Convergence tricks

C. Pressure (SIMPLE) see Patankar (1980)

Momentum equations take the form:

$$A_p^u u_p = \sum_d A_d^u u_d + D^u (P_w - P_p) + S_u^u \qquad \text{(similar eqn. for } v_p\text{)}$$

This would be great if we knew what P field satisfied continuity, but we don't!

$$\underset{\text{true}}{P} = \underset{\text{guessed}}{P^*} + \underset{\text{correction}}{P'}$$
$$u = u^* + u'$$
etc.

Based on our best guess for the pressure field, then, u momentum:

$$A_p^u u_p^* = \sum_d A_d u_d^* + D^u (P_w^* - P_p^*) + S_u^u$$

Subtracting, to get terms like $u - u^* (= u_c^c)$,

$$A_p^u (u_p - u_p^*) = \sum_d A_d (u_d - u_d^*) + D^u \left[(P_w - P_w^*) - (P_p - P_p^*) \right] \qquad \text{(}S_u\text{ term goes away)}$$

OR

$$A_p^u u_p' = \underbrace{\sum_d A_d u_p'}_{\text{assumed to be small!}} + D^u (P_w' - P_p')$$

so
$$u_p' = \frac{D^u}{A_p^u} (P_w' - P_p')$$

Therefore,

$$u_p = u_p^* + u_p' = u_p^* + \frac{D^u}{A_p^u} (P_w' - P_p')$$

Use continuity equation:

$$\rho_e u_E A_e - \rho_w u_p A_w + \rho_n V_N A_n - \rho_s V_p A_s = 0 \qquad \text{in 2-D}$$

substitute $u_p = u_p^* + (\ \ )$ expressions

$$\rightarrow \boxed{A_p^{p'} P_p' = \sum A_d^{p'} P_d' + S_u^{p'}}$$

1. Guess p^*

2. Solve momentum eqns. to get u^*, v^*, etc. based on p^*

3. Solve continuity eqn. to get p'

4. Correct to get $u = u^* + u'$, etc.

5. Under-relax

6. Move on to other variables

→ other extensions, like SIMPLE, SIMPLER, etc.

Lecture 25

Turbulence

What Questions Do I Want to Answer?

1. What is turbulence?
2. How does it affect combustion?
3. How is it modeled in combustion systems

Turbulence

A. What is turbulence?

- random motion (chaotic)
- fluid has more energy than it knows what to do with
- Examples? (Ask students)
  - flame from campfire
  - clouds
  - jet plume
  - Hedman's burner
  - picture of gas flame by CE

  } see powerpoint slide
- Scale (micro $\xrightarrow{\text{continuous}}$ macro) all in same system
  → length scale

B. How does turbulence affect combustion?

1. Flame length (what would you think?)

$$\begin{cases} \text{laminar} & L \propto \dfrac{d^2 u}{\mathcal{D}} \\[2ex] \text{turbulent} & L \propto \dfrac{d^2 u}{\mathcal{D}_{turb}} \propto \dfrac{d^2 u}{d u} \propto d \quad !!! \end{cases}$$

In other words, the turbulent flame length is <u>only</u> dependent on jet nozzle diameter!

2. Affects mixing
   a. flame speed (see vugraf)
   b. concept of "wrinkled flame front"
   c. reaction zone thick compared to laminar flame

C. How does combustion affect turbulence?

→ tends to dampen turbulence

(densities are low due to high T, therefore $\rho = \bar{\rho} + \rho'$ effects are small)

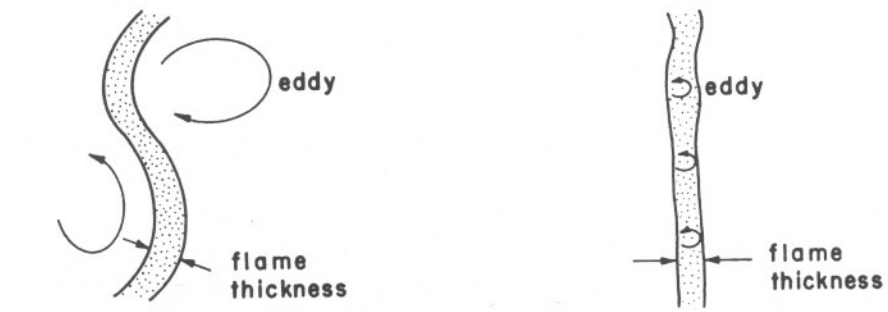


Figure 7.10 Effect of the scale of turbulence on the structure of the flame front.

