

## Class 7

# PI Control

## Chapter 7

1. What is the main point of this chapter?
2. This chapter describes the FOPDT model as a linear model. Clearly, the response is not linear with time. In what way is the model linear?
3. What is the difference between a self-regulating and non-self-regulating process? Qualitatively, sketch the response of each to a step change in input.

## Run Pumped Tank

In Manual Mode, change controller output

## Overdamped Process Model Forms

- Here we focus on the control of temperature, pressure, level, flow, density, concentration, etc. where the process streams are comprised of gases, liquids, slurries and melts
- These are *overdamped* processes because there is no natural tendency to oscillate
- They also tend to be *self regulating* because they seek a steady state operating level if all variables are held constant

## Self Regulating Overdamped Models

Constants

- First Order Plus Dead Time (FOPDT)

$$\tau_p \frac{dy(t)}{dt} + y(t) = K_p u(t - \theta_p)$$

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- Second Order Plus Dead Time (SOPDT)

$$\tau_{p1} \tau_{p2} \frac{d^2 y(t)}{dt^2} + (\tau_{p1} + \tau_{p2}) \frac{dy(t)}{dt} + y(t) = K_p u(t - \theta_p)$$

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- Second Order Plus Dead Time with Lead Time (SOPDT w/ L)

$$\tau_{p1} \tau_{p2} \frac{d^2 y(t)}{dt^2} + (\tau_{p1} + \tau_{p2}) \frac{dy(t)}{dt} + y(t) = K_p \left[ u(t - \theta_p) + \tau_L \frac{du(t - \theta_p)}{dt} \right]$$

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## Which One to Use?

One more adjustable parameter is better for data interpolation but makes it more risky to extrapolate beyond the bounds of the original data used to fit the model

*Choose the simplest model that fits of your data  
because it will provide the "safest" extrapolation*

## The PI Controller

- "Ideal" form of the PI Controller

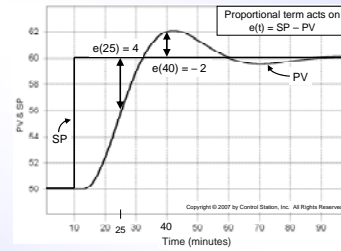
$$CO = CO_{bias} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt$$

where:

CO = controller output signal  
 CO<sub>bias</sub> = controller bias or null value  
 PV = measured process variable  
 SP = set point  
 e(t) = controller error = SP - PV  
 K<sub>c</sub> = controller gain (a tuning parameter)  
 τ<sub>I</sub> = controller **reset time** (a tuning parameter)

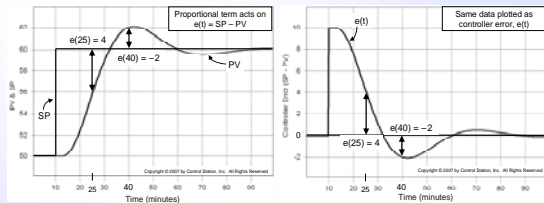
- τ<sub>I</sub> is in denominator so smaller values provide a larger weighting to the integral term
- τ<sub>I</sub> has units of time, and therefore is always positive

## Function of the Proportional Term



- The proportional term, K<sub>c</sub> · e(t), immediately impacts CO<sub>bias</sub> based on the size of e(t) at a particular time t
- The past history and current trajectory of the controller error have no influence on the proportional term computation

## Control Calculation is Based on Error, e(t)

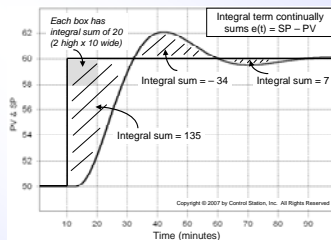


- Here is identical data plotted two ways
- To the right is a plot of error, where: e(t) = SP - PV
- Error e(t) continually changes size and sign with time

## Function of the Integral Term

- The integral term continually sums up error, e(t)
- Through constant summing, integral action accumulates influence based on how long and how far the measured PV has been from SP over time.
- Even a small error, if it persists, will have a sum total that grows over time and the amount added to CO<sub>bias</sub> will similarly grow.
- The continual summing of integration starts from the moment the controller is put in automatic

