Homework 2

Ch En 374 – Fluid Mechanics

Due date: 20 Sept. 2018

Survey Question

Please report how long it took you to complete this assignment (in hours) in the "Notes" section when you turn your assignment in on Learning Suite.

Practice Problems

1. [Lecture 5 – Units and Dimensionless groups]. Answer the following questions:

   (a) The viscosity of an unknown fluid is 10.3 cP (centipoise) at room temperature. What is the viscosity in Pa·s and lbm/(ft hr)? Is viscosity a primary or secondary dimension in the SI system of units?

   (b) The Reynolds number, Capillary number and Fanning friction factor are ratios of what stresses? Give your answer in words not in mathematical symbols. (Note, we will talk about the friction factor in Lecture 7).

   (c) What stress or stresses are dominant if the Reynolds number and the capillary number are both big?

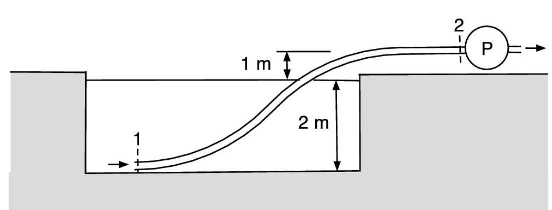
2. [Lecture 6 – Dimensional Analysis]. A simple salad dressing can be prepared by adding one part vinegar to three parts vegetable oil and beating the mixture vigorously using a whisk. The result is an emulsion in which vinegar droplets are dispersed in the oil. The palatability of the dressing depends on the volume, V, of an average vinegar droplet. If it supposed that V depends on the whisking velocity (hand speed) U, the diameter D of the whisk wires, the oil viscosity μ and the oil–water surface tension γ, identify a set of dimensionless groups that might be used to quantify this process.

3. [Lecture 7 – Pipe Flow]. Suppose a food manufacturer wants to fill milk jugs from a large tank at a rate of 15 gal/min. The available space will allow for a tube that is 10 feet long with a 1/4-inch inner diameter, and the outlet of the tube will deliver milk at atmospheric pressure. Assuming that there is no change in elevation and that the density and viscosity of milk are similar to water, what is the pressure at the inlet of the tube at the bottom of the tank? Give your answer in units of psig (psi above atmospheric pressure).

Challenge Problems

4. Bubbles tend to form wherever the pressure in a liquid falls below its vapor pressure (P_v), a phenomenon called *cavitation*. Shock waves created by the rapid collapse of such bubbles can be very damaging to equipment. A system where this might be a concern is shown below. To empty a swimming pool for repairs, a pump with a flow rate $Q = 3.93 \times 10^{-3}\text{m}^3/\text{s}$ is to be connected to 50 m of rubber hose with a diameter of 5 cm. The hose is hydrodynamically smooth.

   (a) When the pool is nearly empty, the pressure at point 1 is the same as the atmosphere, $P_1 = 1.01 \times 10^5$ Pa. Neglecting effects from the change in elevation, calculate P_2, the pressure just before the pump intake. You should find that $P_1 > P_2 > P_v$, where $P_v = 2339$ Pa is the vapor pressure of water at 20°C.



(b) Now find the maximum flow rate, Q, that can be safely used before the onset of cavitation. If you use a numerical tool like Excel or Python, be sure to turn in a printout of your spreadsheet or Jupyter notebook along with the rest of your assignment.

5. Consider the following experimental data which shows the average velocity and pressure drop for water at 60°F ($\mu = 2.34 \times 10^{-5}$ lbf s/ft^2, $\rho = 1.94$ slug/ft^3) in a 5-ft length of a smooth-walled pipe with an inner diameter of 0.496 inches.

   Using your favorite numerical tool, plot the friction factor versus the Reynolds number for the given data. Then fit the data to find an empirical correlation for $f(Re)$. (Again, turn in a printout of any spreadsheet or Jupyter notebook that you use.) How does the empirical fit that you found compare to the correlations in your textbook? *Hint: A power-law is easy to fit on a log-log plot, because it is linear.*

Velocity (ft/s)	Pressure Drop (lbf/ft^2)
1.17	6.26
1.95	15.6
2.91	30.9
5.84	106
11.13	329
16.92	681
23.34	1200
28.73	1730

6. Later in the course we will encounter the Navier–Stokes equation,

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right) = -\nabla P + \mu\nabla^2\boldsymbol{v} \tag{1}$$

   which is a differential expression of the conservation of momentum for a fluid. It is a vector equation, and in Cartesian coordinates the x-component can be written as

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}\right) \tag{2}$$

   Consider the case where there are no velocity gradients in the y or z directions,

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x}\right) = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 v_x}{\partial x^2} \tag{3}$$

   and the boundary conditions dictate a length scale, D, and a velocity scale, U.

   (a) How many variables and dimensional constants are contained in Eq. 3 and the boundary conditions? Given that there are three independent dimensions (M, L, T), how many dimensionless groups does the Pi Theorem predict?

   (b) Using a length scale $L = D$, a time scale $T = D/U$ and a mass scale $M = \rho D^3$ define new dimensionless variables for x, t, v_x, and P. (These are four of your dimensionless groups.)

   (c) Substitute your new variables into Eq. 3 and re-arrange until the equation is dimensionless. Identify the final dimensionless group(s) and verify the prediction of the Pi Theorem.