Class 4

Transient & Steady-State Balances

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HW Mistakes

- Finding the weight in lbf from lbm. Many students forgot to convert to lb_f using $32.2\,\frac{lb_m ft}{lb_f s^2}$ after calculating m*g
- Not recognizing that A*exp(a) = A*e^a
  - Many students incorrectly interpreted A*exp(a) as A^a

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Math Refresher

Definite integrals

We often use **definite integrals** rather than expand with constants and set boundary conditions. The beginning values are at the start of the integral and the end values are at the end of the integral.

$$\int_3^5 x dx = \frac{x^2}{2}\bigg]_3^5 = \frac{5^2}{2} - \frac{3^2}{2} = \frac{(25-9)}{2} = \frac{16}{2} = 8$$

Remember that

$$\ln a - \ln b = \ln \frac{a}{b},$$

and

$$\int \frac{1}{(a+bx)} dx = \frac{1}{b} ln(a+bx),$$

so

$$\int_{x_1}^{x_2} \frac{1}{(a+bx)} dx = \frac{1}{b}[ln(a+bx_2) - ln(a+bx_1)] = \frac{1}{b} ln\left(\frac{a+bx_2}{a+bx_1}\right)$$

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Classification of Balances

- Continuous
- Batch
- Semibatch

- Steady-state
- Transient

Give me an example of each…

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General Balance Equation

Accumulation = In – Out + Generation - Consumption

How do you remember this equation?

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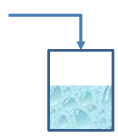
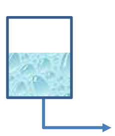
Take terms 2 at a time

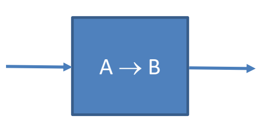


Semibatch
Transient
In = Accumulation



Semibatch
Transient
Accumulation = -Out

Continuous
Steady-state
In = Out



Continuous (100% Conversion of A)
Steady-state
In_A = $Consumption_A$
$Generation_B$ = Out_B

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Cross Out Terms That Are Zero

Steady State

Accumulation = In – Out + Generation - Consumption

At steady state, all time-dependent terms are zero

Mass Balance

Accumulation = In – Out + Generation - Consumption

You cannot generate or destroy mass unless you have a nuclear reaction

Mole Balance

Accumulation = In – Out + Generation - Consumption

Generation and consumption relate to chemical reaction terms

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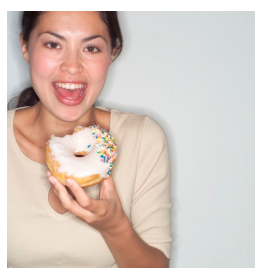
Example: Donut Eatery



In

Consumption

Out

So:

In = Out + Consumption

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Transient Balances

$$accum = in - out + gen - cons$$

- If we sum over time, then the change in the system has to equal the total amount that came in minus the amount that went out, adjusted for generation and consumption

- If we pour a glass of milk, then the "accumulation" is the amount that ended up in the glass
- The "in" term is the amount that we poured into the glass
- The "out" term might be the amount sipped through a straw

If we are interested in the time-dependent behavior, we want the amount of change per time

Change in our variable y per time is $\frac{\Delta y}{\Delta t}$ which in the limit becomes $\frac{dy}{dt}$

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Transient Balances

$$accum = in - out + gen - cons$$

The accumulation term always involves a derivative: $accum = \frac{d(?)}{dt} or \frac{\Delta y}{\Delta t}$

What are the units of the denominator of the derivative term $\frac{d}{dt}$? dt has units of time

Example: total mass

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

Other relationships:

$$m = \rho V \qquad mass = \frac{mass}{volume} \times volume$$

$$\dot{m} = \rho v A = \rho \dot{V} \qquad \frac{mass}{time} = \frac{mass}{volume} \times \frac{volume}{time}$$

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Separate and Integrate

(Solve the ODE)

- **A. Separate** means put one variable on left side and the other variable on the right side.
- Example: $\frac{dy}{dt} = f(y) \cdot f(t)$
- Split the *dy* and the *dt* (like a quotient), and then put *dt* on the RHS.
- Also put all terms with a *y* on the LHS.

$$\frac{dy}{f(y)} = f(t) \cdot dt$$

More info in Chapter 10 (10.1-10.2)

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Integrate

- **B. Integrate** both sides between limits that correspond to each other.

$$\int_{y_0}^{y_f} \frac{dy}{f(y)} = \int_{t_0}^{t_f} f(t)dt$$

Boundary conditions:
$y = y_0$ at $t = t_0$
$y = y_f$ at $t = t_f$

- Example: $\frac{dy}{dt} = -yt^2$
- Separate: $\frac{dy}{y} = -t^2 dt$
- Integrate: $\int_{y_0}^{y_f} \frac{dy}{y} = -\int_{t_0}^{t_f} t^2 dt$
- or $\ln\left(\frac{y_f}{y_0}\right) = -\frac{1}{3}\left(t_f^3 - t_0^3\right)$

$$y_f = y_0 e^{-\frac{1}{3}\left(t_f^3 - t_0^3\right)}$$

Note that I used definite integrals here. This is MUCH easier and faster. Please learn how to do this!!!!

