\[ Y_{sp} \rightarrow K_m \rightarrow Y_s \rightarrow E \rightarrow G_c \rightarrow P \rightarrow G_v \rightarrow u \rightarrow G_p \rightarrow Y \rightarrow Y_d \rightarrow Y \]

Wanted: \( Y / Y_{sp} \) and \( Y / D \)

\[ Y = Y_D + Y_P \]
\[ Y_D = D G_L \]
\[ Y_P = U G_p \]

so \[ Y = D G_L + U G_p \]
\[ U = P G_v \]
\[ P = E G_c \]
\[ E = Y_{sp} - Y_m \]

\[ \tilde{Y}_{sp} = Y_{sp} K_m \]
\[ Y_m = Y G_m \]

Combining:
\[ Y = D G_L + (Y_{sp} K_m - Y G_m) G_c G_v G_p \]

\[ Y = \frac{G_L}{1 + G_m G_c G_v G_p} D + \frac{K_m G_c G_v G_p}{1 + G_m G_c G_v G_p} Y_{sp} \]

\[ \frac{Y}{D} \text{ transfer function} \]
\[ \frac{Y}{Y_{sp}} \text{ transfer function} \]

regulator problem (disturbance change)

servo problem (set-point change)
Shortcut Method for SIMPLE loops

\[ \frac{Z}{Z_i} = \frac{\Pi_f}{1 + \Pi_e} \]

= forward path from \( Z_i \) to \( Z \)

= every transfer function in the feedback loop

So for the general case,

\[ \Pi_f = \text{path from } V_{sp} \text{ to } Y \]

\[ = k_m G_c G_r G_p \]

\[ \Pi_e = G_m G_c G_r G_p \]

\[ \frac{Y}{V_{sp}} = \frac{k_m G_c G_r G_p}{1 + G_m G_c G_r G_p} \]

\[ \frac{Y}{V_{sp}} = \frac{G_l}{1 + G_m G_c G_r G_p} \]

--- Denominator shorthand

\[ G_{ol} = G_m G_c G_r G_p \]

open loop

So denominator = \( 1 + G_{ol} \)

This is not the same as open loop in Control Station

\[ G_{ol} \text{ comes from } \]

open loop test
in Control Station

(openly change \( U \), see what \( Y \) does)
Derive the servo problem transfer function for Fig 11.12

Wanted: \[ \frac{Y}{Y_{sp}} \]

Note inner loop:

\[ Y_1 = \frac{G_{c1} G_{c2} G_1}{1 + \text{loop}} = \frac{G_{c1} G_{c2} G_1}{1 + G_{m2} G_{c2} G_1} = G_{IL} \]

Now do shortcut again

\[ \frac{Y}{Y_{sp}} = \frac{G_{m1} G_{IL} G_2 G_3}{1 + G_{m1} G_{IL} G_2 G_3} \]

Next substitute for \( G_{IL} \)

\[ \frac{Y}{Y_{sp}} = \frac{km}{1 + G_{m1} \left( \frac{G_{c1} G_{c2} G_1}{1 + G_{m2} G_{c2} G_1} \right) G_2 G_3} \]

→ rearrange (multiply top & bottom by \( 1 + G_{m2} G_{c2} G_1 \))

\[ \frac{Y}{Y_{sp}} = \frac{km G_{c1} G_{c2} G_1 G_2 G_3}{1 + G_{m2} G_{c2} G_1 + G_{m1} G_{c1} G_{c2} G_1 G_2 G_3} \]
Cautions:

1. Shortcut method only for simple feedback loop
   (must have \[ - \rightarrow D \rightarrow \cdots \rightarrow D \rightarrow \text{form} \])

2. Know how to work both ways!
   (shortcut and long hand)

Homework

Hint:

Stability → check poles
alternative: all polynomial coefficients in characteristic
  (but quite a good) equation must be positive!

\[ a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0 \]

all of these must be positive

Example

Characteristic Eqn: \[ 1 + k_c \left( G_1 G_2 + G_3 \right) = 0 \]

\[ 1 + k_c \left[ \left( \frac{10}{s-2} \right) + \frac{2}{3s+1} \right] = 0 \]

What range of \( k_c \) keeps things stable?

\[ (s-2)(3s+1) + k_c \left[ 10(3s+1) + 2(s-2) \right] = 0 \]

\[ 3s^2 - s - 2 + k_c (32s + 6) = 0 \]

\[ 2s^2 + (32k_c - 5)s + (6k_c - 2) = 0 \]

\[ k_c \geq \frac{5}{32} = .156 \quad k_c \geq \frac{2}{c} = \frac{4}{3} = .833 \]

greater of two, so keep \( k_c > .833 \)

for closed loop stability