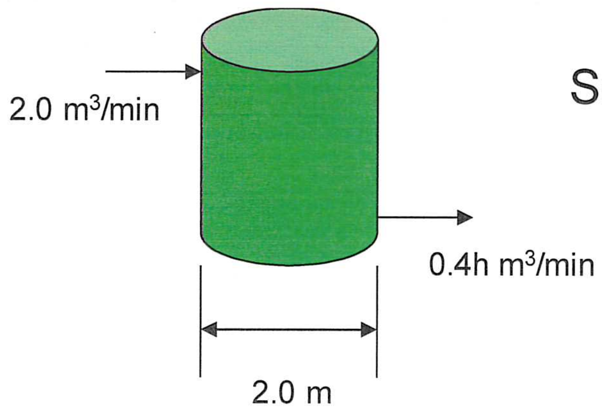
A storage tank that is 2.0 m in diameter is being filled at the rate of 2.0 m^3/min. When the height of the liquid is 2 m in the tank, the bottom of the tank springs a leak. The rate of leaking is proportional to the head of fluid so that it is leaking at a rate of 0.4*h* m^3/min, where *h* is in m. Plot the height of the liquid as a function of time. What is the steady state height of the fluid in the tank?



Set up balance

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$m = \rho V = \rho A h$$

$$\dot{m}_{in} = \rho 2 \ m^3/min$$

$$\dot{m}_{out} = \rho 0.4h \ m^3/min$$

$$\rho A \frac{dh}{dt} = 2\rho - 0.4\rho h$$

$$A = \frac{\pi d^2}{4}$$

$$\frac{dh}{dt} = \frac{2 - .4h}{\pi d^2/4} = \frac{2 - .4h}{\pi r^2} \qquad r = 1 \ m$$

@ steady state, $\frac{dh}{dt} = 0$

$$2 - .4h = 0 \qquad 2 = .4h$$

$$h = \frac{2}{.4} = 5 \ m$$

$$\int_2^h \frac{dh}{2 - .4h} = \int_0^t \frac{4}{\pi d^2} t$$

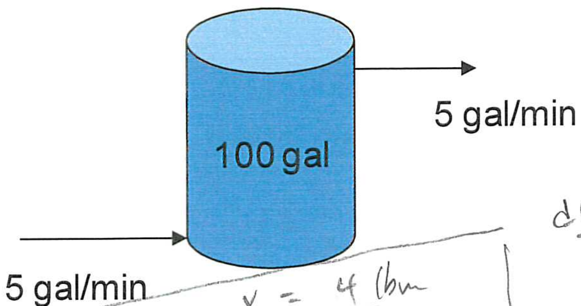
$$-\frac{1}{.4} \ln \frac{2 - .4h}{2 - (.4)(2)} = \frac{4}{\pi d^2} t = \frac{t}{\pi r^2}$$

$$2 - .4h = 1.2 e^{-t/\pi r^2}$$

$$h = \frac{2 - 1.2 e^{-t/\pi r^2}}{.4}$$

A tank holds 100 gal of a salt-water solution in which 4.0 lbm of salt are dissolved. Water runs into the tank at the rate of 5 gal/min and salt solution overflows at the same rate. If the mixing in the tank is adequate to keep the concentration of salt in the tank spatially uniform at any time, how much salt is in the tank at the end of 50 min? Assume that the density of the salt solution is essentially the same as that of pure water.

Set up balance



$$x_0 = \frac{4\ lbm}{m_{H_2O} + m_{s,0}}$$

$$m_{s,0} = (100\ gal)\left(\frac{ft^3}{7.48052\ gal}\right)\left(\frac{62.43\ lbm}{gal}\right)$$

$$= 834.6\ lbm$$

$$x_0 = \frac{4}{4+834.6} = 0.00477$$

$$\frac{dm_s}{dt} = \cancel{\dot{m}_{s_{in}}}^{\,0} - \dot{m}_{s_{out}}$$

$$m_s = x_s m$$

$$\dot{m}_{s_{out}} = x_s \dot{m}$$

$$\frac{d x_s m}{dt} = -x_s \dot{m}$$

$$m \frac{dx_s}{dt} = -x_s \dot{m}$$

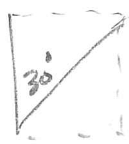
$$\int_{x_s=.00477}^{x_s} \frac{dx_s}{x_s} = \int_0^t -\frac{\dot{m}}{m}\, dt$$

$$\ln \frac{x_s}{4/104} = -\frac{\dot{m}}{m} t$$

$$x_s = .00477\, e^{-\frac{\dot{m} t}{m}}$$

How does the volumetric flow rate change with time in an oil funnel with an angle of 30° from vertical in the cone? The volumetric flow rate out is a function of the discharge coefficient, the square root of the pressure drop (see equation).

$$\dot{V}_{out} = c_v \sqrt{\frac{\Delta P}{S.G.}}$$



Set up balance

$$\frac{dm}{dt} = \cancelto{0}{\dot{m}_{in}} - \dot{m}_{out}$$

$$m = \rho V = \rho \frac{A h}{3}$$

$$A = \pi r^2$$

$$\tan 30^\circ = \frac{r}{h} \qquad r = h \tan 30^\circ$$

$$A = \pi (h \tan 30^\circ)^2$$

$$\dot{V}_{out} = c_v \sqrt{\frac{\Delta P}{SG}}$$

$$\Delta P = \rho g h$$

$$\dot{m}_{out} = \rho c_v \sqrt{\frac{\rho g h}{SG}}$$

$$\frac{d\left(\cancel{\rho}\, \pi (h \cdot \tan 30^\circ)^2\, h/3\right)}{dt} = -\cancel{\rho} c_v \sqrt{\frac{\rho g h}{SG}}$$

$$\frac{\pi (\tan 30^\circ)^2}{3} \frac{dh^3}{dt} = -c_v \sqrt{\frac{\rho g h}{SG}}$$

$$\frac{\pi (\tan 30^\circ)^2}{\cancel{3}} \cancel{3} h^2 \frac{dh}{dt} = -c_v \sqrt{\frac{\rho g h}{SG}}$$

$$a\, h^2 \frac{dh}{dt} = -b\sqrt{h}$$

$$h^{3/2} \frac{dh}{dt} = \frac{-b}{a}$$

$$\int_{h_1}^{h_2} h^{3/2}\, dh = \int_{t_1}^{t_2} \frac{-b}{a}\, dt$$

$$\frac{2}{5}\left(h_2^{5/2} - h_1^{5/2}\right) = -\frac{b}{a}(t_2 - t_1)$$

(a) A kettle containing 3.00 L of water at a temperature of 18°C is placed on an electric stove and begins to boil in four minutes. What is the average rate at which heat is added to the water during this period?

Set up balance

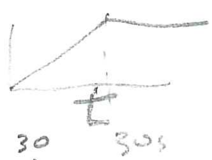
$$\frac{dH}{dt} = \dot{Q} \qquad (H_{liq} = U_{liq})$$

$$m C_p \frac{dT}{dt} = \dot{Q}$$

or

$$m C_v \frac{dT}{dt} = \dot{Q}$$

(b) Suppose that it takes 30 s for the stove heating element to reach full power. What is the time-dependent temperature of the water in the kettle?

$\dot{Q} = \dot{Q}$ 

$$\int_0^t m C_p\, dT = \int_0^{30} Q_1(t)\, dt + \int_{30}^{t} \overset{\text{constant}}{\dot{Q}_2}\, dt$$