Mechanical Energy Balance

- Special Case
- No temperature change, no chemical rxn
  ($\Delta U \cong 0$, $\Delta H \cong 0$)
- velocity, pressure, friction are important

$\Rightarrow$ magic (energy & momentum equations)

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z + \hat{F} = -\frac{W_s}{\dot{m}}$$

$\frac{\Delta P}{\rho}$ ↑ change in pressure; $\frac{\Delta u^2}{2}$ ↑ velocity; $g\Delta z$ ↑ height; $\hat{F}$ ↑ friction; $\frac{W_s}{\dot{m}}$ ↑ shaft work done by system

If $\hat{F} = 0$
$W_s = 0$

$\Rightarrow$ <u>Bernoulli's Equ.</u>

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z = 0$$

or

$$\frac{P_2 - P_1}{\rho} + \frac{u_2^2 - u_1^2}{2} + g(z_2 - z_1) = 0$$

<u>Other Relationships</u>

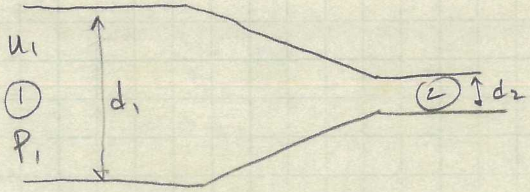
$$\boxed{\dot{V} = u A}$$

$\frac{m^3}{s}$ $\quad \frac{m}{s}$ $\quad m^2$

$$\boxed{\dot{m} = \rho u A}$$

$\frac{g}{s}$ $\quad \frac{g}{m^3}$ $\quad \frac{m}{s}$ $\quad m^2$

Examples



Find A. $P_2 - P_1$

B. u_2

Mass Balance

$$\dot{m} = \text{constant} = \rho A_1 u_1 = \rho A_2 u_2$$

$$A_1 = \pi \frac{d_1^2}{4}$$

$$A_2 = \pi \frac{d_2^2}{4}$$

$$\cancel{\rho \pi} \frac{d_1^2}{\cancel{4}} u_1 = \cancel{\rho \pi} \frac{d_2^2}{\cancel{4}} u_2$$ (if densities are equal)

so $$\boxed{u_2 = u_1 \left(\frac{d_1}{d_2}\right)^2}$$

what about pressure?

Bernoulli's Eqn.

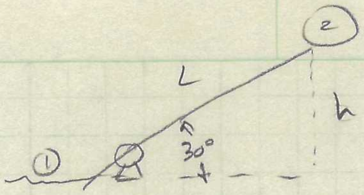
$$\frac{P_2 - P_1}{\rho} + \frac{u_2^2 - u_1^2}{2} + \cancelto{0}{g(z_2 - z_1)} = 0$$

$$P_2 = P_1 - \frac{\rho}{2}\left(u_2^2 - u_1^2\right)$$

$$= P_1 - \frac{\rho}{2}\left(\left[u_1\left(\frac{d_1}{d_2}\right)^2\right]^2 - u_1^2\right)$$

$$\boxed{P_2 = P_1 - \frac{\rho}{2}\left(u_1^2\right)\left(\left[\frac{d_1}{d_2}\right]^4 - 1\right)}$$

7.55



$$\sin 30^\circ = \frac{h}{L} = 0.5 \Rightarrow L = 2h$$

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z + \hat{F} = -\frac{W_s}{\dot{m}} \qquad F = 0.41 \overset{ft}{L} \ \frac{ft\,lb_f}{lb_m}$$

1 → surface of lake $\quad u_1 = 0$

2 → atmospheric discharge $\quad P_2 = P_1$

$$\frac{u_2^2}{2} + gh + \underbrace{(.041)(2h)}_{\hat{F}\ \frac{ft\,lb_f}{lb_m}} = -\frac{W_s}{\dot{m}}$$