(taken from kuo)

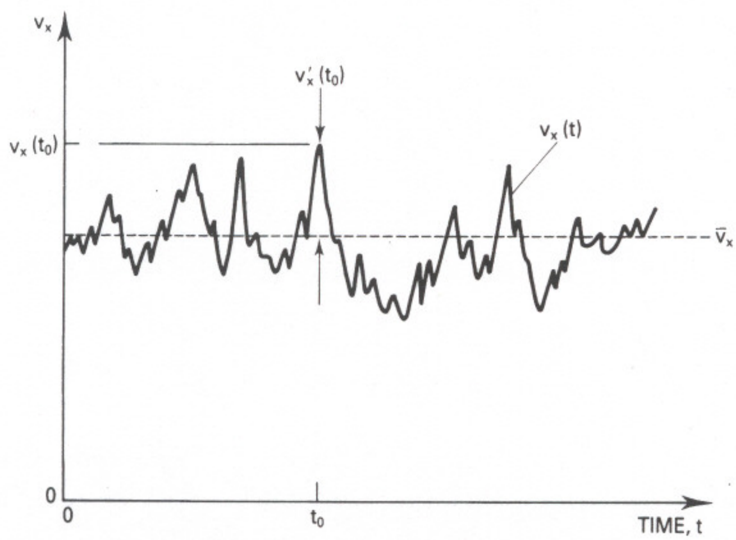


FIGURE 11.1
Velocity as a function of time at a fixed point in a turbulent flow.

(taken from Turns)

from Lewis & von Elbe, Combustion, Flames and Explosions of Gases (3rd Ed., 1987)