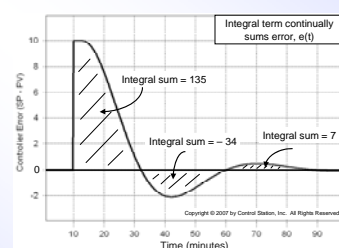
## Integral Term Continually Sums the Value: SP - PV



- The integral is the sum of the area between SP and PV
- At t = 32 min, when the PV first reaches the SP, the integral is:

$$\int_{0 \text{ min}}^{32 \text{ min}} e(t) dt = 135$$

## Integral of Error is the Same as Integral of: SP - PV



- At t = 60 min, the total integral is: 135 - 34 = 101
- When the dynamics have ended, e(t) is constant at zero and the total integral has a final residual value: 135 - 34 + 7 = 108

## Advantage of PI Control – No Offset

- The PI controller stops computing changes in CO when  $e(t)$  equals zero for a sustained period

$$CO = CO_{bias} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt$$

- At that point, the proportional term equals zero, and the integral term may have a residual value

$$CO = CO_{bias} + 0 + \frac{K_c}{\tau_I} (108)$$

*Integral acts as  
"moving bias" term*

- This residual value, when added to  $CO_{bias}$ , essentially creates an overall "moving bias" that tracks changes in operating level
- This moving bias eliminates offset, making PI control the **most widely used industry algorithm**

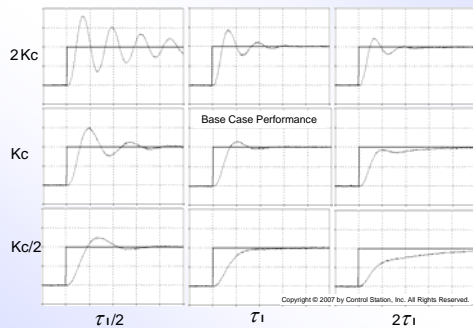
## Disadvantages of PI Control - Interaction

- Integral action tends to increase the oscillatory or rolling behavior of the PV
- There are *two* tuning parameters ( $K_c$  and  $\tau_I$ ) and they interact with each other

$$CO = CO_{bias} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt$$

- This interaction can make it challenging to arrive at "best" tuning values

## PI Controller Tuning Guide (Figure 8.9)



## Reset Time vs Reset Rate

- Manufacturers confuse tuning and implementation by using different names and units for the same controller parameter:
  - Some use proportional band (PB) instead of controller gain ( $K_c$ )
  - Some use reset rate ( $\tau_R$ ) instead of reset time ( $\tau_I$ ), where:

$$\tau_R = \frac{1}{\tau_I}$$

- Reset rate ( $\tau_R$ ) has units of 1/time or sometimes repeats/minute

*Know your manufacturer  
before we start tuning a controller*

## Integral Action and Reset Windup

- The math makes it possible for the error sum (the integral) to grow very large.

$$CO = CO_{bias} + K_c \cdot e(t) + \frac{K_c}{\tau_I} \int e(t) dt$$

*integral*

- The integral term can grow so large that the total CO signal stops making sense (it can be signaling for a valve to be open 120% or negative 15%)
- "Windup" is when the CO grows to exceed the valve limits because the integral has reached a huge positive/negative value
- It is associated with the integral term, so it is called *reset windup*
- The controller can't regulate the process until the error changes sign and the integral term shrinks sufficiently so that the CO value again makes sense (moves between 0 – 100%).

## Reset Windup and Jacketing Logic

- Industrial controllers employ jacketing logic to halt integration when the CO reaches a maximum or minimum value
- Beware if you program your own controller because reset windup is a trap that novices fall into time and again
- If two controllers trade off regulation of a single PV (e.g. select control; override control), jacketing logic must instruct the inactive controller to stop integrating. Otherwise, that controller's integral term can wind up.

