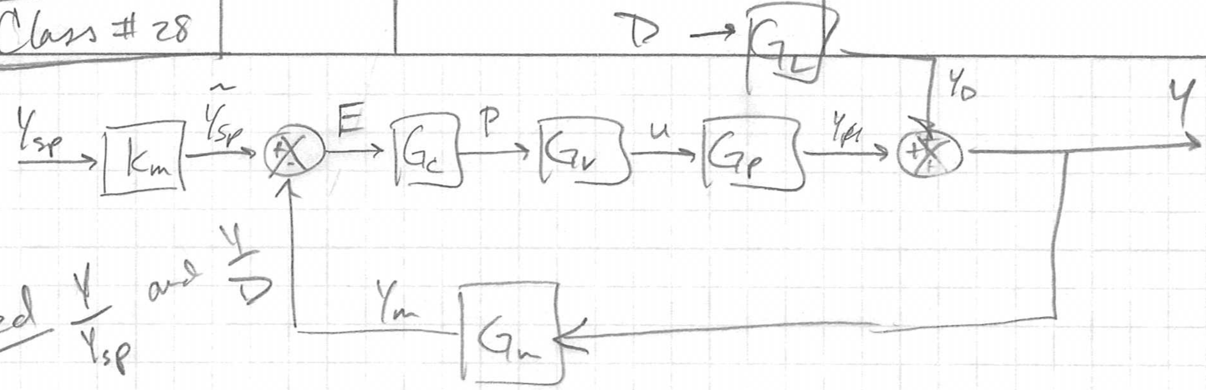
Class # 28






Wanted $\frac{Y}{Y_{sp}}$ and $\frac{Y}{D}$

$$Y = Y_D + Y_P$$

$$Y_D = D G_L \qquad Y_p = u G_p$$

so

$$Y = D G_L + u G_p$$

$$u = P G_v$$

$$P = E G_c$$

$$E = \tilde{Y}_{sp} - Y_m$$

$$Y = D G_L + (\tilde{Y}_{sp} - Y_m) G_c G_v G_p$$

$$\tilde{Y}_{sp} = Y_{sp} K_m \qquad Y_m = Y G_m$$

Combining: $Y = D G_L + (Y_{sp} K_m - Y G_m) G_c G_v G_p$

combine

$$Y(1 + G_m G_c G_v G_p) = D G_L + Y_{sp} K_m G_c G_v G_p$$

$$Y = \underbrace{\frac{G_L}{1 + G_m G_c G_v G_p}}_{\frac{Y}{D} \text{ transfer function}} D + \underbrace{\frac{K_m G_c G_v G_p}{1 + G_m G_c G_v G_p}}_{\frac{Y}{Y_{sp}} \text{ transfer function}} Y_{sp}$$

regulator problem
(disturbance change)

servo problem
(setpoint change)

Shortcut Method for SIMPLE loops

$$\frac{Z}{Z_i} = \frac{\pi_f}{1 + \pi_e}$$

$\pi_f$ — forward path from $Z_i$ to $Z$

$\pi_e$ — every transfer function in the feedback loop

So for the general case,

$$\pi_f = \text{path from } Y_{sp} \text{ to } Y$$

$$= k_m G_c G_v G_p$$

$$\pi_e = G_m G_c G_v G_p$$

$$\frac{Y}{Y_{sp}} = \frac{k_m G_c G_v G_p}{1 + G_m G_c G_v G_p} \qquad \text{same!}$$

