The critical Reynolds number is given to be \( \text{Re}_{cr} = 5 \times 10^5 \). The distance from the leading edge of the plate where the flow becomes turbulent is the distance \( x_{cr} \) where the Reynolds number becomes equal to the critical Reynolds number,

\[
\text{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} \quad \Rightarrow \quad x_{cr} = \frac{\nu \text{Re}_{cr}}{V} = \frac{(1.562 \times 10^{-5} \text{ m}^2/\text{s})(5 \times 10^5)}{8 \text{ m/s}} = 0.976 \text{ m}
\]

The thickness of the boundary layer at that location is obtained by substituting this value of \( x \) into the laminar boundary layer thickness relation,

\[
\delta_{v,x} = \frac{4.91x}{\text{Re}_{x}^{1/2}} \quad \Rightarrow \quad \delta_{v,cr} = \frac{4.91x_{cr}}{\text{Re}_{cr}^{1/2}} = \frac{4.91(0.976 \text{ m})}{(5 \times 10^5)^{1/2}} = 0.00678 \text{ m} = 0.678 \text{ cm}
\]

**Discussion** When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.

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11-58

**Solution** Water is flowing over a long flat plate with a specified velocity. The distance from the leading edge of the plate where the flow becomes turbulent, and the thickness of the boundary layer at that location are to be determined.

**Assumptions** 1 The flow is steady and incompressible. 2 The critical Reynolds number is \( \text{Re}_{cr} = 5 \times 10^5 \). 3 The surface of the plate is smooth.

**Properties** The density and dynamic viscosity of water at 1 atm and 25°C are \( \rho = 997 \text{ kg/m}^3 \) and \( \mu = 0.891 \times 10^{-3} \text{ kg/m}\cdot\text{s} \).

**Analysis** The critical Reynolds number is given to be \( \text{Re}_{cr} = 5 \times 10^5 \). The distance from the leading edge of the plate where the flow becomes turbulent is the distance \( x_{cr} \) where the Reynolds number becomes equal to the critical Reynolds number,

\[
\text{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} \quad \Rightarrow \quad x_{cr} = \frac{\mu \text{Re}_{cr}}{\rho V} = \frac{(0.891 \times 10^{-3} \text{ kg/m}\cdot\text{s})(5 \times 10^5)}{(997 \text{ kg/m}^3)(8 \text{ m/s})} = 0.056 \text{ m}
\]

The thickness of the boundary layer at that location is obtained by substituting this value of \( x \) into the laminar boundary layer thickness relation,

\[
\delta_{v,x} = \frac{5x}{\text{Re}_{x}^{1/2}} \quad \Rightarrow \quad \delta_{v,cr} = \frac{4.91x_{cr}}{\text{Re}_{cr}^{1/2}} = \frac{4.91(0.056 \text{ m})}{(5 \times 10^5)^{1/2}} = 0.00039 \text{ m} = 0.39 \text{ mm}
\]

Therefore, the flow becomes turbulent after about 5.6 cm from the leading edge of the plate, and the thickness of the boundary layer at that location is 0.39 mm.

**Discussion** When the flow becomes turbulent, the boundary layer thickness starts to increase, and the value of its thickness can be determined from the boundary layer thickness relation for turbulent flow.
Solution  

Wind is blowing parallel to the side wall of a house. The drag force acting on the wall is to be determined for two different wind velocities.

Assumptions  
1. The flow is steady and incompressible.  
2. The critical Reynolds number is $Re_{cr} = 5 \times 10^5$.  
3. Air is an ideal gas.  
4. The wall surface is smooth (the actual wall surface is usually very rough).  
5. The wind blows parallel to the wall.

Properties  
The density and kinematic viscosity of air at 1 atm and 5°C are $\rho = 1.269 \text{ kg/m}^3$ and $\nu = 1.382 \times 10^{-5} \text{ m}^2/\text{s}$.

Analysis  
The Reynolds number is

$$Re_L = \frac{VL}{\nu} = \left[\frac{(55/3.6) \text{ m/s}}{10 \text{ m}}\right] \left[\frac{1.105 \times 10^7}{1.382 \times 10^{-3} \text{ m}^2/\text{s}}\right] = 1.105 \times 10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient is

$$C_f = \frac{0.074}{Re_L^{1/5}} \frac{1742}{1.105 \times 10^7} = \frac{0.074}{(1.105 \times 10^7)^{1/5}} \frac{1742}{1.105 \times 10^7} = 0.002730$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a flat plate, the drag force acting on the wall surface is

$$F_D = C_f A \frac{\rho V^2}{2} = 0.00273 \times (10 \times 4 \text{ m}^2) \left(\frac{1.269 \text{ kg/m}^3}(55/3.6 \text{ m/s})^2\right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 16.17 \text{ N} \approx 16 \text{ N}$$

(b) When the wind velocity is doubled to 110 km/h, the Reynolds number becomes

$$Re_L = \frac{VL}{\nu} = \left[\frac{(110/3.6) \text{ m/s}}{10 \text{ m}}\right] \left[\frac{1.105 \times 10^7}{1.382 \times 10^{-3} \text{ m}^2/\text{s}}\right] = 2.211 \times 10^7$$

which is greater than the critical Reynolds number. Thus we have combined laminar and turbulent flow, and the friction coefficient and the drag force become

$$C_f = \frac{0.074}{Re_L^{1/5}} \frac{1742}{2.211 \times 10^7} = \frac{0.074}{(2.211 \times 10^7)^{1/5}} \frac{1742}{2.211 \times 10^7} = 0.002435$$

$$F_D = C_f A \frac{\rho V^2}{2} = 0.002435 \times (10 \times 4 \text{ m}^2) \left(\frac{1.269 \text{ kg/m}^3}(110/3.6 \text{ m/s})^2\right) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 57.70 \text{ N} \approx 58 \text{ N}$$

Treating flow over the side wall of a house as flow over a flat plate is not quite realistic. When flow hits a bluff body like a house, it separates at the sharp corner and a separation bubble exists over most of the side panels of the house. Therefore, flat plat boundary layer equations are not appropriate for this problem, and the entire house should considered in the solution instead.

Discussion  
Note that the actual drag will probably be much higher since the wall surfaces are typically very rough. Also, we can solve this problem using the turbulent flow relation (instead of the combined laminar-turbulent flow relation) without much loss in accuracy. Finally, the drag force nearly quadruples when the velocity is doubled. This is expected since the drag force is proportional to the square of the velocity, and the effect of velocity on the friction coefficient is small.
Solution

The weight of a thin flat plate exposed to air flow on both sides is balanced by a counterweight. The mass of the counterweight that needs to be added in order to balance the plate is to be determined.

Assumptions
1. The flow is steady and incompressible.
2. The critical Reynolds number is \( \text{Re}_{cr} = 5 \times 10^5 \).
3. Air is an ideal gas.
4. The surfaces of the plate are smooth.

Properties
The density and kinematic viscosity of air at 1 atm and 25°C are \( \rho = 1.184 \, \text{kg/m}^3 \) and \( \nu = 1.562 \times 10^{-5} \, \text{m}^2/\text{s} \).

Analysis
The Reynolds number is
\[
\text{Re} = \frac{VL}{\nu} = \frac{(10 \, \text{m/s})(0.5 \, \text{m})}{1.562 \times 10^{-5} \, \text{m}^2/\text{s}} = 3.201 \times 10^5
\]
which is less than the critical Reynolds number of \( 5 \times 10^5 \). Therefore the flow is laminar. The average friction coefficient, drag force and the corresponding mass are
\[
C_f = \frac{1.328}{\text{Re}^{0.5}} = \frac{1.328}{(3.201 \times 10^5)^{0.5}} = 0.002347
\]
\[
F_D = C_f \frac{\rho V^2 A}{2} = (0.002347)((2 \times 0.5 \times 0.5) \, \text{m}^2)(1.184 \, \text{kg/m}^3)(10 \, \text{m/s})^2 = 0.0695 \, \text{kg} \cdot \text{m/s}^2 = 0.0695 \, \text{N}
\]
The mass whose weight is 0.0695 N is
\[
m = \frac{F_D}{g} = \frac{0.0695 \, \text{kg} \cdot \text{m/s}^2}{9.81 \, \text{m/s}^2} = 0.0071 \, \text{kg} = 7.1 \, \text{g}
\]
Therefore, the mass of the counterweight must be 7.1 g to counteract the drag force acting on the plate.

Discussion
Note that the apparatus described in this problem provides a convenient mechanism to measure drag force and thus drag coefficient.
Flow across Cylinders and Spheres

11-61C
Solution We are to discuss why flow separation is delayed in turbulent flow over circular cylinders.

Analysis Flow separation in flow over a cylinder is delayed in turbulent flow because of the extra mixing due to random fluctuations and the transverse motion.

Discussion As a result of the turbulent mixing, a turbulent boundary layer can resist flow separation better than a laminar boundary layer can, under otherwise similar conditions.

11-62C
Solution We are to discuss how pressure drag and friction drag differ in flow over blunt bodies.

Analysis Friction drag is due to the shear stress at the surface whereas pressure drag is due to the pressure differential between the front and back sides of the body because of the wake that is formed in the rear.

Discussion For a blunt or bluff body, pressure drag is usually greater than friction drag, while for a well-streamlined body, the opposite is true. For the case of a flat plate aligned with the flow, all of the drag is friction drag.

11-63C
Solution We are to discuss why the drag coefficient suddenly drops when the flow becomes turbulent.

Analysis Turbulence moves the fluid separation point further back on the rear of the body, reducing the size of the wake, and thus the magnitude of the pressure drag (which is the dominant mode of drag). As a result, the drag coefficient suddenly drops. In general, turbulence increases the drag coefficient for flat surfaces, but the drag coefficient usually remains constant at high Reynolds numbers when the flow is turbulent.

Discussion The sudden drop in drag is sometimes referred to as the drag crisis.
Solution  A spherical dust particle is suspended in the air at a fixed point as a result of an updraft air motion. The magnitude of the updraft velocity is to be determined using Stokes law.

Assumptions  1 The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). 2 The updraft is steady and incompressible. 3 The buoyancy force applied by air to the dust particle is negligible since \( \rho_{\text{air}} \ll \rho_{\text{dust}} \) (besides, the uncertainty in the density of dust is greater than the density of air). (We will solve the problem without utilizing this assumption for generality).

Properties  The density of dust is given to be \( \rho_s = 2.1 \text{ g/cm}^3 = 2100 \text{ kg/m}^3 \). The density and dynamic viscosity of air at 1 atm and 25°C are \( \rho_f = 1.184 \text{ kg/m}^3 \) and \( \mu = 1.849 \times 10^{-5} \text{ kg/m·s} \).

Analysis  The terminal velocity of a free falling object is reached (or the suspension of an object in a flow stream is established) when the drag force equals the weight of the solid object less the buoyancy force applied by the surrounding fluid,

\[
F_D = W - F_B \quad \text{where} \quad F_D = 3\pi \mu V D \quad \text{(Stokes law)}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V
\]

Here \( V = \pi D^3/6 \) is the volume of the sphere. Substituting,

\[
3\pi \mu V D = \rho_s g V - \rho_f g V \quad \rightarrow \quad 3\pi \mu V D = (\rho_s - \rho_f) g \frac{\pi D^3}{6}
\]

Solving for the velocity \( V \) and substituting the numerical values, the updraft velocity is determined to be

\[
V = \frac{g D^2 (\rho_s - \rho_f)}{18 \mu} = \frac{(9.81 \text{ m/s}^2)(0.0001 \text{ m})^2(2100-1.184) \text{ kg/m}^3}{18(1.849 \times 10^{-5} \text{ kg/m·s})} = 0.6186 \text{ m/s} \approx 0.62 \text{ m/s}
\]

The Reynolds number in this case is

\[
\text{Re} = \frac{\rho V D}{\mu} = \frac{(1.184 \text{ kg/m}^3)(0.619 \text{ m/s})(0.0001 \text{ m})}{1.849 \times 10^{-5} \text{ kg·m/s}} = 4.0
\]

which is in the order of 1. Therefore, the creeping flow idealization and thus Stokes law is applicable, and the value calculated is valid.

Discussion  Flow separation starts at about \( \text{Re} = 10 \). Therefore, Stokes law can be used as an approximation for Reynolds numbers up to this value, but this should be done with care.
Solution  
A pipe is exposed to high winds. The drag force exerted on the pipe by the winds is to be determined.

Assumptions  
1. The outer surface of the pipe is smooth so that Fig. 11-34 can be used to determine the drag coefficient. 
2. Air flow in the wind is steady and incompressible. 
3. The turbulence in the wind is not considered. 
4. The direction of wind is normal to the pipe.

Properties  
The density and kinematic viscosity of air at 1 atm and 10°C are \( \rho = 1.246 \text{ kg/m}^3 \) and \( \nu = 1.426 \times 10^{-5} \text{ m}^2/\text{s} \).

Analysis  
Noting that \( D = 0.05 \text{ m} \) and \( 1 \text{ m/s} = 3.6 \text{ km/h} \), the Reynolds number for flow over the pipe is

\[
\text{Re} = \frac{VD}{\nu} = \frac{(50/3.6 \text{ m/s})(0.05 \text{ m})}{1.426 \times 10^{-5} \text{ m}^2/\text{s}} = 0.4870 \times 10^5
\]

The drag coefficient corresponding to this value is, from Fig. 11-34, \( C_D = 1.0 \). Also, the frontal area for flow past a cylinder is \( A = LD \). Then the drag force becomes

\[
F_D = C_D A \frac{1}{2} \rho V^2 = 1.0(1 \times 0.05 \text{ m}^2) \left( \frac{1.246 \text{ kg/m}^3)(50/3.6 \text{ m/s})^2}{2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 6.01 \text{N} \text{ (per m length)}
\]

Discussion  
Note that the drag force acting on a unit length of the pipe is equivalent to the weight of 0.6 kg mass. The total drag force acting on the entire pipe can be obtained by multiplying the value obtained by the pipe length. It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.
11-66

Solution  Spherical hail is falling freely in the atmosphere. The terminal velocity of the hail in air is to be determined.

Assumptions  1 The surface of the hail is smooth so that Fig. 11-34 can be used to determine the drag coefficient. 2 The variation of the air properties with altitude is negligible. 3 The buoyancy force applied by air to hail is negligible since $\rho_{\text{air}} \ll \rho_{\text{hail}}$ (besides, the uncertainty in the density of hail is greater than the density of air). 4 Air flow over the hail is steady and incompressible when terminal velocity is established. 5 The atmosphere is calm (no winds or drafts).

Properties  The density and kinematic viscosity of air at 1 atm and 5°C are $\rho = 1.269 \text{ kg/m}^3$ and $\nu = 1.382 \times 10^{-5} \text{ m}^2/\text{s}$. The density of hail is given to be 910 kg/m$^3$.

