General Statistics

Ch En 475
Unit Operations

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Quantifying variables (i.e. answering a question with a number)

1. Directly measure the variable.
   - referred to as "measured" variable

   ex. Temperature measured with thermocouple

2. Calculate variable from "measured" or "tabulated" variables
   - referred to as "calculated" variable

   ex. Flow rate $\dot{m} = \rho A v$ (measured or tabulated)

Each has some error or uncertainty

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Outline

1. Error of Measured Variables
2. Comparing Averages of Measured Variables

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1. Error of Measured Variable

Several measurements are obtained for a single variable (i.e. T).

Questions

- What is the true value?
- How confident are you?
- Is the value different on different days?

Some definitions:

$\bar{x}$ = sample mean
s = sample standard deviation

μ = exact mean
σ = exact standard deviation

As the sampling becomes larger:

$\bar{x} \rightarrow \mu$ $s \rightarrow \sigma$

t chart   z chart

not valid if bias exists (i.e. calibration is off)

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How do you determine the error?

- Let's assume "normal" Gaussian distribution
- For small sampling: s is known
- For large sampling: σ is assumed

$$\bar{x} = \sum_i \frac{x_i}{n}$$

small: $$s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$$

we'll pursue this approach

Use t tables for this approach

large (n>30): $$\sigma^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$$

Don't often have this much data

Use z tables for this approach

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Example

n	Temp
1	40.1
2	39.2
3	43.2
4	47.2
5	38.6
6	40.4
7	37.7

$$\bar{x} = \frac{(40.1 + 39.2 + 43.2 + 47.2 + 38.6 + 40.4 + 37.7)}{7} = 40.9$$

$$s^2 = \frac{1}{7-1}\begin{bmatrix}(40.1-40.9)^2 + (39.2-40.9)^2 + \\ (43.2-40.9)^2 + (47.2-40.9)^2 + \\ (38.6-40.9)^2 + (40.4-40.9)^2 + \\ (37.7-40.9)^2\end{bmatrix} = 10.7$$

$$s = 3.27$$

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Standard Deviation Summary

(normal distribution)

40.9 ± (3.27) 1s: 68.3% of data are within this range

40.9 ± (3.27x2) 2s: 95.4% of data are within this range

40.9 ± (3.27x3) 3s: 99.7% of data are within this range

If normal distribution is questionable, use Chebyshev's inequality:

At least 50% of the data are within 1.4 s from the mean.
At least 75% of the data are within 2 s from the mean.
At least 89% of the data are within 3 s from the mean.

Note: The above ranges don't state how accurate the mean is - only the % of data within the given range

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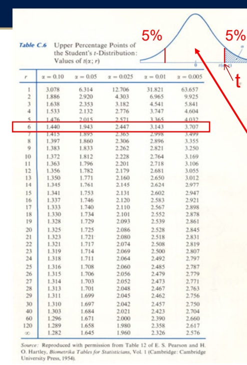
Student t-test (gives confidence of where μ (not data) is located)



measured mean

$\mu = \bar{x} \pm t \cdot \left(\frac{s}{\sqrt{n}}\right)$ where $t = f\left(\frac{\alpha}{2}, n-1\right)$

true mean

α=1- confidence

r = n-1 = 6

2-tail

$\mu = 40.9 \pm ?$

Conf.	α/2	t	+-
90%	.05	1.943	?
95%	.025	2.447	?
99%	.005	3.707	?

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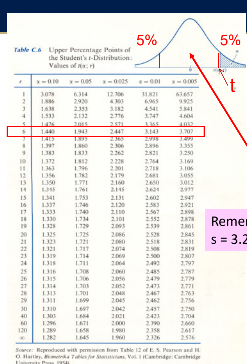

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r = n-1 = 6

2-tail

$\mu = 40.9 \pm ?$

Remember
s = 3.27

Conf.	α/2	t	+-
90%	.05	1.943	2.40
95%	.025	2.447	3.02
99%	.005	3.707	4.58

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t-test in Excel

- The one-tailed t-test function in Excel is:
  =T.INV(α,r)
  - Remember to put in α/2 for tests (i.e., 0.025 for 95% confidence interval)
- The two-tailed t-test function in Excel is:
  =T.INV.2T(α,r)

Where

- α is the probability
  - (i.e, .05 for 95% confidence interval for 2-tailed test) and
- r is the value of the degrees of freedom

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T-test Summary

μ= exact mean
40.9 is sample mean

40.9 ± 2.4 90% confident μ is somewhere in this range

40.9 ± 3.0 95% confident μ is somewhere in this range

40.9 ± 4.6 99% confident μ is somewhere in this range

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Outline

1. Error of Measured Variables
2. Comparing Averages of Measured Variables

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Comparing averages of measured variables

Experiments were completed on two separate days.

Day 1:	$\bar{x}_1 = 40.9$	$s_{x1} = 3.27$	$n_{x1} = 7$
Day 2:	$\bar{x}_2 = 37.2$	$s_{x2} = 2.67$	$n_{x2} = 9$

When comparing means at a given confidence level (e.g. 95%), is there a difference between the means?

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Comparing averages of measured variables

New formula:

Step 1 (compute T)

$$T = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_{x1}-1)s_{x1}^2 + (n_{x2}-1)s_{x2}^2}{n_{x1}+n_{x2}-2}\left(\frac{1}{n_{x1}} + \frac{1}{n_{x2}}\right)}}$$

For this example, $T = 2.5$

Larger $|T|$: More likely different

Step 2 Compute net r

r	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921

← $r = n_{x1}+n_{x2}-2$

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Comparing averages of measured variables

Step 3 Compute net t from net r

r	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
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Step 4 Compare $|T|$ with t

At a given confidence level (e.g. 95% or α=0.05), there is a difference if:

$$|T| > t\left(\frac{\alpha}{2}, r\right)$$

2-tail

T t

2.5 > 2.145

95% confident there is a difference!

(but not 98% confident)

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Example (Students work in Class)

1. Calculate $\bar{x}$ and s for both sets of data
2. Find range in which 95.4% of the data fall (for each set).
3. Determine range for μ for each set at 95% probability
4. At the 95% confidence level are the pressures different each day?

Data points	Pressure Day 1	Pressure Day 2
1	750	730
2	760	750
3	752	762
4	747	749
5	754	-

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