HW Assignment 21

7.16. A horizontal cylinder equipped with a frictionless piston contains 785 cm³ of steam at 400 K and 125 kPa. A total of 83.8 joules of heat is transferred to the steam, causing the steam temperature to rise and the cylinder volume to increase. A constant restraining force is maintained on the piston throughout the expansion, so that the pressure exerted by the piston on the steam remains constant at 125 kPa.

![Diagram of steam system](image)

The specific enthalpy of steam at 125 kPa varies with temperature approximately as

\[ \dot{H}(J/mol) = 34,980 + 35.5T(K) \]

(a) Taking the steam as the system, convince yourself that \( Q = \Delta H \) for this process—that is, the four conditions specified in part (a) of Problem 7.15 are applicable. Then prove that the final steam temperature is 480 K. Finally, calculate (i) the final cylinder volume, (ii) the expansion work done by the steam, and (iii) \( \Delta U(J) \).

(b) Which of the specified conditions of Problem 7.15 would have been only an approximation if the cylinder were not horizontal?

7.21 is to be done in the workbook pages which are included in this document.

7.22. Steam at 250°C and 7.00 bar absolute is expanded through a nozzle to 200°C and 4.00 bar. Negligible heat is transferred from the steam to its surroundings. The approach velocity of the steam is negligible. The specific enthalpy of steam is 2974 J/kg at 250°C and 7 bar and 2860 kJ/kg at 200°C and 4 bar. Use the open-system energy balance to calculate the exit steam velocity.

Hint - There is a typo in the problem statement. The first enthalpy should have units of kJ/kg, not J/kg.

7.28 is to be done in the workbook pages which are included in this document.

Hint - Assume that the local atmospheric pressure is 1 bar.
(c) Suppose instead that the process is adiabatic. Is the final temperature greater than, equal to, or less than 30°C? Explain your reasoning.

Solution
Here is the way to think about it.

- When you push the piston to the right against the restraining force of the gas in the cylinder, you are transferring energy to the gas as work.
- The transferred energy must go somewhere, since energy is conserved. The only things that can happen to it are (a) being converted to kinetic or potential energy, (b) being transferred back out of the system as heat, or (c) being converted to internal energy.
- We ruled out (a) in the beginning, and if the process is adiabatic we know that no heat can be transferred, so that the work that went into the gas must go to raise the internal energy of the gas. We have seen that \( U \) increases with increasing temperature. If \( U \) is higher than it was initially, then \( T \) must also be higher, and so the final temperature \( T \) must be greater than 30°C.

In short, **compressing a gas adiabatically raises its temperature**. If we knew exactly how \( \dot{U} \) depended on \( T \) (which we will later in the text), we could calculate \( \Delta U \) from the energy balance equation and then calculate how much \( T \) rises, but since we don’t yet know the dependence we have done all we can.

**Exercise**

Explain why the converse of the behavior just described is also true: If a gas expands against a restraining force, either the gas gets colder or heat must be added to the gas to keep its temperature constant.

---

**PROBLEM 7.21**
The specific enthalpy of liquid \( n \)-hexane at 1 atm varies linearly with temperature and equals 25.8 \( \text{kJ/kg} \) at 30°C and 129.8 \( \text{kJ/kg} \) at 90°C.

(a) Determine the equation that relates \( \dot{H} \) (kJ/kg) to \( T \) (°C) and calculate the reference temperature on which the given enthalpies are based. Then derive an equation for \( \dot{U} \) (kJ/kg) at 1 atm.

**Solution**
The task is to determine \( a \) and \( b \) in the equation

\[
\dot{H} = aT + b
\]

\((T_1 = 30, \dot{H}_1 = 25.8), (T_2 = 50, \dot{H}_2 = 129.8)\)

You can substitute the given values and solve for \( a \) and \( b \) algebraically as shown in Chapter 2, or you can use E-Z. Solve to solve the following equations for \( a \) and \( b \):

\[
25.8 = a \cdot 30 + b
\]

\[
129.8 = a \cdot 50 + b
\]

Try it, and prove that

\[
\dot{H}(\text{kJ/kg}) = 5.22T(\degree \text{C}) - 130.2
\]

By definition, at the reference temperature \( T \) = 0. Thus, from the formula just derived,

\[
T_{\text{ref}} = ______
\]

(7.21-1)

