# Turbulent Flames

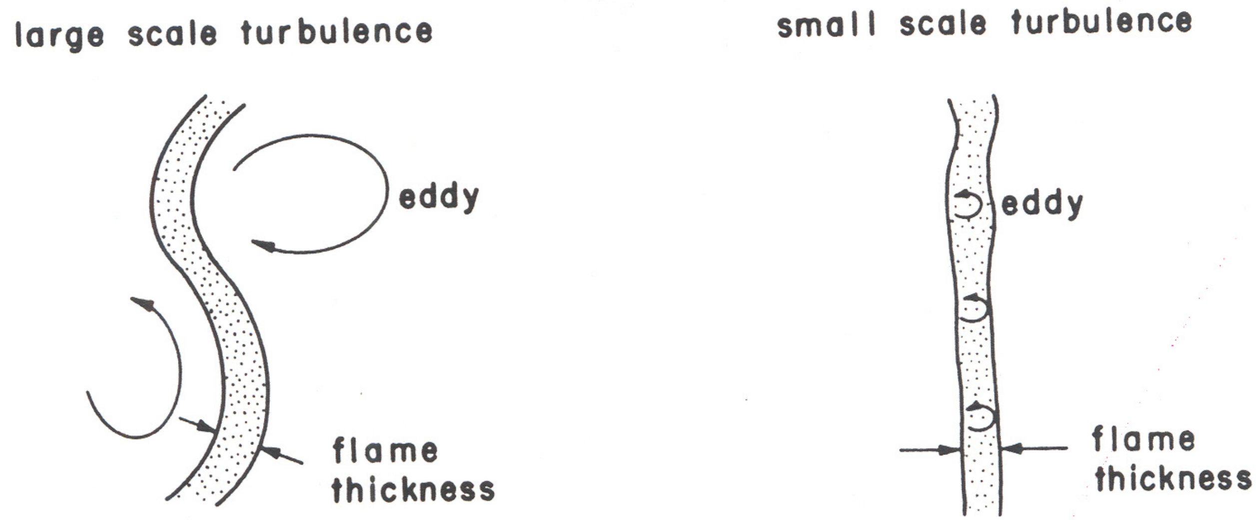



**Figure 7.10** Effect of the scale of turbulence on the structure of the flame front.

From Kuo's book

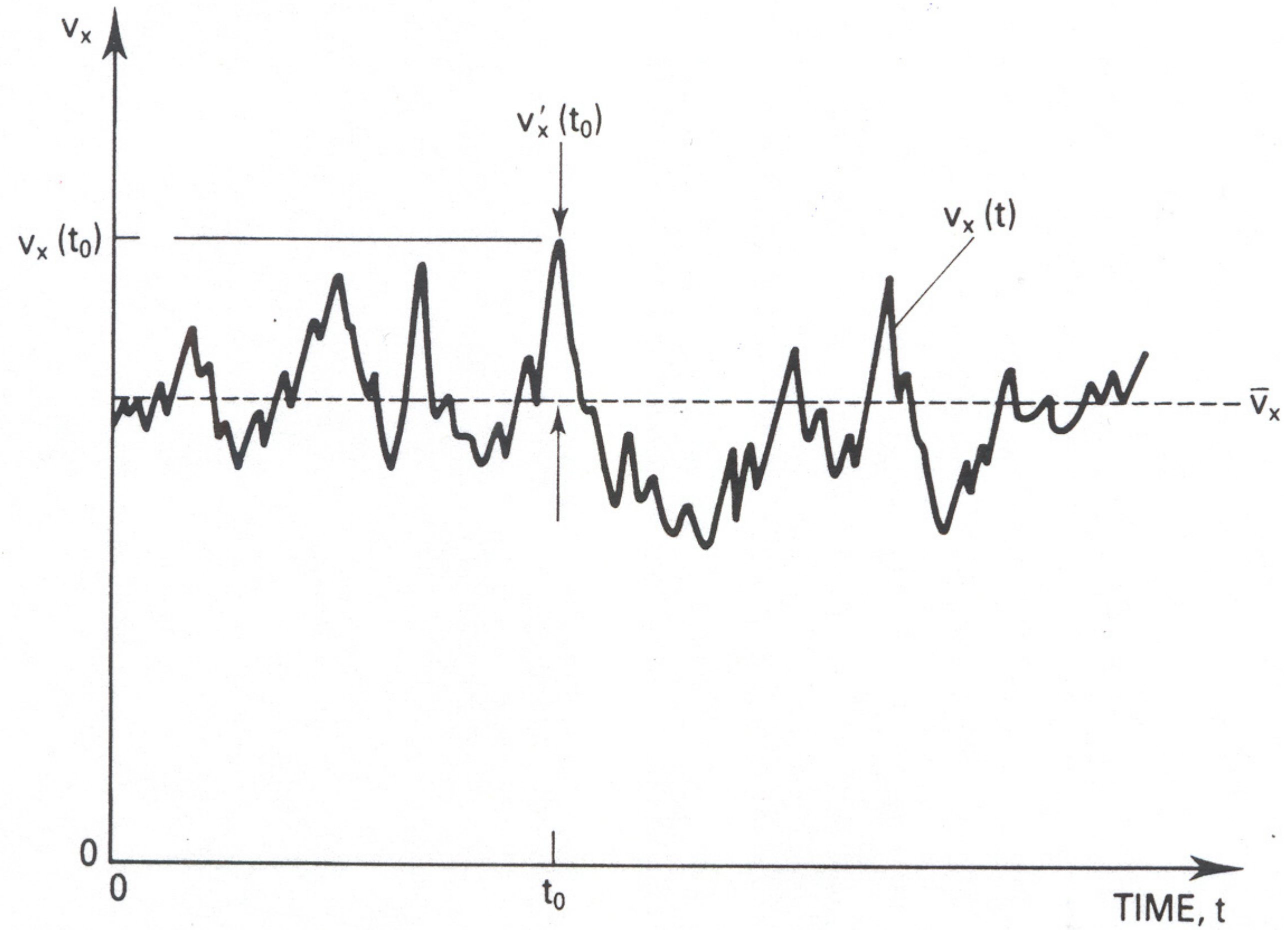



**FIGURE 11.1**
Velocity as a function of time at a fixed point in a turbulent flow