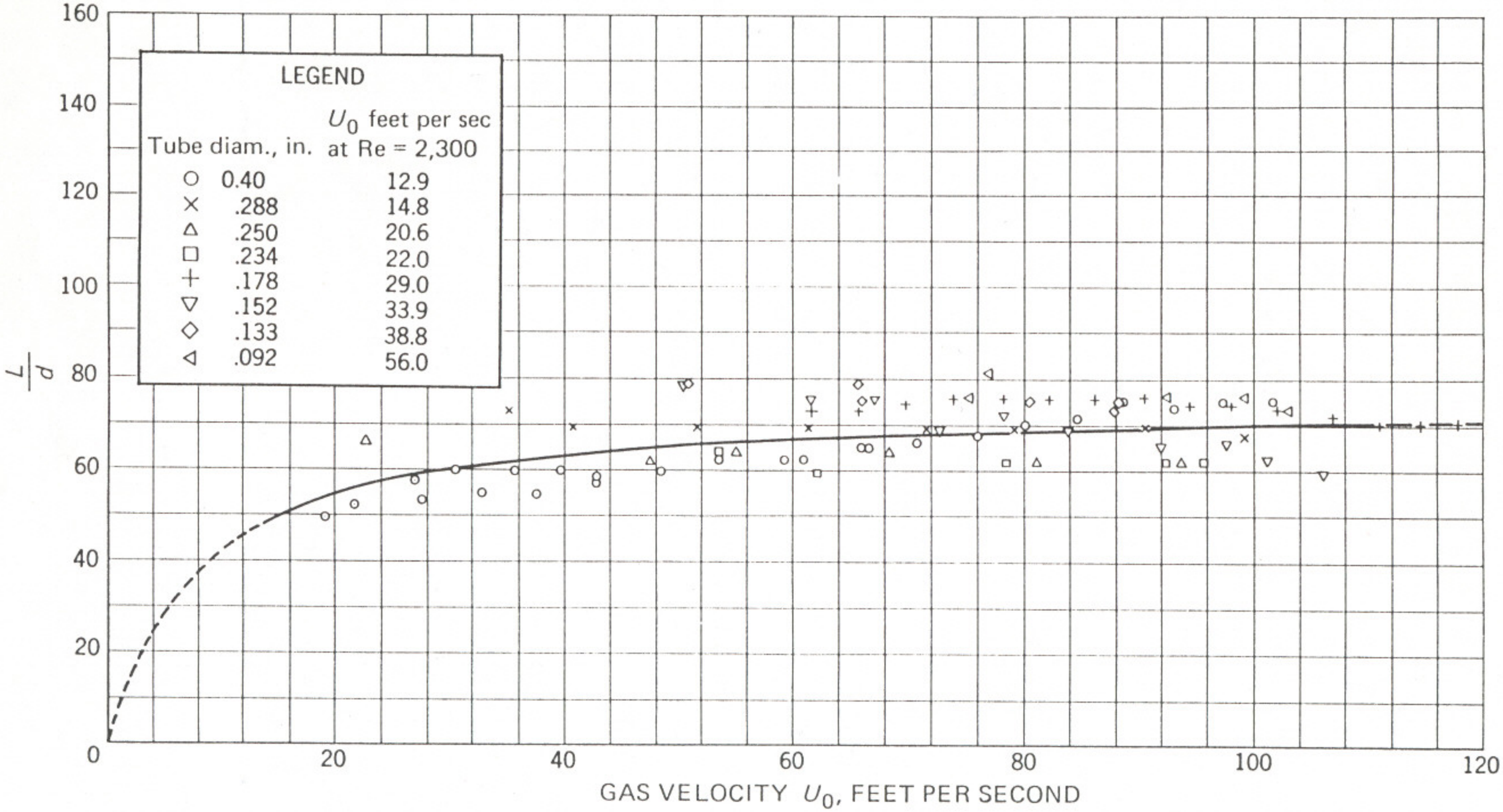


FIG. 267. Constancy of ratio L/d (flame length/port diameter) in turbulent flames. Mixture of 50% Newark, Delaware, city gas and 50% air. U_0, gas velocity at port. Flames burning free in air (Wohl, Gazley, and Kapp[3]).

Approaches to Modeling Turbulence (CFD)

A. Direct Numerical Simulation

→ Now possible for lazy turbulence in a box (transient)

→ used to generate "data" to test other models

B. Boussinesq analogy to laminar viscosity

$$-\overline{u'v'} = -\nu_g^t\left(\frac{\partial \bar{u}}{\partial r} + \frac{\partial \bar{v}}{\partial z}\right) - \underbrace{\frac{2}{3}\left(\nu_g^t \nabla u + k\right)\delta_{ij}}_{\text{sometimes ignored}}$$

i.e., use a turbulent viscosity ν_g^t to get Reynolds stress from gradients in mean velocity

Most popular ⇒ $k-\varepsilon$, where $\nu_g^t = c_\mu k^2/\varepsilon$

$$\underset{\text{transient}}{\frac{\partial k}{\partial t}} + \underset{\text{convection}}{\vec{\bar{u}}\cdot\vec{\nabla}k} = \underset{\text{diffusion}}{\vec{\nabla}\cdot\left(\frac{\nu_g^t}{\sigma_k}\vec{\nabla}k\right)} - \underset{\text{production}}{\overline{u_i' u_j'}\left(\frac{\partial \bar{u}_i}{\partial x_j}\right)} - \underset{\text{dissipation}}{\epsilon}$$

$k = \frac{1}{2}\overline{u_i' u_i'}$

Smoot & Smith (10.8)

$$\frac{\partial \epsilon}{\partial t} + \vec{\bar{u}}\cdot\vec{\nabla}\varepsilon = \vec{\nabla}\cdot\left(\frac{\nu_g^t}{\sigma_\varepsilon}\vec{\nabla}\varepsilon\right) + c_1\left(\frac{\epsilon}{k}\right)P - c_2\left(\frac{\epsilon}{k}\right)^2$$

P = production term from above

Smoot & Smith 10.10

bottom line: (1) fairly easy to use, does as good as most other models over a broad range

(2) can be adjusted for swirl

(3) 5 constants required

(4) non-linear k-ε (add extra terms) used in PCGC-3

(5) RNG-k-ε (add extra terms) used in Fluent

(6) need boundary conditions
(flows become laminar near the wall, so most use universal law of the wall parameters u^+ & y^+ related by logarithmic correlation)

3. Reynolds Stress Model

→ Don't use Boussinesq idea of turbulent viscosity

→ transport equations for Reynolds stress terms

$$\frac{\partial}{\partial t} \overline{u_i' u_j'} + u_k \frac{\partial}{\partial x_k} \left(\overline{u_i' u_j'} \right) = \ldots$$

→ some approximations involved still

→ Transport equations for $\overline{u'u'}$, $\overline{u'v'}$, $\overline{v'v'}$, $\overline{u'w'}$, $\overline{v'w'}$, $\overline{w'w'}$ — 6

ε — 1

$\overline{7}$ 7

plus use relationship

$$k = \frac{1}{2} \cdot \overline{u_i' u_i'}$$

→ Boundary conditions required for $\overline{u_i' u_i'}$, etc.

