

## **Chapters 5-6**

**(where we are going)**

- Ideal gases and liquids (today)
  - Densities
  - Partial pressures
- Non-ideal gases (next time)
  - Eqns. of state
  - Reduced pressures and temperatures
  - Compressibility charts (z)
- Vapor-liquid systems (Ch. 6)
  - Vapor pressure
  - Humidity
  - Raoult's law
  - Bubble and dew points
  - Solid-solid & liquid-liquid systems

1

## **Class 15**

- Properties of Single-Phase Materials
  - Connection to Material Balances
  - Ideal Assumptions
  - Liquids- density of mixtures
  - Ideal Gases
  - Standard conditions

2

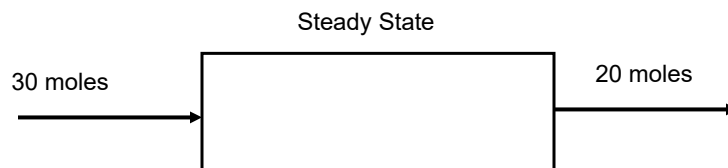
**Is the following possible?**



**Not at Steady State!**

3

**Is the following possible?**

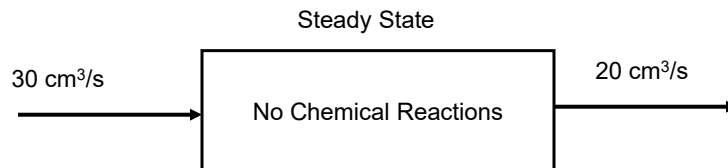


**Yes!**

**Only if reactions occur!**

4

## Is the following possible?

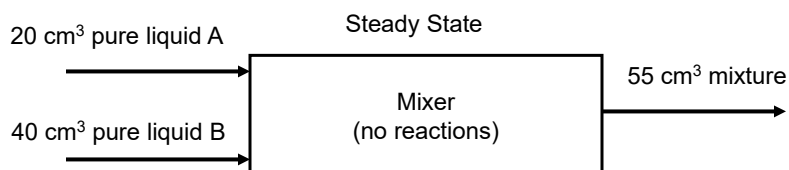


**Yes!**

**Ideal Gas, change in temperature or pressure**

5

## Is the following possible?



**Yes!**

**Even for liquid systems!**

6

## Need Relationships Between Variables

- Not easy to measure molar flow rates
- What can we easily measure?
  - Mass flow rate
  - Volumetric flow rate
  - Composition
  - Pressure
  - Temperature
- Relate measured variables to desired quantities

7

## Liquid Mixtures



- In general, liquid volume is not conserved
$$V = V_1 + V_2 + V^E$$
- Contraction or expansion occurs because of the difference in interactions between like and unlike molecules
- $V^E$  typically small. Assume  $V^E = 0$  for this course.
- This implies that liquid volumes are additive

$$\frac{1}{\bar{\rho}} = \sum_{i=1}^n \frac{x_i}{\rho_i}$$

$x_i$  = mass fraction of species  $i$

8

## Mass and Mole Fractions Have Units!

- $x_i = \frac{\text{mass of species } i}{\text{mass of mixture}}$  (same with moles)

- Look at units of density equation

$$\frac{1}{\rho} = \sum_{i=1}^n \frac{x_i}{\rho_i}$$

- $\frac{1}{\text{kg/m}^3} = \frac{\frac{\text{kg of } i}{\text{kg of mix}}}{\frac{\text{kg of } i}{\text{m}^3}}$

9

## Example

- What is the density of a mixture containing 20 wt% A ( $\rho_A = 1 \text{ g/cm}^3$ ) and the balance B ( $\rho_B = 2 \text{ g/cm}^3$ )?