Analysis  The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_D = W - F_B \quad \text{where} \quad F_D = C_D A \frac{\rho_f V^2}{2}, \quad W = mg = \rho_s g V = \rho_s g (\pi D^3/6), \quad \text{and} \quad F_B = 0$$

and $A = \pi D^2/4$ is the frontal area. Substituting and simplifying,

$$C_D A \frac{\rho_f V^2}{2} = W \quad \rightarrow \quad C_D \frac{\pi D^2}{4} \frac{\rho_f V^2}{2} = \rho_s g \frac{\pi D^3}{6} \quad \rightarrow \quad C_D \rho_f \frac{V^2}{3} = \rho_s g \frac{4D}{3}$$

Solving for $V$ and substituting,

$$V = \sqrt{\frac{4g\rho_f D}{3C_D \rho_f}} = \sqrt{\frac{4(9.81 \text{ m/s}^2)(910 \text{ kg/m}^3)(0.008 \text{ m})}{3C_D (1.269 \text{ kg/m}^3)}} \quad \rightarrow \quad V = \frac{8.662}{\sqrt{C_D}} \quad \text{(1)}$$

The drag coefficient $C_D$ is to be determined from Fig. 11-34, but it requires the Reynolds number which cannot be calculated since we do not know velocity. Therefore, the solution requires a trial-error approach. First we express the Reynolds number as

$$\text{Re} = \frac{V D}{\nu} = \frac{V(0.008 \text{ m})}{1.382 \times 10^{-5} \text{ m}^2/\text{s}} \quad \rightarrow \quad \text{Re} = 578.9V \quad \text{(2)}$$

Now we choose a velocity in m/s, calculate the Re from Eq. 2, read the corresponding $C_D$ from Fig. 11-34, and calculate $V$ from Eq. 1. Repeat calculations until the assumed velocity matches the calculated velocity. With this approach the terminal velocity is determined to be

$$V = 13.7 \text{ m/s}$$

The corresponding Re and $C_D$ values are $\text{Re} = 7930$ and $C_D = 0.40$. Therefore, the velocity of hail will remain constant when it reaches the terminal velocity of 13.7 m/s = 49 km/h.

Discussion  The simple analysis above gives us a reasonable value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air properties with altitude, and by considering the uncertainty in the drag coefficient (a hail is not necessarily spherical and smooth).
A pipe is crossing a river while remaining completely immersed in water. The drag force exerted on the pipe by the river is to be determined.

**Assumptions**
1. The outer surface of the pipe is smooth so that Fig. 11-34 can be used to determine the drag coefficient.
2. Water flow in the river is steady.
3. The turbulence in water flow in the river is not considered.
4. The direction of water flow is normal to the pipe.

**Properties**
The density and dynamic viscosity of water at 70°F are \( \rho = 62.30 \text{ lbm/ft}^3 \) and \( \mu = 2.36 \text{ lbm/ft\cdoth} = 6.556 \times 10^{-4} \text{ lbm/ft\cdots} \).

**Analysis**
Noting that \( D = 1.2 \text{ in} = 0.1 \text{ ft} \), the Reynolds number for flow over the pipe is

\[
Re = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{(62.30 \text{ lbm/ft}^3)(10 \text{ ft/s})(0.1 \text{ ft})}{6.556 \times 10^{-4} \text{ lbm/ft\cdots}} = 9.50 \times 10^4
\]

The drag coefficient corresponding to this value is, from Fig. 11-34, \( C_D = 1.1 \). Also, the frontal area for flow past a cylinder is \( A = LD \). Then the drag force acting on the cylinder becomes

\[
F_D = C_D A \frac{\rho V^2}{2} = 1.1 \times (140 \times 0.1 \text{ ft}^2) \left( \frac{62.30 \text{ lbm/ft}^3}{32.2 \text{ lbm/ft\cdots}^2} \right) = 1490 \text{ lbf}
\]

**Discussion**
Note that this force is equivalent to the weight of a 1490 lbm mass. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a mass of 1490 lbm on the pipe supported at its ends 70 ft apart. The necessary precautions should be taken if the pipe cannot support this force. Also, the fluctuations in water flow may reduce the drag coefficients by inducing turbulence and delaying flow separation.
Chapter 11 External Flow: Drag and Lift

Solution Dust particles that are unsettled during high winds rise to a specified height, and start falling back when things calm down. The time it takes for the dust particles to fall back to the ground and their velocity are to be determined using Stokes law.

Assumptions 1 The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). 2 The atmosphere is calm during fall back (no winds or drafts). 3 The initial transient period during which the dust particle accelerates to its terminal velocity is negligible. 4 The buoyancy force applied by air to the dust particle is negligible since $\rho_{\text{air}} \ll \rho_{\text{dust}}$ (besides, the uncertainty in the density of dust is greater than the density of air). (We will solve this problem without utilizing this assumption for generality).

Properties The density of dust is given to be $\rho_{\text{dust}} = 1.6 \text{ g/cm}^3 = 1600 \text{ kg/m}^3$. The density and dynamic viscosity of air at 1 atm and 30°C are $\rho_{\text{air}} = 1.164 \text{ kg/m}^3$ and $\mu = 1.872 \times 10^{-5} \text{ kg/m·s}$.

Analysis The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the surrounding fluid, $F_D = W - F_B$ where $F_D = 3\pi \mu V D$ (Stokes law), $W = \rho_s g V$, and $F_B = \rho_f g V$

Here $V = \pi D^3/6$ is the volume of the sphere. Substituting,

$$3\pi \mu V D = \rho_s g V - \rho_f g V \rightarrow 3\pi \mu V D = (\rho_s - \rho_f) g \frac{\pi D^3}{6}$$

Solving for the velocity $V$ and substituting the numerical values, the terminal velocity is determined to be

$$V = \frac{g D^2 (\rho_s - \rho_f)}{18 \mu} = \frac{(9.81 \text{ m/s}^2)(6 \times 10^{-5} \text{ m})^2(1600 - 1.164) \text{ kg/m}^3}{18(1.872 \times 10^{-5} \text{ kg/m·s})} = 0.1676 \text{ m/s} \approx 0.168 \text{ m/s}$$

Then the time it takes for the dust particle to travel 200 m at this velocity becomes

$$\Delta t = \frac{L}{V} = \frac{200 \text{ m}}{0.1676 \text{ m/s}} = 1194 \text{ s} = 19.9 \text{ min}$$

The Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1.164 \text{ kg/m}^3)(0.1676 \text{ m/s})(6 \times 10^{-5} \text{ m})}{1.872 \times 10^{-5} \text{ kg·m/s}} = 0.625$$

which is in the order of 1. Therefore, the creeping flow idealization and thus Stokes law is applicable.

Discussion Note that the dust particle reaches a terminal velocity of 0.168 m/s, and it takes about 20 min to fall back to the ground. The presence of drafts in air may significantly increase the settling time.
Solution  A cylindrical log suspended by a crane is subjected to normal winds. The angular displacement of the log and the tension on the cable are to be determined.

Assumptions  1 The surfaces of the log are smooth so that Fig. 11-34 can be used to determine the drag coefficient (not a realistic assumption). 2 Air flow in the wind is steady and incompressible. 3 The turbulence in the wind is not considered. 4 The direction of wind is normal to the log, which always remains horizontal. 5 The end effects of the log are negligible. 6 The weight of the cable and the drag acting on it are negligible. 7 Air is an ideal gas.

Properties  The dynamic viscosity of air at 5°C (independent of pressure) is \( \mu = 1.754 \times 10^{-5} \text{ kg/m·s} \). Then the density and kinematic viscosity of air are calculated to be

\[
\rho = \frac{P}{RT} = \frac{88 \text{ kPa}}{(0.287 \text{ kPa·m}^3/\text{kg·K})(278 \text{ K})} = 1.103 \text{ kg/m}^3
\]

\[
v = \frac{\mu}{\rho} = \frac{1.754 \times 10^{-5} \text{ kg/m·s}}{1.103 \text{ kg/m}^3} = 1.590 \times 10^{-5} \text{ m}^2/\text{s}
\]

Analysis  Noting that \( D = 0.2 \text{ m} \) and \( 1 \text{ m/s} = 3.6 \text{ km/h} \), the Reynolds number is

\[
Re = \frac{VD}{\nu} = \frac{(40/3.6 \text{ m/s})(0.2 \text{ m})}{1.590 \times 10^{-5} \text{ m}^2/\text{s}} = 1.398 \times 10^5
\]

The drag coefficient corresponding to this value is, from Fig. 11-34, \( C_D = 1.2 \). Also, the frontal area for flow past a cylinder is \( A = LD \). Then the total drag force acting on the log becomes

\[
F_D = C_D A \frac{DV^2}{2} = 1.2(2 \times 0.2 \text{ m}^2) \frac{(1.103 \text{ kg/m}^3)(40/3.6 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg·m/s}^2} \right) = 32.7 \text{ N}
\]

The weight of the log is

\[
W = mg = \rho g V = \rho g \frac{\pi D^2 L}{4} = (513 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \frac{\pi(0.2 \text{ m})^2(2 \text{ m})}{4} \left( \frac{1 \text{ N}}{1 \text{ kg·m/s}^2} \right) = 316 \text{ N}
\]

Then the resultant force acting on the log and the angle it makes with the horizontal become

\[
F_{\log} = R = \sqrt{W^2 + F_D^2} = \sqrt{32.7^2 + 316^2} = 318 \text{ N}
\]

\[
\tan \theta = \frac{W}{F_D} = \frac{316}{32.7} = 9.66 \quad \rightarrow \quad \theta = 84^\circ
\]

Drawing a free body diagram of the log and doing a force balance will show that the magnitude of the tension on the cable must be equal to the resultant force acting on the log. Therefore, the tension on the cable is 318 N and the cable makes 84° with the horizontal.

Discussion  Note that the wind in this case has rotated the cable by 6° from its vertical position, and increased the tension action on it somewhat. At very high wind speeds, the increase in the cable tension can be very significant, and wind loading must always be considered in bodies exposed to high winds.
11-70

**Solution** Wind is blowing across the wire of a transmission line. The drag force exerted on the wire by the wind is to be determined.

**Assumptions** 1 The wire surfaces are smooth so that Fig. 11-34 can be used to determine the drag coefficient. 2 Air flow in the wind is steady and incompressible. 3 The turbulence in the wind is not considered. 4 The direction of wind is normal to the wire.

**Properties** The density and kinematic viscosity of air at 1 atm and 15°C are \( \rho = 1.225 \text{ kg/m}^3 \) and \( \nu = 1.470 \times 10^{-5} \text{ m}^2/\text{s} \).

**Analysis** Noting that \( D = 0.006 \text{ m} \) and 1 m/s = 3.6 km/h, the Reynolds number for flow is

\[
Re = \frac{VD}{\nu} = \frac{(65/3.6 \text{ m/s})(0.006 \text{ m})}{1.470 \times 10^{-5} \text{ m}^2/\text{s}} = 7.370 \times 10^3
\]

The drag coefficient corresponding to this value is, from Fig. 11

\[C_D = \frac{240 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} = 240 \text{ N}\]

Therefore, the drag force acting on the wire is 240 N, which is equivalent to the weight of about 24 kg mass hanging on the wire.

**Discussion** It should be kept in mind that wind turbulence may reduce the drag coefficients by inducing turbulence and delaying flow separation.

---

11-71E

**Solution** A person extends his uncovered arms into the windy air outside. The drag force exerted on both arms by the wind is to be determined.

**Assumptions** 1 The surfaces of the arms are smooth so that Fig. 11-34 can be used to determine the drag coefficient. 2 Air flow in the wind is steady and incompressible. 3 The turbulence in the wind is not considered. 4 The direction of wind is normal to the arms. 5 The arms can be treated as 2-ft-long and 4-in.-diameter cylinders with negligible end effects.

**Properties** The density and kinematic viscosity of air at 1 atm and 60°F are \( \rho = 0.07633 \text{ lbm/ft}^3 \) and \( \nu = 0.5718 \text{ ft}^2/\text{h} = 1.588 \times 10^{-4} \text{ ft}^2/\text{s} \).

**Analysis** Noting that \( D = 0.3333 \text{ ft} \) and 1 mph = 1.4667 ft/s, the Reynolds number for flow over the arm is

\[
Re = \frac{VD}{\nu} = \frac{(25 \times 1.4667 \text{ ft/s})(0.3333 \text{ ft})}{1.588 \times 10^{-4} \text{ ft}^2/\text{s}} = 7.697 \times 10^4
\]

The drag coefficient corresponding to this value is, from Fig. 11

\[C_D = \frac{2.12 \text{ lbf}}{1 \text{ lbm} \cdot \text{ft}^2/\text{s}^2} = 2.12 \text{ lbf}\]

**Discussion** Note that this force is equivalent to the weight of 1 lbm mass. Therefore, the drag force the wind exerts on the arms of this person is equivalent to hanging 0.5 lbm of mass on each arm. Also, it should be kept in mind that the wind turbulence and the surface roughness may affect the calculated result significantly.
Solution  

A ping-pong ball is suspended in air by an upward air jet. The velocity of the air jet is to be determined, and the phenomenon that the ball returns to the center of the air jet after a disturbance is to be explained.

**Assumptions** 1 The surface of the ping-pong ball is smooth so that Fig. 11-34 can be used to determine the drag coefficient. 2 Air flow over the ball is steady and incompressible.

**Properties** The density and kinematic viscosity of air at 1 atm and 25°C are $\rho = 1.184$ kg/m$^3$ and $\nu = 1.562 \times 10^{-5}$ m$^2$/s.

**Analysis** The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object less the buoyancy force applied by the fluid, $F_D = W - F_B$ where $F_D = C_D \frac{\rho f V^2}{2}$, $W = mg$, and $F_B = \rho_f g V$

Here $A = \pi D^2/4$ is the frontal area and $V = \pi D^3/6$ is the volume of the sphere. Also, $W = mg = (0.0031 \text{kg})(9.81 \text{m/s}^2) = 0.03041 \text{kg} \cdot \text{m/s}^2 = 0.03041 \text{N}$

$F_B = \rho_f g \frac{\pi D^3}{6} = (1.184 \text{kg/m}^3)(9.81 \text{m/s}^2) \pi \frac{(0.042 \text{m})^3}{6} = 0.000451 \text{kg} \cdot \text{m/s}^2 = 0.000451 \text{N}$

Substituting and solving for $V$,

$$C_D \frac{\pi D^2 \rho_f V^2}{4} = W - F_B \rightarrow V = \sqrt{\frac{8(W - F_B)}{\pi D^2 C_D \rho_f}} = \sqrt{\frac{8(0.03041 - 0.000451) \text{kg} \cdot \text{m/s}^2}{\pi (0.042 \text{m})^2 C_D (1.184 \text{kg/m}^3)}} \rightarrow V = \frac{6.044}{\sqrt{C_D}} \quad (1)$$

The drag coefficient $C_D$ is to be determined from Fig. 11-34, but it requires the Reynolds number which cannot be calculated since we do not know velocity. Therefore, the solution requires a trial-error approach. First we express the Reynolds number as

$$\text{Re} = \frac{VD}{\nu} = \frac{V(0.042 \text{m})}{1.562 \times 10^{-5} \text{m}^2/\text{s}} \rightarrow \text{Re} = 2689V \quad (2)$$

Now we choose a velocity in m/s, calculate the Re from Eq. 2, read the corresponding $C_D$ from Fig. 11-34, and calculate $V$ from Eq. 1. Repeat calculations until the assumed velocity matches the calculated velocity. With this approach the velocity of the fluid jet is determined to be

$$V = 9.2 \text{ m/s}$$

The corresponding Re and $C_D$ values are Re = 24,700 and $C_D = 0.43$. Therefore, the ping-pong ball will remain suspended in the air jet when the air velocity reaches 9.2 m/s = 33.1 km/h.

