Homework 1

Ch En 374 – Fluid Mechanics

Due date: 13 Sept. 2019

Survey Question

Please report how long it took you to complete this assignment (in hours) in the "Notes" section when you turn your assignment in on Learning Suite.

Practice Problems

1. [Lecture 2 – Nonlinear equations] Use the nonlinear equation

$$x^3 - 19x + 30 = 0$$

to do the following.

   (a) Use Newton's method and iterate by hand starting with $x_0 = 4$ as an initial guess to find one of the roots.

   (b) Write a Python code in a Jupyter notebook that uses Newton's method to find *all* of the non-imaginary roots. Turn in a printout of your notebook for this part of the problem. *Hint: How many roots will there be for a cubic equation? Make a plot if you are unsure.*

2. [Lecture 2 and 3 – Vector operations] Do the following:

   (a) Evaluate $\nabla \times \nabla f$ where $f(x, y, z) = 4xy - 3xz^2$.

   (b) Evaluate $\nabla \boldsymbol{g}(x, y, z)$ where $\boldsymbol{g}(x, y, z) = \left[2y^3,\, -3x^2y,\, 5xz\right]^T$.

   (c) Find $h(x, y, z)$ if $\nabla h = \left[4yz,\, 4xz - 4yz,\, 4xy - 2y^2\right]^T$. *Hint: Don't forget the integration constant.*

3. [Lecture 4 – Continuum approximation and fluid properties] Answer the following:

   (a) The diameter of a red blood cell is on the order of 5 to 10 μm. If we wanted to model the dynamics of a fluid flowing around a red blood cell, would we need to consider molecular scale phenomena or is continuum fluid mechanics good enough? Why or why not?

   (b) About how much more dense is water than air at room temperature and atmospheric pressure?

   (c) About how much more viscous is water than air at room temperature and atmospheric pressure?

   (d) Write the mathematical expression for Newton's law of viscosity. Also, write what this equation is saying in words.

   (e) You have perhaps done the elementary school experiment where you gradually add water on top of a penny, forming a spherical droplet. (If you haven't then go do it!) This phenomena is of course due to surface tension. Find three additional examples of phenomena caused by surface tension.

Challenge Problems

4. The density of a fluid depends on its temperature and pressure, $\rho = \rho(P, T)$. In this problem we are going to see how sensitive the density of gases and liquids are to T and P.

   (a) The ideal gas equation, $\rho = P/(RT)$, is an equation of state for gases valid at high temperatures and low pressures.

i. Calculate the percent change of the density, $(\rho_2 - \rho_1)/\rho_1$, of an ideal gas at constant pressure between 100°C (state 1) and 20°C (state 2).

ii. Calculate the percent change of the density of an ideal gas at constant temperature between 14 psi (state 1) and 21 psi (state 2).

(b) Recall in class we introduced the isobaric expansivity and the isothermal compressibility,

$$\beta \equiv -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$
$$\kappa \equiv \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T.$$

For a liquid, assuming constant expansivity and compressibility is often a good approximation. For example, the isothermal compressibility of water is about $\kappa = 46.5 \times 10^{-6}\text{atm}^{-1}$ and the isobaric expansivity is about $\beta = 2.07 \times 10^{-4} K^{-1}$.

i. Calculate the percent change of the density of water at constant pressure between 100°C (state 1) and 20°C (state 2).

ii. Calculate the percent change of the density of water at constant temperature between 14 psi (state 1) and 21 psi (state 2).

(c) Assuming *incompressibility* (i.e. constant density with respect to pressure change) simplifies many equations in fluid dynamics. For which type of fluid is this approximation valid (gases or liquids)? Justify your answer based on your work above.

5. The van der Waals equation of state,

$$\frac{\rho}{1 - b\rho} - \frac{a\rho^2}{RT} = \frac{P}{RT}$$

describes the behavior of gases better than the ideal gas equation of state (ρ here is a molar density). For butane, $a = 1.3701 \times 10^7$ atm cm^6 mol^{-2} and $b = 116.4$ cm^3 mol^{-1}. Find two of the roots of the van der Waals equation of state at 1 atm and 325 K. Use the ideal gas density as your guess value for one of the roots and $0.9/b$ as your guess value for the other one. You may do this problem by hand or in a Python Jupyter notebook. *Hint: Be careful with units!*

6. Follow the instructions below:

(a) Evaluate: $\nabla \cdot (\nabla \times \boldsymbol{g})$ where $\boldsymbol{g}(x, y, z) = \left[-2x^3 z,\, z^2,\, xy\right]^T$.
Hint: Is the result a tensor, vector or scalar? Why?

(b) Evaluate: $\displaystyle\int_0^{2\pi}\int_0^4\int_0^2 \frac{z}{2 + r}\, dr dz d\theta$

(c) Recall that in component notation, we can represent the x-unit vector as $\boldsymbol{e}_x = [1, 0, 0]^T$, the y-unit vector as $\boldsymbol{e}_y = [0, 1, 0]^T$ and the z-unit vector as $\boldsymbol{e}_z = [0, 0, 1]^T$. We can use these unit vectors to "pick out" a component of a vector via a dot product. For example, $\boldsymbol{e}_x \cdot \boldsymbol{v} = v_x$. With this in mind, evaluate $\boldsymbol{e}_y \cdot \boldsymbol{\tau} \cdot \boldsymbol{e}_z$ to find the yz-component of the stress tensor,

$$\boldsymbol{\tau} = \begin{bmatrix} xy & 2y^2 & xz^2 \\ 2y & x^2 y & -2yz^2 \\ 3xz & -z & x^2 \end{bmatrix}$$

Hint: Recall that matrix multiplication and the dot-product are equivalent: $\boldsymbol{a} \cdot \boldsymbol{b} = a^T b$