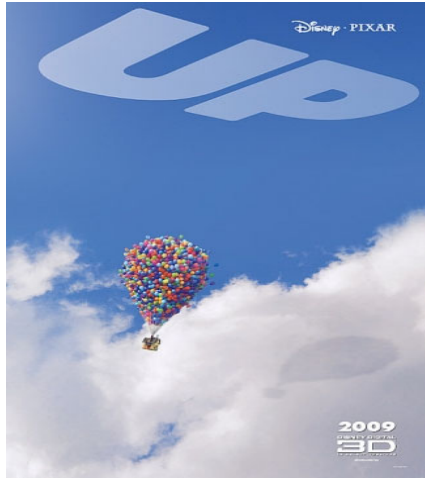
$$\frac{1}{\rho} = \frac{0.2}{\rho_A} + \frac{0.8}{\rho_B} = \frac{0.2}{1} + \frac{0.8}{2}$$

Therefore,  $\rho = 1.67 \text{ g/cm}^3$

If you used  $\rho = \sum x_i \rho_i$ , you would have calculated  
 $\rho = 0.2 \cdot 1 + 0.8 \cdot 2 = 1.8 \text{ g/cm}^3 \dots \text{Oops!}$

10

# Gases



- **Equation of State (EOS)**
  - relates amount (moles) to  $V, T, P$
- Many different equations of state
  - represent behavior of real systems
- **Ideal Gas Law**
  - Simplest equation of state
  - Most widely used

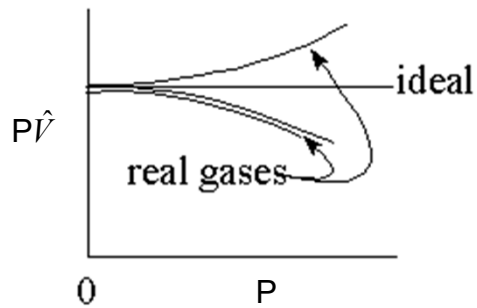
11

## Ideal Gas

- Equation of State:  **$PV = nRT$**
- Also  $P\hat{V} = RT$  (i.e.,  $\hat{V} = V/n$ )  $\hat{V}$  = specific volume
- Assumptions
  - No interaction between molecules
  - Molecules have zero volume
- Valid at low concentration ( $n/V$  is small)
  - Best at high or low pressures?
  - Best at high or low temperatures?

12

# Ideal vs. Real Gases



From p. 221, ideal when:

$$\hat{V}_{ideal} > 5L/mol \quad (\text{diatomic gases})$$

$$\hat{V}_{ideal} > 20L/mol \quad (\text{other gases})$$

$$\hat{V}_{ideal} = \frac{RT}{P}$$

Does this make sense?

13

# Ideal Gas Constant

- In Chemistry and Chemical Engineering, **R** is universal (independent of species)
  - Nice table in the back cover of the book for different units
- In Mechanical Engineering, the MW of the species is included in **R**, so there is a different value of **R** for each species

14

# Ideal Gas Constant

## THE GAS CONSTANT

8.314 m <sup>3</sup> ·Pa/(mol·K)	}	For ideal gas eqn
0.08314 L·bar/(mol·K)		
0.08206 L·atm/(mol·K)		
62.36 L·mm Hg/(mol·K)		
0.7302 ft <sup>3</sup> ·atm/(lb-mole·°R)		
10.73 ft <sup>3</sup> ·psia/(lb-mole·°R)		
8.314 J/(mol·K)	}	For Energy
1.987 cal/(mol·K)		
1.987 Btu/(lb-mole·°R)		

15

## from CRC Handbook

Values of Gas Constant,  $R = \frac{PV}{nT}$

Absolute Pressure

Volume	Temp.	moles	Atm	psia	mm Hg	cm Hg	in Hg	in H <sub>2</sub> O	ft H <sub>2</sub> O
ft <sup>3</sup>	°K	gm	0.00290	0.0426	2.20	0.220	0.0867	1.18	0.0982
		lb	1.31	19.31	999	99.9	39.3	535	44.6
	°R	gm	0.00161	0.02366	1.22	0.122	0.0482	0.655	0.0546
		lb	0.730	10.73	555	55.5	21.8	297	24.8
cm <sup>3</sup>	°K	gm	82.05	1206	62,400	6240	2450	33,400	2780
		lb	37,200	547,000	2.83 × 10 <sup>7</sup>	2.83 × 10 <sup>6</sup>	1.11 × 10 <sup>6</sup>	1.51 × 10 <sup>7</sup>	1.26 × 10 <sup>6</sup>
	°R	gm	45.6	670	34,600	3460	1360	18,500	1550
		lb	20,700	304,000	1.57 × 10 <sup>7</sup>	1.57 × 10 <sup>6</sup>	619,000	8.41 × 10 <sup>6</sup>	701,000
liters	°K	gm	0.08205	1.206	62.4	6.24	2.45	33.4	2.78
		lb	37.2	547	28,300	2830	1113	15,140	1262
	°R	gm	0.0456	0.670	34.6	3.46	1.36	18.5	1.55
		lb	20.7	304	15,700	1570	619	8410	701

