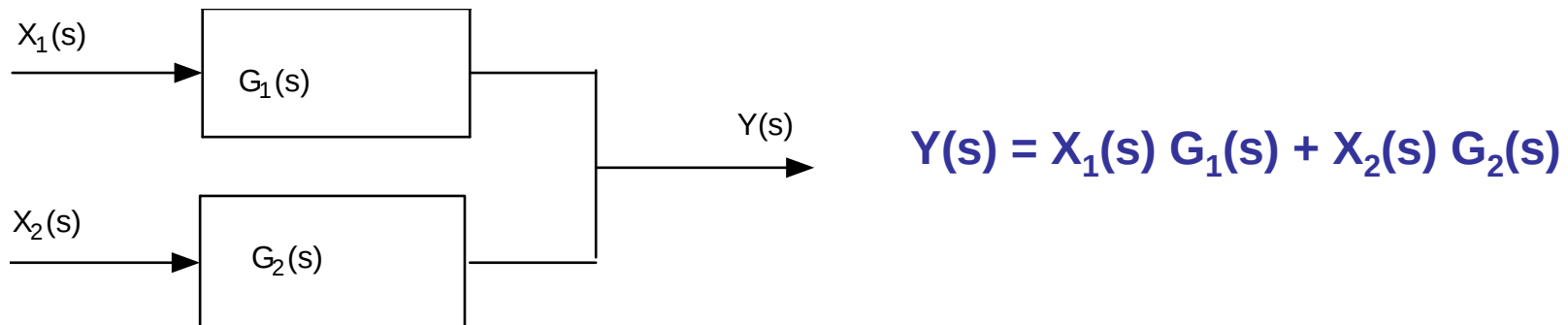


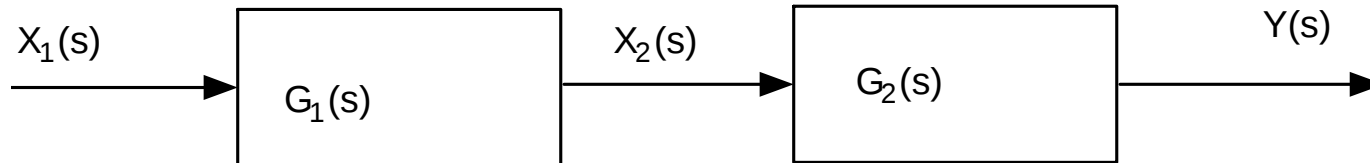
Transfer Functions

- A. Relate single input to single output.
- B. Represent an algebraic relationship in s domain.
- C. Can be combined to give the total system behavior.
- D. Convenient to use with block diagrams.
- E. Require initial conditions to be 0. (use deviation variables)
- F. Additive property (parallel process)



Transfer Functions (cont.)

G. Multiplicative property (series process)



$$\begin{aligned} Y(s) &= X_2(s) G_2(s) \\ X_2(s) &= X_1(s) G_1(s) \\ \text{So } Y(s) &= X_1(s) G_1(s) G_2(s) \end{aligned}$$

H. $\frac{K}{0}$ is the limit of $G(s)$ as s approaches 0 (for a unit step change) when the limit exists (see pg. 84).

I. Find the gain for the following transfer functions:

$$G_1(s) = \frac{1}{\tau s + 1} = 1$$

$$G_1(s) = \frac{a + bs}{\tau s + 1} = a$$

$$G_2(s) = \frac{8 + 2s}{(s + 3)(s + 2)} = \frac{8}{(3)(2)} = \frac{4}{3}$$

Group Activity

1. Find $C'(s)/C'_i(s)$ $\frac{dC'}{dt} = \frac{q}{V}C'_i - \frac{q}{V}C'$

$$sC'(s) = \frac{q}{V}C'_i(s) - \frac{q}{V}C'(s)$$

$$\left(s + \frac{q}{V}\right)C'(s) = \frac{q}{V}C'_i(s)$$

$$\left(\frac{V}{q}s + 1\right)C'(s) = C'_i(s)$$

$$\frac{C'(s)}{C'_i(s)} = \frac{1}{\left(\frac{V}{q}s + 1\right)}$$

$$\tau_p = \frac{V}{q}$$

$$K_p = 1$$

Group Activity

2. Find $C'(s)/C'_i(s)$

$$\frac{dC'}{dt} = \frac{q}{V} C'_i - \left(\frac{q}{V} + 2k_2 \bar{C} \right) C'$$

$$sC'(s) = \frac{q}{V} C'_i(s) - \frac{1}{\beta} C'(s)$$

$$1/\beta$$

$$(\beta s + 1)C'(s) = \beta \frac{q}{V} C'_i(s)$$

$$\frac{C'(s)}{C'_i(s)} = \frac{\beta \frac{q}{V}}{(\beta s + 1)}$$

$$\tau_p = \beta$$

$$K_p = \beta \frac{q}{V}$$

Group Activity

3. Find $T'(s)/T_i'(s)$ and $T'(s)/Q'(s)$

$$\frac{dT'}{dt} = \frac{q}{V}(T_i' - T') + \frac{Q'}{\rho V C_p}$$

$$\beta = \frac{q}{V}$$

$$\alpha = \frac{1}{\rho V C_p}$$

$$\frac{dT'}{dt} = \beta(T_i' - T') + \alpha Q'$$

$$sT'(s) = \beta T_i'(s) - \beta T'(s) + \alpha Q'(s)$$

$$(s + \beta)T'(s) = \beta T_i'(s) + \alpha Q'(s)$$

$$\left(\frac{1}{\beta}s + 1\right)T'(s) = T_i'(s) + \frac{\alpha}{\beta}Q'(s)$$

$$\frac{T'(s)}{T_i'(s)} = \frac{1}{\left(\frac{1}{\beta}s + 1\right)}$$

$$\frac{T'(s)}{Q'(s)} = \frac{\frac{\alpha}{\beta}}{\left(\frac{1}{\beta}s + 1\right)}$$