→ Stability & convergence problems
(Fluent suggests using k-ε first)

4. Algebraic stress model

→ more empiricism, less PDE's than Reynolds Stress Model

→ EXAMPLES (from Sloan)

GOOD REFERENCES

A. Sloan PhD Dissertation, ChE, BYU, 1984.

B. Sloan, et al., PECS, 12, 163-250 (1986)

Recommendations

General Purpose: (a) Stick with k-ε

(b) Combustion not as bad as with cold-flow

(c) keep doing research

where C_3 and C_4 are empirical constants whose values are $C_3 = 1.8$ and $C_4 = 0.60$, $P = \frac{1}{2}P_{ii}$, and

$$P_{ij} = -\overline{u_i'u_k'}\ \frac{\partial u_j}{\partial x_k} - \overline{u_j'u_k'}\ \frac{\partial u_i}{\partial x_k} \tag{6.2-17}$$

Finally, the dissipation term in Equation 6.2-14 is assumed to be isotropic and is approximated via the scalar dissipation rate[46]:

$$2\nu\overline{\frac{\partial u_i'}{\partial x_k}\ \frac{\partial u_j'}{\partial x_k}} = \frac{2}{3}\delta_{ij}\epsilon \tag{6.2-18}$$

The dissipation rate, ϵ, is computed via the transport equation, 6.2-6.

When the RSM is active, FLUENT thus solves:

- Six transport equations of the form 6.2-14 for the Reynolds stresses $\overline{u_i'u_j'}$ (or four transport equations in 2D Cartesian cases where the $\overline{v'w'}$ and $\overline{u'w'}$ terms are not active).
- The transport equation for ϵ, Equation 6.2-6.

In addition, FLUENT derives the turbulent kinetic energy, by definition, as:

$$k = \frac{1}{2}(\overline{u_i'^2}) \tag{6.2-19}$$

Thus no transport equation is solved for k when the RSM is active in FLUENT.

Boundary Conditions for the Reynolds Stresses

At flow inlets FLUENT requires values for each Reynolds stress, $\overline{u_i'u_j'}$, and for the turbulent dissipation rate, ϵ. These quantities can be input directly or derived from the turbulence intensity and characteristic length, as described below. At walls, FLUENT computes the near-wall values of the Reynolds stresses and ϵ from wall functions. In the orthogonal coordinate system uniquely defined by the wall normal vector (η) and the streamwise tangent vector (τ), the production tensor P_{ij} has the following components:

$$P_{\tau\tau} = 2P, \quad P_{\tau\eta} = \pm P$$

$$P_{\eta\eta} = P_{\lambda\lambda} = P_{\tau\lambda} = P_{\eta\lambda} = 0 \tag{6.2-20}$$

where λ is the vector normal to both η and τ, i.e. $\lambda = \tau \times \eta$. To determine the isotropic turbulence production term P, it is assumed that the production and dissipation of turbulence near the

from FLUENT Manual

$$\underbrace{\frac{\partial}{\partial t}(\overline{u_i'u_j'}) + u_k \frac{\partial}{\partial x_k}(\overline{u_i'u_j'}) =}_{\text{Convective Transport}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[(\overline{u_i'u_j'u_k'}) + \overline{\frac{p}{\rho}(\delta_{kj}u_i' + \delta_{ik}u_j')} - \nu\frac{\partial}{\partial x_k}(\overline{u_i'u_j'})\right]}_{\text{Diffusive Transport}}$$

$$\underbrace{-\left[\overline{u_i'u_k'}\ \frac{\partial u_j}{\partial x_k} + \overline{u_j'u_k'}\ \frac{\partial u_i}{\partial x_k}\right]}_{P_{ij} = \text{Production}} \tag{6.2-14}$$

$$\underbrace{+\overline{\frac{p}{\rho}\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right]}}_{\phi_{ij} = \text{Pressure-Strain}} \quad \underbrace{-2\nu\overline{\left[\frac{\partial u_i'}{\partial x_k}\ \frac{\partial u_j'}{\partial x_k}\right]}}_{\epsilon_{ij} = \text{Dissipation}}$$

$$\underbrace{-2\Omega_k\left[\overline{u_j'u_m'}\epsilon_{ikm} + \overline{u_i'u_m'}\epsilon_{jkm}\right]}_{R_{ij} = \text{Rotational Term}}$$

The individual Reynold's stresses are then substituted into the turbulent flow momentum equation, 6.2-2.

