# Homework 12

Ch En 263 – Numerical Tools

Due: 15 Apr. 2024

## Instructions

- Complete the problems below and submit the following files to Learning Suite:
  - Excel portion: submit a workbook named `LastName_FirstName_HW12.xlsx` where each worksheet tab is named "Problem_1", "Problem_2", etc.
  - Python portion: submit a separate file for each problem named `LastName_FirstName_HW12_ProblemXX.py` where XX is the problem number.

## Problems

1. Solve the IVP from Lab 21

$$\frac{dy}{dx} = x^2 y^{1/2}$$
$$y(0) = 1$$

in Python using the Explicit Euler method using four different values of $\Delta x = [10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}]$. (*Hint: This is probably easiest by done writing a function to solve the IVP that takes* $\Delta x$ *as an argument.*) Calculate the error $\epsilon = ||y_{\text{Euler}} - y_{\text{exact}}||$ between each of your four numerical solutions and the exact solution. Make a plot of $\log(\epsilon)$ versus $\log(\Delta x)$. What do you notice about the plot?

2. A continuous stirred tank reactor reacts chemical species A according to the following rate equation:

$$\frac{dA}{dt} = \frac{A_{in} - A}{\tau} - kA^2$$
$$A(0) = A_0.$$

   (a) Solve this equation in Excel using the Explicit Euler method. Plot $A(t)$ out to $t = 15$ using $A_{in} = 1$, $A_0 = 0$, $\tau = 2$, and $k = 0.1$. Use $\Delta t = 0.1$.

   (b) What is the long-time (steady state) value of $A$ from your solution?

   (c) In the above equation, set $dA/dt = 0$ and solve for $A$. This is the analytic steady state value of $A$. How does it compare to the long-time solution from the rate equation?

3. The temperature change of a coal particle in a furnace is given by the following rate equation

$$\frac{dT}{dt} = \frac{hA}{mc_p}(T_f - T) + \frac{\sigma A}{mc_p}(T_f^4 - T^4).$$

The initial particle temperature is $T_0 = 500$ K. In addition, the following data are given

| Variable | Value | Units |
|---|---|---|
| $D$ | 100 | $\mu$m |
| $\rho_p$ | 1000 | kg/m$^3$ |
| $c_p$ | 1380 | J/kg·K |
| $k$ | 0.1 | W/m·K |
| $Nu$ | 2 | – |
| $\sigma$ | 5.67E-8 | W/m$^2$K$^4$ |
| $T_f$ | 1500 | K |
| $t_{\text{end}}$ | 0.05 | s |

Also, the area, mass and Nusselt number $Nu$ are given, respectively, by:

$$A = \frac{\pi}{4} D^2,$$

$$m = \frac{\pi}{6} D^3 \rho_p,$$

$$\text{Nu} = \frac{hD}{k}.$$

Solve the rate equation for the particle temperature as a function of time using `solve_ivp`. Plot your solution and label and format your plot (including units).

4. A harmonic oscillator obeys the second-order differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where $m$ is the mass, $k$ is the spring constant and $c$ is a constant characterizing a damping force. Re-arrange the harmonic oscillator equation as a system of first order rate equations. Solve the resulting system of equations using `solve_ivp` with $m = 1$ kg, $k = 1$ kg/s$^2$ and $c = 0.5$ kg/s. Additionally assume that the initial velocity is zero, $x_0 = 1$ m and $t_{\text{end}} = 10$ s. Plot (and label) the velocity and the position as a function of time.