Dynamic Behavior and Stability of Closed-Loop Control Systems

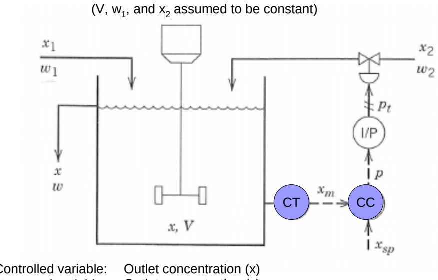
- We now want to consider the dynamic behavior of processes that are operated using feedback control.
- The combination of the process, the feedback controller, and the instrumentation is referred to as a *feedback control loop* or a *closed-loop system.*

Block Diagram Representation

To illustrate the development of a block diagram, we return to a previous example, the stirred-tank blending process considered in earlier chapters.

Composition control system for a stirred-tank blending process (Fig. 11.1)

(V, w_1, and x_2 assumed to be constant)



Controlled variable: Outlet concentration (x)
Measured variable: Outlet concentration (x)
Manipulated variable: Flow rate (w_2)
Disturbance variable: Inlet concentration (x_1)

Process

In section 4.3 the approximate dynamic model of a stirred-tank blending system was developed:

$$\rho V \frac{dx}{dt} = w_1 x_1 + w_2 x_2 - (w_1 + w_2) x$$

variables

$$f = w_1 x_1 + w_2 x_2 - (w_1 + w_2) x$$

$$\left[\frac{\partial f}{\partial x_1}\right]_{ss} = \overline{w}_1$$

$$\left[\frac{\partial f}{\partial w_2}\right]_{ss} = \overline{x}_2 - \overline{x} = 1 - \overline{x} \qquad (x_2 \equiv 1)$$

$$\left[\frac{\partial f}{\partial x}\right]_{ss} = -(\overline{w}_1 + \overline{w}_2)$$

Combining partial fractions and deviation variables,

$$\rho V \frac{dx'}{dt} = \overline{w}_1 x_1' + (1 - \overline{x}) w_2' - (\overline{w}) x'$$

$$\left(\frac{\rho V}{\overline{w}} s + 1\right) X'(s) = \frac{\overline{w}_1}{\overline{w}} X_1'(s) + \frac{(1 - \overline{x})}{\overline{w}} W_2'(s)$$

$$X'(s) = \frac{K_1}{\tau s + 1} X_1'(s) + \frac{K_2}{\tau s + 1} W_2'(s)$$

where

$$\tau = \frac{V\rho}{\overline{w}}, \quad K_1 = \frac{\overline{w}_1}{\overline{w}}, \quad and \quad K_2 = \frac{1 - \overline{x}}{\overline{w}}$$

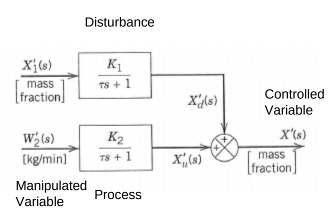
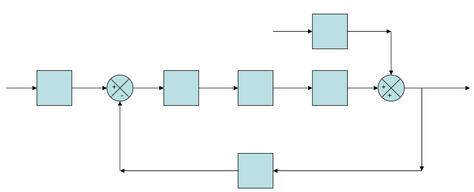


Figure 11.2 Block diagram of the process.

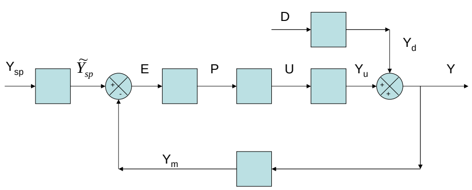
Wanted:

Transfer function for each piece of equipment



Please try to label variables, then transfer functions

Standard Labels



Definitions

$Y =$ controlled variable

$U =$ manipulated variable

$D =$ disturbance variable (also referred to as *load variable)*

$P =$ controller output

$E =$ error signal

$Y_m =$ measured value of Y

$Y_{sp} =$ set point

$\tilde{Y}_{sp} =$ internal set point (used by the controller)

$Y_u =$ change in Y due to U

$Y_d =$ change in Y due to D

$G_c =$ controller transfer function

$G_v =$ transfer function for final control element (including K_{IP}, if required)

