General Balance Equation
Accumulation = In – Out + Generation - Consumption

Steady State
Accumulation = In – Out + Generation - Consumption
At steady state, all time-dependent terms are zero

Mass Balance
Accumulation = In – Out + Generation - Consumption
You cannot generate or destroy mass unless you have a nuclear reaction

Mole Balance
Accumulation = In – Out + Generation - Consumption
Generation and consumption relate to chemical reaction terms

Cross Out Terms That Are Zero

Example: Donut Eatery

So:
In = Out + Consumption

Separate and Integrate (Solve the ODE)

• A. Separate means put one variable on left side and the other variable on the right side.

• Example: \( \frac{dy}{dt} = f(y) \cdot f(t) \)

• Split the \( dy \) and the \( dt \) (like a quotient), and put \( dt \) on the RHS.

• Also put all terms with a \( y \) on the LHS.

Integrate

• B. Integrate both sides between limits that correspond to each other.

\[ \int_y^{y_f} \frac{dy}{f(y)} = \int_t^{t_f} f(t)dt \]

Boundary conditions:
\( y = y_0 \) at \( t = t_0 \)
\( y = y_f \) at \( t = t_f \)

Example:
\( \frac{dy}{dt} = -yt^2 \)

Separate:
\( \frac{dy}{y} = -t^2dt \)

Integrate:
\( \int_y^{y_f} \frac{dy}{y} = \int_{t_0}^{t_f} -t^2dt \)

or
\( \ln \left( \frac{y_f}{y_0} \right) = -\frac{1}{3} \left( t_f^3 - t_0^3 \right) \)

Note that I used definite integrals here. This is MUCH easier and faster. Please learn how to do this!!!

More info in Chapter 11 (11.1-11.2)
Example: Air Quality in Utah Valley
• Let $\rho_p = \text{concentration of pollutants (kg of pollutants/m}^3)$
• Assume valley is a vessel
• Wind comes along
• Perfectly mixed ($\rho_p = \rho_{p,\text{out}}$)
• Find $\rho_p$ as a function of time

\[ \frac{d \rho_p}{dt} = \rho_{p,\text{in}} - \rho_{p,\text{out}} \]

Species Balance on “p”
Accumulation = In – Out + Generation - Consumption

\[ \frac{dm_p}{dt} = \dot{m}_{p,\text{in}} - \dot{m}_{p,\text{out}} \sim \Delta m_p \]

$\dot{m}_p = \rho_p V_{\text{valley}}$

\[ \dot{m}_p = \rho_p V_f \]

So the balance becomes:

\[ \frac{d \rho_p}{dt} = \rho_{p,\text{in}} - \rho_{p,\text{out}} - V_f (0 - \rho_p) \]

Now Separate!

\[ \frac{\rho_p}{V} \frac{d \rho_p}{dt} = -\rho_p \dot{V}_f \]

\[ \frac{d \rho_p}{\rho_p} = \frac{\dot{V}_f}{V} dt \]

Now Integrate

\[ \int \frac{d \rho_p}{\rho_p} = \frac{\dot{V}_f}{V} t \]

\[ \ln \left( \frac{\rho_{p,f}}{\rho_{p,i}} \right) = \frac{\dot{V}_f}{V} t \]

\[ \rho_{p,f} = \rho_{p,i} e^{\frac{\dot{V}_f}{V} t} \]

Consistency of Units
• In any equation, each term must have the same units (as we mentioned before)
• Consider the species mass balance equation:

\[ \frac{d (\text{mass})}{dt} = \dot{m}_{a,\text{in}} - \dot{m}_{a,\text{out}} \]

• What should be the units where the “?” occurs?

\[ \text{Ans: mass of component } a, \text{ NOT mass of a per time} \]

• Getting the units right will help in determining what goes inside the derivative term
  – Only possible terms inside derivative are: $m$, $m_i$, $n$, $n_i$

Integral Balances
• Commonly used on batch processes
• Balance initial and final states
• No time dependence
• No in and out streams

Initial input + generation + final output + consumption

\[ \begin{align*}
80\% \text{ A} & \quad \rightarrow \quad 40\% \text{ A} \\
20\% \text{ B} & \quad \rightarrow \quad 40\% \text{ B} \\
0\% \text{ C} & \quad \rightarrow \quad 20\% \text{ C}
\end{align*} \]

Note: Total mass is constant in a batch process
Example of Integral Balances

- Total mass balance
  \[ m_0 = 10 \text{ kg} = m_1 + m_2 \]
- Balance on species A
  \[ x_{A,0} m_0 = x_{A,1} m_1 + x_{A,2} m_2 \]
  or
  \[ 0.4 (10 \text{ kg}) = 0.65 m_1 + 0.15 m_2 \]

Let it sit

Let's find the masses of phase 1 and phase 2

### Algebra

\[
\begin{align*}
10 &= m_1 + m_2 \\
4 &= 0.65 m_1 + 0.15 m_2 \\
1.5 &= 0.15 m_1 + 0.15 m_2 \\
2.5 &= 0.5 m_1
\end{align*}
\]

So \( m_1 = 2.5 / 0.5 = 5 \text{ kg} \),
and therefore \( m_2 = 10 - 5 = 5 \text{ kg} \)

### Special Problem 4.1

Click to see problem

Accum = in – out + gen - cons