From Turns book

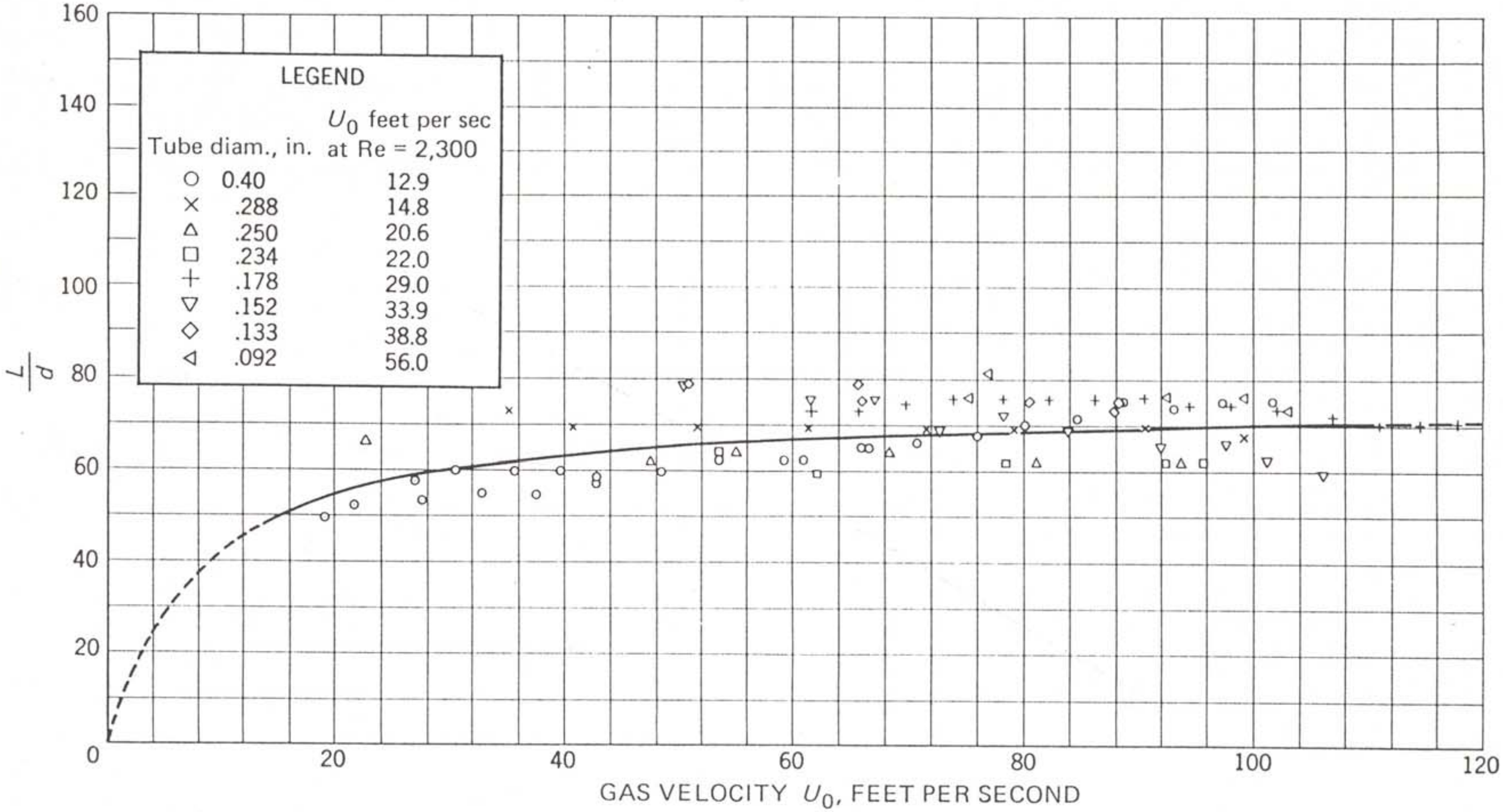



FIG. 267. Constancy of ratio $L/d$ (flame length/port diameter) in turbulent flames. Mixture of 50% Newark, Delaware, city gas and 50% air. $U_0$, gas velocity at port. Flames burning free in air (Wohl, Gazley, and Kapp[3]).

