Homework 14

Ch En 263 – Numerical Tools

Due: 08 Dec. 2025

Instructions

- Complete the problems below and submit the following files to Learning Suite:
  - Excel portion: submit a workbook named `LastName_FirstName_HW14.xlsx` where each worksheet tab is named "Problem_1", "Problem_2", etc.
  - Python portion: submit a separate file for each problem named `LastName_FirstName_HW14_ProblemXX.py` where XX is the problem number.

Problems

1. Solve the IVP from Lab 21

$$\frac{dy}{dx} = x^2 y^{1/2}$$
$$y(0) = 1$$

   in Python using the Explicit Euler method using four different values of $\Delta x = [10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}]$. (*Hint: This is probably easiest by done writing a function to solve the IVP that takes Δx as an argument.*) Calculate the error $\epsilon = ||y_{\text{Euler}} - y_{\text{exact}}||$ between each of your four numerical solutions and the exact solution. Make a plot of $\log(\epsilon)$ versus $\log(\Delta x)$. What do you notice about the plot?

2. A continuous stirred tank reactor reacts chemical species A according to the following rate equation:

$$\frac{dA}{dt} = \frac{A_{in} - A}{\tau} - kA^2$$
$$A(0) = A_0.$$

   (a) Solve this equation in Excel using the Explicit Euler method. Plot $A(t)$ out to $t = 15$ using $A_{in} = 1$, $A_0 = 0$, $\tau = 2$, and $k = 0.1$. Use $\Delta t = 0.1$.

   (b) What is the long-time (steady state) value of A from your solution?

   (c) In the above equation, set $dA/dt = 0$ and solve for A. This is the analytic steady state value of A. How does it compare to the long-time solution from the rate equation?

3. The temperature change of a coal particle in a furnace is given by the following rate equation

$$\frac{dT}{dt} = \frac{hA}{mc_p}(T_f - T) + \frac{\sigma A}{mc_p}(T_f^4 - T^4).$$

   The initial particle temperature is $T_0 = 500$ K. In addition, the following data are given

Variable	Value	Units
D	100	μm
ρ_p	1000	kg/m^3
c_p	1380	J/kg·K
k	0.1	W/m·K
Nu	2	–
σ	5.67E-8	W/m^{2}K^4
T_f	1500	K
t_{end}	0.05	s

Also, the area, mass and Nusselt number Nu are given, respectively, by:

$$A = \frac{\pi}{4}D^2,$$

$$m = \frac{\pi}{6}D^3\rho_p,$$

$$\text{Nu} = \frac{hD}{k}.$$

Solve the rate equation for the particle temperature as a function of time using `solve_ivp`. Plot your solution and label and format your plot (including units).

4. A harmonic oscillator obeys the second-order differential equation

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$

where m is the mass, k is the spring constant and c is a constant characterizing a damping force. Re-arrange the harmonic oscillator equation as a system of first order rate equations. Solve the resulting system of equations using `solve_ivp` with $m = 1$ kg, $k = 1$ kg/s^2 and $c = 0.5$ kg/s. Additionally assume that the initial velocity is zero, $x_0 = 1$ m and $t_{\text{end}} = 10$ s. Plot (and label) the velocity and the position as a function of time.