### Conversion Factors and Constants

1 lb. = 453.59 gm  
 1 atm = 14.696 psia  
 1 atm = 760 mm Hg  
 1 atm = 76 cm Hg  
 1 atm = 29.921 in Hg  
 1 atm = 406.79 in H<sub>2</sub>O  
 1 atm = 33.90 ft H<sub>2</sub>O  
 359.0 ft<sup>3</sup>/lb mole  
 22,414 cm<sup>3</sup>/gm mole  
 1 inch = 2.54 cm  
 Std. temp. = 273.16°K or 491.69°R  
 28.31605 liters = 1 ft<sup>3</sup>  
 $R = 8.31432 \pm 0.00034 \times 10^7 \text{ erg } ^\circ\text{K}^{-1} \text{ mol}^{-1}$

$$8314.3 \frac{\text{N}\cdot\text{m}}{\text{kgmol}\cdot\text{K}} \text{ or } \frac{\text{J}}{\text{kgmol}\cdot\text{K}}$$

$$8.3143 \frac{\text{N}\cdot\text{m}}{\text{gmol}\cdot\text{K}} \text{ or } \frac{\text{J}}{\text{gmol}\cdot\text{K}}$$

16

# Gas Concentration & Density

- Concentration = moles/volume =  $n/V$

$$C = \frac{n}{V} = \frac{P}{RT} \quad \text{or} \quad C_i = \frac{y_i P}{RT}$$

- Density = mass/volume =  $n \cdot MW/V = C \cdot MW$

$$\rho = \frac{n \cdot MW}{V} = \frac{P \cdot MW}{RT} \quad \text{or} \quad \rho_i = \frac{y_i P \cdot MW_i}{RT}$$

Sometimes  $\rho_i$  is called the mass concentration

17

# Application Example

- Suppose the tires on your car are inflated to 32 psig at 25°C (77°F). What is the tire pressure at -10°C (14°F)? Assume that you are at sea level. Would this make a difference in your answer?

Concept: # moles do not change!

$$n = \frac{P_1 V}{RT_1} = \frac{P_2 V}{RT_2} \quad \longrightarrow \quad P_2/T_2 = P_1/T_1$$

Must use absolute P and T

$$\begin{aligned} P_1 &= 32 \text{ psig} + 14.7 \text{ psia} = 46.7 \text{ psia} \\ P_2 &= P_1(T_2/T_1) = 46.7 \text{ psia} * (263 \text{ K}/298 \text{ K}) = 41.2 \text{ psia} \\ 41.2 - 14.7 &= 26.5 \text{ psig} \end{aligned}$$

If  $P_{\text{atm}} = 12.5$ ,  $P_2 = 39.3 \text{ psia} = 26.8 \text{ psig}$

Time Saver!

18

# Tom Brady & Deflate-Gate

- $T = 72^{\circ}\text{F}$  indoors ( $532^{\circ}\text{R}$ )
- $T = 10^{\circ}\text{F}$  Outdoors ( $470^{\circ}\text{R}$ )
- $V_{\text{gas}} = \text{constant}$
- Regulation: 12.5 to 13.5 psig



Concepts: # moles do not change!  
 $V_{\text{gas}}$  constant

- $\frac{n}{V_{\text{gas}}} = \text{constant} = \frac{P_1}{RT_1} = \frac{P_2}{RT_2}$

- $P_{\text{out}} = P_{\text{indoor}} \left( \frac{T_{\text{out}}}{T_{\text{indoors}}} \right)$

$$12.5 \text{ psig} = 27.2 \text{ psia} \left( \frac{470^{\circ}\text{R}}{532^{\circ}\text{R}} \right) = 24.0 \text{ psia} = 9.3 \text{ psig}$$

- 11.9 psig if starting pressure was 13.5 psig

19

## Caution

- Only take ratios of values in absolute units
- **NEVER** take ratios of:
  - Temperatures in  $^{\circ}\text{C}$  or  $^{\circ}\text{F}$   
 (instead use  $\text{K}$  or  $^{\circ}\text{R}$ )
  - Gauge pressures  
 (instead use absolute pressures)