# Turbulence Closure Problem

Let the instantaneous velocity be broken up into a mean and a fluctuating component:

$$u = \overline{u} + u'$$

The momentum equation can then be written as:

$$\frac{\partial}{\partial t}\left(\overline{u}_j + u'_j\right) + \frac{\partial}{\partial x_i}\left[\left(\overline{u}_j + u'_j\right)\left(\overline{u}_j + u'_j\right)\right] = \frac{\partial}{\partial x_i}\left[-\delta_{ij}\left(\overline{P} + P'\right) + \left(\overline{\tau}_{ij} + \tau'_{ij}\right)\right]$$

where body forces have been neglected, and where

$$\overline{\tau}_{ij} \equiv \nu\left[\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)\right] \qquad \tau'_{ij} \equiv \nu\left[\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)\right]$$

Substituting and then averaging the entire equation over time gives the following equation:

$$\frac{\partial}{\partial t}\left(\overline{u}_j\right) + \frac{\partial}{\partial x_i}\left(\overline{u}_i\overline{u}_j + \overline{u'_i u'_j}\right) = \frac{\partial}{\partial x_i}\left[-\delta_{ij}\overline{P} + \overline{\tau}_{ij}\right]$$

The turbulence closure problem is how to model $\overline{u'_i u'_j}$ .

# Boussinesq Hypothesis

The most convenient way to model the Reynolds stress is defining an effective turbulent viscosity ($\Gamma$), as follows:

$$-\overline{u_i' u_j'} = \Gamma\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$$

This effective turbulent viscosity is then modeled by one of several models, as follows.

A. Prandtl mixing length

$$\Gamma = C\left(\overline{u'u'}\right)^{0.5} l_m$$

where C is a constant, $l_m$ is a mixing length that is specified, and $\left(\overline{u'u'}\right)^{0.5}$ is the turbulence intensity that is specified.

B. k-ε model

$$\Gamma = C_\mu k^2 / \varepsilon$$

where $C_\mu$ is a constant, $k$ is the turbulent kinetic energy $\overline{u'u'}/2$, and ε is the rate of dissipation of $k$. Transport equations have been developed for both $k$ and ε.

# Reynolds Stress Model

The exact transport equations for the transport of the Reynolds stresses, $\rho\overline{u_i'u_j'}$, may be written as follows:

$$\underbrace{\frac{\partial}{\partial t}(\rho\,\overline{u_i'u_j'})}_{\text{Local Time Derivative}} + \underbrace{\frac{\partial}{\partial x_k}(\rho u_k \overline{u_i'u_j'})}_{C_{ij}\equiv\text{Convection}} = \underbrace{-\frac{\partial}{\partial x_k}\left[\rho\,\overline{u_i'u_j'u_k'} + \overline{p\left(\delta_{kj}u_i' + \delta_{ik}\right.}\right.}_{D_{T,ij}\equiv\text{Turbulent Diffusion}}$$

$$+ \underbrace{\frac{\partial}{\partial x_k}\left[\mu\frac{\partial}{\partial x_k}(\overline{u_i'u_j'})\right]}_{D_{L,ij}\equiv\text{Molecular Diffusion}} \underbrace{-\rho\left(\overline{u_i'u_k'}\frac{\partial u_j}{\partial x_k} + \overline{u_j'u_k'}\frac{\partial u_i}{\partial x_k}\right)}_{P_{ij}\equiv\text{Stress Production}} \underbrace{-\rho\beta(g_i\overline{u_j'\theta}\,-}_{G_{ij}\equiv\text{Buoyancy}}$$

$$+ \underbrace{\overline{p\left(\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right)}}_{\phi_{ij}\equiv\text{Pressure Strain}} - \underbrace{2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}}_{\epsilon_{ij}\equiv\text{Dissipation}}$$

$$\underbrace{-2\rho\Omega_k\left(\overline{u_j'u_m'}\epsilon_{ikm} + \overline{u_i'u_m'}\epsilon_{jkm}\right)}_{F_{ij}\equiv\text{Production by System Rotation}} + \underbrace{S_{\text{user}}}_{\text{User-Defined Source Term}} \quad (11.6\text{-}1)$$

Idea:
Write transport equations for each of the 6 Reynolds stress terms

Boxes represent terms that must be modeled

$$-\underbrace{\frac{\partial}{\partial x_k}\left[\overline{(u_i'u_j'u_k')} + \overline{\frac{p}{\rho}(\delta_{kj}u_i' + \delta_{ik}u_j')} - \nu\frac{\partial}{\partial x_k}(\overline{u_i'u_j'})\right]}_{\text{Diffusive Transport}} = \frac{\partial}{\partial x_k}\left(\frac{\nu_t}{\sigma_k}\ \frac{\partial(\overline{u_i'u_j'})}{\partial x_k}\right) \tag{6.2-15}$$

Secondly, the pressure-strain term is approximated as[25]:

$$\overline{\frac{p}{\rho}\left[\frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i}\right]} = -C_3\frac{\epsilon}{k}\left[\overline{u_i'u_j'} - \frac{2}{3}\delta_{ij}k\right] - C_4\left[P_{ij} - \frac{2}{3}\delta_{ij}P\right] \tag{6.2-16}$$

where $C_3$ and $C_4$ are empirical constants whose values are $C_3 = 1.8$ and $C_4 = 0.60$, $P = \frac{1}{2}P_{ii}$, and

$$P_{ij} = -\overline{u_i'u_k'}\ \frac{\partial u_j}{\partial x_k} - \overline{u_j'u_k'}\ \frac{\partial u_i}{\partial x_k} \tag{6.2-17}$$