From Eq. (7.4-7) on p. 321 of the text,

\[
\dot{U} = \dot{H} - P\dot{V}
\]

Remember that the specific volume \( (\dot{V}) \) (volume/mass) of a substance is the inverse of the density \( (\text{mass/volume}) \). From Table B.1 in the text, the specific gravity of liquid hexane is 0.659. Fill in the blanks, using the specific gravity and conversion factors from the inside front cover of the text.

\[
\dot{U} (\text{kJ/kg}) \quad \dot{H} - P\dot{V}
\]

\[
= (5.22T - 130.2) \left( \frac{\text{kJ}}{\text{kg}} \right) (1 \text{ atm}) \left( \frac{\text{N/m}^2}{\text{atm}} \right) \left( \frac{\text{m}^3}{\text{N} \cdot \text{m}} \right) \left( \frac{J}{\text{m}^3} \right) \left( \frac{1 \text{ kJ}}{10^3 \text{ J}} \right)
\]

\[
\Rightarrow \dot{U}(\text{kJ/kg}) = 5.22(T(\degree \text{C})) - 130.4
\]
(b) Calculate the average heat transfer rate required to cool 20.0 kg of liquid \( n \)-hexane from 80°C to 20°C in 5 min.

**Solution**

Since we are cooling a fixed mass of hexane and not a continuously flowing stream, this is a closed system. The closed system energy balance equation simplifies as shown below:

\[
Q - W = \Delta U + \Delta E_k + \Delta E_p
\]

\[
\begin{align*}
W &= 0 \quad \text{(Why?)} \\
\Delta E_k &= 0 \\
\Delta E_p &= 0
\end{align*}
\]

\[
Q(kJ) = \Delta U(kJ) = m(\text{kg}) \left[ \dot{U}_{\text{final}} - \dot{U}_{\text{initial}} \right] \left( \frac{\text{kJ}}{\text{kg}} \right)
\]

We can substitute the given value of \( m \) and the values of \( \dot{U} \) at the initial and final temperatures determined from the equation of Part (a) to calculate the total amount of heat transferred, \( Q \), and then divide \( Q \) by the cooling time to determine the average heat transfer rate.

\[
Q(kJ) = (20.0 \text{ kg}) \left[ (5.2 \times 130.4) - (5.2 \times -130.4) \right] \left( \frac{\text{kJ}}{\text{kg}} \right) = -6240 \text{ kJ}
\]

\[
\Rightarrow \text{ Average rate of heat removal (kW)} = \frac{-6240 \text{ kJ}}{5 \text{ min}}
\]
PROBLEM 7.28

Saturated steam at a gauge pressure of 2.0 bar is to be used to heat a stream of ethane. The ethane enters a heat exchanger at 16°C and 1.5 bar (gage) at a rate of 795 m³/min and is heated at constant pressure to 93°C. The steam condenses and leaves the exchanger as a liquid at 27°C. The specific enthalpy of ethane at the given pressure is 941 kJ/kg at 16°C and 1073 kJ/kg at 93°C.

The heat exchanger may be shown schematically as follows:

![Diagram of heat exchanger](image)

(For simplicity, we have assumed atmospheric pressure is 1 bar when converting the given gauge pressures to absolute pressures.)

The hot steam passes on one side of a metal barrier that is a good conductor of heat, and the cold ethane passes on the other side. Heat is transferred through the barrier from the steam to the ethane at the rate of $\dot{Q}$ (kW). The heat transferred to the ethane raises its temperature to 93°C. The heat transferred from the steam causes the steam to condense and the liquid condensate temperature to drop.

**Question**

How do we know the steam condenses immediately rather than being cooled to a lower temperature in the vapor phase? Answer below.

\[(7.28-1)\]

Following is some information that will help you understand how this process works. (The better you understand it, the less trouble you’re likely to have with the calculations.) The problem solution begins on the page after the second set of asterisks.

** ***

- Heat exchange is one of the most common operations in chemical processes, for obvious economic reasons. In the process shown, it is necessary to heat the ethane stream. If we did so by feeding the ethane through a gas-fired, oil-fired, or electrical heater, we would have to pay a high cost for the fuel or power consumption. If elsewhere in the process there is a hot stream that is to be either cooled down or emitted to the atmosphere, a heat exchanger can be used as a preheater for the ethane. Now, instead of having to pay for the energy that heats the cold stream, we recover energy that might otherwise be wasted by transferring it from the hot stream. (The heat exchanger certainly costs something to buy and maintain, but so would the heater.) If we were going to cool the hot stream anyway, the heat exchanger also saves us from having to pay the cost of the cooling.