20

# Standard Conditions

- Provide a reference point for reporting gas flows
- Convenient for use in calculations with ideal gas law
- Units generally reported by flow meters

## SI

$$T_s = 0^\circ\text{C} = 273\text{K}$$

$$P_s = 1 \text{ atm}$$

$$V_s = 0.022415 \text{ m}^3 \text{ (22.415 L/mol)}$$

$$n_s = 1 \text{ gmol}$$

## American Engineering

$$T_s = 492^\circ\text{R (i.e., } 32^\circ\text{F)}$$

$$P_s = 1 \text{ atm}$$

$$\hat{V}_s = 359 \text{ ft}^3/\text{lb-mol}$$

$$n_s = 1 \text{ lb-mol}$$

Used frequently  
by author

## Society of Petroleum Engineers

(common industrial standard)

$$T_s = 60^\circ\text{F}$$

$$P_s = 14.7 \text{ psia}$$

21

# Gas Flows under Standard Conditions

- **SCM**: standard cubic meters ( $\text{m}^3$  STP)
- **SCF**: standard cubic feet ( $\text{ft}^3$  STP)
- **SCFM**: standard cubic feet per minute ( $\text{ft}^3/\text{min}$  STP)
- **SCMH**: standard cubic meters per hour ( $\text{m}^3/\text{hr}$  STP)
- **SCFH**: standard cubic feet per hour ( $\text{ft}^3/\text{hr}$  STP)
- **SCCM**: standard cubic centimeters per minute ( $\text{cm}^3/\text{min}$  STP)
- **SLPM**: standard liters per minute ( $\text{lit}/\text{min}$  STP)

22

## Gas Flows under Standard Conditions

- **SCM**: standard cubic meters ( $\text{m}^3$  STP)
- **SCF**: standard cubic feet ( $\text{ft}^3$  STP)
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- **SCFH**: standard cubic feet per hour ( $\text{ft}^3/\text{hr}$  STP)
- **SCCM**: standard cubic centimeters per minute ( $\text{cm}^3/\text{min}$  STP)
- **SLPM**: standard liters per minute ( $\text{lit}/\text{min}$  STP)

Frequently used in Dr. Fletcher's experience

23

## Standard Conditions and Ideal Gas Calculations

$$PV = nRT$$

Concepts:  $n$  does not change!  
 $\dot{n}$  does not change!

$$P_s \hat{V}_s = RT_s \quad (\text{s} = \text{standard state})$$

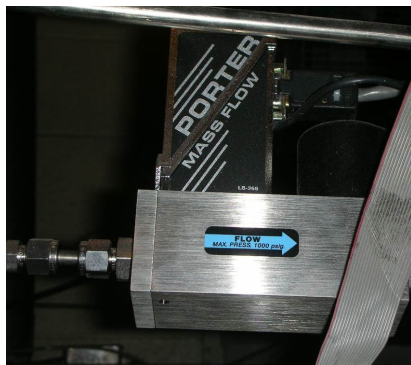
$$\frac{P\hat{V}}{RT} = \frac{P_s \hat{V}_s}{RT_s} \quad \text{so} \quad \frac{P\hat{V}}{T} = \frac{P_s \hat{V}_s}{T_s} \quad \frac{P\dot{V}}{T} = \frac{P_s \dot{V}_s}{T_s}$$

Given standard volume or volumetric flow rate, use ratios to convert to actual volume or volumetric flow rate

24

## Flow Meters Often Report Standard Conditions

- Examples from my labs



Mass flow controller  
(slpm)



Rotameters  
(slpm or sccm)

25

## Example

- The flow rate of a stream at 285°F and 1.3 atm is measured with an orifice meter. The calibration chart for the meter indicates that the flow is  $3.95 \times 10^3$  SCFM. Calculate the molar flow rate and the true volumetric flow rate of the stream.