$$\frac{Y}{D} = \frac{G_L}{1 + G_m G_c G_v G_p} \qquad !!$$

— Denominator shorthand

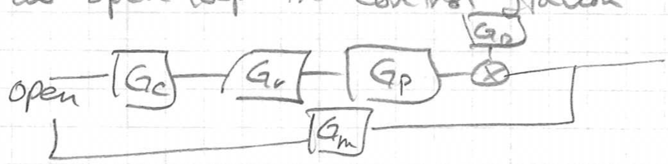
$$G_{OL} = G_m G_c G_v G_p$$

$G_{OL}$: open loop

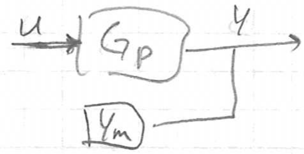
so denominator $= 1 + G_{OL}$

This is not the same as open loop in Control Station

$G_{OL}$ comes from



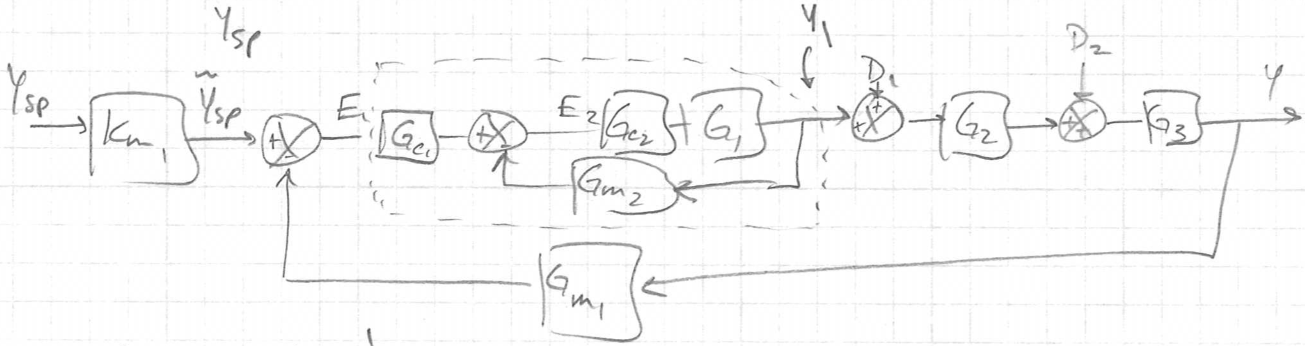

open loop Test in Control station




(manually change u, see what Y does)

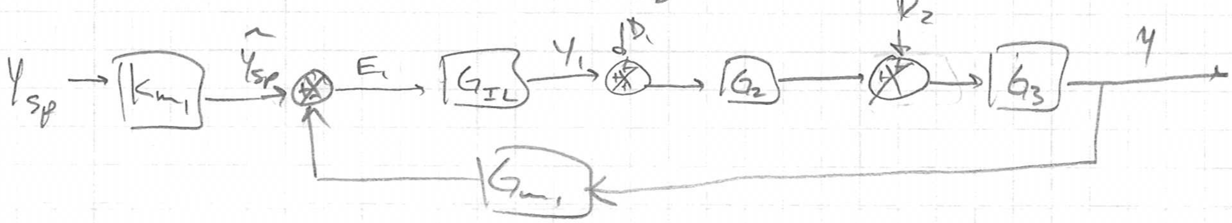
Derive the servo problem transfer function for Fig 11.12

Wanted: $\frac{Y}{Y_{sp}}$




Note inner loop!

$$\frac{Y_1}{E_1} = \frac{\text{direct}}{1+\text{loop}} = \frac{G_{c1}G_{c2}G_1}{1+G_{m2}G_{c2}G_1} = G_{IL}$$




— Now do shortcut again

$$\frac{Y}{Y_{sp}} = \frac{\text{direct}}{1+\text{loop}} = \frac{K_{m1}G_{IL}G_2G_3}{1+G_{m1}G_{IL}G_2G_3}$$

— Next substitute for $G_{IL}$

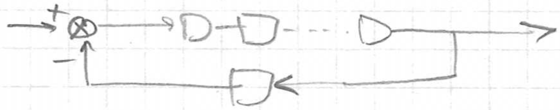
$$\frac{Y}{Y_{sp}} = \frac{K_m \frac{G_{c1}G_{c2}G_1}{1+G_{m2}G_{c2}G_1} G_2G_3}{1+G_{m1}\left(\frac{G_{c1}G_{c2}G_1}{1+G_{m2}G_{c2}G_1}\right)G_2G_3}$$

→ rearrange (multiply top & bottom by $1+G_{m2}G_{c2}G_1$

$$\frac{Y}{Y_{sp}} = \frac{K_m G_{c1}G_{c2}G_1G_2G_3}{1+G_{m2}G_{c2}G_1+G_{m1}G_{c1}G_{c2}G_1G_2G_3}$$

## Cautions:

1. Shortcut method only for simple feeback loop

(must have  form)

2. Know how to work both ways!

(shortcut and long hand)

## Homework Hint:

Stability → check poles

alternative: all polynomial coefficients in characteristic equation must be positive!

(not quite as good)

$$a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + a_0$$

↑ all of these must be positive

## Example

Characteristic Eqn: $1 + K_c (G_1 G_2 + G_3) = 0$

$$1 + K_c \left[ \left( \frac{10}{s-2} \right) + \frac{2}{3s+1} \right] = 0$$

What range of $K_c$ keeps things stable?

$$(s-2)(3s+1) + K_c \left[ 10(3s+1) + 2(s-2) \right] = 0$$

$$3s^2 - 5s - 2 + K_c (32s + 6) = 0$$

$$3s^2 + (32K_c - 5)s + (6K_c - 2) = 0$$

$K_c > \frac{5}{32} = .156$ $\qquad$ $K_c > \frac{2}{6} = \frac{1}{3} = .333$

greater of two, so keep $K_c > .333$ for closed loop stability