Finally, the dissipation term in Equation 6.2-14 is assumed to be isotropic and is approximated via the scalar dissipation rate[46]:

$$2\nu\overline{\frac{\partial u_i'}{\partial x_k}\ \frac{\partial u_j'}{\partial x_k}} = \frac{2}{3}\delta_{ij}\epsilon \tag{6.2-18}$$

# Algebraic Stress Model

Table 7. Algebraic stress model in cylindrical coordinates

$\overline{v'v'}$ Equation

$$\left[1+\frac{4}{3}\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{v}}{\partial x}\right]\overline{v'v'}=\frac{2}{3}k+\frac{2}{3}\lambda\frac{k}{\varepsilon}\left\{\left(\frac{\partial\bar{u}}{\partial x}\right)\overline{u'u'}+\frac{\partial\bar{u}}{\partial r}-2\frac{\partial\bar{v}}{\partial x}\right\}\overline{v'u'}$$

$$+\left(\frac{\bar{v}}{r}\right)\overline{w'w'}+\left[\frac{\partial\bar{w}}{\partial r}+\frac{\bar{w}}{r}(2+3\beta)\right]\overline{v'w'}+\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{w'u'} \qquad (76)$$

$\overline{u'u'}$ Equation:

$$\left[1+\frac{4}{3}\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{u}}{\partial x}\right]\overline{u'u'}=\frac{2}{3}k+\frac{2}{3}\lambda\frac{k}{\varepsilon}\left\{\left(\frac{\partial\bar{v}}{\partial r}\right)\overline{v'v'}+\left(\frac{\partial\bar{v}}{\partial x}-2\frac{\partial\bar{u}}{\partial r}\right)\overline{v'u'}\right.$$

$$\left.+\left(\frac{\bar{v}}{r}\right)\overline{w'w'}+\left(\frac{\partial\bar{w}}{\partial r}-\frac{\bar{w}}{r}\right)\overline{v'w'}+\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{w'u'}\right\} \qquad (77)$$

$\overline{v'u'}$ Equation:

$$\left[1-\lambda\left(\frac{k}{\varepsilon}\right)\frac{\bar{v}}{r}\right]\overline{v'u'}=\lambda\left(\frac{k}{\varepsilon}\right)\left[-\left(\frac{\partial\bar{u}}{\partial r}\right)\overline{v'v'}-\left(\frac{\partial\bar{v}}{\partial x}\right)\overline{u'u'}+\left(\frac{\bar{w}}{r}\right)(1+\beta)\overline{w'u'}\right] \qquad (78)$$

$\overline{w'w'}$ Equation:

$$\left[1+\frac{4}{3}\lambda\left(\frac{k}{\varepsilon}\right)\frac{\bar{v}}{r}\right]\overline{w'w'}=\frac{2}{3}k+\frac{2}{3}\lambda\left(\frac{k}{\varepsilon}\right)\left\{\left(\frac{\partial\bar{v}}{\partial r}\right)\overline{v'v'}+\left(\frac{\partial\bar{u}}{\partial x}\right)\overline{u'u'}+\left(\frac{\partial\bar{v}}{\partial x}+\frac{\partial\bar{u}}{\partial r}\right)\overline{v'u'}\right.$$

$$\left.-\left[2\frac{\partial\bar{w}}{\partial r}+(1+3\beta)\frac{\bar{w}}{r}\right]\overline{v'w'}-2\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{w'u'}\right\} \qquad (79)$$

$\overline{v'w'}$ Equation:

$$\left[1-\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{u}}{\partial x}\right]\overline{v'w'}=\lambda\left(\frac{k}{\varepsilon}\right)\left\{-\left(\frac{\partial\bar{w}}{\partial r}+\beta\frac{\bar{w}}{r}\right)\overline{v'v'}-\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{v'u'}+\left[(1+\beta)\frac{\bar{w}}{r}\right]\overline{w'w'}-\left(\frac{\partial\bar{v}}{\partial x}\right)\overline{w'u'}\right\} \qquad (80)$$

$\overline{w'u'}$ Equation:

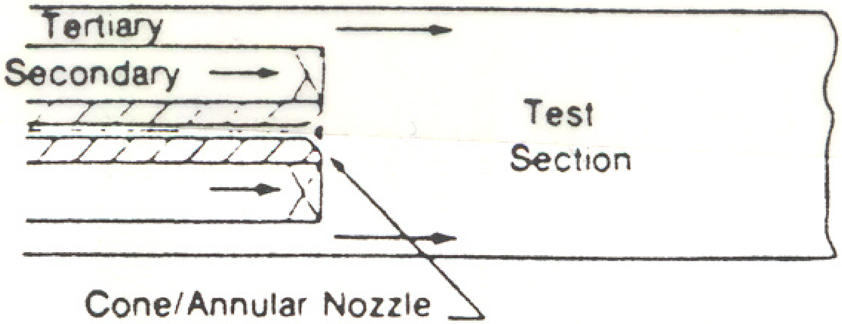
$$\left[1-\lambda\left(\frac{k}{\varepsilon}\right)\frac{\partial\bar{v}}{\partial r}\right]\overline{w'u'}=\lambda\left(\frac{k}{\varepsilon}\right)\left\{-\left(\frac{\partial\bar{w}}{\partial x}\right)\overline{u'u'}-\left(\frac{\partial\bar{w}}{\partial r}+\beta\frac{\bar{w}}{r}\right)\overline{v'u'}-\left(\frac{\partial\bar{u}}{\partial r}\right)\overline{v'w'}\right] \qquad (81)$$

where $\beta=\psi/(1-C_2)$ (82)