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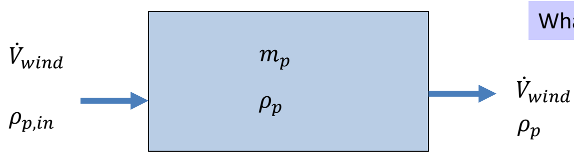
Example: Air Quality in Utah Valley

- Let m_p = mass of pollutants (kg of pollutants)
- Let ρ_p = mass concentration of pollutants (kg of p/m^3)
- Assume valley is a vessel
- Wind comes along
- Perfectly mixed ($\rho_p = \rho_{p,out}$)
- Find ρ_p as a function of time

What does the box represent?



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Species Balance on "p"

Accumulation = In – Out + Generation - Consumption

$$\frac{dm_p}{dt} = \dot{m}_{p,in} - \dot{m}_{p,out} \qquad = \frac{\Delta m_p}{\Delta t}$$

$$m_p = \rho_p V_{valley}$$

$$\dot{m}_p = \rho_p v A = \rho_p \dot{V}$$

So the balance becomes:

Assume $\rho_{p,in} = 0$

$$\frac{d\rho_p V}{dt} = \rho_{p,in}\dot{V}_{wind} - \rho_p \dot{V}_{wind} = \dot{V}_w\left(0 - \rho_p\right)$$

$$V\frac{d\rho_p}{dt} = -\rho_p \dot{V}_w$$

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Now Separate!

$$V\frac{d\rho_p}{dt} = -\rho_p \dot{V}_w$$

⇩

$$\frac{d\rho_p}{\rho_p} = -\frac{\dot{V}_w}{V}dt$$

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Now Integrate

$$\int_{\rho_{p,0}}^{\rho_{p,f}} \frac{d\rho_p}{\rho_p} = -\frac{\dot{V}_w}{V}\int_0^t dt$$

$$\ln\left(\frac{\rho_{p,f}}{\rho_{p,0}}\right) = -\frac{\dot{V}_w}{V}t$$

$$\rho_{p,f} = \rho_{p,0}e^{-\frac{\dot{V}_w}{V}t}$$

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Consistency of Units

- In any equation, each term must have the same units (as we mentioned before)
- Consider the species mass balance equation:

$$\frac{d(?)}{dt} = \dot{m}_{a,in} - \dot{m}_{a,out}$$

- What should be the units where the "?" occurs?

Ans: mass of component "a" (m_a), NOT total mass of "a" per time ($\dot{m}_a$)

- Getting the units right will help in determining what goes inside the derivative term
  - Only possible terms inside derivative are: m, m_i, n, n_i

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Example: Leaking Reactor

- A laboratory reactor leaks SO_2 at a rate of 0.1 mol/hr. The room volume is 1000 m^3. If the air/SO_2 gas leaves the room at the same rate as the leak (0.1 mol/hr), how long will it take for the room to reach the toxic limit of 1 ppm ($y_{SO2} = 1\times10^{-6}$)?
- If the room is ventilated at 10 room volumes per hour, how will y_{SO2} change with time?

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See Handwritten Notes

See Excel Sheet

Both a pdf file with my notes and an Excel sheet are posted online.

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Integral Balances

- Commonly used on batch processes
- Balance initial and final states
- No time dependence
- No in and out streams

80% A 20% B 0% C	→	40% A 40% B 20% C

Balance on A
0.8 m = 0.4 m + consumption
Or
Consumption = 0.8m – 0.4m = 0.4 m

Initial input + generation = final output + consumption
Note: Total mass is constant in a batch process

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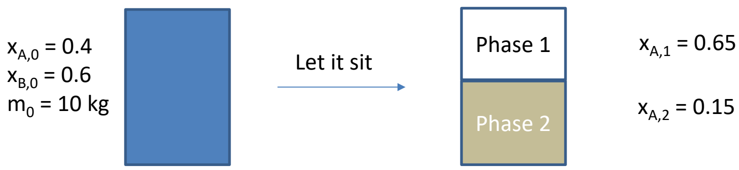
Example of Integral Balances



Find masses of phase 1 and phase 2

- Total mass balance

$$m_0 = 10 \text{ kg} = m_1 + m_2$$

- Balance on species A

$$x_{A,0}m_0 = x_{A1}m_1 + x_{A2}m_2$$

or

$$0.4\ (10 \text{ kg}) = 0.65\ m_1 + 0.15\ m_2$$

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Algebra

$$10 = m_1 + m_2$$

$$4 = 0.65\ m_1 + 0.15\ m_2$$

Multiply by 0.15

$$1.5 = 0.15\ m_1 + 0.15\ m_2$$

Now subtract to eliminate m_2

$$2.5 = 0.5\ m_1$$

So $m_1 = 2.5/0.5 = 5$ kg,

and therefore $m_2 = 10 – 5 = 5$ kg

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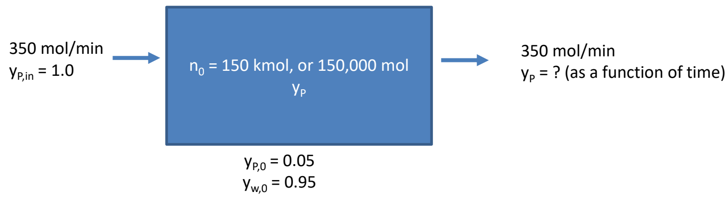
HW 5.3

Click to see problem



Accum = in – out + gen - cons

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