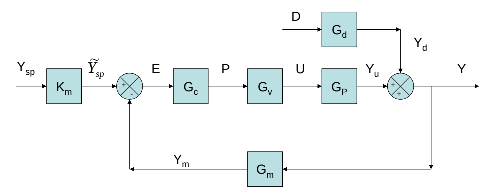
$G_p =$ process transfer function

$G_d =$ disturbance transfer function

$G_m =$ transfer function for measuring element and transmitter

$K_m =$ steady-state gain for G_m

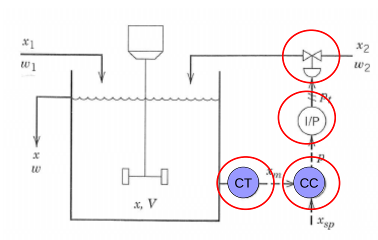
Transfer Functions



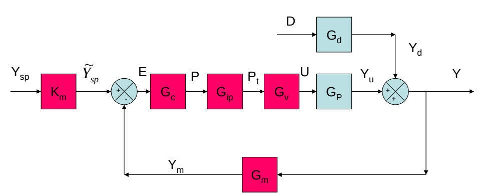
Now back to our problem

(Blending Tank)

Need transfer functions for:



Modified Block Diagram



Notes:

1. All variables are in deviation variables except E
2. All variables are in Laplace coordinates (i.e., Y'(s))
3. Pink boxes need transfer functions

Composition Sensor-Transmitter (Analyzer)

We assume that the dynamic behavior of the composition sensor-transmitter can be approximated by a first-order transfer function:

G_m
$$\frac{X'_m(s)}{X'(s)} = \frac{K_m}{\tau_m s + 1} \qquad (11\text{-}3)$$

Controller

Suppose that an electronic proportional plus integral controller is used. From Chapter 8, the controller transfer function is

G_c
$$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) \qquad (11\text{-}4)$$

where $P'(s)$ and $E(s)$ are the Laplace transforms of the controller output $p'(t)$ and the error signal $e(t)$. Note that p' and e are electrical signals that have units of mA, while K_c is dimensionless. The error signal is expressed as

$$e(t) = \tilde{x}'_{sp}(t) - x'_m(t) \qquad (11\text{-}5)$$

or after taking Laplace transforms,

$$E(s) = \tilde{X}'_{sp}(s) - X'_m(s) \qquad (11\text{-}6)$$

The symbol $\tilde{x}'_{sp}(t)$ denotes the *internal set-point* composition expressed as an equivalent electrical current signal. This signal is used internally by the controller. $\tilde{x}'_{sp}(t)$ is related to the actual composition set point $x'_{sp}(t)$ by the composition sensor-transmitter gain K_m:

$$\tilde{x}'_{sp}(t) = K_m x'_{sp}(t) \qquad (11\text{-}7)$$

Thus

K_m

$$\frac{\tilde{X}'_{sp}(s)}{X'_{sp}(s)} = K_m \qquad (11\text{-}8)$$

Current-to-Pressure (I/P) Transducer

Because transducers are usually designed to have linear characteristics and negligible (fast) dynamics, we assume that the transducer transfer function merely consists of a steady-state gain K_{IP}:

G_{ip}

$$\frac{P'_t(s)}{P'(s)} = K_{IP} \qquad (11\text{-}9)$$

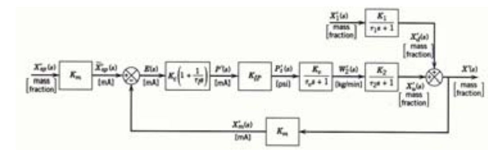
Control Valve

As discussed in Section 9.2, control valves are usually designed so that the flow rate through the valve is a nearly linear function of the signal to the valve actuator. Therefore, a first-order transfer function usually provides an adequate model for operation of an installed valve in the vicinity of a nominal steady state. Thus, we assume that the control valve can be modeled as

G_v

$$\frac{W'_2(s)}{P'_t(s)} = \frac{K_v}{\tau_v s + 1} \qquad (11\text{-}10)$$

Block diagram for the entire blending process composition control system (Fig 11.7)



Done!