Idea:
Write algebraic equations for each of the Reynolds stress terms

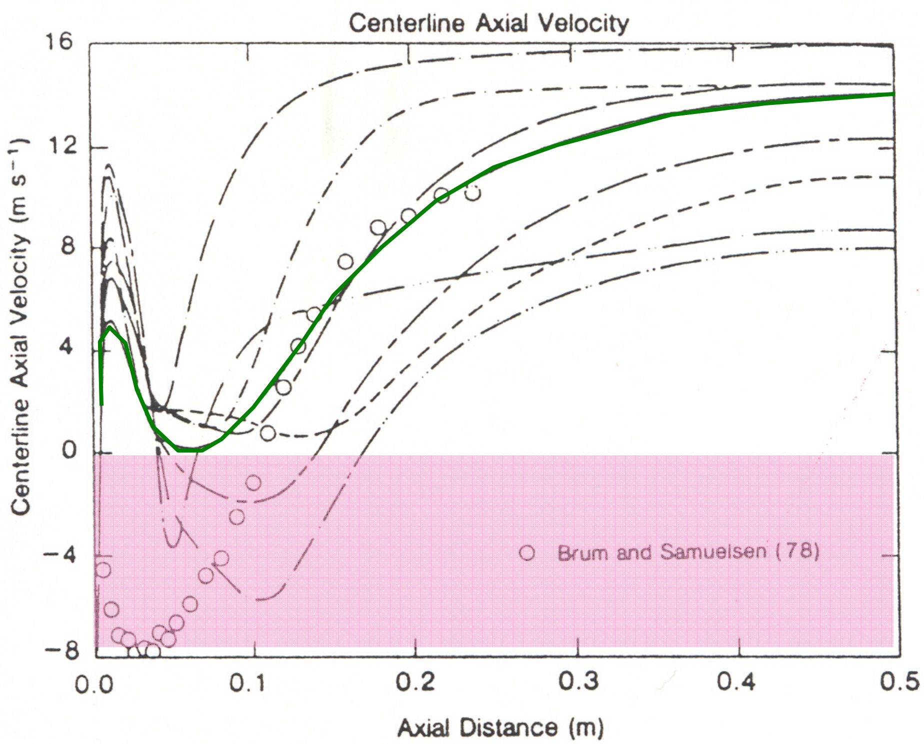
Note:
No terms with $\frac{\partial}{\partial x}\left(\overline{u'u'}\right)$

# Non-reacting swirling flow




Cases 1 and 2—Brum and Samuelson[78]

Standard k-ε




Negative centerline velocity region

FIG. 4. Comparison of predicted and measured centerline axial and velocity profiles for Case 1 (data from Brum and Samuelsen[78]; legend supplied by Table 18).

TABLE 18. Legend description for case studies

| | | Case reference | | | |
|---|---|---|---|---|---|
| | | Brum and Samuelsen[78] | | Yoon[71] | Roback and Johnson[164] |
| Turbulence model description/legend | Equations of reference | 1 | 4 | 5 | 6 |
| Standard $k$–$\varepsilon$ model<br>———— | 34–42 | | | | |
| LPS gradient Richardson no.*<br>—··—··— | 108–110 | $C_{gs}=0.10$<br>MTS | $C_{gs}=0.005$<br>TTS | $C_{gs}=0.03$<br>MTS | $C_{gs}=0.005$<br>TTS |
| Rodi flux Richardson no.*<br>—·—·— | 114, 115 | $C_{fs}=0.90$<br>MTS | | | $C_{fs}=0.90$<br>MTS |
| "Boysan" Richardson no.<br>—···—···— | 109, 112 | $C_{gs}=0.20$ | | | |
| Modified $C_\mu$ coefficient<br>— — — — — | 134 | $C_a=0.03$<br>$C_b=2.63$ | | $C_a=0.03$<br>$C_b=2.63$ | |
| Gibson–Launder ASM*<br>(I) ——— ——— ——— | 76–99 | $C_1=2.5$<br>$C_2=0.55$ | $C_1=2.5$<br>$C_2=0.55$ | $C_1=2.5$<br>$C_2=0.55$ | $C_1=2.5$<br>$C_2=0.55$ |
| Gibson–Launder ASM<br>(II) —-—-—-—-— | 76–99 | $C_1=2.0$<br>$C_2=0.40$ | | | |
| Gibson–Launder ASM<br>(III) added convection<br>- - - - - - - - | 76–99 | $\psi=0.40$<br>$C_1=2.5$<br>$C_2=0.55$ | $\psi=0.40$<br>$C_1=2.5$<br>$C_2=0.55$ | | |

*MTS = Mean flow time scale in the denominator of the Richardson number.
TTS = Turbulence time scale in the denominator of the Richardson number.
ASM = Algebraic stress model.