- We are assuming that the outside of the heat exchanger is perfectly insulated, so that all the heat given up by the steam goes into the ethane. In a real heat exchanger, there would
always be some heat transferred through the outer wall. If the insulation is good, however, this leakage would be negligible compared to the heat transfer through the barrier, and so the values we are about to calculate would be quite accurate.

- Our flowchart shows countercurrent flow, meaning that the hot and cold streams flow in opposite directions through the unit. Co-current flow, in which the streams flow in the same direction, would not work for this process. Can you figure out why? (We'll supply the answer later in the problem.)

- In a real heat exchanger the steam and ethane streams would not simply flow on opposite sides of a wall. For example, one of the fluids might flow through the inside of a bundle of many narrow tubes mounted in a large cylindrical shell while the other might flow through this shell (on the outside of the tubes). The rate of heat transfer is proportional to the surface area of the interface between the two fluids. The large surface/volume ratio of a collection of small tubes means that a much smaller (in volume) and less expensive unit is required to get the same rate of heat transfer. For a picture of what this would look like, see the figure attached to workbook Problem 4.36 or the section on shell-and-tube heat exchangers in the Visual Encyclopedia of Chemical Engineering Equipment on the CD that came with the text.

- The description above applies to a "shell-and-tube" heat exchanger. There are many different configurations of shell-and-tube heat exchangers—they vary primarily in differences in the complexity of the flow on the shell side of the heat exchanger. There are other types of heat exchangers as well. For example, look up the "plate-and-frame" type exchanger on the internet and compare its contacting configuration to that of the shell-and-tube exchanger. Each type of heat exchanger has its own design nuances with higher levels of design complexity accompanying more complex flow contacting patterns.

***

(a) How much energy (kW) must be transferred to the ethane to heat it from 16°C to 93°C?

**Solution**

There are two different process subsystems that we could write energy balances on in addition to the complete heat exchanger: the ethane heating process and the steam condensation and cooling process. The flowcharts for the subsystems appear as follows:

![Flowchart](attachment:image.png)

Look at the three flowcharts (the complete heat exchanger, the ethane heating process, and the steam cooling and condensation process), and answer the following questions about the degree-of-freedom analyses.
Energy and Energy Balances 7-9

Ethane Heating Process Questions (fill in the blanks) (7.28-2)

Q: Why can’t you count any material balances for any of the processes?
A: 

Q: Why doesn’t \( \dot{Q} \) count as an unknown variable for the DOF analysis of the exchanger?
A: 

Q: How many degrees of freedom does each process system have?
A: Overall exchanger \( \) variables (___) – 1 gas eq. of state – 1 energy balance = 0 DF
Ethane heating \( \) variables (___) – (___) – (___) = 0 DF
Steam condensation/cooling \( \) variables (___) – (___) – (___) = 1 DF

Q: What strategy should be followed to calculate all unknown variables?
A: (i) 

(ii) 

The open system energy balance on the ethane heating process is written below. Using information on the flowchart, fill in the correct values for the gas variables (and conversion factors).

\[
m_c \left( \frac{\text{kg} \text{C}_2\text{H}_6}{\text{min}} \right) = n_c \left( \frac{\text{kmol} \text{C}_2\text{H}_6}{\text{min}} \right) \times 30.07 \left( \frac{\text{kg} \text{C}_2\text{H}_6}{\text{kmol}} \right) = \left( \frac{\dot{P} \dot{V}}{RT} \right) (30.07) = \frac{\text{bar}}{\text{K}} \frac{\text{m}^3}{\text{min}} \frac{\text{L}}{1} \frac{\text{mol} \cdot \text{K}}{g} \frac{1 \text{kg}}{10^3 \text{g}} \\
= 2.487 \times 10^3 \text{ kg C}_2\text{H}_6/\text{min} \tag{7.28-3}
\]