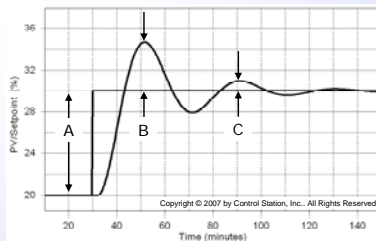
## Evaluating Controller Performance

- Bioreactors can't tolerate sudden operating changes because the fragile living cell cultures could die.
  - » "good" control means PV moves *slowly*
- Packaging/filling stations can be unreliable. Upstream process must ramp back quickly if a container filling station goes down.
  - » "good" control means PV moves *quickly*
- The operator or engineer defines what is good or best control performance based on their knowledge of:
  - goals of production
  - capabilities of the process
  - impact on down stream units
  - desires of management

## Many Performance Analysis Methods

- Classical "PV Response to Set Point Step" Analysis
  - Rise Time
  - Settling Time
  - Peak Overshoot Ratio
- Diagnostic Measures
  - Auto and Cross Correlation
  - Power Spectrum (Spectral Density)
- Performance Monitoring Indexes (Use Moving Window)
  - Moving Average Window
  - Relative Variance and Standard Deviation

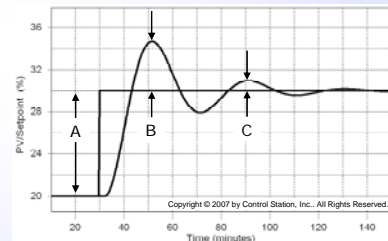
## Classical Analysis - Peak Related Criteria



- Peak Overshoot Ratio (POR) =  $B/A$
- Decay Ratio =  $C/B$

## Classical Analysis - Peak Related Criteria

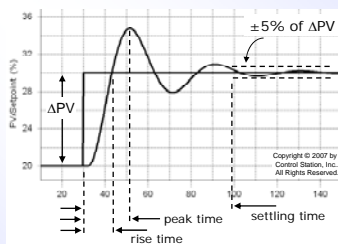
- $A = (30 - 20) = 10\%$
- $B = (34.5 - 30) = 4.5\%$
- $C = (31 - 30) = 1\%$



Here:

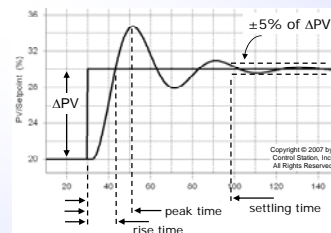
- POR =  $4.5/10 = 0.45$  or 45%
- Decay ratio =  $1/4.5 = 0.22$  or 22%

## Classical Analysis - Time Related Criteria



- The clock for time related events begins when the SP is stepped

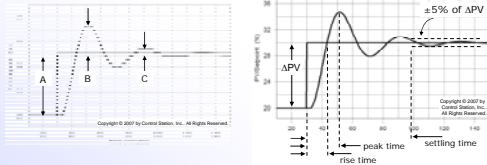
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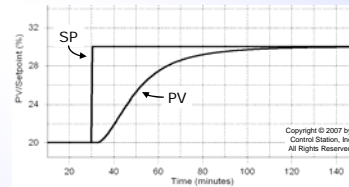
- $t_{rise} = 43 - 30 = 13 \text{ min}$
- $t_{peak} = 51 - 30 = 21 \text{ min}$
- $t_{settle} = 100 - 30 = 70 \text{ min}$

## Classical Analysis Note

- The classical criteria are not independent:
  - if decay ratio is large, then likely will have a long settling time
  - if rise time is long, then likely will have a long peak time

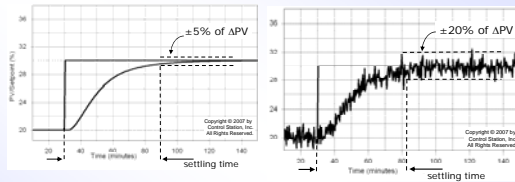


## Classical Analysis – What If No Peaks?



- Old rule-of-thumb is to design for a 10% POR and/or a 25% decay ratio (called a quarter decay)
- Yet many modern operations want no PV overshoot at all, making  $B = C = 0$
- With no peaks, the classical criteria are of limited value

## Settling Time Doesn't Use Peaks – But is Noise Specific



- Settling time is the time it takes for the PV to enter and remain within a band of operation - no peaks required
- A process with more noise must have a wider settling band