from Sloan, et al.,

Non-reacting

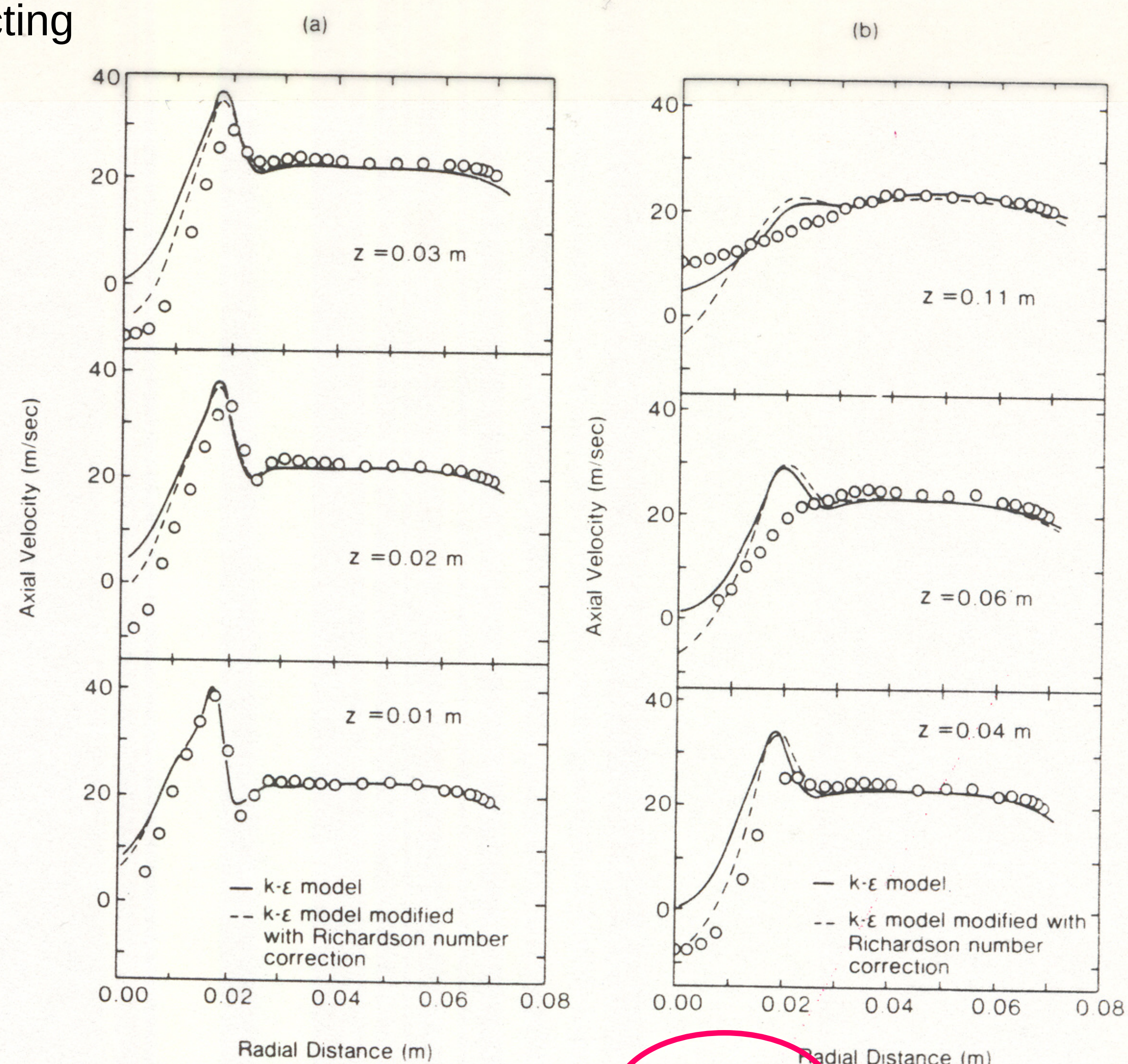



**Figure 10.4.** Comparison of predicted and measured axial velocity. (Data from Vu and Gouldin, 1982; prediction from Sloan, 1984). Conditions were those for a nonreacting coaxial, counterswirl jet. The swirl number of the inner jet was 0.49 and that of the annulus was −0.51.