\[
\dot{Q} - \dot{W} = \Delta H + \Delta E_k + \Delta E_p \\
\dot{W} = 0 \quad \text{(Why?)} \quad \text{(___)} \\
\Delta E_k = 0 \quad \text{(___)} \\
\Delta E_p = 0 \quad \text{(___)} \\
\dot{Q} = \Delta H = \dot{m}_c \left[ \dot{H}_\text{out} - \dot{H}_\text{in} \right] = \frac{\text{kg}}{\text{min}} \left( \frac{\text{___} - \text{___}}{} \right) \text{kJ} \text{1 min} \frac{\text{1 kW}}{1 \text{kJ/s}} = \frac{\text{kg}}{60 \text{ s}} = \frac{\text{kJ}}{60 \text{ s}} \tag{7.28-4}
\]
(b) Assuming that all the energy transferred from the steam goes to heat the ethane, at what rate in m³/s must steam be supplied to the exchanger? If the assumption is incorrect, would the calculated value be too high or too low?

**Strategy**

An energy balance on the steam condensation/cooling process is required. We know the flow rate of the ethane and its temperature change so we can calculate the amount of energy needed to heat this gas. All the energy is assumed to come from the steam. From the conditions of the steam, we can determine how much energy per kg of steam is released when the steam condenses and cools and, therefore, how much steam is required to deliver the computed amount of energy to the ethane.

In the energy balance equation, we will need values for the specific enthalpy of steam at the inlet condition (saturated, 2.0 bar) and of liquid water at the outlet condition (liquid, 27°C), and to convert the mass flow rate of the inlet steam to the corresponding volumetric flow rate we’ll need the specific volume of the steam.

**Q:** Where in the book can we find the required values of \( \hat{H} \) and \( \hat{V} \) for the inlet steam?

**A:**

**Q:** Where in the book can we find the required value of \( \hat{H} \) for the outlet liquid?

**A:**

**Q:** The value of \( \hat{H} \) in Table B.5 for liquid water at 27°C applies to a pressure that is probably different from the outlet water pressure in the process (which we don’t know). Why doesn’t it matter?

**A:**

The open system energy balance for this system is (fill in the blanks):

\[
\dot{Q} - \dot{W}_i = \Delta \hat{H} + \Delta \hat{E}_k + \Delta \hat{E}_p
\]

\[
\begin{align*}
\dot{W}_i & = 0 \quad \text{(no moving parts)} \\
\Delta \hat{E}_k & = 0 \quad \text{(neglect kinetic energy change from inlet to outlet)} \\
\Delta \hat{E}_p & = 0 \quad \text{(neglect vertical displacement between inlet and outlet)}
\end{align*}
\]

\[
\dot{Q} = \Delta \hat{H} = \dot{m}_w \left( \hat{H}_{out} - \hat{H}_{in} \right)
\]

\[
\Rightarrow - \quad \frac{\dot{Q}}{\text{kW}} = \dot{m}_w (\text{kg/s}) \cdot \frac{\text{(_____ - _____)}}{\text{kJ/kg}} \cdot \frac{1 \text{ kW}}{1 \text{ kJ/s}}
\]

\[
\Rightarrow \dot{m}_w = \quad \text{kg/s} \quad \Rightarrow \dot{V}_{\text{steam}} = \dot{m}_w \left( \frac{\text{kg}}{\text{s}} \right) \left( \frac{\text{m}^3}{\text{kg}} \right) = \left( \frac{\text{kg}}{\text{s}} \right) \left( \frac{\text{m}^3}{\text{kg}} \right) = \quad \text{m}^3/\text{s}
\]

If heat is lost to the surroundings, the calculated value of \( \dot{m}_w \) is too _______.

**Explain:**

(c) Should the heat exchanger be set up for cocurrent or countercurrent flow? Explain. (*Hint:* Remember that heat always flows from a higher temperature to a lower temperature.)

**Solution**

The heat exchanger should use **countercurrent flow**. For cocurrent flow the exchanger would appear as follows:

\[
\begin{align*}
\dot{m}_w & [\text{kg H}_2\text{O(y)/min}] & & \dot{m}_w & [\text{kg H}_2\text{O(l)/min}] \\
3 \text{ bar, sat'd} & & 27^\circ\text{C} \\
\dot{m}_w & [\text{kg C}_2\text{H}_6/\text{min}] & & \dot{m}_w & [\text{kg C}_2\text{H}_6/\text{min}] \\
16^\circ\text{C}, 2.5 \text{ bar} & & 93^\circ\text{C}, 2.5 \text{ bar}
\end{align*}
\]

The problem with this setup is ____________________________

__________________________

__________________________

__________________________

__________________________

(7.28-8)