Class # 29

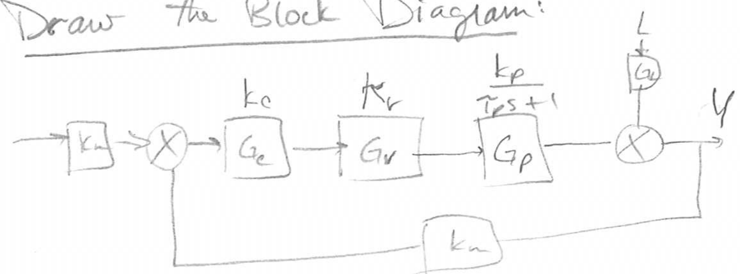
Closed Loop Systems

① PROPORTIONAL CONTROL

Simple Case

1 - Proportional Control
2 - Neglect valve dynamics
3 - Neglect measurement dynamics
4 - First order process
5 - pneumatic controller

A. Draw the Block Diagram:



B. What is Y/Y_{sp}? (shortcut method)

$$Y/Y_{sp} = \frac{k_m k_c k_v \frac{k_p}{\tau_p s+1}}{1 + k_m k_c k_v \frac{k_p}{\tau_p s+1}}$$

rearrange (multiply top & bottom by $\tau_p s+1$)

$$Y/Y_{sp} = \frac{k_m k_c k_v k_p}{\tau_p s + 1 + k_m k_c k_v k_p}$$

rearrange (divide by $(1+k_m k_c k_v k_p)$ to get $\tau' s+1$ form)

$$Y/Y_{sp} = \frac{\frac{k_m k_c k_v k_p}{1+k_m k_c k_v k_p}}{\frac{\tau_p}{1+k_m k_c k_v k_p} s + 1} = \frac{\frac{K_{OL}}{1+K_{OL}}}{\frac{\tau_p}{1+k_{OL}} s + 1} = \frac{K'}{\tau' s+1}$$

redefine terms

$$k_{OL} = k_m k_c k_v k_p$$

C. How does the magnitude of τ' compare to τ_p? why?

ans: if k_{OL} is finite, $\tau' < \tau_p$

How is τ' affected by k_c?

ans. - as $k_c \uparrow$, $\tau' \downarrow$ (faster response)

D. What is the time domain solution to a step change in Y_{sp}?

$$Y(s) = \left(\frac{K'}{\tau' s+1}\right)\left(\frac{M}{s}\right)$$

$$Y(t) = K'M\left(1 - e^{-t/\tau'}\right)$$

E. What is the steady-state value of Y_{sp}? (M)

What is the steady-state value of Y? $K'M = \frac{K_{OL}}{1+K_{OL}} M$

What is offset?

$$\text{offset} = Y_{sp}(\infty) - Y(\infty) = M - K'M = M(1-K')$$

$$= M\left(1 - \frac{K_{OL}}{1+K_{OL}}\right) = M\left(\frac{1+K_{OL} - K_{OL}}{1+K_{OL}}\right)$$

$$= M\left(\frac{1}{1+K_{OL}}\right)$$ see Eq. 11-43

F. So for proportional control, how does K_c affect offset?

As $K_c \uparrow$, offset $\downarrow$

G. Summary

As $K_c \uparrow$, offset $\downarrow$, $\tau' \uparrow$

see Figure 11.18

H. Same thing happens for $\frac{C}{L}$ (responses to step changes in Load)

② PI control

$$G_c = k_c\left(1 + \frac{1}{\tau_I s}\right)$$

A. What is Y/Y_{sp}?

$$\frac{Y}{Y_{sp}} = \frac{k_m\, k_c\left(1+\frac{1}{\tau_I s}\right) k_v\, k_p/(\tau_p s+1)}{1 + k_m\, k_c\left(1+\frac{1}{\tau_I s}\right) k_v \frac{k_p}{\tau_p s+1}}$$