**Discussion**

1 If the ball is pushed to the side by a finger, the ball will come back to the center of the jet (instead of falling off) due to the Bernoulli effect. In the core of the jet the velocity is higher, and thus the pressure is lower relative to a location away from the jet.

2 Note that this simple apparatus can be used to determine the drag coefficients of certain objects by simply measuring the air velocity, which is easy to do.

3 This problem can also be solved roughly by taking $C_D = 0.5$ from Table 11-2 for a sphere in laminar flow, and then verifying that the flow is laminar.
11-73C
Solution We are to discuss why the contribution of viscous effects to lift of airfoils is usually negligible.

Analysis The contribution of viscous effects to lift is usually negligible for airfoils since the wall shear is nearly parallel to the surfaces of such devices and thus nearly normal to the direction of lift.

Discussion However, viscous effects are extremely important for airfoils at high angles of attack, since the viscous effects near the wall (in the boundary layer) cause the flow to separate and the airfoil to stall, losing significant lift.

11-74C
Solution We are to discuss the lift and drag on a symmetrical airfoil at 5° angle of attack.

Analysis When air flows past a symmetrical airfoil at an angle of attack of 5°, both the (a) lift and (b) drag acting on the airfoil are nonzero.

Discussion Because of the lack of symmetry with respect to the free-stream flow, the flow is different on the top and bottom surfaces of the airfoil, leading to lift. There is drag too, just as there is drag even at zero angle of attack.

11-75C
Solution We are to define and discuss stall.

Analysis The decrease of lift with an increase in the angle of attack is called stall. When the flow separates over nearly the entire upper half of the airfoil, the lift is reduced dramatically (the separation point is near the leading edge). Stall is caused by flow separation and the formation of a wide wake region over the top surface of the airfoil. Commercial aircraft are not allowed to fly at velocities near the stall velocity for safety reasons. Airfoils stall at high angles of attack (flow cannot negotiate the curve around the leading edge). If a plane stalls, it loses much of its lift, and it can crash.

Discussion At angles of attack above the stall angle, the drag also increases significantly.

11-76C
Solution We are to discuss the lift and drag on a nonsymmetrical airfoil at zero angle of attack.

Analysis When air flows past a nonsymmetrical airfoil at zero angle of attack, both the (a) lift and (b) drag acting on the airfoil are nonzero.

Discussion Because of the lack of symmetry, the flow is different on the top and bottom surfaces of the airfoil, leading to lift. There is drag too, just as there is drag even on a symmetrical airfoil.
We are to discuss the lift and drag on a symmetrical airfoil at zero angle of attack.

When air flows past a symmetrical airfoil at zero angle of attack, (a) **the lift is zero**, but (b) **the drag acting on the airfoil is nonzero**.

In this case, because of symmetry, there is no lift, but there is still skin friction drag, along with a small amount of pressure drag.

---

We are to discuss which increases at a greater rate – lift or drag – with increasing angle of attack.

Both the lift and the drag of an airfoil increase with an increase in the angle of attack, but in general, **the lift increases at a much higher rate than does the drag**.

In other words, the lift-to-drag ratio increases with increasing angle of attack – at least up to the stall angle.

---

We are to why flaps are used on aircraft during takeoff and landing.

Flaps are used at the leading and trailing edges of the wings of large aircraft during takeoff and landing to alter the shape of the wings to **maximize lift and to enable the aircraft to land or takeoff at low speeds**. **An aircraft can take off or land without flaps, but it can do so at very high velocities**, which is undesirable during takeoff and landing.

In simple terms, the planform area of the wing increases as the flaps are deployed. Thus, even if the lift coefficient were to remain constant, the actual lift would still increase. In fact, however, flaps **increase the lift coefficient** as well, leading to even further increases in lift. Lower takeoff and landing speeds lead to shorter runway length requirements.

---

We are to discuss the lift on a spinning and non-spinning ball.

When air is flowing past a spherical ball, **the lift exerted on the ball is zero if the ball is not spinning**, and it is **nonzero if the ball is spinning about an axis normal to the free stream velocity** (no lift is generated if the ball is spinning about an axis parallel to the free stream velocity).

In the parallel spinning case, however, a **side force** would be generated (e.g., a curve ball).

---

We are to discuss the effect of wing tip vortices on drag and lift.

The effect of wing tip vortices is to **increase drag (induced drag) and to decrease lift**. This effect is also due to the **downwash**, which **causes an effectively smaller angle of attack**.

Induced drag is a three-dimensional effect; there is no induced drag on a 2-D airfoil since there are no tips.
Chapter 11 External Flow: Drag and Lift

11-82C
Solution  We are to discuss induced drag and how to minimize it.

Analysis  Induced drag is the additional drag caused by the tip vortices. The tip vortices have a lot of kinetic energy, all of which is wasted and is ultimately dissipated as heat in the air downstream. Induced drag can be reduced by using long and narrow wings, and by modifying the geometry of the wing tips.

Discussion  Birds are designed with feathers that fan out at the tips of their wings in order to reduce induced drag.

11-83C
Solution  We are to explain why some airplane wings have endplates or winglets.

Analysis  Endplates or winglets are added at the tips of airplane wings to reduce induced drag. In short, the winglets disrupt flow from the high pressure lower part of the wing to the low pressure upper part of the wing, thereby reducing the strength of the tip vortices.

Discussion  Comparing a wing with and without winglets, the one with winglets can produce the same lift with less drag, thereby saving fuel or increasing range. Winglets are especially useful in glider planes because high lift with small drag is critical to their operation (and safety!).

11-84C
Solution  We are to discuss how flaps affect the lift and drag of airplane wings.

Analysis  Flaps increase both the lift and the drag of the wings. But the increase in drag during takeoff and landing is not much of a concern because of the relatively short time periods involved. This is the penalty we pay willingly to take off and land at safe speeds.

Discussion  Note, however, that the engine must operate at nearly full power during takeoff to overcome the large drag.
**Solution**  The wing area, lift coefficient at takeoff settings, the cruising drag coefficient, and total mass of a small aircraft are given. The takeoff speed, the wing loading, and the required power to maintain a constant cruising speed are to be determined.

**Assumptions**  1 Standard atmospheric conditions exist. 2 The drag and lift produced by parts of the plane other than the wings are not considered.

**Properties**  The density of standard air at sea level is \( \rho = 1.225 \text{ kg/m}^3 \).

**Analysis**  

(a) An aircraft will takeoff when lift equals the total weight. Therefore,

\[
W = F_L \implies W = \frac{1}{2} C_L \rho V^2 A \implies V = \sqrt{\frac{2W}{\rho C_L A}}
\]

Substituting, the takeoff speed is determined to be

\[
V_{\text{takeoff}} = \sqrt{\frac{2mg}{\rho C_{L,\text{takeoff}} A}} = \sqrt{\frac{2(4000 \text{ kg})(9.81 \text{ m/s}^2)}{(1.225 \text{ kg/m}^3)(0.45)(35 \text{ m}^2)}} = 63.8 \text{ m/s} = 230 \text{ km/h}
\]

(b) Wing loading is the average lift per unit planform area, which is equivalent to the ratio of the lift to the planform area of the wings since the lift generated during steady cruising is equal to the weight of the aircraft. Therefore,

\[
F_{\text{loading}} = \frac{F_L}{A} = \frac{W}{A} = \frac{(4000 \text{ kg})(9.81 \text{ m/s}^2)}{35 \text{ m}^2} = 1121 \text{ N/m}^2
\]

(c) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force, which is

\[
F_D = C_D A \frac{\rho V^2}{2} = (0.035)(35 \text{ m}^2) \frac{(1.225 \text{ kg/m}^3)(300/3.6 \text{ m/s})^2}{2} \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right) = 5.211 \text{ kN}
\]

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

\[
\text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (5.211 \text{ kN})(300/3.6 \text{ m/s}) \left( \frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}} \right) = 434 \text{ kW}
\]

Therefore, the engines must supply 434 kW of propulsive power to overcome the drag during cruising.

**Discussion**  The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.
Solution  The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the weight of the aircraft is increased by 10% as a result of overloading is to be determined.

Assumptions  1 The atmospheric conditions (and thus the properties of air) remain the same. 2 The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same.

Analysis  An aircraft will takeoff when lift equals the total weight. Therefore,

\[ W = F_L \quad \rightarrow \quad W = \frac{1}{2} C_L \rho V^2 A \quad \rightarrow \quad V = \sqrt[2]{\frac{2W}{\rho C_L A}} \]

We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and area remain constant, the ratio of the velocities of the overloaded and fully loaded aircraft becomes

\[ \frac{V_2}{V_1} = \sqrt[2]{\frac{2W_2}{\rho C_L A}} \frac{\sqrt[2]{W_1}}{\sqrt[2]{W_1}} \quad \rightarrow \quad V_2 = V_1 \sqrt[2]{\frac{W_2}{W_1}} \]

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

\[ V_2 = V_1 \sqrt[2]{\frac{1.1W_1}{W_1}} = (260 \text{ km/h})\sqrt[2]{1.1} = 273 \text{ km/h} \]

Discussion  A similar analysis can be performed for the effect of the variations in density, lift coefficient, and planform area on the takeoff velocity.
The takeoff speed and takeoff time of an aircraft at sea level are given. The required takeoff speed, takeoff time, and the additional runway length required at a higher elevation are to be determined.

Assumptions
1. Standard atmospheric conditions exist. 2. The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane and the planform area remain constant. 3. The acceleration of the aircraft during takeoff remains constant.

Properties
The density of standard air is \( \rho_1 = 1.225 \text{ kg/m}^3 \) at sea level, and \( \rho_2 = 1.048 \text{ kg/m}^3 \) at 1600 m altitude.

Analysis
(a) An aircraft will takeoff when lift equals the total weight. Therefore,

\[
W = F_L \rightarrow W = \frac{1}{2} C_L \rho V^2 A \rightarrow V = \sqrt{\frac{2W}{\rho C_L A}}
\]

We note that the takeoff speed is inversely proportional to the square root of air density. When the weight, lift coefficient, and area remain constant, the ratio of the speeds of the aircraft at high altitude and at sea level becomes

\[
\frac{V_2}{V_1} = \sqrt{\frac{\rho_1}{\rho_2}} \rightarrow V_2 = V_1 \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{1.225}{1.048}} = 238 \text{ km/h}
\]

Therefore, the takeoff velocity of the aircraft at higher altitude is 238 km/h.

(b) The acceleration of the aircraft at sea level is

\[
a = \frac{\Delta V}{\Delta t} = \frac{220 \text{ km/h} - 0 \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)}{15 \text{ s}} = 4.074 \text{ m/s}^2
\]

which is assumed to be constant both at sea level and the higher altitude. Then the takeoff time at the higher altitude becomes

\[
a = \frac{\Delta V}{\Delta t} \rightarrow \Delta t = \frac{\Delta V}{a} = \frac{238 \text{ km/h} - 0 \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)}{4.074 \text{ m/s}^2} = 16.2 \text{ s}
\]

(c) The additional runway length is determined by calculating the distance traveled during takeoff for both cases, and taking their difference:

\[
L_1 = \frac{1}{2} a t_1^2 = \frac{1}{2} (4.074 \text{ m/s}^2) (15 \text{ s})^2 = 458 \text{ m}
\]

\[
L_2 = \frac{1}{2} a t_2^2 = \frac{1}{2} (4.074 \text{ m/s}^2) (16.2 \text{ s})^2 = 535 \text{ m}
\]

\[
\Delta L = L_2 - L_1 = 535 - 458 = 77 \text{ m}
\]

Discussion
Note that altitude has a significant effect on the length of the runways, and it should be a major consideration on the design of airports. It is interesting that a 1.2 second increase in takeoff time increases the required runway length by about 100 m.
11-88E

**Solution**  The rate of fuel consumption of an aircraft while flying at a low altitude is given. The rate of fuel consumption at a higher altitude is to be determined for the same flight velocity.

**Assumptions**  1 Standard atmospheric conditions exist. 2 The settings of the plane during takeoff are maintained the same so that the drag coefficient of the plane and the planform area remain constant. 3 The velocity of the aircraft and the propulsive efficiency remain constant. 4 The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

**Properties**  The density of standard air is \( \rho_1 = 0.05648 \) lbm/ft\(^3\) at 10,000 ft, and \( \rho_2 = 0.02866 \) lbm/ft\(^3\) at 30,000 ft altitude.

**Analysis**  When an aircraft cruises steadily (zero acceleration) at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force. Also, power is force times velocity (distance per unit time), and thus the propulsive power required to overcome drag is equal to the thrust times the cruising velocity. Therefore,

\[
\eta_{\text{prop}} = \text{Thrust} \times V = F_D V = C_D A \frac{\rho V^2}{2} V = C_D A \frac{\rho V^3}{2}
\]

The propulsive power is also equal to the product of the rate of fuel energy supplied (which is the rate of fuel consumption times the heating value of the fuel, \( \dot{m}_{\text{fuel}} HV \)) and the propulsive efficiency. Then,

\[
\eta_{\text{prop}} \dot{m}_{\text{fuel}} HV = \frac{C_D A \rho V^3}{2} = \frac{\rho_2}{\rho_1} \dot{m}_{\text{fuel}, 1}HV
\]

We note that the rate of fuel consumption is proportional to the density of air. When the drag coefficient, the wing area, the velocity, and the propulsive efficiency remain constant, the ratio of the rates of fuel consumptions of the aircraft at high and low altitudes becomes

\[
\frac{\dot{m}_{\text{fuel}, 2}}{\dot{m}_{\text{fuel}, 1}} = \frac{C_D A \rho_2 V^3 / 2 \eta_{\text{prop}} HV}{C_D A \rho_1 V^3 / 2 \eta_{\text{prop}} HV} = \frac{\rho_2}{\rho_1} \rightarrow \dot{m}_{\text{fuel}, 2} = \dot{m}_{\text{fuel}, 1} \frac{\rho_2}{\rho_1} = (7 \text{ gal/min}) \frac{0.02866}{0.05648} = 3.55 \text{ gal/min}
\]

**Discussion**  Note the fuel consumption drops by half when the aircraft flies at 30,000 ft instead of 10,000 ft altitude. Therefore, large passenger planes routinely fly at high altitudes (usually between 30,000 and 40,000 ft) to save fuel. This is especially the case for long flights.
**Solution**

The takeoff speed of an aircraft when it is fully loaded is given. The required takeoff speed when the aircraft has 100 empty seats is to be determined.

**Assumptions**

1. The atmospheric conditions (and thus the properties of air) remain the same.
2. The settings of the plane during takeoff are maintained the same so that the lift coefficient of the plane remains the same.
3. A passenger with luggage has an average mass of 140 kg.