FLUENT approximates several of the terms in Equation 6.2-14 in order to close the equation set. First, the diffusive transport term is described using a scalar diffusion coefficient[24]:

$$\underbrace{-\frac{\partial}{\partial x_k}\left[(\overline{u_i'u_j'u_k'}) + \overline{\frac{p}{\rho}(\delta_{kj}u_i' + \delta_{ik}u_j')} - \nu\frac{\partial}{\partial x_k}(\overline{u_i'u_j'})\right]}_{\text{Diffusive Transport}} = \frac{\partial}{\partial x_k}\left(\frac{\nu_t}{\sigma_k}\ \frac{\partial(\overline{u_i'u_j'})}{\partial x_k}\right) \tag{6.2-15}$$

Secondly, the pressure-strain term is approximated as[25]:

$$\overline{\frac{p}{\rho}\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right]} = -C_3\frac{\epsilon}{k}\left[\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}k\right] - C_4\left[P_{ij} - \frac{2}{3}\delta_{ij}P\right] \tag{6.2-16}$$

from FLUENT manual

TABLE 7. Algebraic stress model in cylindrical coordinates

$\overline{v'v'}$ Equation

$$\left[1+\frac{4}{3}\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{v}}{\partial x}\right]\overline{v'v'}=\frac{2}{3}k+\frac{2}{3}\lambda\frac{k}{\varepsilon}\left\{\left(\frac{\partial\bar{u}}{\partial x}\right)\overline{u'u'}+\frac{\partial\bar{u}}{\partial r}-2\frac{\partial\bar{v}}{\partial x}\right\}\overline{v'u'}$$

$$+\left(\frac{\bar{v}}{r}\right)\overline{w'w'}+\left[\frac{\partial\bar{w}}{\partial r}+\frac{\bar{w}}{r}(2+3\beta)\right]\overline{v'w'}+\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{w'u'} \qquad (76)$$

$\overline{u'u'}$ Equation:

$$\left[1+\frac{4}{3}\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{u}}{\partial x}\right]\overline{u'u'}=\frac{2}{3}k+\frac{2}{3}\lambda\frac{k}{\varepsilon}\left\{\left(\frac{\partial\bar{v}}{\partial r}\right)\overline{v'v'}+\left(\frac{\partial\bar{v}}{\partial x}-2\frac{\partial\bar{u}}{\partial r}\right)\overline{v'u'}\right.$$

$$\left.+\left(\frac{\bar{v}}{r}\right)\overline{w'w'}+\left(\frac{\partial\bar{w}}{\partial r}-\frac{\bar{w}}{r}\right)\overline{v'w'}+\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{w'u'}\right\} \qquad (77)$$

$\overline{v'u'}$ Equation:

$$\left[1-\lambda\left(\frac{k}{\varepsilon}\right)\frac{\bar{v}}{r}\right]\overline{v'u'}=\lambda\left(\frac{k}{\varepsilon}\right)\left[-\left(\frac{\partial\bar{u}}{\partial r}\right)\overline{v'v'}-\left(\frac{\partial\bar{v}}{\partial x}\right)\overline{u'u'}+\left(\frac{\bar{w}}{r}\right)(1+\beta)\overline{w'u'}\right] \qquad (78)$$

$\overline{w'w'}$ Equation:

$$\left[1+\frac{4}{3}\lambda\left(\frac{k}{\varepsilon}\right)\frac{\bar{v}}{r}\right]\overline{w'w'}=\frac{2}{3}k+\frac{2}{3}\lambda\left(\frac{k}{\varepsilon}\right)\left\{\left(\frac{\partial\bar{v}}{\partial r}\right)\overline{v'v'}+\left(\frac{\partial\bar{u}}{\partial x}\right)\overline{u'u'}+\left(\frac{\partial\bar{v}}{\partial x}+\frac{\partial\bar{u}}{\partial r}\right)\overline{v'u'}\right.$$

$$\left.-\left[2\frac{\partial\bar{w}}{\partial r}+(1+3\beta)\frac{\bar{w}}{r}\right]\overline{v'w'}-2\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{w'u'}\right\} \qquad (79)$$

$\overline{v'w'}$ Equation:

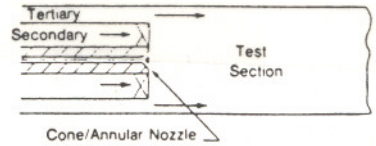
$$\left[1-\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{u}}{\partial x}\right]\overline{v'w'}=\lambda\left(\frac{k}{\varepsilon}\right)\left\{-\left(\frac{\partial\bar{w}}{\partial r}+\beta\frac{\bar{w}}{r}\right)\overline{v'v'}-\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{v'u'}+\left[(1+\beta)\frac{\bar{w}}{r}\right]\overline{w'w'}-\left(\frac{\partial\bar{v}}{\partial x}\right)\overline{w'u'}\right\} \qquad (80)$$

$\overline{w'u'}$ Equation:

$$\left[1-\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{v}}{\partial r}\right]\overline{w'u'}=\lambda\left(\frac{k}{\varepsilon}\right)\left\{-\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{u'u'}-\left(\frac{\partial\bar{w}}{\partial r}+\beta\frac{\bar{w}}{r}\right)\overline{v'u'}-\left(\frac{\partial\bar{u}}{\partial r}\right)\overline{v'w'}\right] \qquad (81)$$

where $\beta=\psi/(1-C_2)$ (82)

from Sloan, et al., PECS, 1984.