$\frac{u_2^2}{2} = ?$ need u_2

$$\dot{V} = (95\ gal/min)\left(\frac{ft^3}{7.4805\ gal}\right)\left(\frac{min}{60\,s}\right) = 0.212\ \frac{ft^3}{s}$$

$$uA = \dot{V}$$

$$A = \pi\frac{d^2}{4} = \left(\pi\frac{(1.049\,in)^2}{4}\right)\left(\frac{ft}{12\,in}\right)^2 = 6\times10^{-3}\ ft^2$$

$$u_2 = \frac{.212\ ft^3/s}{6\times10^{-3}\ ft^2} = 35.3\ ft/s$$

$$\dot{m} = \rho A u_2 = \left(62.4\ \frac{lb_m}{ft^3}\right)\left(6\times10^{-3}\ ft^2\right)\left(35.3\ ft/s\right) = 13.2\ \frac{lb_m}{s}$$

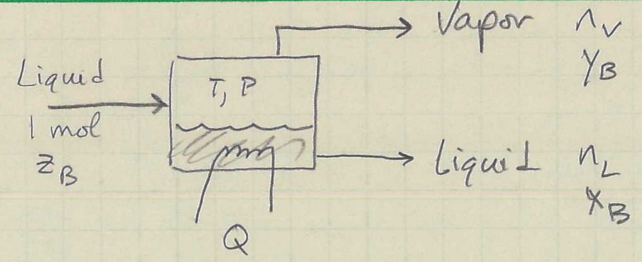
$$\left(32.2\ \frac{ft\,lb_m}{lb_f\text{-}s^2}\right)$$

$$\frac{(35.3\ ft/s)^2}{2} + \left(32.2\ \frac{ft}{s^2}\right)h + \left(.082\,h\ \frac{ft\,lb_f}{lb_m}\right)\left(32.2\ \frac{ft\text{-}lb_m}{lb_f\text{-}s^2}\right) = \left(\frac{-8\ hp}{13.2\ \frac{lb_m}{s}}\right)\left(\frac{.7376\ \frac{ft\,lb_f}{s}}{1.34\times10^{-3}\ hp}\right)$$

$$623 + 32.2h + 2.64h = 10{,}742$$

$$\boxed{h = 290\ ft!}$$

(7.51)



(a) unknowns $n_V, n_L, y_B, x_B, Q, T, P = 7$

species balances (benz, tol) $= 2$

Other Equations

(1) Raoult's Law

$$x_B P_B^* = y_B P_{tot} \quad 1$$

$$x_T P_T^* = y_T P_{tot} \quad 2$$

(2) Energy Balance $\quad 3 \Rightarrow 3$

$\overline{\quad\quad}$ 2 DoF

But if T & P are known, DoF = 0

(b) $T = 90°C$, $P_{tot} = 652$ mm Hg

$$P_{tot} = P_B + P_T = x_B P_B^* + x_T P_T^*$$

So $$P_{tot} = x_B P_B^* + (1 - x_B) P_T^*$$

rearranging, $$P_{tot} = x_B (P_B^* - P_T^*) + P_T^*$$

so $$x_B = \frac{P_{tot} - P_T^*}{P_B^* - P_T^*}$$

then, $$y_B = \frac{x_B P_B^*}{P_{tot}}$$

Total mole balance

$$1 = n_V + n_L$$

Benzene balance

$$0.5 = x_B n_L + y_B n_V = x_B n_L + y_B (1 - n_L)$$
$$= y_B + n_L (x_B - y_B)$$

So $$n_L = \frac{0.5 - y_B}{x_B - y_B}$$

Energy Balance

$$Q = \sum n_i \hat{H}_{i\,out} - \sum n_i \hat{H}_{i,in}$$

See spreadsheet

7.51 (cont.)

make a linear correlation for $\hat{H}$ for each species in each phase

$$y = mx + b$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

then use one data point to get $b = y_1 - m x_1$

$$Q = n_v\left(y_B \hat{H}_{B,v} + y_T \hat{H}_{T,v}\right) + n_L\left(x_B \hat{H}_{B,L} + x_T \hat{H}_{T,L}\right)$$
$$- n_f\left(z_B H_{B,f} - z_T H_{T,F}\right) \quad \Rightarrow n_f = 1$$

(b) See spreadsheet

(c) If P is too low, all vapor

If P is too high, all liquid

(d) See spreadsheet as P↑, n_L↑, so less energy req'd to produce vapor

(e) See spreadsheet