$$\dot{n} = \frac{P\dot{V}}{RT} = \frac{P_s\dot{V}_s}{RT_s}$$

so

$$\begin{aligned}\dot{V} &= \dot{V}_s \left( \frac{P_s}{P} \right) \left( \frac{T}{T_s} \right) = 3.95 \times 10^3 \text{ SCFM} \left( \frac{1 \text{ atm}}{1.3 \text{ atm}} \right) \left( \frac{745^\circ \text{R}}{492^\circ \text{R}} \right) \\ &= 4.60 \times 10^3 \text{ ft}^3/\text{min}\end{aligned}$$

Absolute T's



26

## Gas Mixtures



- **Partial pressure** of component A

$$P_A = y_A P_{\text{tot}}$$

- Dalton's Law

total P = sum of partial pressures

Example: At sea level, what is the partial pressure of O<sub>2</sub>?

$$P_{\text{O}_2} = y_{\text{O}_2} P_{\text{tot}} = 0.21 \times 1.0 \text{ atm} = 0.21 \text{ atm}$$

- Good for both ideal and non-ideal gases!

27



## Gas Mixtures (cont.)



- In LaBarge, Wyoming, there is a natural gas well for ExxonMobil
- Gas composition coming from the well is:
  - $y_{\text{CO}_2} = 0.70$
  - $y_{\text{CH}_4} = 0.25$
  - $y_{\text{H}_2\text{S}} = 0.05$
- If the total pressure in the well is 20 atm, what is the partial pressure of CH<sub>4</sub>?

$$\begin{aligned} P_{\text{CH}_4} &= y_{\text{CH}_4} P_{\text{tot}} \\ &= (0.25)(20 \text{ atm}) \\ &= 5 \text{ atm} \end{aligned}$$

28

# Ideal Gas Mixtures

- Amagat's Law  
partial volumes add up to total volume

$$V_A + V_B + \dots = V_{\text{tot}}$$

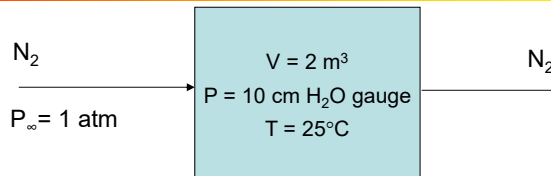
- Volume fraction = mole fraction  
percent by volume = mole percent

$$V_A = y_A V_{\text{tot}}$$

**Good only for ideal gases!**

29

## Problem 5.13 (purging a box)



Replace volume every 5 minutes

Find  $\dot{m}_{N_2}$

$$P_{\text{gauge}} = (0.1 \text{ m H}_2\text{O}) \left( \frac{1 \text{ atm}}{10.333 \text{ m H}_2\text{O}} \right) = 0.0097 \text{ atm}$$

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atm}} = 1.0097 \text{ atm}$$

(a) Calculate  $\dot{m}_{N_2}$  using the ideal gas equation (easy)

(b) Calculate  $\dot{m}_{N_2}$  using 22.4 L/mol at standard conditions (this way is confusing)

$$\dot{V} = \frac{V}{\text{time}} = \frac{2 \text{ m}^3}{5 \text{ min}} = 0.4 \frac{\text{m}^3}{\text{min}} = 400 \text{ L/min}$$

$$\dot{m} = \rho \dot{V} \quad \rho = \frac{P \cdot MW}{RT} = \frac{1.0097 \text{ atm} \cdot 28 \frac{\text{g}}{\text{mol}}}{\frac{0.08205 \text{ L} \cdot \text{atm}}{\text{K} \cdot \text{mol}} \cdot 298 \text{ K}} = 1.156 \text{ g/L}$$

$$\dot{m} = (400 \text{ L/min})(1.156 \text{ g/L}) = 462.5 \text{ g/min}$$

30

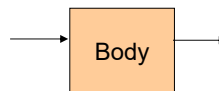
## Review

- Ideal liquid mixtures (volume is additive)
  - How to get an average liquid density
- Ideal Gas Law
  - Gas constant in different units
  - Use absolute units of T & P
- Standard Conditions
- Definition of Partial Pressure

31

## HW Hints

- 14.1 – Ideal gas & MW
- 14.2 – Ideal gas & safety
- 14.3 – Avg MW, mass fraction
- 14.4 – breathing
  - Find  $\dot{V}_{in}$ , convert to  $\dot{m}_{in}$
  - N<sub>2</sub> balance, find  $\dot{m}_{out}$  and  $\dot{V}_{out}$



32