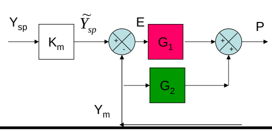
What about PID with derivative on measurement?

$$P(t) = K_c\left[e(t) + \frac{1}{\tau_I}\int_0^t e(t)\right] - K_C\tau_D\frac{dY_m}{dt}$$

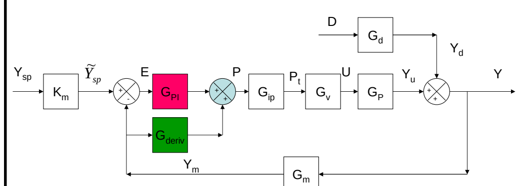
$$P'(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)E(s) - K_c\tau_D s Y_m(s)$$

$G_1(s)$ $\qquad$ $G_2(s)$

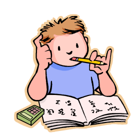
$$E(s) = \tilde{Y}'_{sp} - Y'_m$$



PID w/Derivative on Measurement



Your Homework Problem



11.11

Problem 11.11

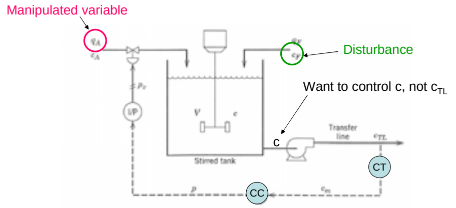


Figure E11.11

11.11 A mixing process consists of a single stirred-tank instrumented as shown in Fig. E11.11. The concentration of a single species A in the feed stream varies. The controller attempts to compensate for this by varying the flow rate of pure A through the control valve. The transmitter dynamics are negligible.

(a) Draw a block diagram for the controlled process.
(b) Derive a transfer function for each block in your block diagram.

1. First identify the controlled variable, manipulated variable, and disturbance variable.

Prob. 11.11

Process

(i) The volume is constant (5 m^3).
(ii) The feed flow rate is constant ($\bar{q}_F = 7$ m^3/min).
(iii) The flow rate of the A stream varies but is small compared to $\bar{q}_F$ ($\bar{q}_A = 0.5$ m^3/min).
(iv) $\bar{c}_F = 50$ kg/m^3 and $\bar{c}_A = 800$ kg/m^3.
(v) All densities are constant and equal.

Transfer Line

(i) The transfer line is 20 m long and has 0.5 m inside diameter.
(ii) Pump volume can be neglected.

Composition Transmitter Data

c (kg/m^3)	c_m (mA)
0	4
200	20

Transmitter dynamics are neglible.

PID Controller

(i) Derivative on measurement only (cf. Eq. 8-17)
(ii) Direct or reverse acting, as required
(iii) Current (mA) input and output signals

I/P Transducer Data

p (mA)	p_t (psig)
4	3
20	15

Control Valve

An equal percentage valve is used, which has the following relation:

$$q_A = 0.17 + 0.03\,(20)^{[illegible]}$$

For a step change in input pressure, the valve requires approximately 1 min to move to its new position.

Problem 11.11

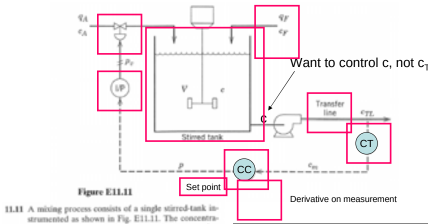


Figure E11.11

11.11 A mixing process consists of a single stirred-tank instrumented as shown in Fig. E11.11. The concentration of a single species A in the feed stream varies. The controller attempts to compensate for this by varying the flow rate of pure A through the control valve. The transmitter dynamics are negligible.

(a) Draw a block diagram for the controlled process.
(b) Derive a transfer function for each block in your block diagram.

2. Draw a block diagram similar to the one we did in class. My diagram has 9 boxes for transfer functions, including the unit conversion on the set point (K_m). Also, you will have a transfer function G_{TL} for the transport delay in the transfer line. The time delay box should be in the feedback loop, since it only represents the delay in measurement.