General Balance Equation

Accumulation = In – Out + Generation - Consumption

Cross Out Terms That Are Zero

Steady State

Accumulation = In – Out + Generation - Consumption

At steady state, all time-dependent terms are zero

Mass Balance

Accumulation = In – Out + Generation - Consumption

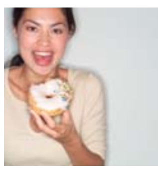
You cannot generate or destroy mass unless you have a nuclear reaction

Mole Balance

Accumulation = In – Out + Generation - Consumption

Generation and consumption relate to chemical reaction terms

Example: Donut Eatery



In Consumption Out

So:
In = Out + Consumption

Transient Balances

$$accum = in - out + gen - cons$$

The accumulation term always involves a derivative: $accum = \frac{d(?)}{dt}$ or $\frac{\Delta x}{\Delta t}$

What are the units of the denominator of the derivative term $\frac{d}{dt}$? dt has units of time

Example: total mass

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

Other relationships:

$$m = \rho V$$

$$\dot{m} = \rho v A = \rho \dot{V}$$

Separate and Integrate

(Solve the ODE)

- **A. Separate** means put one variable on left side and the other variable on the right side.
- Example: $\frac{dy}{dt} = f(y) \cdot f(t)$
- Split the *dy* and the *dt* (like a quotient), and put *dt* on the RHS.
- Also put all terms with a *y* on the LHS.

$$\frac{dy}{f(y)} = f(t) \cdot dt$$

More info in Chapter 11 (11.1-11.2)

Integrate

- **B. Integrate** both sides between limits that correspond to each other.

$$\int_{y_0}^{y_f} \frac{dy}{f(y)} = \int_{t_0}^{t_f} f(t) dt$$

Boundary conditions:
$y = y_0$ at $t = t_0$
$y = y_f$ at $t = t_f$

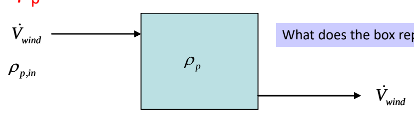
- Example: $\frac{dy}{dt} = -yt^2$
- Separate: $\frac{dy}{y} = -t^2 dt$
- Integrate: $\int_{y_0}^{y_f} \frac{dy}{y} = -\int_{t_0}^{t_f} t^2 dt$
- or $\ln\left(\frac{y_f}{y_0}\right) = -\frac{1}{3}\left(t_f^3 - t_0^3\right)$

$$y_f = y_0 e^{-\frac{1}{3}\left(t_f^3 - t_0^3\right)}$$

Note that I used definite integrals here.
This is MUCH easier and faster.
Please learn how to do this!!!!

Example: Air Quality in Utah Valley

- Let ρ_p = concentration of pollutants (kg of pollutants/m^3)
- Assume valley is a vessel
- Wind comes along
- Perfectly mixed ($\rho_p = \rho_{p,out}$)
- Find ρ_p as a function of time



What does the box represent?

Species Balance on "p"

Accumulation = In – Out + Generation - Consumption

$$\frac{dm_p}{dt} = \dot{m}_{p,in} - \dot{m}_{p,out} \qquad = \frac{\Delta m_p}{\Delta t}$$

$$m_p = \rho_p V_{valley}$$

$$\dot{m}_p = \rho_p v A = \rho_p \dot{V}$$

So the balance becomes:

Assume $\rho_{p,in} = 0$

$$\frac{d\rho_p V}{dt} = \rho_{p,in}\dot{V}_{wind} - \rho_p \dot{V}_{wind} = \dot{V}_w(0 - \rho_p)$$

$$V\frac{d\rho_p}{dt} = -\rho_p \dot{V}_w$$

Now Separate!

$$V\frac{d\rho_p}{dt} = -\rho_p \dot{V}_w$$

$$\Downarrow$$

$$\frac{d\rho_p}{\rho_p} = -\frac{\dot{V}_w}{V}dt$$

Now Integrate

$$\int_{\rho_{p,0}}^{\rho_{p,f}} \frac{d\rho_p}{\rho_p} = -\frac{\dot{V}_w}{V}\int_0^t dt$$

$$\ln\left(\frac{\rho_{p,f}}{\rho_{p,0}}\right) = -\frac{\dot{V}_w}{V}t$$

$$\rho_{p,f} = \rho_{p,0} e^{-\frac{\dot{V}_w}{V}t}$$

Consistency of Units

- In any equation, each term must have the same units (as we mentioned before)
- Consider the species mass balance equation:

$$\frac{d(?)}{dt} = \dot{m}_{a,in} - \dot{m}_{a,out}$$

- What should be the units where the "?" occurs?

Ans: mass of component a, NOT mass of a per time

- Getting the units right will help in determining what goes inside the derivative term
  – Only possible terms inside derivative are: m, m_i, n, n_i

Integral Balances

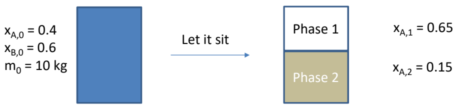
- Commonly used on batch processes
- Balance initial and final states
- No time dependence
- No in and out streams

80% A 20% B 0% C	→	40% A 40% B 20% C

Balance on A
0.8 m = 0.4 m + consumption
Or
Consumption = 0.8 – 0.4 = 0.4 m

Initial input + generation = final output + consumption
Note: Total mass is constant in a batch process

Example of Integral Balances



Find masses of phase 1 and phase 2

- Total mass balance

$$m_0 = 10 \text{ kg} = m_1 + m_2$$

- Balance on species A

$$x_{A,0}m_0 = x_{A1}m_1 + x_{A2}m_2$$

or

$$0.4\ (10 \text{ kg}) = 0.65\ m_1 + 0.15\ m_2$$

Algebra

$10 = m_1 + m_2$

$4 = 0.65\ m_1 + 0.15\ m_2$

$1.5 = 0.15\ m_1 + 0.15\ m_2$

$2.5 = 0.5\ m_1$

Multiply by 0.15

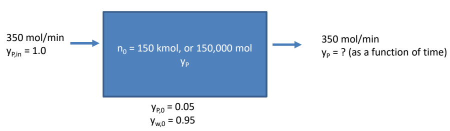
Now subtract to eliminate m_2

So $m_1 = 2.5/0.5 = 5$ kg,

and therefore $m_2 = 10 – 5 = 5$ kg

Special Problem 4.1

Click to see problem



Accum = in – out + gen - cons