Non-reacting

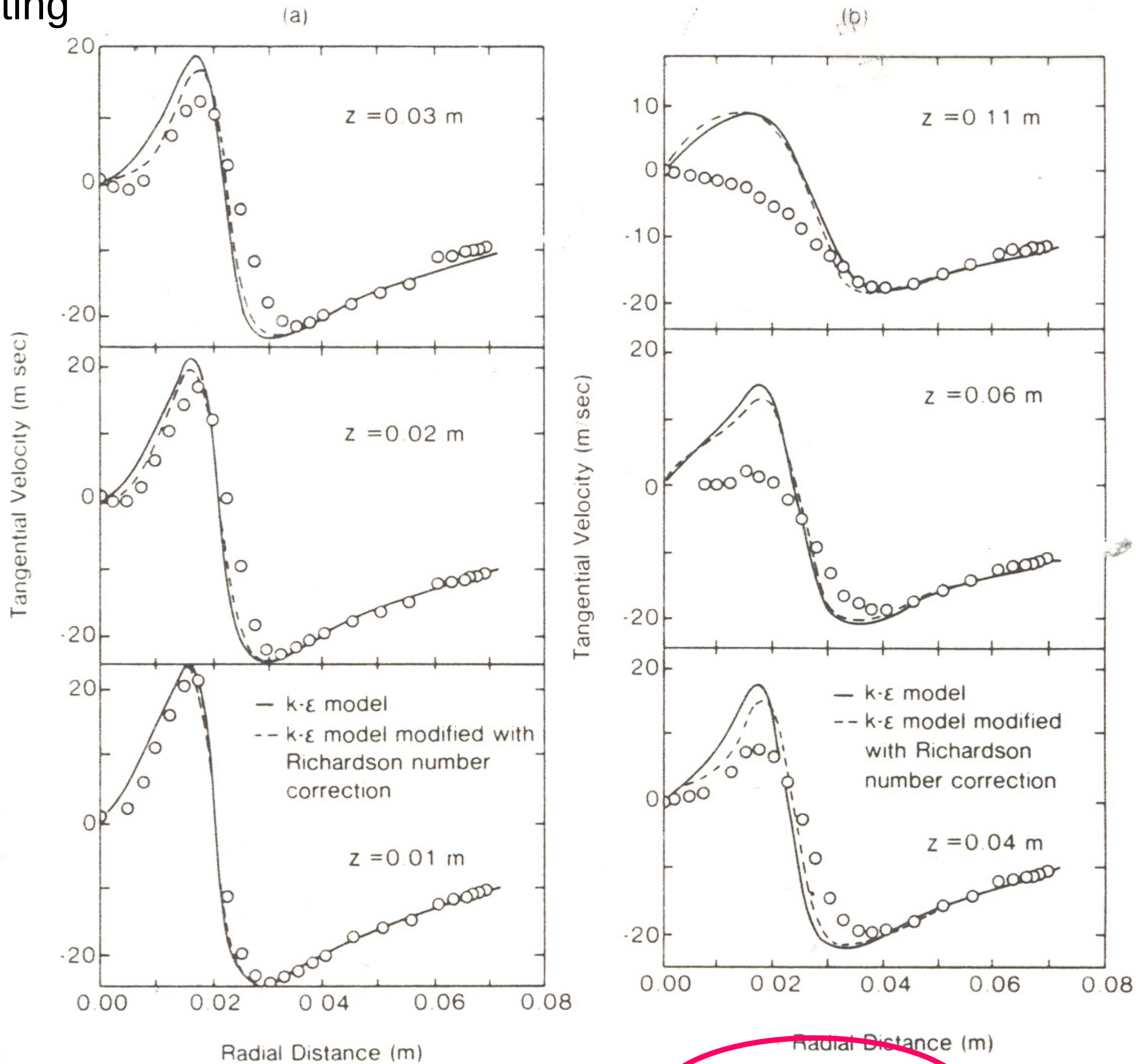



**Figure 10.5.** Comparison of predicted and measured tangential velocity. (Data from Vu and Gouldin, 1982; predictions from Sloan, 1984.) Conditions are those of Figure 10.4 and discussed further in text.