B. Reduce (multiply by $\tau_p s + 1$)

$$\frac{Y}{Y_{sp}} = \frac{k_m\, k_c\, k_v\, k_p\left(1+\frac{1}{\tau_I s}\right)}{\tau_p s + 1 + k_m k_c k_v k_p\left(1+\frac{1}{\tau_I s}\right)}$$

C. Multiply top & bottom by $\tau_I s$

$$\frac{Y}{Y_{sp}} = \frac{k_m k_c k_v k_p(\tau_I s + 1)}{\tau_p \tau_I s^2 + \tau_I s + k_m k_c k_v k_p(\tau_I s + 1)}$$

$$= \frac{k_m k_c k_v k_p(\tau_I s + 1)}{\tau_p \tau_I s^2 + \tau_I(1 + k_m k_c k_v k_p)s + \underbrace{k_m k_c k_v k_p}_{\text{needs to be } 1}}$$

D. Place in standard 2nd order form (divide by $k_m k_c k_v k_p$)

$$\frac{Y}{Y_{sp}} = \frac{\tau_I s + 1}{\underbrace{\frac{\tau_p \tau_I}{k_m k_c k_v k_p}}_{(\tau')^2} s^2 + \underbrace{\tau_I\frac{(1 + k_m k_c k_v k_p)}{k_m k_c k_v k_p}}_{2\tau'\zeta} s + 1}$$

$$\tau' = \sqrt{\frac{\tau_p \tau_I}{k_m k_c k_v k_p}} \qquad \zeta = \frac{\frac{\tau_I(1 + k_m k_c k_v k_p)}{k_m k_c k_v k_p}}{2\sqrt{\frac{\tau_p \tau_I}{k_m k_c k_v k_p}}}$$

$$= \frac{(1 + k_m k_c k_v k_p)}{2\sqrt{k_m k_c k_v k_p}}\sqrt{\frac{\tau_I}{\tau_p}}$$

E. Stability?

ans: since ζ is positive, zeroes are negative, so stable roots!

(if $\zeta < 1$, underdamped)

② PI control (cont.)

F. So what did the addition of PI control do to the system?

→ 2^{nd} order

G. What is the steady-state value of Y in response to a unit step change in Y_{sp}?

$$Y(s) = \frac{(\tau_I s + 1)}{\tau'^2 s^2 + 2\tau'\zeta s + 1} \quad \frac{M}{s}$$

$$\lim_{s \to 0} s\,Y(s) = M$$

(no offset!)

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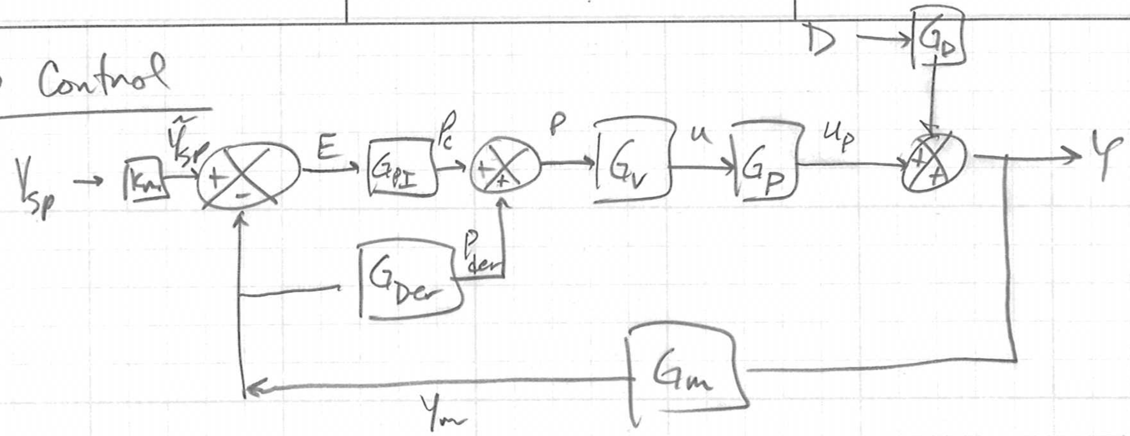
PID Control



