ChE 273 Class 34

Transient Balances

A. General Balance Equation

(1) Fill up a cup of water (during filling period)

$$\frac{dm}{dt} = \text{accum} = \text{in}$$

(2) Drain a milk jug

$$\frac{dm}{dt} = \text{accum} = -\text{out}$$

$$\boxed{\text{accum} = \text{in} - \text{out} + \text{gen} - \text{cons}}$$

Always has a $\frac{d(\ )}{dt}$ term

B. Example: Tank of gasoline, driving steady on freeway

20 miles/gal
70 miles/hr

$$\dot{V}_{out} = \text{Flow out} = \frac{70 \text{ miles/hr}}{20 \text{ miles/gal}} = 3.5 \text{ gal/hr}$$

$$\text{accum} = \rho \frac{dV}{dt} = -\rho \dot{V}_{out}$$

$$\frac{dV}{dt} = -\dot{V}_{out}$$

separate and integrate

$$\int_{V_0}^{V(t)} \frac{-dV}{\dot{V}_{out}} = \int_0^t dt$$

$$-\frac{(V - V_0)}{\dot{V}_{out}} = t$$

30 gallon tank, $V(t) = 0$ $\quad \frac{35 \text{ gal}}{3.5 \text{ gal/hr}} = 10 \text{ hrs}$

- show sheet with transient equations
- show practice ppt slides
    - have them come up with starting equations

Example 1

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\cancel{\rho} \frac{dV}{dt} = \cancel{\rho} \dot{V}_{in} - \cancel{\rho} \dot{V}_{out}$$

$$V = (\pi r^2) h$$

$$r^2 \pi \frac{dh}{dt} = 2 - \dot{V}_{out}$$

$$\dot{V}_{out} = 0.4h$$

$$r^2 \pi \frac{dh}{dt} = 2 - 0.4h \qquad \boxed{@ \text{ s.s. } \frac{dh}{dt} \to 0 \quad h = \frac{2}{.4} = 5 \, m}$$

$r = 1 \, m$

$$\int_2^h \frac{dh}{2 - 0.4h} = \int_0^t \frac{1}{\pi r^2} dt$$

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx)$$

$$\left(-\frac{1}{.4} \ln 2 - 0.4h\right) - \left(-\frac{1}{.4} \ln(1.2)\right) = \frac{1}{\pi} t$$

$$\frac{1}{.4} \ln \frac{1.2}{2 - .4h} = \frac{1}{\pi} t$$

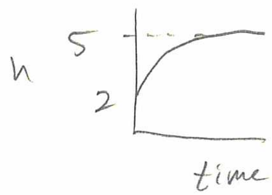
$$\ln \frac{1.2}{2 - .4h} = \frac{.4}{\pi} t$$

$$\frac{1.2}{2 - .4h} = e^{\frac{.4t}{\pi}}$$

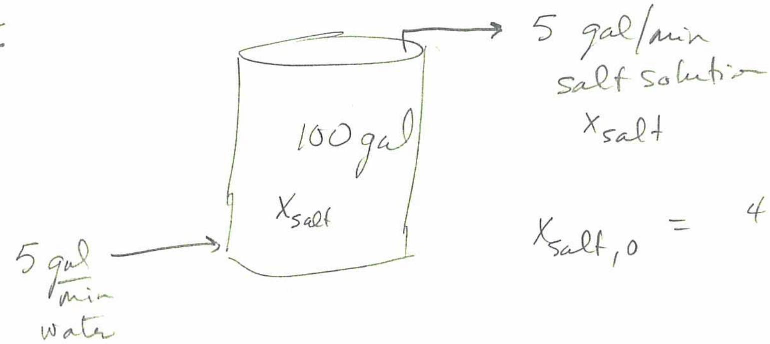
$$1.2 e^{-\frac{.4t}{\pi}} = 2 - .4h$$

$$.4h = \left(2 - 1.2 e^{-.4t/\pi}\right)$$

$$h = \frac{1}{.4}\left(2 - 1.2 e^{-.4t/\pi}\right)$$



Example 2



$$x_{salt,0} = 4\ \frac{lb_m\ salt}{gal\ of\ solution}$$

Species mass balance

$$\frac{d\, x_s m}{dt} = \cancel{(x_s \dot{m})_{in}}^{\,0} - (x_s \dot{m})_{out}$$

$$m_s \frac{dx_s}{dt} = -x_s \dot{m}_{out}$$

$$\int_{x_{s,0}}^{x_{s,f}} \frac{dx_s}{x_s} = -\int_0^t \frac{\dot{m}_{out}}{m}\, dt$$

$$\ln \frac{x_{s,f}}{x_{s,0}} = -\frac{\dot{m}_{out}}{m} t$$

$$x_{s,f} = x_{s,0}\, e^{-\frac{\dot{m}_{out}}{m} t}$$

Example 3

(a) for liquid, $H \simeq U$

$$\frac{dU}{dt} = \cancel{\dot{H}_{in}}^{0} - \cancel{\dot{H}_{out}}^{0} + \dot{Q} - \cancel{\dot{W}_s}^{0}$$

$$\frac{d m \hat{U}}{dt} = \dot{Q}$$

$$m \frac{d\hat{U}}{dt} = \dot{Q}$$

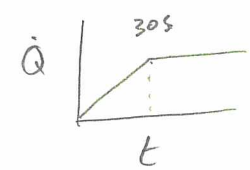
$$m C_v \frac{dT}{dt} = \dot{Q} \qquad \text{but } C_v \cong C_p \text{ for liquid}$$