Cases 1 and 2—Brum and Samuelson[78]

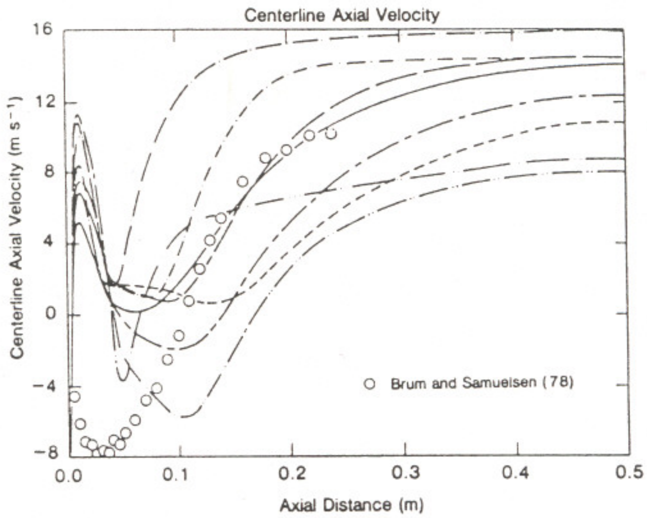


Fig. 4. Comparison of predicted and measured centerline axial and velocity profiles for Case 1 (data from Brum and Samuelsen[78]; legend supplied by Table 18).

Table 18. Legend description for case studies

		Case reference			
		Brum and Samuelsen[78]	Yoon[71]		Roback and Johnson[164]
Turbulence model description/legend	Equations of reference	1	4	5	6
Standard k–ε model ———	34–42				
LPS gradient Richardson no.* —··—··—	108–110	$C_{gs}=0.10$ MTS	$C_{gs}=0.005$ TTS	$C_{gs}=0.03$ MTS	$C_{gs}=0.005$ TTS
Rodi flux Richardson no.* —·—·—	114, 115	$C_{fs}=0.90$ MTS			$C_{fs}=0.90$ MTS
"Boysan" Richardson no. —···—···—	109, 112	$C_{gs}=0.20$			
Modified C_μ coefficient – – – – – –	134	$C_a=0.03$ $C_b=2.63$		$C_a=0.03$ $C_b=2.63$	
Gibson–Launder ASM* (I) ——— ——— ———	76–99	$C_1=2.5$ $C_2=0.55$	$C_1=2.5$ $C_2=0.55$	$C_1=2.5$ $C_2=0.55$	$C_1=2.5$ $C_2=0.55$
Gibson–Launder ASM (II) —-—-—-—-—	76–99	$C_1=2.0$ $C_2=0.40$			
Gibson–Launder ASM (III) added convection - - - - - - - -	76–99	$\psi=0.40$ $C_1=2.5$ $C_2=0.55$	$\psi=0.40$ $C_1=2.5$ $C_2=0.55$		

*MTS = Mean flow time scale in the denominator of the Richardson number.
TTS = Turbulence time scale in the denominator of the Richardson number.
ASM = Algebraic stress model.