**Analysis**

An aircraft will takeoff when lift equals the total weight. Therefore,

\[ W = F_L \rightarrow W = \frac{1}{2} C_L \rho V^2 A \rightarrow V = \sqrt{\frac{2W}{\rho C_L A}} \]

We note that the takeoff velocity is proportional to the square root of the weight of the aircraft. When the density, lift coefficient, and wing area remain constant, the ratio of the velocities of the under-loaded and fully loaded aircraft becomes

\[ \frac{V_2}{V_1} = \sqrt{\frac{2W_2}{\rho C_L A}} \cdot \frac{\sqrt{W_1}}{\sqrt{W_2}} = \sqrt{\frac{m_2 g}{m_1 g}} = \sqrt{\frac{m_2}{m_1}} \rightarrow V_2 = V_1 \sqrt{\frac{m_2}{m_1}} \]

where \( m_2 = m_1 - m\text{ unused capacity} = 400,000 \text{ kg} - (140 \text{ kg/passanger}) \times (100 \text{ passengers}) = 386,000 \text{ kg} \)

Substituting, the takeoff velocity of the overloaded aircraft is determined to be

\[ V_2 = V_1 \sqrt{\frac{m_2}{m_1}} = (250 \text{ km/h}) \sqrt{\frac{386,000}{400,000}} = 246 \text{ km/h} \]

**Discussion**

Note that the effect of empty seats on the takeoff velocity of the aircraft is small. This is because the most weight of the aircraft is due to its empty weight (the aircraft itself rather than the passengers and their luggage.)
Solution  The previous problem is reconsidered. The effect of empty passenger count on the takeoff speed of the aircraft as the number of empty seats varies from 0 to 500 in increments of 50 is to be investigated.

Analysis  The EES Equations window is printed below, along with the tabulated and plotted results.

\[ m_{\text{passenger}} = 140 \text{ "kg"} \]
\[ m_1 = 400000 \text{ "kg"} \]
\[ m_2 = m_1 - N_{\text{empty}} \times m_{\text{passenger}} \]
\[ V_1 = 250 \text{ "km/h"} \]
\[ V_2 = V_1 \times \sqrt{\frac{m_2}{m_1}} \]

<table>
<thead>
<tr>
<th>Empty seats</th>
<th>( m_{\text{airplane,1}} ) kg</th>
<th>( m_{\text{airplane,1}} ) kg</th>
<th>( V_{\text{takeoff}} ) m/s</th>
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<tr>
<td>500</td>
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</table>

Discussion  As expected, the takeoff speed decreases as the number of empty seats increases. On the scale plotted, the curve appears nearly linear, but it is not; the curve is actually a small portion of a square-root curve.
Solution

A tennis ball is hit with a backspin. It is to be determined if the ball will fall or rise after being hit.

Assumptions

1. The outer surface of the ball is smooth enough for Fig. 11-53 to be applicable.
2. The ball is hit horizontally so that it starts its motion horizontally.

Properties

The density and kinematic viscosity of air at 1 atm and 25°C are \( \rho = 1.184 \text{ kg/m}^3 \) and \( \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s} \).

Analysis

The ball is hit horizontally, and thus it would normally fall under the effect of gravity without the spin. The backspin will generate a lift, and the ball will rise if the lift is greater than the weight of the ball. The lift can be determined from

\[ F_L = C_L \frac{\rho V^2}{2}, \]

where \( A \) is the frontal area of the ball, \( A = \pi D^2 / 4 \). The regular and angular velocities of the ball are

\[ V = (105 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 29.17 \text{ m/s} \quad \text{and} \quad \omega = \left( \frac{4200 \text{ rev/min}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 440 \text{ rad/s} \]

Then,

\[ \frac{\omega D}{2V} = \frac{(440 \text{ rad/s})(0.064 \text{ m})}{2(29.17 \text{ m/s})} = 0.483 \text{ rad} \]

From Fig. 11-53, the lift coefficient corresponding to this value is \( C_L = 0.095 \). Then the lift acting on the ball is

\[ F_L = (0.095) \frac{\pi(0.064 \text{ m})^2}{4} (1.184 \text{ kg/m}^3)(29.17 \text{ m/s})^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 0.15 \text{ N} \]

The weight of the ball is \( W = mg = (0.057 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 0.56 \text{ N} \)

which is more than the lift. Therefore, the ball will drop under the combined effect of gravity and lift due to spinning after hitting, with a net force of 0.56 - 0.15 = 0.41 N.

Discussion

The Reynolds number for this problem is

\[ \text{Re}_L = \frac{V D}{\nu} = \frac{(29.17 \text{ m/s})(0.064 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 1.20 \times 10^5 \]

which is close enough to \( 6 \times 10^4 \) for which Fig. 11-53 is prepared. Therefore, the result should be close enough to the actual answer.
11-92E

Solution  A spinning ball is dropped into a water stream. The lift and drag forces acting on the ball are to be determined.

Assumptions  1 The outer surface of the ball is smooth enough for Fig. 11-53 to be applicable. 2 The ball is completely immersed in water.

Properties  The density and dynamic viscosity of water at 60°F are \( \rho = 62.36 \text{ lbm/ft}^3 \) and \( \mu = 2.713 \text{ lbm/ft·s} \).

Analysis  The drag and lift forces can be determined from

\[
F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad F_L = C_L A \frac{\rho V^2}{2}
\]

where \( A \) is the frontal area of the ball, which is \( A = \pi D^2 / 4 \), and \( D = 2.4/12 = 0.2 \text{ ft} \). The Reynolds number and the angular velocity of the ball are

\[
Re = \frac{\rho V D}{\mu} = \frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})(0.2 \text{ ft})}{7.536 \times 10^{-4} \text{ lbm·ft/s}} = 6.62 \times 10^4
\]

\[
\omega = (500 \text{ rev/min}) \left( \frac{2\pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 52.4 \text{ rad/s}
\]

and

\[
\frac{\omega D}{2V} = \frac{(52.4 \text{ rad/s})(0.2 \text{ ft})}{2(4 \text{ ft/s})} = 1.31 \text{ rad}
\]

From Fig. 11-53, the drag and lift coefficients corresponding to this value are \( C_D = 0.56 \) and \( C_L = 0.35 \). Then the drag and the lift acting on the ball are

\[
F_D = (0.56) \frac{\pi(0.2 \text{ ft})^2}{4} \frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})^2}{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm·ft/s}^2} \right) = 0.27 \text{ lbf}
\]

\[
F_L = (0.35) \frac{\pi(0.2 \text{ ft})^2}{4} \frac{(62.36 \text{ lbm/ft}^3)(4 \text{ ft/s})^2}{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm·ft/s}^2} \right) = 0.17 \text{ lbf}
\]

Discussion  The Reynolds number for this problem is \( 6.62 \times 10^4 \) which is close enough to \( 6 \times 10^4 \) for which Fig. 11-53 is prepared. Therefore, the result should be close enough to the actual answer.

11-93

Solution  An airfoil has a given lift-to-drag ratio at 0° angle of attack. The angle of attack that will raise this ratio to 80 is to be determined.

Analysis  The ratio \( C_L/C_D \) for the given airfoil is plotted against the angle of attack in Fig. 11-43. The angle of attack corresponding to \( C_L/C_D = 80 \) is \( \theta = 3^\circ \).

Discussion  Note that different airfoils have different \( C_L/C_D \) vs. \( \theta \) charts.
Chapter 11 External Flow: Drag and Lift

Solution The wings of a light plane resemble the NACA 23012 airfoil with no flaps. Using data for that airfoil, the takeoff speed at a specified angle of attack and the stall speed are to be determined.

Assumptions 1 Standard atmospheric conditions exist. 2 The drag and lift produced by parts of the plane other than the wings are not considered.

Properties The density of standard air at sea level is \( \rho = 1.225 \text{ kg/m}^3 \). At an angle of attack of 5°, the lift and drag coefficients are read from Fig. 11-45 to be \( C_L = 0.6 \) and \( C_D = 0.015 \) [Note: Student values may differ significantly because these values are very hard to read from the plots]. The maximum lift coefficient is \( C_{L,\text{max}} = 1.52 \) and it occurs at an angle of attack of 15°.

Analysis An aircraft will takeoff when lift equals the total weight. Therefore,

\[
W = F_L \Rightarrow W = \frac{1}{2} C_L \rho V^2 A \Rightarrow V = \sqrt{\frac{2W}{\rho C_L A}}
\]

Substituting, the takeoff speed is determined to be

\[
V_{\text{takeoff}} = \sqrt{\frac{2(11,000 \text{ N})}{(1.225 \text{ kg/m}^3)(0.6)(39 \text{ m}^2)}} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 27.70 \text{ m/s} \approx 99.7 \text{ km/h}
\]

since 1 m/s = 3.6 km/h. The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

\[
V_{\text{min}} = \sqrt{\frac{2W}{\rho C_{L,\text{takeoff}} A}} = \sqrt{\frac{2(11,000 \text{ N})}{(1.225 \text{ kg/m}^3)(1.52)(39 \text{ m}^2)}} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 17.41 \text{ m/s} \approx 62.7 \text{ km/h}
\]

Discussion The “safe” minimum velocity to avoid the stall region is obtained by multiplying the stall velocity by 1.2:

\[
V_{\text{min, safe}} = 1.2V_{\text{min}} = 1.2 \times 17.41 \text{ m/s} = 20.9 \text{ m/s} = 75.2 \text{ km/h}
\]

Note that the takeoff velocity decreased from 107 km/h at an angle of attack of 5° to 80.8 km/s under stall conditions with a safety margin.
Solution  The total mass, wing area, cruising speed, and propulsive power of a small aircraft are given. The lift and drag coefficients of this airplane while cruising are to be determined.

Assumptions  1 Standard atmospheric conditions exist. 2 The drag and lift produced by parts of the plane other than the wings are not considered. 3 The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties  The density of standard air at an altitude of 4000 m is $\rho = 0.819 \text{ kg/m}^3$.

Analysis  Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity. Also, when the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. Then,

$$W_{\text{prop}} = \text{Thrust} \times \text{Velocity} = F_D V$$

$$F_D = \frac{W_{\text{prop}}}{V} = \frac{190 \text{ kW}}{280/3.6 \text{ m/s}} \left( \frac{1000 \text{ N} \cdot \text{m/s}}{1 \text{ kW}} \right) = 2443 \text{ N}$$

Then the drag coefficient becomes

$$F_D = C_D A \frac{\rho V^2}{2} \Rightarrow C_D = \frac{2F_D}{\rho AV^2} = \frac{2(2443 \text{ N})}{(0.819 \text{ kg/m}^3)(42 \text{ m}^2)(280/3.6 \text{ m/s})^2} \left( \frac{1 \text{ kg} \cdot \text{m}^2}{1 \text{ N}} \right) = 0.0235$$

An aircraft cruises at constant altitude when lift equals the total weight. Therefore,

$$W = F_L = \frac{1}{2} C_L \rho V^2 A \Rightarrow C_L = \frac{2W}{\rho V^2 A} = \frac{2(1800 \text{ kg})(9.81 \text{ m}^2/\text{s}^2)}{(0.819 \text{ kg/m}^3)(42 \text{ m}^2)(280/3.6 \text{ m/s})^2} = 0.17$$

Therefore, the drag and lift coefficients of this aircraft during cruising are 0.0235 and 0.17, respectively, with a $C_L/C_D$ ratio of 7.2.

Discussion  The drag and lift coefficient determined are for cruising conditions. The values of these coefficient can be very different during takeoff because of the angle of attack and the wing geometry.
Solution

The mass, wing area, the maximum (stall) lift coefficient, the cruising speed and the cruising drag coefficient of an airplane are given. The safe takeoff speed at sea level and the thrust that the engines must deliver during cruising are to be determined.

Assumptions
1. Standard atmospheric conditions exist
2. The drag and lift produced by parts of the plane other than the wings are not considered.
3. The takeoff speed is 20% over the stall speed.
4. The fuel is used primarily to provide propulsive power to overcome drag, and thus the energy consumed by auxiliary equipment (lights, etc) is negligible.

Properties
The density of standard air is $\rho_1 = 1.225 \text{ kg/m}^3$ at sea level, and $\rho_2 = 0.312 \text{ kg/m}^3$ at 12,000 m altitude. The cruising drag coefficient is given to be $C_D = 0.03$. The maximum lift coefficient is given to be $C_{L,max} = 3.2$.

Analysis
(a) An aircraft will takeoff when lift equals the total weight. Therefore,

$$ W = F_L \implies W = \frac{1}{2} C_L \rho V^2 A \implies V = \sqrt{\frac{2W}{\rho C_L A}} = \sqrt{\frac{2mg}{\rho C_L A}} $$

The stall velocity (the minimum takeoff velocity corresponding the stall conditions) is determined by using the maximum lift coefficient in the above equation,

$$ V_{min} = \sqrt{\frac{2mg}{\rho_1 C_{L,max} A}} = \sqrt{\frac{2(50,000 \text{ kg})(9.81 \text{ m/s}^2)}{(1.225 \text{ kg/m}^3)(3.2)(300 \text{ m}^2)}} = 28.9 \text{ m/s} = 104 \text{ km/h} $$

since 1 m/s = 3.6 km/h. Then the “safe” minimum velocity to avoid the stall region becomes

$$ V_{min, safe} = 1.2V_{min} = 1.2 \times (28.9 \text{ m/s}) = 34.7 \text{ m/s} = 125 \text{ km/h} $$

(b) When the aircraft cruises steadily at a constant altitude, the net force acting on the aircraft is zero, and thus the thrust provided by the engines must be equal to the drag force, which is

$$ F_D = C_D A \frac{\rho_2 V^2}{2} = (0.03)(300 \text{ m}^2) \frac{(0.312 \text{ kg/m}^3)(700 / 3.6 \text{ m/s})^2}{2} \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right) = 53.08 \text{ kN} $$

Noting that power is force times velocity, the propulsive power required to overcome this drag is equal to the thrust times the cruising velocity,

$$ \text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (53.08 \text{ kN})(700 / 3.6 \text{ m/s}) \left(\frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}}\right) = 10,300 \text{ kW} $$

Therefore, the engines must supply 10,300 kW of propulsive power to overcome drag during cruising.

Discussion
The power determined above is the power to overcome the drag that acts on the wings only, and does not include the drag that act on the remaining parts of the aircraft (the fuselage, the tail, etc). Therefore, the total power required during cruising will be greater. The required rate of energy input can be determined by dividing the propulsive power by the propulsive efficiency.
11-97

Solution  A blimp connected to the ground by a rope is subjected to parallel winds. The rope tension when the wind is at a specified value is to be determined.

Assumptions  1 The blimp can be treated as an ellipsoid. 2 The wind is steady and turbulent, and blows parallel to the ground.

Properties  The drag coefficient for an ellipsoid with $L/D = 8/3 = 2.67$ is $C_D = 0.1$ in turbulent flow (Table 11-2). We take the density of air to be 1.20 kg/m$^3$.