Non-reacting

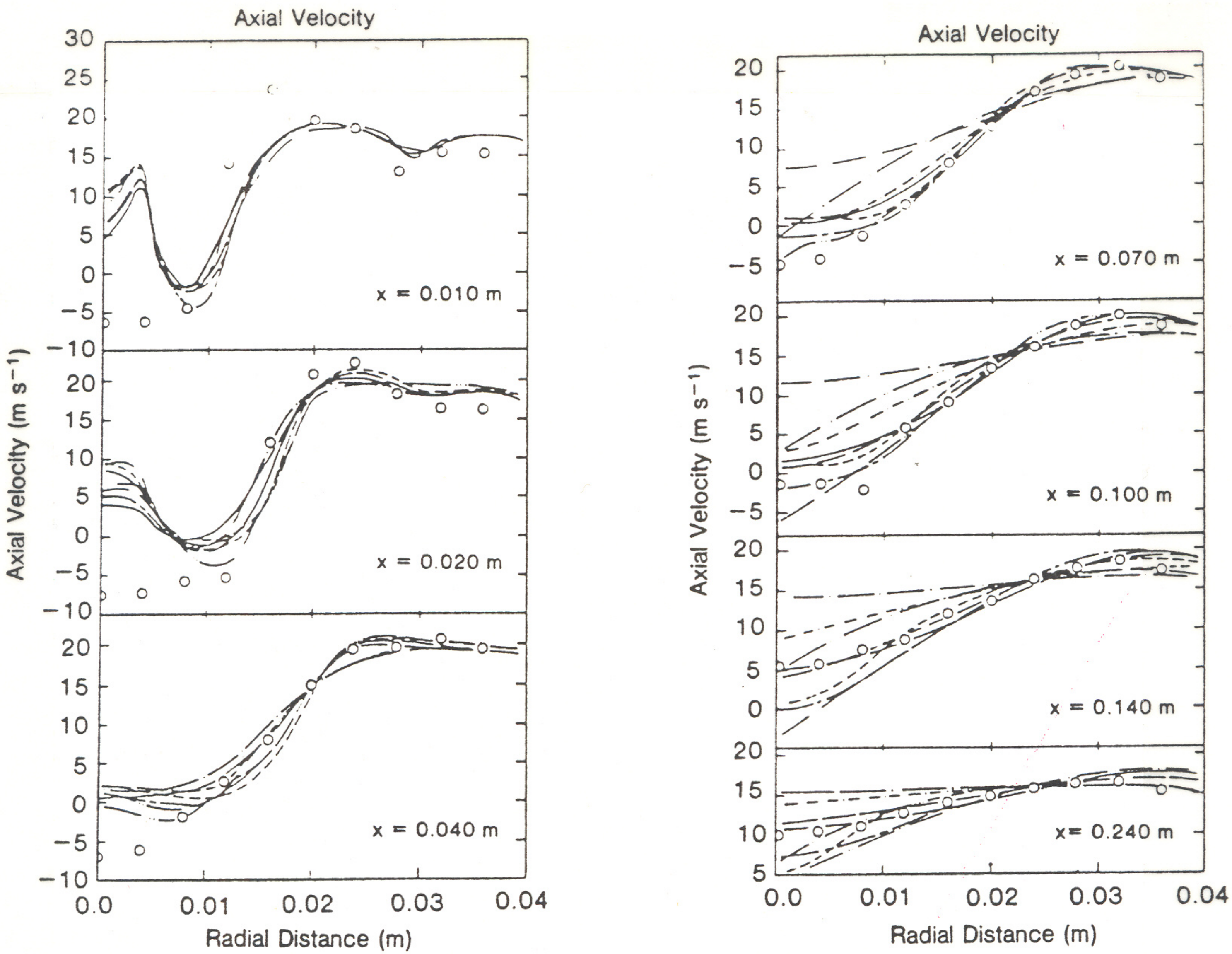



FIG. 5. Comparison of predicted and measured axial velocity profiles for Case 1 (data from Brum and Samuelsen[78]; legend supplied by Table 18).

Reacting

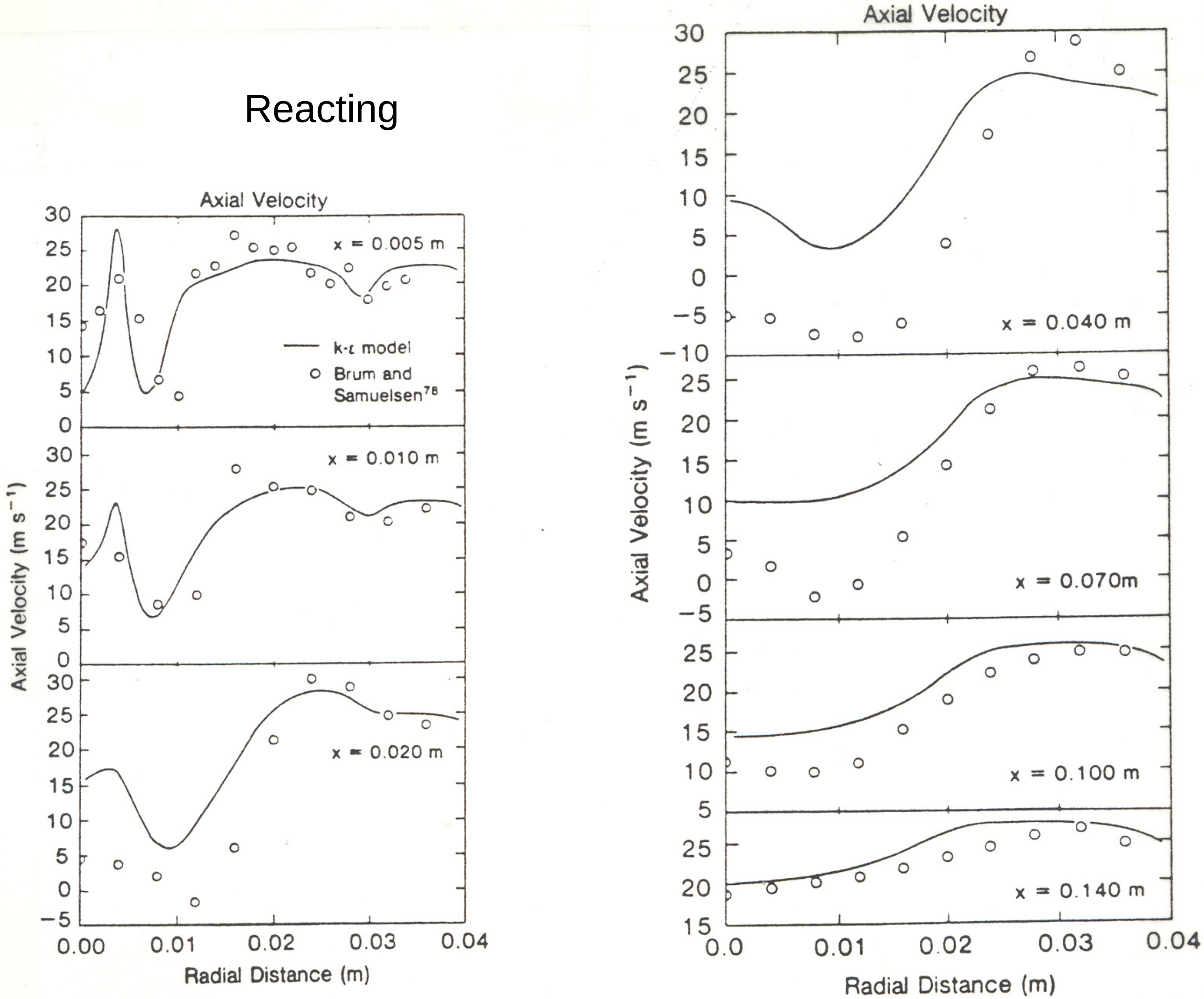



FIG. 8. Comparison of predicted and measured axial velocity profiles for Case 2 (data from Brum and Samuelsen[78]).

From Sloan et al., PECS, 1984