from Sloan, et al., PECS, 1984

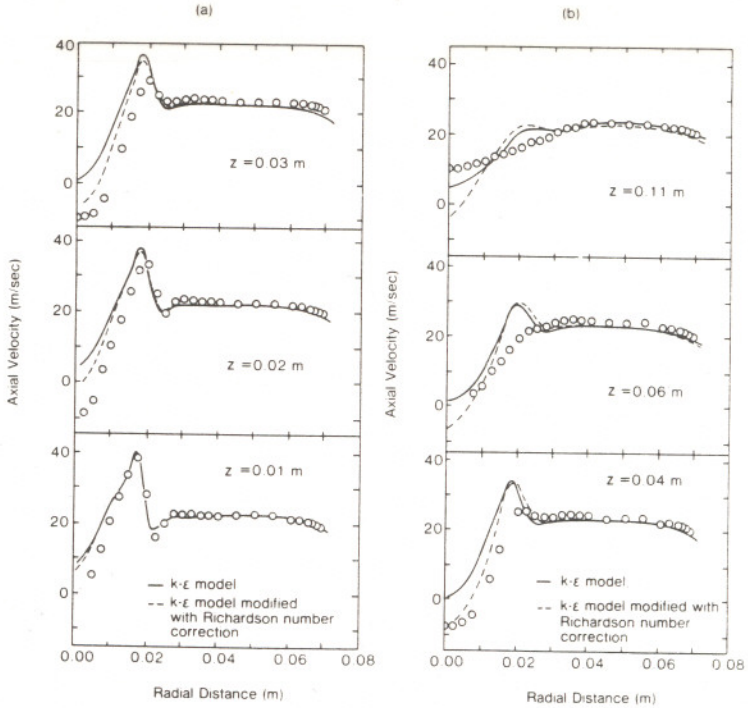


Figure 10.4. Comparison of predicted and measured axial velocity. (Data from Vu and Gouldin, 1982; prediction from Sloan, 1984). Conditions were those for a nonreacting coaxial, counterswirl jet. The swirl number of the inner jet was 0.49 and that of the annulus was −0.51.

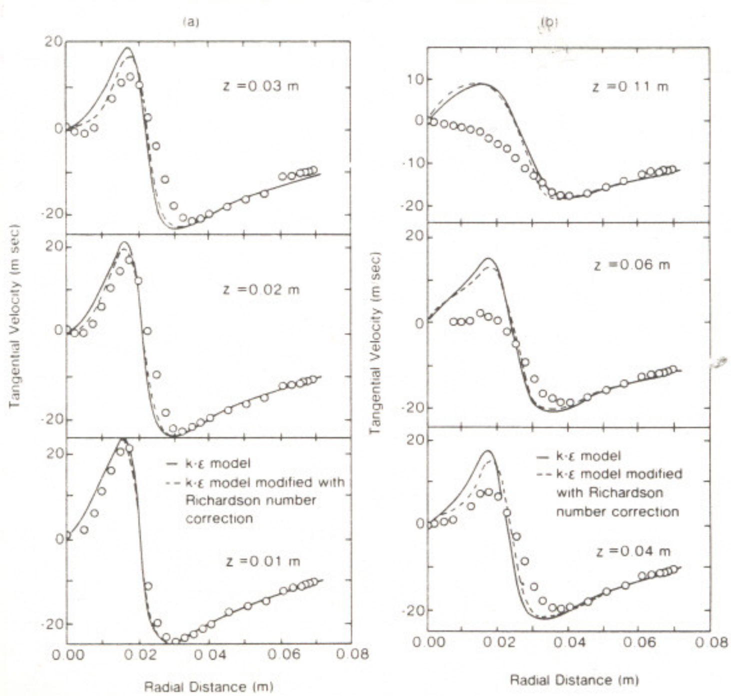


Figure 10.5. Comparison of predicted and measured tangential velocity. (Data from Vu and Gouldin, 1982; predictions from Sloan, 1984.) Conditions are those of Figure 10.4 and discussed further in text.

from Smoot & Smith, Coal Comb. & Gasif., Plenum (1985)