Analysis  The frontal area of an ellipsoid is $A = \pi D^2/4$. Noting that 1 m/s = 3.6 km/h, the drag force acting on the blimp becomes

$$F_D = C_D A \frac{\rho V^2}{2} = (0.1)\left[\pi (3 \text{ m})^2 / 4\right] \left(\frac{1.20 \text{ kg/m}^3 \times 50 / 3.6 \text{ m/s}}{2}\right) = 81.8 \text{ N}$$

This is the horizontal component of the rope tension. The vertical component is equal to the rope tension when there is no wind, and is given to be $F_B = 120 \text{ N}$. Knowing the horizontal and vertical components, the magnitude of rope tension becomes

$$T = \sqrt{F_D^2 + F_B^2} = \sqrt{(81.8 \text{ N})^2 + (120 \text{ N})^2} = 145 \text{ N}$$

with  \( \theta = \arctan \left( \frac{F_B}{F_D} \right) = \arctan \left( \frac{120 \text{ N}}{81.8 \text{ N}} \right) = 55.7^\circ \).

Discussion  Note that the drag force acting on the blimp is proportional to the square of the blimp diameter. Therefore, the blimp diameter should be minimized to minimize the drag force.
Solution
A large spherical tank located outdoors is subjected to winds. The drag force exerted on the tank by the winds is to be determined.

Assumptions
1. The outer surfaces of the tank are smooth.
2. Air flow in the wind is steady and incompressible, and flow around the tank is uniform.
3. Turbulence in the wind is not considered.
4. The effect of any support bars on flow and drag is negligible.

Properties
The density and kinematic viscosity of air at 1 atm and 25°C are \( \rho = 1.184 \text{ kg/m}^3 \) and \( \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s} \).

Analysis
Noting that \( D = 1.2 \text{ m} \) and \( 1 \text{ m/s} = 3.6 \text{ km/h} \), the Reynolds number for the flow is
\[
Re = \frac{VD}{\nu} = \frac{(48/3.6 \text{ m/s})(1.2 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 1.024 \times 10^6
\]
The drag coefficient for a smooth sphere corresponding to this Reynolds number is, from Fig. 11-36, \( C_D = 0.14 \). Also, the frontal area for flow past a sphere is \( A = \pi D^2/4 \).

Then the drag force becomes
\[
F_D = C_D \frac{A \rho V^2}{2} = 0.14[\pi(1.2 \text{ m})^2 / 4](1.184 \text{ kg/m}^3)(48/3.6 \text{ m/s})^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m}/\text{s}^2} \right) = 16.7 \text{ N}
\]

Discussion
Note that the drag coefficient is very low in this case since the flow is turbulent (Re > 2 \times 10^5). Also, it should be kept in mind that wind turbulence may affect the drag coefficient.
A rectangular advertisement panel attached to a rectangular concrete block by two poles is to withstand high winds. For a given maximum wind speed, the maximum drag force on the panel and the poles, and the minimum length \( L \) of the concrete block for the panel to resist the winds are to be determined.

**Assumptions**
1. The flow of air is steady and incompressible.
2. The wind is normal to the panel (to check for the worst case).
3. The flow is turbulent so that the tabulated value of the drag coefficients can be used.

**Properties**
In turbulent flow, the drag coefficient is \( C_D = 0.3 \) for a circular rod, and \( C_D = 2.0 \) for a thin rectangular plate (Table 11-2). The densities of air and concrete block are given to be \( \rho = 1.30 \text{ kg/m}^3 \) and \( \rho_c = 2300 \text{ kg/m}^3 \).

**Analysis**
(a) The drag force acting on the panel is

\[
F_{D,\text{panel}} = C_D A \frac{\rho V^2}{2} = (2.0)(2 \times 4 \text{ m}^2) \frac{(1.30 \text{ kg/m}^3)(150 / 3.6 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 18,000 \text{ N}
\]

(b) The drag force acting on each pole is

\[
F_{D,\text{pole}} = C_D A \frac{\rho V^2}{2} = (0.3)(0.05 \times 4 \text{ m}^2) \frac{(1.30 \text{ kg/m}^3)(150 / 3.6 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 68 \text{ N}
\]

Therefore, the drag force acting on both poles is \( 68 \times 2 = 136 \text{ N} \). Note that the drag force acting on poles is negligible compared to the drag force acting on the panel.

(c) The weight of the concrete block is

\[
W = mg = \rho g V = (2300 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(L \times 4 \text{ m} \times 0.15 \text{ m}) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 13,540L \text{ N}
\]

Note that the resultant drag force on the panel passes through its center, the drag force on the pole passes through the center of the pole, and the weight of the panel passes through the center of the block. When the concrete block is first tipped, the wind-loaded side of the block will be lifted off the ground, and thus the entire reaction force from the ground will act on the other side. Taking the moment about this side and setting it equal to zero gives

\[
\sum M = 0 \rightarrow F_{D,\text{panel}} \times (1 + 4 + 0.15) + F_{D,\text{pole}} \times (2 + 0.15) - W \times (L / 2) = 0
\]

Substituting and solving for \( L \) gives

\[
18,000 \times 5.15 + 136 \times 2.15 - 13,540L \times L / 2 = 0 \rightarrow L = 3.70 \text{ m}
\]

Therefore, the minimum length of the concrete block must be \( L = 3.70 \).

**Discussion**
This length appears to be large and impractical. It can be reduced to a more reasonable value by (a) increasing the height of the concrete block, (b) reducing the length of the poles (and thus the tipping moment), or (c) by attaching the concrete block to the ground (through long nails, for example).
Solution  

The bottom surface of a plastic boat is approximated as a flat surface. The friction drag exerted on the bottom surface of the boat by water and the power needed to overcome it are to be determined.

Assumptions  
1. The flow is steady and incompressible.  
2. The water is calm (no significant currents or waves).  
3. The water flow is turbulent over the entire surface because of the constant agitation of the boat.  
4. The bottom surface of the boat is a flat surface, and it is smooth.

Properties  
The density and dynamic viscosity of water at 15°C are \( \rho = 999.1 \text{ kg/m}^3 \) and \( \mu = 1.138 \times 10^{-3} \text{ kg/m} \cdot \text{s} \).

Analysis  
The Reynolds number at the end of the bottom surface of the boat is

\[
\text{Re}_L = \frac{\rho VL}{\mu} = \frac{(999.1 \text{ kg/m}^3)(45/3.6 \text{ m/s})(2 \text{ m})}{1.138 \times 10^{-3} \text{ kg/m} \cdot \text{s}} = 2.195 \times 10^7
\]

The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes

\[
C_f = \frac{0.074}{(2.195 \times 10^7)^{1/5}} = 0.002517
\]

\[
F_D = C_f A \frac{DV^2}{2} = (0.002517)(1.5 \times 2 \text{ m}^2) \frac{(999.1 \text{ kg/m}^3)(45/3.6 \text{ m/s})^2}{2} \left( \frac{1 \text{ N}}{1 \text{ kg/m} \cdot \text{s}^2} \right) = 589.4 \text{ N} \approx 589 \text{ N}
\]

Noting that power is force times velocity, the power needed to overcome this drag force is

\[
\text{\(P_{\text{drag}}\)} = F_D V = (589.4 \text{ N})(45/3.6 \text{ m/s}) \left( \frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) = 7.37 \text{ kW}
\]

Discussion  

Note that the calculated drag force (and the power required to overcome it) is relatively small. This is not surprising since the drag force for blunt bodies (including those partially immersed in a liquid) is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.
Solution  The previous problem is reconsidered. The effect of boat speed on the drag force acting on the bottom surface of the boat and the power needed to overcome as the boat speed varies from 0 to 100 km/h in increments of 10 km/h is to be investigated.

Analysis  The EES Equations window is printed below, along with the tabulated and plotted results.

rho=999.1 "kg/m^3"
mu=1.138E-3 "m^2/s"
V=Vel/3.6 "m/s"
L=2 "m"
W=1.5 "m"
A=L*W

Re=rho*L*V/mu
Cf=0.074/Re^0.2
g=9.81 "m/s^2"
F=Cf*A*(rho*V^2)/2 "N"
P_drag=F*V/1000 "kW"

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Discussion  The curves look similar at first glance, but in fact $F_{\text{drag}}$ increases like $V^2$, while $F_{\text{drag}}$ increases like $V^3$. 
Solution

The chimney of a factory is subjected to high winds. The bending moment at the base of the chimney is to be determined.

Assumptions

1. The flow of air in the wind is steady, turbulent, and incompressible.
2. The ground effect on the wind and the drag coefficient is negligible (a crude approximation) so that the resultant drag force acts through the center of the side surface.
3. The edge effects are negligible and thus the chimney can be treated as a 2-D long cylinder.

Properties

The drag coefficient for a long cylindrical bar in turbulent flow is 0.3 (Table 11-2). The density of air at 20°C and 1 atm is $\rho = 1.204 \text{ kg/m}^3$.

Analysis

Noting that the frontal area is $DH$ and 1 m/s = 3.6 km/h, the drag force becomes

$$F_D = C_D A \frac{\rho V^2}{2}$$

$\quad \quad = (0.3)(1.1 \times 20 \text{ m}^2)(1.204 \text{ kg/m}^3)(110/3.6 \text{ m/s})^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right)$

$\quad \quad = 3710 \text{ N}$

The drag force acts through the mid height of the chimney. Noting that moment is force times moment arm, the wind-induced bending moment at the base of the chimney becomes

$$M_{Bending} = M_B = F_D \times h/2 = (3710 \text{ N})(20/2 \text{ m}) = 3.71 \times 10^4 \text{ N} \cdot \text{m}$$

Discussion

Forces and moments caused by winds can be very high. Therefore, wind loading is an important consideration in the design of structures.
Chapter 11  
External Flow: Drag and Lift

11-103E  
Solution  
The passenger compartment of a minivan is modeled as a rectangular box. The drag force acting on the top and the two side surfaces and the power needed to overcome it are to be determined.

Assumptions  
1. The air flow is steady and incompressible.  
2. The air flow over the exterior surfaces is turbulent because of constant agitation.  
3. Air is an ideal gas.  
4. The top and side surfaces of the minivan are flat and smooth (in reality they can be rough).  
5. The atmospheric air is calm (no significant winds).

Properties  
The density and kinematic viscosity of air at 1 atm and 80°F are $\rho = 0.07350 \text{ lbm/ft}^3$ and $\nu = 0.6110 \text{ ft}^2/\text{h} = 1.697\times10^{-4} \text{ ft}^2/\text{s}$.

Analysis  
The Reynolds number at the end of the top and side surfaces is

$$Re = \frac{VL}{\nu} = \frac{(50\times1.4667) \text{ ft/s} \times (11 \text{ ft})}{1.697\times10^{-4} \text{ ft}^2/\text{s}} = 4.754\times10^6$$

The air flow over the entire outer surface is assumed to be turbulent. Then the friction coefficient becomes

$$C_f = \frac{0.074}{(4.754\times10^6)^{1/5}} = 0.003418$$

The area of the top and side surfaces of the minivan is

$$A = A_{top} + 2A_{side} = 6\times11 + 2\times4.5\times11 = 165 \text{ ft}^2$$

Noting that the pressure drag is zero and thus $C_D = C_f$ for a plane surface, the drag force acting on these surfaces becomes

$$F_D = C_fA\frac{\rho V^2}{2} = 0.003418 \times (165 \text{ ft}^2) \left(\frac{0.074 \text{ lbm/ft}^3 \times (50 \times 1.4667 \text{ ft/s})^2}{2 \times 32.2 \text{ lbm} \cdot \text{ft/s}^2}\right) = 3.49 \text{ lbf}$$

Noting that power is force times velocity, the power needed to overcome this drag force is

$$P_{\text{drag}} = F_DV = (3.49 \text{ lbf})(50 \times 1.4667 \text{ ft/s}) = 0.347 \text{ kW}$$

Discussion  
Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.
Solution Cruising conditions of a passenger plane are given. The minimum safe landing and takeoff speeds with and without flaps, the angle of attack during cruising, and the power required are to be determined.

Assumptions 1 The drag and lift produced by parts of the plane other than the wings are not considered. 2 The wings are assumed to be two-dimensional airfoil sections, and the tip effects are neglected. 4 The lift and drag characteristics of the wings can be approximated by NACA 23012 so that Fig. 11-45 is applicable.

Properties The densities of air are 0.075 lbm/ft³ on the ground and 0.0208 lbm/ft³ at cruising altitude. The maximum lift coefficients of the wings are 3.48 and 1.52 with and without flaps, respectively (Fig. 11-45).

Analysis (a) The weight and cruising speed of the airplane are

\[ W = mg = (150,000 \text{ lbm})(32.2 \text{ ft/s}^2) \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) = 150,000 \text{ lbf} \]

\[ V = (550 \text{ mph}) \left( \frac{1.4667 \text{ ft/s}}{1 \text{ mph}} \right) = 806.7 \text{ ft/s} \]

Minimum velocity corresponding the stall conditions with and without flaps are

\[ V_{min1} = \sqrt{\frac{2W}{\rho C_{L,\text{max}} A}} = \sqrt{\frac{2(150,000 \text{ lbf})}{(0.075 \text{ lbm/ft}^3)(1.52)(1800 \text{ ft}^2)}} \left( \frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) = 217 \text{ ft/s} \]

\[ V_{min2} = \sqrt{\frac{2W}{\rho C_{L,\text{max}} A}} = \sqrt{\frac{2(150,000 \text{ lbf})}{(0.0208 \text{ lbm/ft}^3)(3.48)(1800 \text{ ft}^2)}} \left( \frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) = 143 \text{ ft/s} \]

The “safe” minimum velocities to avoid the stall region are obtained by multiplying these values by 1.2:

**Without flaps:** \( V_{min1,\text{safe}} = 1.2 \times 217 \text{ ft/s} = 260 \text{ ft/s} = 178 \text{ mph} \)

**With flaps:** \( V_{min2,\text{safe}} = 1.2 \times 143 \text{ ft/s} = 172 \text{ ft/s} = 117 \text{ mph} \)

Since 1 mph = 1.4667 ft/s. Note that the use of flaps allows the plane to takeoff and land at considerably lower velocities, and thus at a shorter runway.

(b) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, \( F_L = W \). Then the lift coefficient is determined to be

\[ C_L = \frac{F_L}{\frac{1}{2} \rho V^2 A} = \frac{150,000 \text{ lbf}}{\frac{1}{2} (0.0208 \text{ lbm/ft}^3)(806.7 \text{ ft/s})^2 (1800 \text{ ft}^2)} \left( \frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) = 0.40 \]

For the case of no flaps, the angle of attack corresponding to this value of \( C_L \) is determined from Fig. 11-45 to be about \( \alpha = 3.5^\circ \).