$$\rho V C_p \frac{dT}{dt} = \dot{Q}$$

$$\int_{T_0}^{T} \rho V C_p \, dT = \int_0^t \dot{Q} \, dt$$

$$\rho V \bar{C}_p (T - T_0) = \dot{Q} t$$

(b) if $\dot{Q}$ known over 1st 30 seconds $= ct$



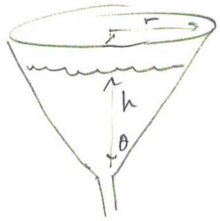
$$\int_{T_0}^{T} \rho V C_p \, dT = \int_0^{30\,s} ct \, dt + \int_{30\,s}^{t} \dot{Q} \, dt$$

for 1st 30 s $\quad \rho V C_p (T - T_0) = c \frac{t^2}{2} \qquad c = \frac{\dot{Q}}{30\,s}$

after 30 s $\quad \rho V C_p (T - T_0) = \frac{c(30)^2}{2} + \dot{Q} t$

Example 4

oil funnel



$$V = \pi r^2 h$$

$$\tan\theta = \frac{r}{h}, \quad \text{so} \quad r = h\tan\theta$$

$$V = \pi (h^2 \tan^2\theta) h = \pi h^3 (\tan\theta)^2$$

If $\theta = 30°$, $\tan\theta = .5774$, $\tan^2\theta = \frac{1}{3}$

so $V = \pi \frac{h^3}{3}$

$$\frac{dm}{dt} = \cancel{\dot{m}_{in}}^{\,0} - \dot{m}_{out}$$

$$\frac{d\rho V}{dt} = -\rho \dot{V}_{out}$$

$$\frac{dV}{dt} = -\dot{V}_{out}$$

however, $\dot{V}_{out} = f(h) \simeq C_v \sqrt{\frac{\Delta P}{S.G.}}$

$$\Delta P = \rho g h$$

$$\dot{V}_{out} = C_v \sqrt{\frac{\rho g h}{S.G.}} = a h^{1/2} \qquad a = C_v \sqrt{\frac{\rho g}{S.G.}}$$

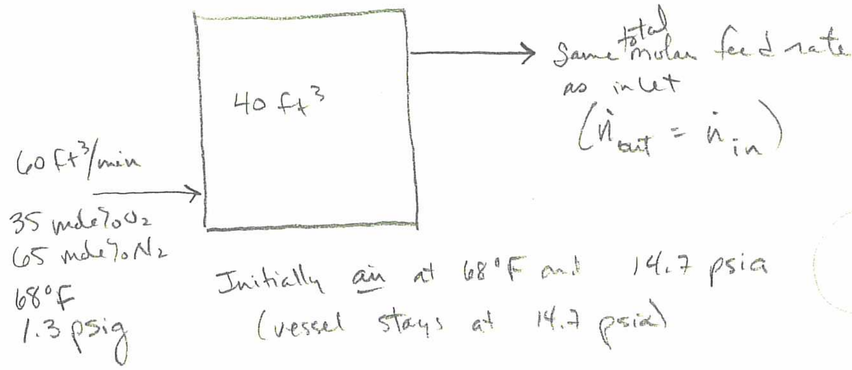
$$\frac{d\left(\frac{\pi h^3}{3}\right)}{dt} = -a h^{1/2}$$

$$3\frac{\pi}{3} h^2 \frac{dh}{dt} = -a h^{1/2}$$

$$\int_{h_1}^{h_2} h^{3/2}\, dh = \int_{t_1}^{t_2} -\frac{a}{\pi}\, dt$$

$$\frac{2}{5}\left(h_2^{5/2} - h_1^{5/2}\right) = -\frac{a}{\pi}(t_2 - t_1)$$

D. Problem 11.12



Initially air at 68°F and 14.7 psia
(vessel stays at 14.7 psia)

Find: Time to raise y_{O_2} in room to 0.27

$$N_{\text{in vessel}} = \frac{PV}{RT} = \frac{(14.7\ \text{psia})(40\ \text{ft}^3)}{\left(10.73\ \frac{\text{psia ft}^3}{°\text{R lb-mol}}\right)(528°\text{R})} = 0.1038\ \text{lb-moles}$$

$$\frac{d(\overbrace{y_{O_2} N_{\text{vessel}}}^{n_{O_2}})}{dt} = \underbrace{\overbrace{.35\,\dot{n}_{in}}^{in}}_{=\ \text{constant}} - \overbrace{y_{O_2}\,\dot{n}_{out}}^{out}$$

$$= (.35 - y_{O_2})\,\dot{n}_{in}$$

$$\dot{n}_{in} = \frac{(16\ \text{psia})(60\ \text{ft}^3/\text{min})}{\left(10.73\ \frac{\text{psia ft}^3}{\text{lbmole}\cdot°\text{R}}\right)(528°\text{R})} = 0.1694\ \frac{\text{lb-mol}}{\text{min}}$$

$$N_{\text{vessel}}\frac{dy_{O_2}}{dt} = (.35 - y_{O_2})\,\dot{n}_{in}$$

$$\int \frac{dx}{a+bx} = \frac{1}{b}\ln(a+bx)$$

$$\int_{.21}^{y_{O_2}} \frac{dy_{O_2}}{.35 - y_{O_2}} = \int_0^t \frac{\dot{n}_{in}}{N_{\text{vessel}}}$$

$$-\ln\frac{.35 - y_{O_2}}{.35 - .21} = \frac{\dot{n}_{in}\,t}{N_{\text{vessel}}} = \frac{.1694}{.1038}\,t = 1.632\,t$$

if $y_{O_2} = .27$, $t = 0.343$ min