Non - Reacting

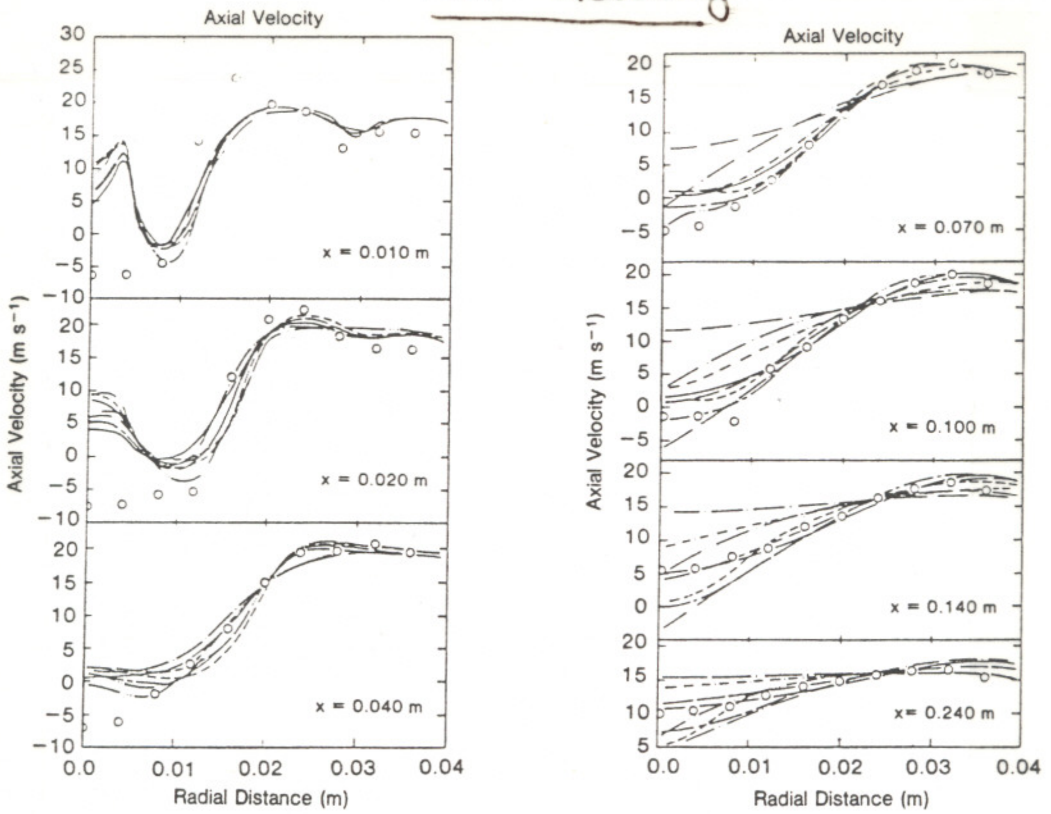


FIG. 5. Comparison of predicted and measured axial velocity profiles for Case 1 (data from Brum and Samuelsen[78]; legend supplied by Table 18).

Reacting

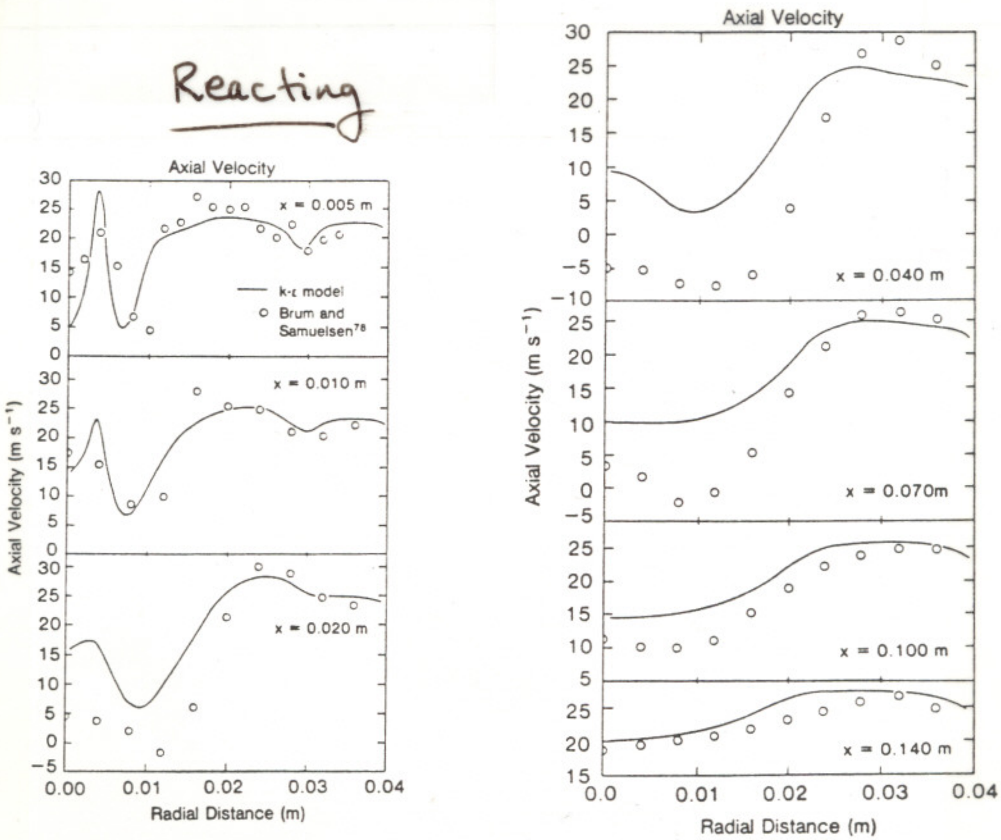


FIG. 8. Comparison of predicted and measured axial velocity profiles for Case 2 (data from Brum and Samuelsen[78]).

from Sloan, et al., PECS, 1984