(c) When aircraft cruises steadily, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 0.40 is \( C_D = 0.015 \) (Fig. 11-45). Then the drag force acting on the wings becomes

\[ F_D = C_D A \frac{\rho V^2}{2} = (0.015)(1800 \text{ ft}^2) \left( \frac{0.0208 \text{ lbm/ft}^3)(806.7 \text{ ft/s})^2}{2} \left( \frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2} \right) \right) = 5675 \text{ lbf} \]

Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity,

\[ \text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (5675 \text{ lbf})(806.7 \text{ ft/s}) \left( \frac{1 \text{ kW}}{737.56 \text{ lbf} \cdot \text{ft/s}} \right) = 6200 \text{ kW} \]

Discussion Note that the engines must supply 6200 kW of power to overcome the drag during cruising. This is the power required to overcome the drag that acts on the wings only, and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc.).
11-105

Solution  
An automotive engine is approximated as a rectangular block. The drag force acting on the bottom surface of the engine is to be determined.

Assumptions  
1 The air flow is steady and incompressible.  
2 Air is an ideal gas.  
3 The atmospheric air is calm (no significant winds).  
4 The air flow is turbulent over the entire surface because of the constant agitation of the engine block.  
5 The bottom surface of the engine is a flat surface, and it is smooth (in reality it is quite rough because of the dirt collected on it).

Properties  
The density and kinematic viscosity of air at 1 atm and 15°C are \( \rho = 1.225 \text{ kg/m}^3 \) and \( \nu = 1.470 \times 10^{-5} \text{ m}^2/\text{s} \).

Analysis  
The Reynolds number at the end of the engine block is
\[
Re_L = \frac{VL}{\nu} = \frac{(120/3.6 \text{ m/s})(0.7 \text{ m})}{1.470 \times 10^{-5} \text{ m}^2/\text{s}} = 1.587 \times 10^6
\]
The flow is assumed to be turbulent over the entire surface. Then the average friction coefficient and the drag force acting on the surface becomes
\[
C_f = \left( \frac{2}{Re_L} \right)^{1/5} = \left( \frac{2}{1.587 \times 10^6} \right)^{1/5} = 0.004257
\]
\[
F_D = C_f A \frac{\rho f V^2}{2} = (0.004257)(0.6 \times 0.7 \text{ m}^2)(1.225 \text{ kg/m}^3)(120/3.6 \text{ m/s})^2 = 1.22 \text{ N}
\]

Discussion  
Note that the calculated drag force (and the power required to overcome it) is very small. This is not surprising since the drag force for blunt bodies is almost entirely due to pressure drag, and the friction drag is practically negligible compared to the pressure drag.

11-106

Solution  
The total weight of a paratrooper and its parachute is given. The terminal velocity of the paratrooper in air is to be determined.

Assumptions  
1 The air flow over the parachute is turbulent so that the tabulated value of the drag coefficient can be used.  
2 The variation of the air properties with altitude is negligible.  
3 The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low air density.

Properties  
The density of air is given to be \( 1.20 \text{ kg/m}^3 \). The drag coefficient of a parachute is \( C_D = 1.3 \).

Analysis  
The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid, which is negligible in this case,
\[
F_D = W - F_B \quad \text{where} \quad F_D = C_D A \frac{\rho f V^2}{2} \quad \text{and} \quad F_B \approx 0
\]

where \( A = \pi D^2/4 \) is the frontal area. Substituting and simplifying,
\[
C_D A \frac{\rho f V^2}{2} = W \quad \Rightarrow \quad C_D \frac{\pi D^2}{4} \frac{\rho f V^2}{2} = W
\]

Solving for \( V \) and substituting,
\[
V = \sqrt{\frac{8W}{C_D \pi D^2 \rho_f}} = \sqrt{\frac{8(950 \text{ N})}{1.3\pi(8 \text{ m})^2(1.20 \text{ kg/m}^3)}} \left( \frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = 4.9 \text{ m/s}
\]

Therefore, the velocity of the paratrooper will remain constant when it reaches the terminal velocity of \( 4.9 \text{ m/s} = 18 \text{ km/h} \).

Discussion  
The simple analysis above gives us a rough value for the terminal velocity. A more accurate answer can be obtained by a more detailed (and complex) analysis by considering the variation of air density with altitude, and by considering the uncertainty in the drag coefficient.
Solution

The water needs of a recreational vehicle (RV) are to be met by installing a cylindrical tank on top of the vehicle. The additional power requirements of the RV at a specified speed for two orientations of the tank are to be determined.

Assumptions

1. The flow of air is steady and incompressible.
2. The effect of the tank and the RV on the drag coefficient of each other is negligible (no interference).
3. The flow is turbulent so that the tabulated value of the drag coefficient can be used.

Properties

The drag coefficient for a cylinder corresponding to \( L/D = 3/0.5 = 6 \) is \( C_D = 0.95 \) when the circular surfaces of the tank face the front and back, and \( C_D = 0.8 \) when the circular surfaces of the tank face the sides of the RV (Table 11-2). The density of air at the specified conditions is

\[
\rho = \frac{P}{RT} = \frac{87 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg}.\text{K})(295 \text{ K})} = 1.028 \text{ kg/m}^3
\]

Analysis

(a) The drag force acting on the tank when the circular surfaces face the front and back is

\[
F_D = C_D A \frac{D V^2}{2} = (0.95)\left(\pi(0.5 \text{ m})^2 / 4\right) \frac{(1.028 \text{ kg/m}^3)(80 \text{ km/h})^2}{2} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 47.35 \text{ N}
\]

Noting that power is force times velocity, the amount of additional power needed to overcome this drag force is

\[
W_{\text{drag}} = F_D V = (47.35 \text{ N})(80/3.6 \text{ m/s}) \left( \frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) = 1.05 \text{ kW}
\]

(b) The drag force acting on the tank when the circular surfaces face the sides of the RV is

\[
F_D = C_D A \frac{D V^2}{2} = (0.8)\left(\pi(0.5 \text{ m})^2 \right) \frac{(1.028 \text{ kg/m}^3)(80 \text{ km/h})^2}{2} \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right)^2 \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 304.6 \text{ N}
\]

Then the additional power needed to overcome this drag force is

\[
W_{\text{drag}} = F_D V = (304.6 \text{ N})(80/3.6 \text{ m/s}) \left( \frac{1 \text{ kW}}{1000 \text{ N} \cdot \text{m/s}} \right) = 6.77 \text{ kW}
\]

Therefore, the additional power needed to overcome the drag caused by the tank is 1.05 kW and 6.77 W for the two orientations indicated.

Discussion

Note that the additional power requirement is the lowest when the tank is installed such that its circular surfaces face the front and back of the RV. This is because the frontal area of the tank is minimum in this orientation.
Solution  
A smooth ball is moving at a specified velocity. The increase in the drag coefficient when the ball spins is to be determined.

Assumptions  
1. The outer surface of the ball is smooth.  
2. The air is calm (no winds or drafts).

Properties  
The density and kinematic viscosity of air at 1 atm and 25°C are \( \rho = 1.184 \text{ kg/m}^3 \) and \( \nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s} \).

Analysis  
Noting that \( D = 0.09 \text{ m} \) and \( 1 \text{ m/s} = 3.6 \text{ km/h} \), the regular and angular velocities of the ball are

\[
V = (36 \text{ km/h}) \left( \frac{1 \text{ m/s}}{3.6 \text{ km/h}} \right) = 10 \text{ m/s} \quad \text{and} \quad \omega = (3500 \text{ rev/min}) \left( \frac{2 \pi \text{ rad}}{1 \text{ rev}} \right) \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 367 \text{ rad/s}
\]

From these values, we calculate the nondimensional rate of rotation and the Reynolds number:

\[
\frac{\omega D}{2V} = \frac{(367 \text{ rad/s})(0.09 \text{ m})}{2(10 \text{ m/s})} = 1.652 \text{ rad} \quad \text{and} \quad \text{Re} = \frac{VD}{\nu} = \frac{(10 \text{ m/s})(0.09 \text{ m})}{1.562 \times 10^{-5} \text{ m}^2/\text{s}} = 5.762 \times 10^4
\]

Then the drag coefficients for the ball with and without spin are determined from Figs. 11-36 and 11-53 to be:

- **Without spin:** \( C_D = 0.50 \) (Fig. 11-36, smooth ball)  
- **With spin:** \( C_D = 0.58 \) (Fig. 11-53)

Then the increase in the drag coefficient due to spinning becomes

\[
\text{Increase in } C_D = \frac{C_D, \text{ spin} - C_D, \text{ no spin}}{C_D, \text{ no spin}} = \frac{0.58 - 0.50}{0.50} = 0.160
\]

Therefore, the drag coefficient in this case increases by about 16% because of spinning.

Discussion  
Note that the Reynolds number for this problem is \( 5.762 \times 10^4 \) which is close enough to \( 6 \times 10^4 \) for which Fig. 11-53 is prepared. Therefore, the result obtained should be fairly accurate.
Solution  A fluid flows over a 2.5-m long flat plate. The thickness of the boundary layer at intervals of 0.25 m is to be determined and plotted against the distance from the leading edge for air, water, and oil.

Assumptions  1 The flow is steady and incompressible. 2 The critical Reynolds number is \( \text{Re}_c = 5 \times 10^5 \). 3 Air is an ideal gas. 4 The surface of the plate is smooth.

Properties  The kinematic viscosity of the three fluids at 1 atm and 20°C are: \( \nu = 1.516 \times 10^{-5} \text{ m}^2/\text{s} \) for air, \( \nu = \frac{\mu}{\rho} = \frac{(1.002 \times 10^{-3} \text{ kg/m/s})}{(998 \text{ kg/m}^3)} = 1.004 \times 10^{-6} \text{ m}^2/\text{s} \) for water, and \( \nu = 9.429 \times 10^{-4} \text{ m}^2/\text{s} \) for oil.

Analysis  The thickness of the boundary layer along the flow for laminar and turbulent flows is given by

Laminar flow: \( \delta_x = \frac{4.91x}{\text{Re}_x^{1/2}} \)

Turbulent flow: \( \delta_x = \frac{0.38x}{\text{Re}_x^{1/5}} \)

(a) AIR: The Reynolds number and the boundary layer thickness at the end of the first 0.25 m interval are

\[
\text{Re}_x = \frac{x \cdot V}{\nu} = \frac{(3 \text{ m/s})(0.25 \text{ m})}{1.516 \times 10^{-5} \text{ m}^2/\text{s}} = 4.95 \times 10^4,
\]

\[
\delta_x = \frac{5x}{\text{Re}_x^{1/2}} = \frac{4.91 \times (0.25 \text{ m})}{(4.95 \times 10^4)^{0.5}} = 5.52 \times 10^{-3} \text{ m}
\]

We repeat calculations for all 0.25 m intervals. The EES Equations window is printed below, along with the tabulated and plotted results.

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Discussion  Note that the flow is laminar for (a) and (c), and turbulent for (b). Also note that the thickness of the boundary layer is very small for air and water, but it is very large for oil. This is due to the high viscosity of oil.
Solution: A fairing is installed to the front of a rig to reduce the drag coefficient. The maximum speed of the rig after the fairing is installed is to be determined.

Assumptions: 1 The rig moves steadily at a constant velocity on a straight path in calm weather. 2 The bearing friction resistance is constant. 3 The effect of velocity on the drag and rolling resistance coefficients is negligible. 4 The buoyancy of air is negligible. 5 The power produced by the engine is used to overcome rolling resistance, bearing friction, and aerodynamic drag.

Properties: The density of air is given to be 1.25 kg/m$^3$. The drag coefficient of the rig is given to be $C_D = 0.96$, and decreases to $C_D = 0.76$ when a fairing is installed. The rolling resistance coefficient is $C_{RR} = 0.05$.

Analysis: The bearing friction resistance is given to be $F_{bearing} = 350$ N. The rolling resistance is

$$F_{RR} = C_{RR}W = 0.05(17,000 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 8339 \text{ N}$$

The maximum drag occurs at maximum velocity, and its value before the fairing is installed is

$$F_{D1} = C_{D1}A \frac{\rho V_1^2}{2} = (0.96)(9.2 \text{ m}^2) \left( \frac{1.25 \text{ kg/m}^3}(110 / 3.6 \text{ m/s})^2 \right) \frac{1 \text{ N}}{2 \text{ (1 kg \cdot m/s}^2)} = 5154 \text{ N}$$

Power is force times velocity, and thus the power needed to overcome bearing friction, drag, and rolling resistance is the product of the sum of these forces and the velocity of the rig,

$$W_{total} = W_{bearing} + W_{drag} + W_{RR} = (F_{bearing} + F_D + F_{RR})V = (350 + 8339 + 5154)(110 / 3.6 \text{ m/s}) \frac{1000 \text{ N} \cdot \text{m/s}}{}$$

$$= 423 \text{ kW}$$

The maximum velocity the rig can attain at the same power of 423 kW after the fairing is installed is determined by setting the sum of the bearing friction, rolling resistance, and the drag force equal to 423 kW,

$$W_{total} = W_{bearing} + W_{drag2} + W_{RR} = (F_{bearing} + F_{D2} + F_{RR})V_2 = \left( 350 + C_{D2}A \frac{\rho V_2^2}{2} + 5154 \right)V_2$$

Substituting the known quantities,

$$(423 \text{ kW}) \left( \frac{1000 \text{ N} \cdot \text{m/s}}{1 \text{ kW}} \right) = \left( 350 N + (0.76)(9.2 \text{ m}^2) \left( \frac{1.25 \text{ kg/m}^3}{2} \right) \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} + 5154 \text{ N} \right)V_2$$

or,

$$423,000 = 5504V_2 + 4.37V_2^3$$

Solving it with an equation solver gives $V_2 = 36.9 \text{ m/s} = 133 \text{ km/h}$.

Discussion: Note that the maximum velocity of the rig increases from 110 km/h to 133 km/h as a result of reducing its drag coefficient from 0.96 to 0.76 while holding the bearing friction and the rolling resistance constant.
We are to estimate how much money is wasted by driving with a tennis ball on a car antenna.

**Properties**

\[ \rho_{\text{fuel}} = 50.2 \text{ lbm/ft}^3, \quadHV_{\text{fuel}} = 1.47 \times 10^7 \text{ ft-lbf/lbm}, \quad \rho_{\text{air}} = 0.07518 \text{ lbm/ft}^3, \quad \mu_{\text{air}} = 1.227 \times 10^{-5} \text{ lbm/ft-s} \]

**Analysis**

First some conversions: 

\[ V = 55 \text{ mph} = 80.667 \text{ ft/s}, \quad D = 2.62 \text{ in} = 0.21833 \text{ ft}, \quad \text{and the total distance traveled in one year} = L = 16,000 \text{ miles} = 8.448 \times 10^7 \text{ ft}. \]

We calculate the Reynolds number,

\[ \text{Re} = \frac{\rho_{\text{air}} V D}{\mu} = \frac{(0.07518 \text{ lbm/ft}^3)(80.667 \text{ ft/s})(0.21833 \text{ ft})}{1.227 \times 10^{-5} \text{ lbm/(ft \cdot s)}} = 107900 \]

At this Reynolds number and for the given roughness ratio, we look up \( C_D \) for a sphere:

\[ C_D \approx 0.505. \]

[Note: student answers may vary because this value is not easily read from the graph, but \( C_D \) should be around 0.5.]