$$Y = G_d D + P\, G_v G_p$$

$$P = P_c + P_{der}$$

$$P_c = G_{PI} E$$

$$P_{der} = Y_m G_{der}$$

$$E = \tilde{Y}_{sp} - Y_m = K_m Y_{sp} - G_m Y$$

so

$$P = G_{PI} K_m Y_{sp} - G_{PI} G_m Y + \underbrace{Y_m}_{G_m Y} G_{Der}$$

$$Y = G_d D + G_v G_p \left(G_{PI} K_m Y_{sp} - G_{PI} G_m Y + G_m G_{der} Y \right)$$

$$Y\left(1 + G_v G_p\left(G_{PI} G_m - G_m G_{der}\right)\right) = G_d D + G_v G_p G_{PI} K_m Y_{sp}$$

$$\frac{Y}{Y_{sp}} = \frac{K_m G_v G_p G_{PI}}{1 + G_v G_p G_m (G_{PI} - G_{der})}$$

$$G_v = K_v$$

$$G_p = 1^{st} \text{ order} = \frac{K_p}{\tau_p s + 1}$$

$$G_{PI} = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

$$G_{der} = -K_c \tau_{der} s$$

$$G_m = K_m$$

$$Y/Y_{sp} = \frac{K_m K_c \left(1 + \frac{1}{\tau_I s}\right) K_v \frac{K_p}{\tau_p s + 1}}{1 + K_m K_d \left(1 + \frac{1}{\tau_I s}\right) K_v \left(\frac{K_p}{\tau_p s + 1}\right) + K_c \tau_{der} s K_m K_v \left(\frac{K_p}{\tau_p s + 1}\right)}$$

next: (a) multiply top & bottom by $\tau_p s + 1$

(b) multiply top & bottom by $\tau_I s$

(a) multiply by $\tau_p s + 1$

$$\frac{Y}{Y_{sp}} = \frac{k_m k_c k_v k_p \left(1 + \frac{1}{\tau_I s}\right)}{(\tau_p s + 1) + k_m k_c k_v k_p \left(1 + \frac{1}{\tau_I s}\right) + k_c k_m k_v k_p \tau_{Der} s}$$

(b) multiply by $\tau_I s$

$$\frac{Y}{Y_{sp}} = \frac{k_m k_c k_v k_p (\tau_I s + 1)}{(\tau_p s + 1)\tau_I s + k_m k_c k_v k_p (\tau_I s + 1) + k_c k_m k_v k_p \tau_{Der} s (\tau_I s)}$$

now group s^2 terms, s terms, etc.

$$\frac{Y}{Y_{sp}} = \frac{k_m k_c k_v k_p (\tau_I s + 1)}{\tau_p \tau_I s^2 + \tau_I s + k_m k_c k_v k_p \tau_I s + k_m k_c k_v k_p + k_c k_m k_v k_p \tau_{Der} \tau_I s^2}$$

also divide by $k_m k_c k_v k_p = k_{OL}$

$$\frac{Y}{Y_{sp}} = \frac{\tau_I s + 1}{\left(\tau_{Der}\tau_I + \frac{\tau_p \tau_I}{k_m k_c k_v k_p}\right) s^2 + \tau_I \left(1 + \frac{1}{k_m k_c k_v k_p}\right) s + 1}$$

$$\frac{Y}{Y_{sp}} = \frac{\tau_I s + 1}{\left(\tau_{Der}\tau_I + \frac{\tau_p \tau_I}{K_{OL}}\right) s^2 + \tau_I \left(1 + \frac{1}{K_{OL}}\right) s + 1}$$

2^{nd} order $\Rightarrow$ get τ^2, ζ, k

TOPS.
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