The additional drag force due to the sphere is

\[ F_D = \frac{1}{2} \rho_{\text{air}} V^2 C_D A \]

where \( A \) is the frontal area of the sphere, \( A = \pi D^2 / 4 \). The work required to overcome this additional drag is force times distance. So, letting \( L \) be the total distance driven in a year,

\[ \text{Work}_{\text{drag}} = F_D L = \frac{1}{2} \rho_{\text{air}} V^2 C_D \left( \pi D^2 / 4 \right) L \]

The energy required to perform this work is much greater than this due to overall efficiency of the car engine, transmission, etc. Thus,

\[ E_{\text{required}} = \frac{\text{Work}_{\text{drag}}}{\eta_{\text{overall}}} = \frac{1}{2} \rho_{\text{air}} V^2 C_D \left( \pi D^2 / 4 \right) \frac{L}{\eta_{\text{overall}}} \]

But the required energy is also equal to the heating value of the fuel \( HV \) times the mass of fuel required. In terms of required fuel volume, volume = mass/density. Thus,

\[ V_{\text{fuel required}} = \frac{m_{\text{fuel required}}}{\rho_{\text{fuel}}} = \frac{E_{\text{required}}}{HV} = \frac{1}{2} \rho_{\text{air}} V^2 C_D \frac{\left( \pi D^2 / 4 \right) L}{\eta_{\text{overall}} HV} \]

The above is our answer in variable form. Finally, we plug in the given values and properties to obtain the numerical answer,

\[ V_{\text{fuel required}} = \frac{0.5(0.07518 \text{ lbm/ft}^3)(80.667 \text{ ft/s})^2(0.505)(\pi(0.21833 \text{ ft})^2 / 4)(8.448 \times 10^7 \text{ ft})}{(50.2 \text{ lbm/ft}^3)(0.308)(1.47 \times 10^7 \text{ ft-lbf/lbm})} \left( \frac{1 \text{ lbf}}{32.174 \text{ lbm \cdot ft/s}^2} \right) \]

\[ = 0.6418 \text{ L} \approx 0.64 \text{ L} \]

which yields \( V_{\text{fuel required}} = 0.05343 \text{ ft}^3 \), which is equivalent to 0.3997 gallons per year. At $4 per gallon, the total cost is about $1.60 per year.

Since the cost is minimal (only about a dollar per year), Janie’s friends should get off her back about this.

**Discussion**

If we use \( C_D \approx 0.50 \), the cost turns out to be $1.58, but we should report our result to only 2 significant digits anyway, so our answer is still $1.6 per year.
Spherical aluminum balls are dropped into glycerin, and their terminal velocities are measured. The velocities are to be compared to those predicted by Stokes law, and the error involved is to be determined.

**Assumptions**

1. The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified).
2. The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body.
3. The tube is long enough to assure that the velocity measured is terminal velocity.

**Properties**

The density of aluminum balls is given to be \( \rho_s = 2700 \text{ kg/m}^3 \). The density and viscosity of glycerin are given to be \( \rho_f = 1274 \text{ kg/m}^3 \) and \( \mu = 1 \text{ kg/m} \cdot \text{s} \).

**Analysis**

The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

\[
F_D = W - F_B \quad \text{where} \quad F_D = 3\pi \mu V D \quad \text{(Stokes law)}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V
\]

Here \( V = \pi D^3/6 \) is the volume of the sphere. Substituting and simplifying,

\[
3\pi \mu V D = \rho_s g V - \rho_f g V \quad \rightarrow \quad 3\pi \mu V D = (\rho_s - \rho_f) g \frac{\pi D^3}{6}
\]

Solving for the terminal velocity \( V \) of the ball gives

\[
V = \frac{gD^2(\rho_s - \rho_f)}{18\mu}
\]

(a) \( D = 2 \text{ mm} \) and \( V = 3.2 \text{ mm/s} \)

\[
V = \frac{(9.81 \text{ m}^2/\text{s}^2)(0.002 \text{ m})^2(2700 - 1274) \text{ kg/m}^3}{18(1 \text{ kg/m} \cdot \text{s})} = 0.00311 \text{ m/s} = 3.11 \text{ mm/s}
\]

Error = \( \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{3.2 - 3.11}{3.2} = 0.029 \) or \( 2.9\% \)

(b) \( D = 4 \text{ mm} \) and \( V = 12.8 \text{ mm/s} \)

\[
V = \frac{(9.81 \text{ m}^2/\text{s}^2)(0.004 \text{ m})^2(2700 - 1274) \text{ kg/m}^3}{18(1 \text{ kg/m} \cdot \text{s})} = 0.0124 \text{ m/s} = 12.4 \text{ mm/s}
\]

Error = \( \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{12.8 - 12.4}{12.8} = 0.029 \) or \( 2.9\% \)

(c) \( D = 10 \text{ mm} \) and \( V = 60.4 \text{ mm/s} \)

\[
V = \frac{(9.81 \text{ m}^2/\text{s}^2)(0.010 \text{ m})^2(2700 - 1274) \text{ kg/m}^3}{18(1 \text{ kg/m} \cdot \text{s})} = 0.0777 \text{ m/s} = 77.7 \text{ mm/s}
\]

Error = \( \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{60.4 - 77.7}{60.4} = -0.287 \) or \( -28.7\% \)

There is a good agreement for the first two diameters. However, the error for the third one is large. The Reynolds number for each case is

\[
(\text{a}) \quad \text{Re} = \frac{\rho V D}{\mu} = \frac{(1274 \text{ kg/m}^3)(0.0032 \text{ m/s})(0.002 \text{ m})}{1 \text{ kg} \cdot \text{m/s}} = 0.008, \quad \text{(b)} \quad \text{Re} = 0.065, \quad \text{and} \quad \text{(c)} \quad \text{Re} = 0.770.
\]

We observe that \( \text{Re} \ll 1 \) for the first two cases, and thus the creeping flow idealization is applicable. But this is not the case for the third case.

**Discussion**

If we used the general form of the equation (see next problem) we would obtain \( V = 59.7 \text{ mm/s} \) for part (c), which is very close to the experimental result (60.4 mm/s).
Spherical aluminum balls are dropped into glycerin, and their terminal velocities are measured. The velocities predicted by a more general form of Stokes law and the error involved are to be determined.

**Assumptions**

1. The Reynolds number is low (of order 1) so that Stokes law is applicable (to be verified).
2. The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body.
3. The tube is long enough to assure that the velocity measured is terminal velocity.

**Properties**

The density of aluminum balls is given to be \( \rho_s = 2700 \text{ kg/m}^3 \). The density and viscosity of glycerin are given to be \( \rho_f = 1274 \text{ kg/m}^3 \) and \( \mu = 1 \text{ kg/m.s} \).

**Analysis**

The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

\[
F_D = W - F_B
\]

where \( F_D = 3\pi \mu DV + \left(9\pi /16\right) \rho_s V^2 D^2 \), \( W = \rho_s g V \), and \( F_B = \rho_f g V \).

Here \( V = \pi D^3 / 6 \) is the volume of the sphere. Substituting and simplifying,

\[
3\pi \mu DV + \left(9\pi /16\right) \rho_s V^2 D^2 = \left(\rho_s - \rho_f\right) g \frac{\pi D^3}{6}
\]

Solving for the terminal velocity \( V \) of the ball gives

\[
V = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]

where \( a = \frac{9\pi}{16} \rho_s D^2 \), \( b = 3\pi \mu D \), and \( c = -\left(\rho_s - \rho_f\right) g \frac{\pi D^3}{6} \).

\( a \) \( D = 2 \text{ mm and } V = 3.2 \text{ mm/s}: a = 0.01909, b = 0.01885, c = -0.0000586 \)

\[
V = \frac{-0.01885 + \sqrt{(0.01885)^2 - 4 \times 0.01909 \times (-0.0000586)}}{2 \times 0.01909} = 0.00310 \text{ m/s} = 3.10 \text{ mm/s}
\]

Error \( \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{3.2 - 3.10}{3.2} = 0.032 \) or 3.2%

\( b \) \( D = 4 \text{ mm and } V = 12.8 \text{ mm/s}: a = 0.07634, b = 0.0377, c = -0.0004688 \)

\[
V = \frac{-0.0377 + \sqrt{(0.0377)^2 - 4 \times 0.07634 \times (-0.0004688)}}{2 \times 0.07634} = 0.0121 \text{ m/s} = 12.1 \text{ mm/s}
\]

Error \( \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{12.8 - 12.1}{12.8} = 0.052 \) or 5.2%

\( c \) \( D = 10 \text{ mm and } V = 60.4 \text{ mm/s}: a = 0.4771, b = 0.09425, c = -0.007325 \)

\[
V = \frac{-0.09425 + \sqrt{(0.09425)^2 - 4 \times 0.4771 \times (-0.007325)}}{2 \times 0.4771} = 0.0597 \text{ m/s} = 59.7 \text{ mm/s}
\]

Error \( \frac{V_{\text{experimental}} - V_{\text{Stokes}}}{V_{\text{experimental}}} = \frac{60.4 - 59.7}{60.4} = 0.012 \) or 1.2%

The Reynolds number for the three cases are

\( \text{Re} = \frac{\rho V D}{\mu} = \frac{(1274 \text{ kg/m}^3)(0.0032 \text{ m/s})(0.002 \text{ m})}{1 \text{ kg} \cdot \text{m/s}} = 0.008 \), \( \text{Re} = 0.065 \), and \( \text{Re} = 0.770 \).

**Discussion**

There is a good agreement for the third case (case \( c \)), but the general Stokes law increased the error for the first two cases (cases \( a \) and \( b \)) from 2.9% and 2.9% to 3.2% and 5.2%, respectively. Therefore, the basic form of Stokes law should be preferred when the Reynolds number is much lower than 1.
11-114

**Solution**  
A spherical aluminum ball is dropped into oil. A relation is to be obtained for the variation of velocity with time and the terminal velocity of the ball. The variation of velocity with time is to be plotted, and the time it takes to reach 99% of terminal velocity is to be determined.

**Assumptions**  
1. The Reynolds number is low (Re \(<<1\)) so that Stokes law is applicable. 2. The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body.

**Properties**  
The density of aluminum balls is given to be \(\rho_s = 2700 \text{ kg/m}^3\). The density and viscosity of oil are given to be \(\varrho_f = 876 \text{ kg/m}^3\) and \(\mu = 0.2177 \text{ kg/m.s}\).

**Analysis**  
The free body diagram is shown in the figure. The net force acting downward on the ball is the weight of the ball less the weight of the fluid and the buoyancy force applied by the fluid,

\[
F_{\text{net}} = W - F_D - F_B \quad \text{where} \quad F_D = 3\pi\mu D \frac{\rho_s}{6} \quad \text{and} \quad F_B = \rho_f g V
\]

where \(F_D\) is the drag force, \(F_B\) is the buoyancy force, and \(W\) is the weight. Also, \(V = \pi D^3/6\) is the volume, \(m\) is the mass, \(D\) is the diameter, and \(V\) the velocity of the ball. Applying Newton’s second law in the vertical direction,

\[
F_{\text{net}} = ma \quad \Rightarrow \quad m_s g - F_D - F_B = m \frac{dV}{dt}
\]

Substituting the drag and buoyancy force relations,

\[
\rho_s \frac{\pi D^3}{6} g - 3\pi\mu D \frac{\rho_s}{6} - \rho_f g \frac{\pi D^3}{6} = \rho_s \frac{\pi D^3}{6} \frac{dV}{dt}
\]

or,

\[
\left( 1 - \frac{\rho_f}{\rho_s} \right) \frac{18\mu}{\rho_s D^2} V = \frac{dV}{dt} \quad \Rightarrow \quad a - bV = \frac{dV}{dt}
\]

where \(a = (1 - \rho_f / \rho_s)\) and \(b = 18\mu/(\rho_s D^2)\). It can be rearranged as \(\frac{dV}{a - bV} = dt\)

Integrating from \(t = 0\) where \(V = 0\) to \(t = t\) where \(V = V\) gives

\[
\int_0^t \frac{dV}{a - bV} = \int_0^t dt \quad \Rightarrow \quad -\ln\left(\frac{a - bV}{b}\right)_0^t = t \quad \Rightarrow \quad \ln\left(\frac{a - bV}{a}\right)_0^t = -bt
\]

Solving for \(V\) gives the desired relation for the variation of velocity of the ball with time,

\[
V = \frac{a}{b} \left( 1 - e^{-bt} \right) \quad \text{or} \quad V = \frac{(\rho_s - \rho_f)gD^2}{18\mu} \left( 1 - e^{-\frac{18\mu}{\rho_s D^2}} \right)
\]

(1)

Note that as \(t \to \infty\), it gives the terminal velocity as \(V_{\text{terminal}} = \frac{a}{b} \frac{(\rho_s - \rho_f)gD^2}{18\mu}\) (2)

The time it takes to reach 99% of terminal velocity can be determined by replacing \(V\) in Eq. 1 by \(0.99V_{\text{terminal}} = 0.99a/b\). This gives \(e^{-bt} = 0.01\) or

\[
t_{99\%} = -\ln(0.01) = -\ln(0.01)\rho_s D^2 = \frac{-\ln(0.01)\rho_s D^2}{18\mu}
\]

(3)

Given values: \(D = 0.003 \text{ m}\), \(\rho_s = 876 \text{ kg/m}^3\), \(\mu = 0.2177 \text{ kg/m.s}\), \(g = 9.81 \text{ m/s}^2\).

Calculation results: \(\text{Re} = 0.50\), \(a = 6.627\), \(b = 161.3\), \(t_{99\%} = 0.029 \text{ s}\), and \(V_{\text{terminal}} = 1.0b = 0.04 \text{ m/s}\).

Continued on next page →
Discussion  The velocity increases rapidly at first, but quickly levels off by around 0.04 s.

<table>
<thead>
<tr>
<th>$t$, s</th>
<th>$V$, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
</tr>
<tr>
<td>0.01</td>
<td>0.033</td>
</tr>
<tr>
<td>0.02</td>
<td>0.039</td>
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<tr>
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<td>0.041</td>
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<td>0.041</td>
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<tr>
<td>0.10</td>
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</table>
Solution  Engine oil flows over a long flat plate. The distance from the leading edge $x_{cr}$ where the flow becomes turbulent is to be determined, and thickness of the boundary layer over a distance of $2x_{cr}$ is to be plotted.

Assumptions  1 The flow is steady and incompressible. 2 The critical Reynolds number is $Re_{cr} = 5 \times 10^5$. 3 The surface of the plate is smooth.

Properties  The kinematic viscosity of engine oil at 40°C is $\nu = 2.485 \times 10^{-4}$ m²/s.

Analysis  The thickness of the boundary layer along the flow for laminar and turbulent flows is given by:

Laminar flow:  $\delta_x = \frac{4.91x}{Re_x^{1/2}}$,  
Turbulent flow:  $\delta_x = \frac{0.38x}{Re_x^{1/5}}$

The distance from the leading edge $x_{cr}$ where the flow turns turbulent is determined by setting Reynolds number equal to the critical Reynolds number,

$Re_{cr} = \frac{Vx_{cr}}{\nu} \rightarrow x_{cr} = \frac{Re_{cr} \nu}{V} = \frac{(5 \times 10^5)(2.485 \times 10^{-4} \text{ m}^2/\text{s})}{6 \text{ m/s}} = 20.7 \text{ m}$

Therefore, we should consider flow over $2 \times 20.7 = 41.4$ m long section of the plate, and use the laminar relation for the first half, and the turbulent relation for the second part to determine the boundary layer thickness. For example, the Reynolds number and the boundary layer thickness at a distance 2 m from the leading edge of the plate are

$Re_x = \frac{Vx}{\nu} = \frac{(6 \text{ m/s})(2 \text{ m})}{2.485 \times 10^{-4} \text{ m}^2/\text{s}} = 48,290$,  
$\delta_x = \frac{4.91x}{Re_x^{1/2}} = \frac{4.91 \times (2 \text{ m})}{(48,290)^{0.5}} = 0.0447 \text{ m}$

Calculating the boundary layer thickness and plotting give:

<table>
<thead>
<tr>
<th>$x$, m</th>
<th>Re</th>
<th>$\delta$, laminar</th>
<th>$\delta$, turbulent</th>
</tr>
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<tbody>
<tr>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>96579</td>
<td>0.0632</td>
<td></td>
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<tr>
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<td>12</td>
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<td>0.1095</td>
<td></td>
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<td>16</td>
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<td></td>
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<td></td>
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<tr>
<td>40</td>
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<td>0.9658</td>
<td></td>
</tr>
</tbody>
</table>

Discussion  Notice the sudden, rapid rise in boundary layer thickness when the boundary layer becomes turbulent.
Solution  A spherical object is dropped into a fluid, and its terminal velocity is measured. The viscosity of the fluid is to be determined.

Assumptions  1. The Reynolds number is low (at the order of 1) so that Stokes law is applicable (to be verified). 2. The diameter of the tube that contains the fluid is large enough to simulate free fall in an unbounded fluid body. 3. The tube is long enough to assure that the velocity measured is the terminal velocity.

Properties  The density of glass ball is given to be \( \rho_s = 2500 \text{ kg/m}^3 \). The density of the fluid is given to be \( \rho_f = 875 \text{ kg/m}^3 \).

Analysis  The terminal velocity of a free falling object is reached when the drag force equals the weight of the solid object, less the buoyancy force applied by the fluid,

\[
F_D = W - F_B \quad \text{where} \quad F_D = 3\pi\mu VD \quad \text{(Stokes law)}, \quad W = \rho_s g V, \quad \text{and} \quad F_B = \rho_f g V
\]

Here \( V = \pi D^3/6 \) is the volume of the sphere. Substituting and simplifying,

\[
3\pi\mu VD = \rho_s g V - \rho_f g V \quad \rightarrow \quad 3\pi\mu VD = (\rho_s - \rho_f) g \frac{\pi D^3}{6}
\]

Solving for \( \mu \) and substituting, the dynamic viscosity of the fluid is determined to be

\[
\mu = \frac{gD^2(\rho_s - \rho_f)}{18V} = \frac{(9.81 \text{ m/s}^2)(0.003 \text{ m})^2(2500 - 875) \text{ kg/m}^3}{18(0.12 \text{ m/s})} = 0.0664 \text{ kg/m \cdot s}
\]

The Reynolds number is

\[
\text{Re} = \frac{\rho V D}{\mu} = \frac{(875 \text{ kg/m}^3)(0.12 \text{ m/s})(0.003 \text{ m})}{0.0664 \text{ kg \cdot m/s}} = 4.74
\]

which is at the order of 1. Therefore, the creeping flow idealization is valid.

Discussion  Flow separation starts at about \( \text{Re} = 10 \). Therefore, Stokes law can be used for Reynolds numbers up to this value, but this should be done with care.
Fundamentals of Engineering (FE) Exam Problems

11-117
Which ones are physical phenomena associated with fluid flow over bodies?
I. Drag force acting on automobiles  
II. The lift developed by airplane wings  
III. Upward draft of rain and snow  
IV. Power generated by wind turbines
(a) I and II  
(b) I and III  
(c) II and III  
(d) I, II, and III  
(e) I, II, III, and IV

Answer (e) I, II, III, and IV

11-118
The sum of the components of the pressure and wall shear forces in the direction normal to the flow is called
(a) Drag  
(b) Friction  
(c) Lift  
(d) Bluff  
(e) Blunt

Answer (c) Lift

11-119
A car is moving at a speed of 70 km/h in air at 20°C. The frontal area of the car is 2.4 m². If the drag force acting on the car in the flow direction is 205 N, the drag coefficient of the car is
(a) 0.312  
(b) 0.337  
(c) 0.354  
(d) 0.375  
(e) 0.391

Answer (d) 0.375

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[ V = 70 \text{ [km/h]} \times \text{Convert(km/h, m/s)} \]
\[ T = 20 \text{ [C]} \]
\[ A = 2.4 \text{ [m}^2\text{]} \]
\[ F_D = 205 \text{ [N]} \]
\[ \rho = 1.204 \text{ [kg/m}^3\text{]} \text{ "at 20 C"} \]
\[ F_D = C_D \times A \times \rho \times V^2 / 2 \]
A person is driving his motorcycle at a speed of 110 km/h in air at 20°C. The frontal area of the motorcycle and driver is 0.75 m². If the drag coefficient under these conditions is estimated to be 0.9, the drag force acting on the car in the flow direction is

(a) 379 N  (b) 220 N  (c) 283 N  (d) 308 N  (e) 450 N

Answer (a) 379 N

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[ V = 110 \text{ km/h} \times \text{Convert(km/h, m/s)} \]
\[ T = 20 \text{ [C]} \]
\[ A = 0.75 \text{ [m}^2\text{]} \]
\[ C_D = 0.9 \]
\[ \rho = 1.204 \text{ [kg/m}^3\text{]} \text{ "at 20 C"} \]
\[ F_D = C_D \times A \times \rho \times V^2/2 \]

The manufacturer of a car reduces the drag coefficient of the car from 0.38 to 0.33 as a result of some modifications in its shape and design. If, on average, the aerodynamic drag accounts for 20 percent of the fuel consumption, the percent reduction in the fuel consumption of the car due to reducing the drag coefficient is

(a) 15%  (b) 13%  (c) 6.6%  (d) 2.6%  (e) 1.3%

Answer (d) 2.6%

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[ C_{D1} = 0.38 \]
\[ C_{D2} = 0.33 \]
\[ f_{\text{drag}} = 0.20 \]
\[ \text{PercentReduction} = f_{\text{drag}} \times (C_{D1} - C_{D2}) / C_{D1} \times \text{Convert(,\%)} \]

The region of flow trailing the body where the effects of the body on velocity are felt is called

(a) Wake  (b) Separated region  (c) Stall  (d) Vortice  (e) Boundary

Answer (a) Wake
The turbulent boundary layer can be considered to consist of four regions. Which one is not one of them?

(a) Buffer layer (b) Overlap layer (c) Transition layer (d) Viscous layer (e) Turbulent layer

Answer (c) Transition layer

Water at 10°C flows over a 1.1-m-long flat plate with a velocity of 0.55 m/s. If the width of the plate is 2.5 m, the drag force acting on the top side of the plate is (Water properties at 10°C are: \( \rho = 999.7 \text{ kg/m}^3 \), \( \mu = 1.307 \times 10^{-3} \text{ kg/m} \cdot \text{s} \).)

(a) 0.46 N  (b) 0.81 N  (c) 2.75 N  (d) 4.16 N  (e) 6.32 N

Answer (b) 0.81 N

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[

t=10 \, [\text{C}]
\]

\[
L=1.1 \, [\text{m}]
\]

\[
V=0.55 \, [\text{m/s}]
\]

\[
W=2.5 \, [\text{m}]
\]

\[
\rho=999.7 \, [\text{kg/m}^3]
\]

\[
\mu=1.307E-3 \, [\text{kg/m} \cdot \text{s}]
\]

\[
\nu=\mu/\rho
\]

\[
Re_L=V*L/\nu
\]

\[
C_f=1.328*Re_L^{-0.5}
\]

\[
A=L*W
\]

\[
F_D=C_f*A*\rho*V^2/2
\]

Water at 10°C flows over a 3.75-m-long flat plate with a velocity of 1.15 m/s. If the width of the plate is 6.5 m, the average friction coefficient over the entire plate is (Water properties at 10°C are: \( \rho = 999.7 \text{ kg/m}^3 \), \( \mu = 1.307 \times 10^{-3} \text{ kg/m} \cdot \text{s} \).)

(a) 0.00508  (b) 0.00447  (c) 0.00302  (d) 0.00367  (e) 0.00315

Answer (e) 0.00315

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
T=10 \, [\text{C}]
\]

\[
L=3.75 \, [\text{m}]
\]

\[
V=1.15 \, [\text{m/s}]
\]

\[
W=6.5 \, [\text{m}]
\]

\[
\rho=999.7 \, [\text{kg/m}^3]
\]

\[
\mu=1.307E-3 \, [\text{kg/m} \cdot \text{s}]
\]

\[
\nu=\mu/\rho
\]

\[
Re_L=V*L/\nu
\]

\[
C_f=0.074/Re_L^{(1/5)}-1742/Re_L
\]

PROPRIETARY MATERIAL. © 2014 by McGraw-Hill Education. This is proprietary material solely for authorized instructor use. Not authorized for sale or distribution in any manner. This document may not be copied, scanned, duplicated, forwarded, distributed, or posted on a website, in whole or part.
Air at 30°C flows over a 3.0-cm outer-diameter, 45-m-long pipe with a velocity of 6 m/s. The drag force exerted on the pipe by the air is (Air properties at 30°C are: \( \rho = 1.164 \text{ kg/m}^3 \), \( \nu = 1.608 \times 10^{-5} \text{ m}^2/\text{s} \)).

(a) 19.3 N  
(b) 36.8 N  
(c) 49.3 N  
(d) 53.9 N  
(e) 60.1 N

Answer (b) 36.8 N

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
\begin{align*}
T &= 30 \degree C \\
D &= 0.03 \text{ [m]} \\
L &= 45 \text{ [m]} \\
V &= 6 \text{ [m/s]} \\
\rho &= 1.164 \text{ [kg/m}^3] \\
\nu &= 1.608 \times 10^{-5} \text{ [m}^2/\text{s]} \\
\text{Re} &= \frac{\rho V D}{\nu} \\
C_D &= 1.3 \text{ "From Fig. 11-34 at the calculated Reynolds number"} \\
A &= L D \\
F_D &= C_D A \rho V^2/2
\end{align*}
\]

A 0.8-m outer-diameter spherical tank is completely submerged in a flowing water stream at a velocity of 2.5 m/s. The drag force acting on the tank is (Water properties are: \( \rho = 998.0 \text{ kg/m}^3 \), \( \mu = 1.002 \times 10^{-3} \text{ kg/m-s} \)).

(a) 878 N  
(b) 627 N  
(c) 545 N  
(d) 356 N  
(e) 220 N

Answer (e) 220 N

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
\begin{align*}
D &= 0.8 \text{ [m]} \\
V &= 2.5 \text{ [m/s]} \\
\rho &= 998 \text{ [kg/m}^3] \\
\mu &= 1.002 \times 10^{-3} \text{ [kg/m-s]} \\
\text{Re} &= \frac{\rho V D}{\mu} \\
C_D &= 0.14 \text{ "From Fig. 11-36 at the calculated Reynolds number"} \\
A &= \pi D^2/4 \\
F_D &= C_D A \rho V^2/2
\end{align*}
\]
Chapter 11 External Flow: Drag and Lift

11-128

An airplane has a total mass of 18,000 kg and a wing planform area of 35 m². The density of air at the ground is 1.2 kg/m³. The maximum lift coefficient is 3.48. The minimum safe speed for takeoff and landing with extending the flaps is

\[
\begin{align*}
(a) \ 305 \ km/h \\
(b) \ 173 \ km/h \\
(c) \ 194 \ km/h \\
(d) \ 212 \ km/h \\
(e) \ 246 \ km/h 
\end{align*}
\]

Answer (d) 212 km/h

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
\begin{align*}
m &= 18000 \ [kg] \\
A &= 35 \ [m^2] \\
rho &= 1.2 \ [kg/m^3] \\
C_{L_{\text{max}}} &= 3.48 \\
g &= 9.81 \ [m/s^2] \\
W &= m \cdot g \\
V_{\text{min}} &= \sqrt{\frac{2 \cdot W}{\rho \cdot C_{L_{\text{max}}} \cdot A}} \\
f_{\text{safety}} &= 1.2 \\
V_{\text{min safe}} &= f_{\text{safety}} \cdot V_{\text{min}} \cdot \text{convert(m/s, km/h)}
\end{align*}
\]

11-129

An airplane has a total mass of 35,000 kg and a wing planform area of 65 m². The airplane is cruising at 10,000 m altitude with a velocity of 1100 km/h. The density of air on cruising altitude is 0.414 kg/m³. The lift coefficient of this airplane at the cruising altitude is

\[
\begin{align*}
(a) \ 0.273 \\
(b) \ 0.290 \\
(c) \ 0.456 \\
(d) \ 0.874 \\
(e) \ 1.22
\end{align*}
\]

Answer (a) 0.273

Solution Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
\begin{align*}
m &= 35000 \ [kg] \\
A &= 65 \ [m^2] \\
V &= 1100 \ [km/h] \cdot \text{Convert(km/h, m/s)} \\
rho &= 0.414 \ [kg/m^3] \\
g &= 9.81 \ [m/s^2] \\
W &= m \cdot g \\
F_{L} &= W \\
C_{L} &= F_{L} / (1/2 \cdot \rho \cdot V^2 \cdot A)
\end{align*}
\]
An airplane is cruising at a velocity of 800 km/h in air whose density is 0.526 kg/m$^3$. The airplane has a wing planform area of 90 m$^2$. The lift and drag coefficients on cruising conditions are estimated to be 2.0 and 0.06, respectively. The power that needs to be supplied to provide enough thrust to overcome wing drag is

\[(a)\ 9760\ kW\quad (b)\ 11,300\ kW\quad (c)\ 15,600\ kW\quad (d)\ 18,200\ kW\quad (e)\ 22,600\ kW\]

**Answer** (c) 15,600 kW

**Solution** Solved by EES Software. Solutions can be verified by copying-and-pasting the following lines on a blank EES screen. (Similar problems and their solutions can be obtained easily by modifying numerical values).

\[
\begin{align*}
V&=800\ [\text{km/h}]\times\text{Convert}(\text{km/h},\ \text{m/s}) \\
\rho&=0.526\ [\text{kg/m}^3] \\
A&=90\ [\text{m}^2] \\
\text{CL}&=2.0 \\
\text{C_D}&=0.06 \\
F_D&=\text{C_D}\times A\times \rho\times V^2/2 \\
\text{Power}&=F_D\times V\times \text{Convert}(W,\ \text{kW})
\end{align*}
\]

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**Design and Essay Problems**

**11-131 to 11-134**

**Solution** Students’ essays and designs should be unique and will differ from each other.